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ON THE DIOPHANTINE EQUATION $\sum_1^m a_i^2 = \sum_{m+1}^{2m-1} a_i^2$

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ABSTRACT:

In this paper, we have discussed the Diophantine equation $\sum_1^m a_i^2 = \sum_{m+1}^{2m-1} a_i^2$, for different values of m and $a_1 < a_2 < a_3 \ldots a_{2m-1}$ are consecutive positive integers.

KEYWORDS: Diophantine equation; consecutive; triplet and integral solution.

ACADEMIC DISCIPLINE: Number Theory.

SUBJECT CLASSIFICATION: 11D45.

TYPE (METHOD/APPROACH): This paper considers a particular Diophantine equation. Its positive integral solutions have been obtained by algebraic method.

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1 INTRODUCTION: Several researchers have discussed the non-linear Diophantine equations. Most famous non-linear Diophantine equations are **Fermat's Last Problem** (1637) and **Beal's Conjecture** (1993). The Pythagorean equation $a^2 + b^2 = c^2$ has infinitely many solutions in positive integers known as Pythagorean triplets (a, b, c). But this equation has exactly one solution in consecutive positive integers a, b, c given by $(a, b, c) = (3,4,5)$.

In this paper, we have generalized this result for the solution of

$$\sum_1^m a_i^2 = \sum_{m+1}^{2m-1} a_i^2 \,, \qquad \ldots(1)$$

for different values of *m* and $a_1 < a_2 < a_3 \ldots a_{2m-1}$ are consecutive positive integers. This may be considered as super Pythagorean equation.

2 ANALYSIS: (A) Diophantine equation $a_1^2 + a_2^2 + a_3^2 = a_4^2 + a_5^2$: For *m*=3 the Diophantine equation (1) reduces to

$$a_1^2 + a_2^2 + a_3^2 = a_4^2 + a_5^2. \qquad \ldots(2)$$

Let $a_1 = n$. Then $a_2 = n + 1$, $a_3 = n + 2$, $a_4 = n + 3$ and $a_5 = n + 4$. Putting these values in (2), we get

$$n^2 + (n + 1)^2 + (n + 2)^2 = (n + 3)^2 + (n + 4)^2$$

or $$n^2 - 8n - 20 = 0. \qquad \ldots(3)$$

Solution of equation (3) is given by $n = 10$ and -2 (discarded). Thus the required solution is given by $a_1 = 10$, $a_2 = 11$, $a_3 = 12$, $a_4 = 13$ and $a_5 = 14$.

(B) Diophantine equation $a_1^2 + a_2^2 + a_3^2 + a_4^2 = a_5^2 + a_6^2 + a_7^2$: For *m*=4 the Diophantine equation (1) reduces to

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 = a_5^2 + a_6^2 + a_7^2. \qquad \ldots(4)$$

Let $a_1 = n$. Then $a_2 = n + 1$, $a_3 = n + 2$, $a_4 = n + 3$, $a_5 = n + 4$, $a_6 = n + 5$ and $a_7 = n + 6$. Putting these values in (4), we get

$$n^2 + (n + 1)^2 + (n + 2)^2 + (n + 3)^2 = (n + 4)^2 + (n + 5)^2 + (n + 6)^2$$

or $$n^2 - 18n - 63 = 0. \qquad \ldots(5)$$

Solution of equation (5) is given by $n = 21$ and -3 (discarded). Thus the required solution is given by $a_1 = 21$, $a_2 = 22$, $a_3 = 23$, $a_4 = 24$, $a_5 = 25$, $a_6 = 26$ and $a_7 = 27$.

(C) Diophantine equation $a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 = a_6^2 + a_7^2 + a_8^2 + a_9^2$: For *m*=5 the Diophantine equation (1) reduces to

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 = a_6^2 + a_7^2 + a_8^2 + a_9^2. \qquad \ldots(6)$$

Let $a_1 = n$. Then $a_2 = n + 1$, $a_3 = n + 2$, $a_4 = n + 3$, $a_5 = n + 4$, $a_6 = n + 5$, $a_7 = n + 6, a_8 = n + 7$ and $a_9 = n + 8$. Putting these values in (6), we get

$$n^2 + (n + 1)^2 + (n + 2)^2 + (n + 3)^2 + (n + 4)^2$$
$$= (n + 5)^2 + (n + 6)^2 + (n + 7)^2 + (n + 8)^2$$

or $$n^2 - 32n - 144 = 0. \qquad \ldots(7)$$

Solution of equation (7) is given by $n = 36$ and -4 (discarded). Thus the required solution is given by $a_1 = 36$, $a_2 = 37$, $a_3 = 38$, $a_4 = 39$, $a_5 = 40$, $a_6 = 41, a_7 = 42, a_8 = 43$ and $a_9 = 44$.

(D) Diophantine equation $a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 = a_7^2 + a_8^2 + a_9^2 + a_{10}^2 + a_{11}^2$: For *m*=6 the Diophantine equation (1) reduces to

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2.$$
$$= a_7^2 + a_8^2 + a_9^2 + a_{10}^2 + a_{11}^2 \qquad \ldots(8)$$

Let $a_1 = n$. Then $a_2 = n + 1$, $a_3 = n + 2$, $a_4 = n + 3$, $a_5 = n + 4$, $a_6 = n + 5$, $a_7 = n + 6$, $a_8 = n + 7$, $a_9 = n + 8$, $a_{10} = n + 9$ and $a_{11} = n + 10$. Putting these values in (8), we get

$$n^2 + (n + 1)^2 + (n + 2)^2 + (n + 3)^2 + (n + 4)^2 + (n + 5)^2$$
$$= (n + 6)^2 + (n + 7)^2 + (n + 8)^2 + (n + 9)^2 + (n + 10)^2$$

or $$n^2 - 50n - 275 = 0. \qquad \ldots(9)$$

Solution of equation (9) is given by $n = 55$ and -5 (discarded). Thus the required solution is given by $a_1 = 55$, $a_2 = 56$, $a_3 = 57$, $a_4 = 58$, $a_5 = 59$, $a_6 = 60, a_7 = 61$, $a_8 = 62$, $a_9 = 63$, $a_{10} = 64$ and $a_{11} = 65$.

(E) Diophantine equation $a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2=a_8^2+a_9^2+a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2$: For *m*=7 the Diophantine equation (1) reduces to

$$a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2.$$
$$=a_8^2+a_9^2+a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2 \quad \ldots(10)$$

Let $a_1=n$. Then $a_2=n+1$, $a_3=n+2$, $a_4=n+3$, $a_5=n+4$, $a_6=n+5$, $a_7=n+6$, $a_8=n+7$, $a_9=n+8$, $a_{10}=n+9$, $a_{11}=n+10$, $a_{12}=n+11$ and $a_{13}=n+12$. Putting these values in (10), we get

$$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2$$
$$=(n+7)^2+(n+8)^2+(n+9)^2+(n+10)^2+(n+11)^2+(n+12)^2$$

or $$n^2-72n-468=0. \quad \ldots(11)$$

Solution of equation (11) is given by $n=78$ and -6 (discarded). Thus the required solution is given by $a_1=78$, $a_2=79$, $a_3=80$, $a_4=81$, $a_5=82$, $a_6=83$, $a_7=84$, $a_8=85$, $a_9=86$, $a_{10}=87$, $a_{11}=88$, $a_{12}=89$ and $a_{13}=90$.

(F) Diophantine equation $a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2=a_9^2+a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2$: For *m*=8 the Diophantine equation (1) reduces to

$$a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2$$
$$=a_9^2+a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2 \quad \ldots(12)$$

Let $a_1=n$. Then $a_2=n+1$, $a_3=n+2$, $a_4=n+3$, $a_5=n+4$, $a_6=n+5$, $a_7=n+6$, $a_8=n+7$, $a_9=n+8$, $a_{10}=n+9$, $a_{11}=n+10$, $a_{12}=n+11$, $a_{13}=n+12$, $a_{14}=n+13$ and $a_{15}=n+14$. Putting these values in (12), we get

$$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2$$
$$+(n+7)^2=(n+8)^2+(n+9)^2+(n+10)^2+(n+11)^2+(n+12)^2$$
$$+(n+13)^2+(n+14)^2$$

or $$n^2-98n-735=0. \quad \ldots(13)$$

Solution of equation (13) is given by $n=105$ and -7 (discarded). Thus the required solution is given by $a_1=105$, $a_2=106$, $a_3=107$, $a_4=108$, $a_5=109$, $a_6=110$, $a_7=111$, $a_8=112$, $a_9=113$, $a_{10}=114$, $a_{11}=115$, $a_{12}=116$, $a_{13}=117$, $a_{14}=118$ and $a_{15}=119$.

(G) Diophantine equation $a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2=a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2$: For *m*=9 the Diophantine equation (1) reduces to

$$a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2$$
$$=a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2 \quad \ldots(14)$$

Let $a_1=n$. Then $a_2=n+1$, $a_3=n+2$, $a_4=n+3$, $a_5=n+4$, $a_6=n+5$, $a_7=n+6$, $a_8=n+7$, $a_9=n+8$, $a_{10}=n+9$, $a_{11}=n+10$, $a_{12}=n+11$, $a_{13}=n+12$, $a_{14}=n+13$, $a_{15}=n+14$, $a_{16}=n+15$ and $a_{17}=n+16$. Putting these values in (14), we get

$$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2$$
$$+(n+7)^2+(n+8)^2=(n+9)^2+(n+10)^2+(n+11)^2+(n+12)^2$$
$$+(n+13)^2+(n+14)^2+(n+15)^2+(n+16)^2$$

or $$n^2-128n-1088=0. \quad \ldots(15)$$

Solution of equation (15) is given by $n=136$ and -8 (discarded). Thus the required solution is given by $a_1=136$, $a_2=137$, $a_3=138$, $a_4=139$, $a_5=140$, $a_6=141$, $a_7=142$, $a_8=143$, $a_9=144$, $a_{10}=145$, $a_{11}=146$, $a_{12}=147$, $a_{13}=148$, $a_{14}=149$, $a_{15}=150$, $a_{16}=151$ and $a_{17}=152$.

(H) Diophantine equation $a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2=a_{11}^2+a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2$: For *m*=10 the Diophantine equation (1) reduces to

$$a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2$$
$$=a_{11}^2+a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2 \quad \ldots(16)$$

Let $a_1=n$. Then $a_2=n+1$, $a_3=n+2$, $a_4=n+3$, $a_5=n+4$, $a_6=n+5$, $a_7=n+6$, $a_8=n+7$, $a_9=n+8$, $a_{10}=n+9$, $a_{11}=n+10$, $a_{12}=n+11$, $a_{13}=n+12$, $a_{14}=n+13$, $a_{15}=n+14$, $a_{16}=n+15$, $a_{17}=n+16$, $a_{18}=n+17$ and $a_{19}=n+18$. Putting these values in (16), we get

$$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2$$
$$+(n+7)^2+(n+8)^2+(n+9)^2+(n+10)^2=(n+11)^2+(n+12)^2$$
$$+(n+13)^2+(n+14)^2+(n+15)^2+(n+16)^2+(n+17)^2+(n+18)^2$$

or $\quad n^2-162n-1539=0.$ …(17)

Solution of equation (17) is given by $n=171$ and -9 (discarded). Thus the required solution is given by $a_1=171$, $a_2=172$, $a_3=173$, $a_4=174$, $a_5=175$, $a_6=176, a_7=177$, $a_8=178$, $a_9=179$, $a_{10}=180$, $a_{11}=181$, $a_{12}=182$, $a_{13}=183$, $a_{14}=184$, $a_{15}=185$, $a_{16}=186$, $a_{17}=187$, $a_{18}=188$ and $a_{19}=189$.

(I) Diophantine equation $a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2+a_{11}^2=a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2+a_{20}^2+a_{21}^2$: For *m*=11 the Diophantine equation (1) reduces to

$$a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2+a_{11}^2$$
$$=a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2+a_{20}^2+a_{21}^2 \quad \text{…(18)}$$

Let $a_1=n$. Then $a_2=n+1$, $a_3=n+2$, $a_4=n+3$, $a_5=n+4$, $a_6=n+5$, $a_7=n+6$, $a_8=n+7$, $a_9=n+8$, $a_{10}=n+9$, $a_{11}=n+10$, $a_{12}=n+11$, $a_{13}=n+12$, $a_{14}=n+13$, $a_{15}=n+14$, $a_{16}=n+15, a_{17}=n+16$, $a_{18}=n+17, a_{19}=n+18, a_{20}=n+19$ and $a_{21}=n+20$. Putting these values in (18), we get

$$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2$$
$$+(n+7)^2+(n+8)^2+(n+9)^2+(n+10)^2=(n+11)^2+(n+12)^2$$
$$+(n+13)^2+(n+14)^2+(n+15)^2+(n+16)^2+(n+17)^2+(n+18)^2+(n+19)^2+(n+20)^2$$

or $\quad n^2-200n-2100=0.$ …(19)

Solution of equation (19) is given by $n=210$ and -10 (discarded). Thus the required solution is given by $a_1=210$, $a_2=211$, $a_3=212$, $a_4=213$, $a_5=214$, $a_6=215, a_7=216$, $a_8=217$, $a_9=218$, $a_{10}=219$, $a_{11}=220$, $a_{12}=221$, $a_{13}=222$, $a_{14}=223$, $a_{15}=224$, $a_{16}=225$, $a_{17}=226$, $a_{18}=227, a_{19}=228$, $a_{20}=229$ and $a_{21}=230$.

(J) Diophantine equation $a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2+a_{11}^2+a_{12}^2=a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2+a_{20}^2+a_{21}^2+a_{22}^2+a_{23}^2$: For *m*=12 the Diophantine equation (1) reduces to

$$a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2+a_{11}^2+a_{12}^2$$
$$=a_{13}^2+a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2+a_{20}^2+a_{21}^2+a_{22}^2+a_{23}^2 \quad \text{…(20)}$$

Let $a_1=n$. Then $a_2=n+1$, $a_3=n+2$, $a_4=n+3$, $a_5=n+4$, $a_6=n+5$, $a_7=n+6$, $a_8=n+7$, $a_9=n+8$, $a_{10}=n+9$, $a_{11}=n+10$, $a_{12}=n+11$, $a_{13}=n+12$, $a_{14}=n+13$, $a_{15}=n+14$, $a_{16}=n+15, a_{17}=n+16$, $a_{18}=n+17, a_{19}=n+18, a_{20}=n+19$, $a_{21}=n+20, a_{22}=n+21$ and $a_{23}=n+22$. Putting these values in (20), we get

$$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2$$
$$+(n+7)^2+(n+8)^2+(n+9)^2+(n+10)^2+(n+11)^2=(n+12)^2$$
$$+(n+13)^2+(n+14)^2+(n+15)^2+(n+16)^2+(n+17)^2+(n+18)^2++(n+19)^2+(n+20)^2+(n+21)^2+(n+22)^2$$

or $\quad n^2-242n-2783=0.$ …(21)

Solution of equation (21) is given by $n=253$ and -11 (discarded). Thus the required solution is given by $a_1=253$, $a_2=254$, $a_3=255$, $a_4=256$, $a_5=257$, $a_6=258, a_7=259$, $a_8=260$, $a_9=261$, $a_{10}=262$, $a_{11}=263$, $a_{12}=264$, $a_{13}=265$, $a_{14}=266$, $a_{15}=267$, $a_{16}=268$, $a_{17}=269$, $a_{18}=270, a_{19}=271$, $a_{20}=272$, $a_{21}=273$, $a_{22}=274$ and $a_{23}=275$.

(K) Diophantine equation $a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2=a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2+a_{20}^2+a_{21}^2+a_{22}^2+a_{23}^2+a_{24}^2+a_{25}^2$: For *m*=13 the Diophantine equation (1) reduces to

$$a_1^2+a_2^2+a_3^2+a_4^2+a_5^2+a_6^2+a_7^2+a_8^2+a_9^2+a_{10}^2+a_{11}^2+a_{12}^2+a_{13}^2$$
$$=a_{14}^2+a_{15}^2+a_{16}^2+a_{17}^2+a_{18}^2+a_{19}^2+a_{20}^2+a_{21}^2+a_{22}^2+a_{23}^2+a_{24}^2+a_{25}^2 \quad \text{…(22)}$$

Let $a_1=n$. Then $a_2=n+1$, $a_3=n+2$, $a_4=n+3$, $a_5=n+4$, $a_6=n+5$, $a_7=n+6$, $a_8=n+7$, $a_9=n+8$, $a_{10}=n+9$, $a_{11}=n+10$, $a_{12}=n+11$, $a_{13}=n+12$, $a_{14}=n+13$, $a_{15}=n+14$, $a_{16}=n+15, a_{17}=n+16$, $a_{18}=n+17, a_{19}=n+18, a_{20}=n+1, a_{21}=n+20, a_{22}=n+21$, $a_{23}=n+22, a_{24}=n+23$ and $a_{25}=n+22$. Putting these values in (22), we get

$$n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2$$
$$+(n+7)^2+(n+8)^2+(n+9)^2+(n+10)^2+(n+11)^2+(n+12)^2$$

$$= (n+13)^2 + (n+14)^2 + (n+15)^2 + (n+16)^2 + (n+17)^2 + (n+18)^2 + +(n+19)^2 + (n+20)^2 + (n+21)^2 + (n+22)^2 + (n+23)^2 + (n+24)^2$$

or $\quad n^2 - 290n - 3624 = 0.$ …(23)

Solution of equation (23) is given by $n = 302$ and -12 (discarded). Thus the required solution is given by $a_1 = 302$, $a_2 = 303$, $a_3 = 304$, $a_4 = 305$, $a_5 = 306$, $a_6 = 307, a_7 = 308$, $a_8 = 309$, $a_9 = 310$, $a_{10} = 311$, $a_{11} = 312$, $a_{12} = 313$, $a_{13} = 314$, $a_{14} = 315$, $a_{15} = 316$, $a_{16} = 317$, $a_{17} = 318$, $a_{18} = 319, a_{19} = 320$, $a_{20} = 321$, $a_{21} = 322$, $a_{22} = 323$, $a_{23} = 324$, $a_{24} = 325$ and $a_{25} = 326$.

(L) Diophantine equation $a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2 + a_8^2 + a_9^2 + a_{10}^2 + a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{14}^2 = a_{15}^2 + a_{16}^2 + a_{17}^2 + a_{18}^2 + a_{19}^2 + a_{20}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{24}^2 + a_{25}^2 + a_{26}^2 + a_{27}^2$: For m=14 the Diophantine equation (1) reduces to

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2 + a_8^2 + a_9^2 + a_{10}^2 + a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{14}^2 = a_{15}^2 + a_{16}^2 + a_{17}^2 + a_{18}^2 + a_{19}^2 + a_{20}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{24}^2 + a_{25}^2 + a_{26}^2 + a_{27}^2 \quad \ldots(24)$$

Let $a_1 = n$. Then $a_2 = n+1$, $a_3 = n+2$, $a_4 = n+3$, $a_5 = n+4$, $a_6 = n+5$, $a_7 = n+6$, $a_8 = n+7$, $a_9 = n+8$, $a_{10} = n+9$, $a_{11} = n+10$, $a_{12} = n+11$, $a_{13} = n+12$, $a_{14} = n+13$, $a_{15} = n+14$, $a_{16} = n+15, a_{17} = n+16$, $a_{18} = n+17, a_{19} = n+18, a_{20} = n+1, a_{21} = n+20, a_{22} = n+21$, $a_{23} = n+22, a_{24} = n+23, a_{25} = n+24, a_{26} = n+25$ and $a_{27} = n+26$. Putting these values in (24), we get

$$n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2 + (n+4)^2 + (n+5)^2 + (n+6)^2$$
$$+(n+7)^2 + (n+8)^2 + (n+9)^2 + (n+10)^2 + (n+11)^2 + (n+12)^2$$
$$+(n+13)^2 = (n+14)^2 + (n+15)^2 + (n+16)^2 + (n+17)^2 + (n+18)^2 + +(n+19)^2 + (n+20)^2 + (n+21)^2 + (n+22)^2 + (n+23)^2 + (n+24)^2 + (n+25)^2 + (n+26)^2$$

or $\quad n^2 - 340n - 4589 = 0.$ …(25)

Solution of equation (25) is given by $n = 353$ and -13 (discarded). Thus the required solution is given by $a_1 = 353$, $a_2 = 354$, $a_3 = 355$, $a_4 = 356$, $a_5 = 357$, $a_6 = 358, a_7 = 359$, $a_8 = 360$, $a_9 = 361$, $a_{10} = 362$, $a_{11} = 363$, $a_{12} = 364$, $a_{13} = 365$, $a_{14} = 366$, $a_{15} = 367$, $a_{16} = 368$, $a_{17} = 369$, $a_{18} = 370, a_{19} = 371$, $a_{20} = 372$, $a_{21} = 373$, $a_{22} = 374$, $a_{23} = 375$, $a_{24} = 376, a_{25} = 377$, $a_{26} = 378$, and $a_{27} = 379$.

3 CONCLUDING REMARKS: In this paper, the Diophantine equation $\sum_1^m a_i^2 = \sum_{m+1}^{2m-1} a_i^2$ has been solved for $m = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13$ and 14. Solutions thus obtained have particular property. This Diophantine equation can further be solved for more values of *m*.

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