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ARE THETA FUNCTIONS THE FOUNDATIONS OF PHYSICS?

J.A.de Wet

Box 514,Plettenberg Bay,6600,South Africa jadew@global.co.za

ABSTRACT

The Jacobi theta functions are essentially rotations in a complex space and as such provide a basis for the lattices of the exceptional Lie algebras E_6, E_8 in complex 3-space and complex 4-space.In this note we will show that a choice of the nome q of the theta functions $\theta_{E_6}, \theta_{E_8}$ leads to the equilateral tritangents of these lattices. Specifically we will find quarter period ratios of the real and complex axes.

In particular E_8 is isomorphic to the binary icosahedral group shown recently to describe Elementary Particle theory and Quantum Gravity so q is fundamental to the structure of space itself.

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1 INTRODUCTION

This note should be read as a supplement to a recent publication[3] where it was shown how quantum gravity and Icosahedral symmetry both employ the same Jacobi Theta function

$$\theta_{E_8} = 1 + 240q^2 + 2160q^4 + 6720q^6 + \ldots. \quad (1)$$

Here N_m = 240 is the number of vertices in the 8 sucessive shells of the toric lattice shown in Fig.2 and $q = exp(i\pi\tau)$ is the elliptic nome $exp(-\pi iK/K)$ with a maximum value of 0.06586 [5],where K and iK=K'are quarter periods. 240 is also the kissing number for a sphere packing of the E_8 lattice in Fig.2 ,where the binary icosahedral group is isomorphic to the exceptional Lie algebra E_8 by the MacKay correspondence[7].Here there are 8 sets of 30 vertices on 4 dual Riemann surfaces but the third coefficient N_m = 2160 is no longer a kissing number between spheres because it belongs to another set of vectors given in ([1],Table 4.10). But N_m remains the number of vectors in sucessive shells of the toric lattice.

In this contribution we will find that the ratio iK/K in the elliptic nome is $\sqrt{(3)}/2$ =sin120 and is the same for the theta functions of the exceptional Lie algebras $E_6 \subset E_8$ and therefore determines the geometry of the equilateral tritangents appearing in Figs.1,2.In this way theta functions determine icosahedral symmetry and the structure of space underlining particle physics and quantum gravity [3].

2 Jacobi Theta Functions

We will employ the Jacobi theta Function

$$\theta = \sum_{n=-\infty}^{\infty} exp(i\pi\tau n^2 + 2i\pi nz) \quad (2)$$

for the $\theta_{E_8}, \theta_{E_7}, \theta_{E_6}$ lattices([1],[4])where $exp(i\pi\tau)$ =q and the dependence on z is carrried by the lattice Γ so (2) reduces to

$$\theta = \sum_{m=1}^{\infty} r_\Gamma(2m)q^{2m} \quad (3)$$

where r_γ is the kissing number N_m =240 in equation(1)according to Lucas Lewark[6].Then the theta series for E_6 is[1]Ch.4

$$\theta_{E_6} = 1 + 72q^2 + 270q^4 + 720q^4 + \ldots \quad (4)$$

Fig.1 shows 12 vertices on 2 Riemann surfaces plus 3 at the origin thus acounting for 27 elementary particles of the Standard Model.Only 27 of 72 vertices rotate into themselves by ω =120 degrees and Coxeter([2] p.119) labels these by $(0,\omega^2,-\omega),(-\omega,0,1)$ in the equilateral tritangents Fig.12.3A which should be replaced by the torus of Fig.1 according to [3].

However if q= $exp(i\pi K/K)$ =0.0658 then it folows that iK/K= $\sqrt{3}/2 = sin\omega$ and it may easily be shown that this value of q remains the same for the powers in equations (1),(4)

The remaining terms of the tori(1),(4) are multiples of the kissing number possibly implying quantum entanglement.

3 Conclusion

We have seen how $\sin\omega$ =sin120 specifies the tritangents in the lattices of E_6, E_8 . Specifically in Section 12.5 of[2] Coxeter associates E_8 with the Witting Polytope shown in the frontispiece and Fig.2.Here there are 27 edges,labeled by ω ,at each vertex which implies quantum entanglement and illustrates how theta functions underlie the elementary particles and nucleons.

There are only 3 complex axes i,j,k which are a basis for the quaternion algebra and E_8 is isomorphic to the binary octahedral group by the Mackay correspondence [7] and is the largest of the Exceptional Lie algebras, There are no more.

Fig.1 Graph ot E_6

Fig.2 Graph of E_8

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