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An Introduction to Fuzzy Edge Coloring

[1]Jahir Hussain R and [2]Kanzul Fathima K S
[1,2] PG and Research Department of Mathematics, Jamal Mohamed College,
Trichy-620020, Tamil Nadu, India, Cell No.: [2]+91 9150198781,
hssn_jhr@yahoo.com and [2]kanzulfathima@gmail.com.

ABSTRACT

In this paper, a new concept of fuzzy edge coloring is introduced. The fuzzy edge coloring is an assignment of colors to edges of a fuzzy graph G. It is proper if no two strong adjacent edges of G will receive the same color. Fuzzy edge chromatic number of G is a least positive integer for which G has a proper fuzzy edge coloring. In this paper, the fuzzy edge chromatic number of different classes of fuzzy graphs and the fuzzy edge chromatic number of fuzzy line graphs are found. Isochromatic fuzzy graph is also defined.

Keywords

Fuzzy edge coloring; fuzzy edge chromatic number; fuzzy bipartite graph; fuzzy cycle; complete fuzzy graph; fuzzy line graph.

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INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has numerous applications in almost all fields. Fuzzy graph coloring is one of the most important concepts in fuzzy graph theory. In particular, fuzzy edge coloring concept can be applied to the problems like job scheduling, register allocation, exam scheduling, time tabling problem, assignment problem etc. In this paper, the fuzzy edge chromatic number of different classes of fuzzy graphs and fuzzy line graphs are discussed. In addition, the isochromatic fuzzy graph is defined and some of the isochromatic fuzzy graphs are presented.

1. PRELIMINARIES

A *fuzzy graph* G is a pair of functions G = (σ,μ) where σ : V→ [0,1], where V is a vertex (node) set and μ :V×V→ [0,1], a symmetric fuzzy relation on σ . The *underlying crisp graph* of G= (σ,μ) is G^* = (V,E), where E ⊆ V×V. *Strength of a path* in a fuzzy graph G is the weight of the weakest arc (edge) in that path. A *weakest arc* is an arc of minimum weight. A *strongest path* between two nodes u,v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by μ^{∞} (u,v). An edge (x,y) is said to be a *strong* if $\mu^{\infty}(x,y) = \mu(x,y)$. A cycle in a fuzzy graph is said to be *fuzzy cycle* if it contains more than one weakest arc. A fuzzy graph G is said to be *strong* if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$, ∀ (x,y)∈E. A fuzzy graph G is said to be *complete* if $\mu(x,y)= \sigma(x) \wedge \sigma(y)$, ∀ x,y∈V.

Two nodes of a fuzzy graph are said to be *fuzzy independent* if there is no strong edge between them. A subset S of V is said to be *fuzzy independent* of G if any two nodes of S are fuzzy independent. A fuzzy graph G is said to be *fuzzy bipartite* if V can be partitioned into two fuzzy independent sets V_1 and V_2. A fuzzy bipartite graph G is said to be *fuzzy complete bipartite* if there exist a strong edge between every pair of vertices of V_1 and V_2. If (x,y) is strong arc then x and y are *strong adjacent*.

Fuzzy coloring is an assignment of colors to vertices of a fuzzy graph G. It is said to be *proper* if every strong adjacent vertices have different colors. *Fuzzy chromatic number* of a fuzzy graph G is a minimum number of colors needed for proper fuzzy coloring of G. It is denoted by χ_f (G). The *strong degree* of a vertex v is the number of vertices that are strong adjacent to v. It is denoted by d^s (v). The *minimum strong degree* of G is δ^s (G) =min {d^s (v) / v∈ V}. The *maximum strong degree* of G is Δ^s (G) =max {d^s (v) / v∈ V}. A fuzzy graph G is said to be *strong regular fuzzy graph* if $\delta^s (G) = \Delta^s (G)$ = k, for some constant k.

2. FUZZY EDGE COLORING

Definition 2.1: Two edges of a fuzzy graph G are *strong adjacent* if they are strong and have a common vertex.

Definition 2.2: Two edges of G are *fuzzy edge independent* if they are not strong adjacent. The fuzzy edge independence number β^1(G) is the number of elements in the maximum fuzzy edge independent set of G.

Definition 2.3: A subset S of E is said to be *fuzzy edge independent* (fuzzy matching) if any two edges of S are fuzzy edge independent.

Definition 2.4: A *fuzzy k-edge coloring* of a fuzzy graph G is an assignment of k colors to edges of G.

Definition 2.5: A fuzzy k-edge coloring is said to *proper* if no two strong adjacent edges have the same color.

Definition 2.6: A *fuzzy edge chromatic number* of a fuzzy graph G is the minimum number of colors needed for proper fuzzy edge coloring. It is denoted by χ_f^1 (G).

Note 2.7: i. Clearly every fuzzy graph G has a proper fuzzy s- edge coloring, where s is the number of strong arcs in G.

ii. If G has proper fuzzy k-edge coloring then G has also proper fuzzy k^1-edge coloring, for every k^1 >k.

Theorem 2.8: *If G is a fuzzy graph such that its underlying crisp graph is a path P_n of length n then χ_f^1 (G) = 2.*

Proof

Let G be a fuzzy graph such that G^* is a path of length n. Clearly every edge of G is strong. So we can give color 1 and 2 alternatively to edges of G. This is a proper fuzzy edge coloring. Hence χ_f^1 (G) = 2.

Theorem 2.9: *The fuzzy edge chromatic number of complement of a complete fuzzy graph is 0.*

Proof

Let G be a complete fuzzy graph.

Since G is complete, $\mu(x,y) = \sigma(x) \wedge \sigma(y)$, ∀ x, y ∈V.

By the definition of complement of a fuzzy graph,

$$\overline{\mu}(x,y) = \sigma(x) \wedge \sigma(y) - \mu(x,y).$$

$$\therefore \overline{\mu}(x,y) = \sigma(x) \wedge \sigma(y) - \sigma(x) \wedge \sigma(y)$$

= 0.

Thus $\bar{G}$ does not have any edge. Hence χ_f^1 (G) = 0.

Theorem 2.10: *If G is a null fuzzy graph then χ_f^1 (G) = 0.*

Proof

Since G does not have any edge, χ_f^1 (G) = 0.

Theorem 2.11: *Let G be a fuzzy graph such that G* is a cycle. If G contains only one weakest arc then χ_f^1 (G) = 2.*

Proof

Let G be a fuzzy graph on a cycle of length n. Then we have two cases.

Case 1: let n≡1 (mod 2). Then n=2m+1, m≥1.

Let $e_1, e_2 \ldots e_n$ be n edges of G. Now assume that G contains only one weakest arc, say e_1. Assign color 1 to $e_1, e_2, e_4, e_6 \ldots e_{2m}$, since e_1 is weakest arc and color 2 to $e_3, e_5, e_7 \ldots e_{2m+1}$. This is a proper fuzzy edge coloring. Hence χ_f^1 (G) = 2.

Case 2: let n≡0 (mod 2). Then n = 2m, m≥1.

Let $e_1, e_2 \ldots e_n$ be edges of G. Now color the edges $e_1, e_3 \ldots e_{2m-1}$ as 1 and $e_2, e_4 \ldots e_{2m}$ as 2. Clearly this coloring is proper. Hence χ_f^1 (G) = 2.

Theorem 2.12: *Let G be a fuzzy cycle of length n. Then* $\chi_f{}^1(G) = \begin{cases} 2, \text{if } n \text{ is even} \\ 3, \text{ if } n \text{ is odd} \end{cases}$

Proof

Case i. Let G be a fuzzy cycle of even length and $e_1, e_2 \ldots e_{2m}$ be edges of G. In G every edge is strong. Now assign color 1 to $e_1, e_2 \ldots e_{2m-1}$ and color 2 to $e_2, e_4 \ldots e_{2m}$. This assignment will give a proper fuzzy edge coloring. Hence χ_f^1 (G) = 2.

Case ii. Let G be a fuzzy cycle of odd length and $e_1, e_2 \ldots e_{2m+1}$ are edges of G. Now give color 1 to $e_1, e_3 \ldots e_{2m-1}$ and 2 to $e_2, e_4 \ldots e_{2m}$. All edges of G are colored except e_{2m+1}. This coloring is proper fuzzy edge coloring. Since e_{2m+1} is strong adjacent to e_1 which is colored as 1and e_{2m} which is colored as 2, we cannot give color 1 or 2 to e_{2m+1}. So assign another color 3 to e_{2m+1}. Thus χ_f^1 (G) =3.

Note 2.13: Since there is at least Δ^S (G) strong edges must have different color for every proper fuzzy edge coloring, χ_f^1 (G) ≥ Δ^S (G).

Theorem 2.14: Let G be a fuzzy graph. Then the maximum number of strong edges in a fuzzy edge independent set of G is $\left\lceil \frac{n-1}{2} \right\rceil$.

Proof

We prove this theorem in two cases.

Case 1. Let G be a fuzzy graph containing only strong edges. Choose any one edge e_1 from G and remove the edges which are strong adjacent to e_1. If the resulting graph will contain at least one edge other than e_1, then go to the next step, otherwise stop. Choose another edge e_2 and remove all edges which are strong adjacent to e_2. If the resulting graph will contain at least one edge other than e_1 and e_2, then continue the process otherwise stop the process. Continuing in this way, the process is terminated when there is no edge in the resulting fuzzy graph or the resulting fuzzy graph will contain K_2. If n is even, the resulting fuzzy graph in the last step will contain K_2 and include this K_2 in the set $\{e_1, e_2 \ldots\}$.

Claim: The number of edges in the set $\{e_1, e_2 \ldots\}$ is at most $\frac{n-1}{2}$.

The number of edges we remove in the first step is at most 2(n–2), in second step is at most 2(n–4) …and in last step is at most 2(n–(n–1)) = 2.

Thus the number of edges in the set $\{e_1, e_2 \ldots\}$

$$\leq \frac{n(n-1)}{2} - 2(n-2) - 2(n-4) - 2(n-6) \ldots - (2(n-(n-1))$$

$$= \frac{n(n-1)}{2} - 2[(n-2) + (n-4) + (n-6) \ldots + (1)]$$

$$= \frac{n(n-1)}{2} - 2[1 + 3 + \cdots + (n-2)] = \frac{n(n-1)}{2} - 2[(\frac{n-1}{2})(\frac{n-1}{2})] = \frac{n-1}{2}, \text{ if n is odd.}$$

If n is even, the number of edges in the set $\{e_1, e_2 \ldots\}$is $\left\lceil \frac{n-1}{2} \right\rceil$.

Case 2: Consider a fuzzy graph with not every edge is strong. Let S be a fuzzy edge independent set of G. Put all edges which are not strong adjacent in S. Then the resulting fuzzy graph G^1 is a fuzzy graph containing only strong edges. Applying case 1 to G^1 we have at most $\left\lceil\frac{n-1}{2}\right\rceil$ strong edges.

Theorem 2.15: If G = K_n is a complete fuzzy graph on n vertices, then $\beta^1(G) = \left\lceil\frac{n-1}{2}\right\rceil$.

Proof

Let S be a maximum fuzzy edge independent set of G. Since G is complete, it have $\frac{n(n-1)}{2}$ strong edges and each edge is strong adjacent to (n–2) + (n–2) = 2(n–2) edges. Choose any one edge in G, say e_1. Put e_1 in S. We know that e_1 is strong adjacent to 2(n–2) edges. To find another edge e_2 which is independent to e_1, remove 2(n–2) strong edges which are strong adjacent to e_1. The resulting fuzzy graph will contain two components, namely K_2 & K_{n-2}. Clearly this K_2 is e_1. Choose any one edge in K_{n-2}, say e_2 and put it in S. Since e_2 is strong adjacent to 2(n–4) edges, remove these edges to find another edge e_3 which is independent to e_1& e_2. Then the resulting fuzzy graph will contain three components, namely K_2 (= e_1), K_2 (= e_2) & K_{n-4}. This process can be repeated until $K_{n-(n-1)} = K_1$ is obtained for n is odd and K_2 is obtained for n is even.

Let n be odd. Then

$$|S| = \frac{n(n-1)}{2} - 2(n-2) - 2(n-4) - 2(n-6) \ldots - (2(n-(n-1))$$
$$= \frac{n(n-1)}{2} - 2[(n-2) + (n-4) + (n-6) \ldots + (1)]$$
$$= \frac{n(n-1)}{2} - 2[1 + 3 + \cdots + (n-2)] = \frac{n(n-1)}{2} - 2[(\frac{n-1}{2})(\frac{n-1}{2})] = \frac{n-1}{2}.$$

If n is even, then $|S| = \left\lceil\frac{n-1}{2}\right\rceil$.

Theorem 2.16: *Let G be a fuzzy graph. Then χ_f^1 (G) ≤ n.*

Proof

For any fuzzy graph G, Δ^S (G) ≤ n–1. Suppose that χ_f^1 (G) > n. Without loss of generality, let χ_f^1 (G) = n+1. Then there exist n+1 maximal fuzzy edge independent sets E_1, E_2 … E_{n+1} in G whose union is E and intersection is empty. By theorem 2.14, the number of strong edges in each maximal fuzzy edge independent set is at most $\left\lceil\frac{n-1}{2}\right\rceil$.

The sum of number of strong edges in each E_i's (x) ≤ $\left\lceil\frac{n-1}{2}\right\rceil + \left\lceil\frac{n-1}{2}\right\rceil + \cdots + \left\lceil\frac{n-1}{2}\right\rceil (n+1) times$.

$$= \left\lceil\frac{n-1}{2}\right\rceil [n+1].$$

If n is odd, then x≤ $\frac{(n-1)(n+1)}{2}$ and if n is even x≤ $\frac{n(n+1)}{2}$. In both cases, we have a contradiction to the number of strong edges in G is at most $\frac{n(n-1)}{2}$. Hence *χ_f^1 (G) ≤ n.*

Theorem 2.17: *For any fuzzy graph G, Δ^S (G) ≤ χ_f^1 (G) ≤ Δ^S (G) +1.*

Proof

By note 2.13, Δ^S (G) ≤ χ_f^1 (G). It is enough to prove that χ_f^1 (G) ≤ Δ^S (G) + 1. Suppose that χ_f^1 (G) > Δ^S (G) + 1. Then by theorem 2.16, n ≥ χ_f^1 (G) >Δ^S (G) + 1. This implies that n > Δ^S (G) + 1. This is a contradiction to Δ^S (G) ≤ n– 1. Hence χ_f^1 (G) ≤ Δ^S (G) + 1.

Theorem 2.18: *The fuzzy edge chromatic number of a fuzzy complete graph on n vertices is n, if n is odd and n –1, if n is even.*

Proof

Let n be odd. By theorem 2.17, χ_f^1 (G) ≤ Δ^S (G) + 1. This implies that χ_f^1 (G) ≤ n–1+ 1= n.

Suppose that χ_f^1 (G) < n. Then G can have proper (n-1)-fuzzy edge coloring c = (E_1, E_2 … E_{n-1}). By theorem 2.15, $|E_1| \le \frac{n-1}{2}$, $|E_2| \le \frac{n-1}{2}$ … $|E_{n-1}| \le \frac{n-1}{2}$.

Now $|E_1| + |E_2| + \cdots + |E_{n-1}| \le \frac{n-1}{2} + \frac{n-1}{2} + \cdots + \frac{n-1}{2}(n-1) times$.

$= \frac{n-1}{2}[1 + 1 + \cdots + 1(n-1) times] = \frac{(n-1)(n-1)}{2}$ which is a contradiction to the number of strong edges in G is $\frac{n(n-1)}{2}$. Hence χ_f^1 (G) = n.

Let n be even. By theorem 2.17, χ_f^1 (G) ≥ n–1. G has proper (n-1)- fuzzy edge coloring c = (E_1,E_2 …E_{n-1}), since $|E| = |E_1| + |E_2| + \cdots + |E_{n-1}| \le \left\lceil\frac{n-1}{2}\right\rceil (n-1) = \frac{n(n-1)}{2}$ is true. Therefore χ_f^1 (G) ≤ n–1. Hence χ_f^1 (G) = n–1.

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Remark 2.19:

- A fuzzy bipartite graph G with bipartition (X, Y) can have at most mn strong edges, where m is the number of elements in X & n is the number of elements in Y.
- Maximum strong degree of a fuzzy bipartite graph G is at most max{m,n}

Proposition 2.20: The maximum number of strong edges in a fuzzy edge independent set of a fuzzy bipartite graph G is min {m, n}.

Theorem 2.21: *If G is a fuzzy bipartite graph then* χ_f^1 *(G) =* Δ^S *(G).*

Proof

Let G be a fuzzy bipartite graph. For any fuzzy graph G, χ_f^1 (G) ≥ Δ^S (G).

Suppose χ_f^1 (G) > Δ^S (G). Then G can have Δ^S +1maximal fuzzy edge independent set $E_1, E_2 \ldots E_{\Delta^S+1}$ whose union is E & intersection is empty. Without loss of generality, m ≤ n.By proposition 2.20, number of strong edges in each E_i is at most m.

Thus the sum of number of strong edges in all E_i ≤ m + m+ … +m (Δ^S (G) + 1) times.

= m (Δ^S (G) + 1) ≤ m(n+1). .

This is a contradiction to the number of strong edges in G is at most mn. Hence χ_f^1 (G) = Δ^S (G).

Corollary 2.22: *If G is complete fuzzy bipartite graph* $K_{m,n}$ *then* χ_f^1 *(G) = max {m,n}.*

Proof

Let G be a complete fuzzy bipartite graph with bipartition X and Y. Here $|X|$ = m and $|Y|$ = n. Since G is fuzzy bipartite, χ_f^1 (G) = Δ^S (G) and since G is complete, there exist a strong edge between every node of X & Y. Thus maximum strong degree Δ^S (G) = max {m,n}. Hence proved.

Corollary 2.23: *Let G be a fuzzy graph such that* G^* *is a star. Then* χ_f^1 *(G) =n.*

Proof

Let G be a fuzzy graph such that G^* is a star. Clearly G is $K_{1,n}$. Thus χ_f^1 (G) = max {1, n}= n.

3. ISOCHROMATIC FUZZY GRAPH

Definition 3.1: A fuzzy graph G is said to be *isochromatic* if χ_f (G) = χ_f^1 (G).

Examples of isochromatic fuzzy graphs

1. A complete fuzzy graph on odd nodes is isochromatic.
2. Let G be a fuzzy bipartite graph. If Δ^S (G) = 2, then G is isochromatic.
3. If G is a complete fuzzy bipartite graph such that Δ^S (G) =2, then G is isochromatic.
4. Every fuzzy cycle is isochromatic.
5. If G is a fuzzy graph on a cycle then G is isochromatic.
6. If G is a fuzzy graph such that G^* is a path of length n then G is isochromatic.

4. FUZZY EDGE CHROMATIC NUMBER OF FUZZY LINE GRAPHS.

Definition 4.1: Let G= (σ,μ) be a fuzzy graph. Then the fuzzy line graph of G is L (G) = (λ,ω) whose vertices are edges of G and two vertices in L (G) are adjacent if the corresponding edges are adjacent in G. The membership values of vertices and edges are given below

λ (x) = μ (x), ∀ vertices in L (G) and

ω (x,y) = λ (x) ∧ λ (y)

= μ (x) ∧ μ (y), ∀ edges in L (G).

The underlying crisp graph of a fuzzy graph G is G^*= (V,E) and of L (G) is $L(G)^* = (V^1,E^1)$ (here V^1 = E).

Theorem 4.2: *If every edge of a fuzzy graph G is strong then* χ_f^1 *(G) =* χ_f *(L (G)).*

Proof

Let G be a fuzzy graph in which every edge of G is strong. By the definition of line graph, every edge in L (G) is also strong.

Let χ_f^1 (G) = k. Then G has proper fuzzy k-edge coloring $\pi = (E_1, E_2 \ldots E_k)$. Assume that 1, 2…k be the colors of edges of G in which no two strong adjacent edges have same color.

Now color the vertices of L (G) such that a vertex in L (G) has color i if the corresponding edge has color i in G. Since two vertices in L (G) are strong adjacent iff the corresponding edges are strong adjacent in G and proper fuzzy k-edge coloring of G gives the proper fuzzy k-vertex coloring of L (G). Now we have to prove that k is minimum number.

Suppose that k is not minimum number. Without loss of generality, let L (G) has fuzzy (k-1)-vertex coloring $\pi^1 = (V_1^1, V_2^1 \ldots V_{k-1}^1)$. We know that "A set S is fuzzy edge independent in G iff S is fuzzy vertex independent in L (G)". Since $V_1^1, V_2^1 \ldots V_{k-1}^1$ are fuzzy vertex independent in L (G) and $V_i^1 = E_i$, where $E_1, E_2 \ldots E_{k-1}$ are fuzzy edge independent set in G, G has fuzzy proper (k-1)-edge coloring, which is a contradiction to χ_f^1 (G) = k. Therefore χ_f (G) = k. Hence χ_f^1 (G) = χ_f (L (G)).

Corollary 4.3: *If G is a strong fuzzy graph then χ_f^1 (G) = χ_f (L (G)).*

Proof

Since every arc of G is strong, by theorem 4.2, χ_f^1 (G) = χ_f (L (G)).

Corollary 4.4: *If G is a complete fuzzy graph then χ_f^1 (G) = χ_f (L (G)).*

Corollary 4.5: *If G is a fuzzy cycle then χ_f^1 (G) = χ_f (L (G)).*

Remark 4.6:

If we consider any fuzzy graph G, every edge needs not to be strong and there exist edge (edges) which is not strong.

Two edges that are not strong adjacent in G must be strong adjacent vertices in L (G), i.e, if e_i and e_j are not strong adjacent in G for $i \neq j$ but the corresponding vertices ($v_i^1 = e_i$ & $v_j^1 = e_j$, $i \neq j$) must be strong adjacent in L (G). Because of the membership values of edges in L (G), every edge in L (G) is strong. Every proper fuzzy vertex coloring of $L(G)$ gives a proper fuzzy edge coloring of G. But converse need not be true.

Let χ_f (L(G)) = k. Then L (G) has k-proper fuzzy coloring. This implies that G has proper fuzzy k- edge coloring. Thus χ_f^1 (G)≤ k = χ_f (L(G)) = k. Thus we have the following proposition.

Proposition 4.7:

For any fuzzy graph G, χ_f^1 (G) $\leq \chi_f$ (L (G)).

Note 4.8: If we redefine the fuzzy line graph of a fuzzy graph G in the following manner then χ_f^1 (G) = χ_f (L (G)).

The fuzzy line graph of a fuzzy graph G is L (G) = (λ,ω) whose vertices are edges of G and two edges of L (G) are adjacent if the corresponding vertices are strong adjacent in G. the membership values of vertices and edges are given by

$\lambda (x) = \mu (x), \forall$ vertices in L (G) and

$\omega (x,y) = \lambda (x) \wedge \lambda (y)$

$= \mu (x) \wedge \mu (y), \forall$ edges in L (G).

Note 4.9: In general for a fuzzy graph G $\chi_f(G) \neq \chi_f{}^1(L(G))$. But there are some fuzzy graphs satisfying this condition.

Theorem 4.10: *If G is a fuzzy cycle then χ_f (G) = χ_f^1 (L (G)).*

Proof

Let G be a fuzzy cycle.

⇒G has more than one weakest arc.

⇒Every edge of G is strong. To prove this, we have to prove the following lemma.

Lemma: *The fuzzy line graph of a fuzzy cycle is again a fuzzy cycle.*

Proof

Let G = $[v_1,v_2 \ldots v_n]$ be the fuzzy cycle of length n and $v_1, v_2 \ldots v_n$ be vertices of G and $e_1 = (v_1, v_2)$, $e_2 = (v_2, v_3) \ldots e_{n-1} = (v_{n-1}, v_n)$ & $e_n = (v_n, v_1)$ be edges of G. Since every edge of G is strong, $\mu (e_i) = \mu^\infty (e_i), \forall i$. Since every edge is strong adjacent to exactly two edges in G, the corresponding vertex in L (G) is strong adjacent to exactly two vertices. Thus the vertices of L (G) are $e_1, e_2 \ldots e_n$ and edges are $e_1^1 = (e_1,e_2)$, $e_2^1 = (e_2,e_{23}) \ldots e_{n-1}^1 = (e_{n-1},e_n)$& $e_n^1 = (e_n,e_1)$ such that

$$\omega(e_i^1) = \lambda(e_i) \wedge \lambda(e_{i+1})$$

$$= \mu(e_i) \wedge \mu(e_{i+1}), \forall i=1, 2 \ldots n \ \& \ (e_{n+1} = e_1).$$

Thus L (G) contains more than one weakest arc and L (G) = [e_1,e_2 … e_n]. Hence L (G) is fuzzy cycle of length n. Hence the lemma.

We know that χ_f (G) = χ_f^1 (G) = 2, if n is even &

χ_f (G) = χ_f^1 (G) = 3, if n is odd.

Hence χ_f (G) = χ_f^1 (L (G)).

Proposition 4.11: *If G is a fuzzy path (fuzzy graph such that the underlying crisp graph of G is a path of length n>2) then* $L(G)$ *is also a fuzzy path of length n*−*1.*

Proposition 4.12: *If G is a fuzzy path of length n>3 then* $\chi_f(G) = \chi_f{}^1(L(G))$

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