

Peristaltic Transport of Couple Stress fluids In a Uniform and Non-Uniform Annulus

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ABSTRACT

The aim of the present investigation is to study the peristaltic transport through the gap between coaxial tubes, where the outer tube is non uniform and the inner tube is rigid. The necessary theoretical results such as viscosity, pressure gradient and friction force on the inner and outer tubes have been obtained in terms of couple stress parameter. Out of these theoretical results the numerical solution of pressure gradient, outer friction, inert friction and flow rate are shown graphically for the better understanding of the problem.

Keyword: Peristaltic transport; Couple Stress Fluids

1 INTRODUCTION

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a mechanism may be involved in swallowing food through the esophagus, in urine transport from the kidney to the bladder through the urethra, in movement of chyme in the gastro-intestinal tract, in the transport of spermatozoa in the ductus efferent of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels, and in the vasomotion of small blood vessel such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

A number of analytical [1-8], numerical and experimental [9-13] studies of peristaltic flows of different fluids have been reported. A summary of most of the investigation reported up to the year 1983, has been presented by Srivastava and Srivastava [14], and some imported contribution of recent year, are reference in Srivastava and Saxsen [15]. Physiological organs are generally observed have the form of a non-uniform duct [16, 17]. In particular, the vas deferens in rhesus monkey is in the form of a diverging tube with a ration of exit to inlet dimensions of approximately four [18]. Hence, peristaltic analysis of a Newtonian fluid in a uniform geometry cannot be applied when explaining the mechanism of transport of fluid in most bio-systems. Recently, Srivastava et al [19] and Srivastava and Srivastava [20] studied peristaltic transport of Newtonian and non-Newtonian fluids in non-uniform geometries. Asha and Rathod [23,24] studied the effect of magnetic on peristaltic motion in uniform and non-uniform annulus. Rathod and Sridhar [25] showed the effect of couple stress fluid on peristaltic transport in a uniform and non- annulus porous media.

With the above discussion in mind, we propose to study the peristaltic transport of a viscous incompressible fluid (creeping flow) through the gap between coaxial tubes, where the outer tube is non-uniform and has a sinusoidal wave travelling down its wall and the inner one is a rigid, uniform tube and moving with a constant velocity. This investigation may have application in many clinical applications such as the endoscopes problem.

2 FORMULATION OF THE PROBLEM

Consider the flow of an incompressible Newtonian fluid through coaxial tubes such that the outer tubes is non-uniform and has a sinusoidal wave traveling down and inner one rigid, and moving with a constant velocity. The geometry of the wall surface is

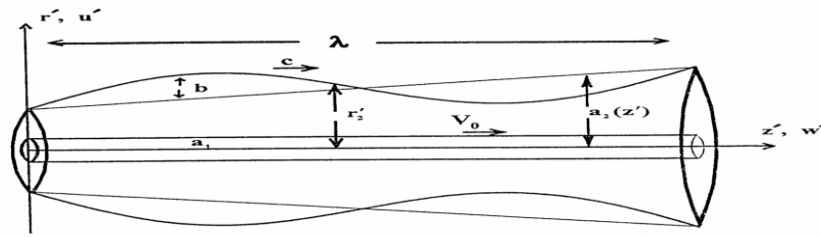
$$r_1' = a_1, \quad (2.1)$$

$$r_2' = a_2 + b \sin\left(\frac{2\pi}{\lambda}(x' - ct)\right) \quad (2.2)$$

With

$$a_2(z') = a_{20} k z'$$

With a_1 is the radius of the inner tube $a_2(z')$ is the radius of the outer tube at axial distance z' from inlet, a_{20} is the radius of the outer tube at the inlet, k ($\ll 1$) is the constant whose magnitude depends on the length of the outer tube, b is the amplitude, λ is the wave length, c is the propagation velocity, and t is the time. We choose a cylindrical coordinate system (r', z') where the z -axis lies along the centreline of the inner and the outer tubes and r' is the distance measured radially.



The equation of motion of the flow in the gap between the inner and the outer tubes are

$$\frac{1}{r} \frac{\partial(r', u')}{\partial r} + \frac{\partial(w')}{\partial z} = 0 \quad (2.3)$$

$$\rho \left\{ \frac{\partial u'}{\partial t} + u' \frac{\partial u'}{\partial r} + w' \frac{\partial u'}{\partial z} \right\} = - \frac{\partial p'}{\partial r} + \mu \left\{ \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(r', u')}{\partial r} \right) + \frac{\partial^2 u'}{\partial z^2} \right\} - \eta \nabla^2 (\nabla^2 (u')) \quad (2.4)$$

$$\rho \left\{ \frac{\partial w'}{\partial t} + u' \frac{\partial w'}{\partial r} + w' \frac{\partial w'}{\partial z} \right\} = - \frac{\partial p'}{\partial z} + \mu \left\{ \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(r', w')}{\partial r} \right) + \frac{\partial^2 w'}{\partial z^2} \right\} - \eta \nabla^2 (\nabla^2 (w')) \quad (2.5)$$

$$\text{Where } \nabla^2 = \left\{ \frac{\partial}{\partial r'} \left[\frac{1}{r'} \left(\frac{\partial(r')}{\partial r'} \right) \right] \right\}$$

Where u' and w' are the velocity components in the r' and w' direction respectively, ρ is the density, p' is the pressure and μ is the viscosity, η is the couple stress parameter.

The boundary conditions are

$$\begin{aligned} u' = 0, \quad w' = V_0, \quad \nabla^2(w') \text{ finite at } r' = r_1 \\ u' = \frac{\partial r_2'}{\partial t}, \quad w' = 0, \quad \nabla^2(w') = 0 \text{ at } r' = r_2 \end{aligned} \quad (2.6)$$

It is convenient to non-dimensionalize the variable appearing in equation (2.1-2.6) and introducing Reynolds number Re , wave number ratio δ , and velocity parameter V_0 as follows:

$$\begin{aligned} z = \frac{z'}{\lambda}, \quad r = \frac{r'}{c}, \quad u = \frac{\lambda u'}{a_{20} c}, \quad p = \frac{a_{20}^2}{\lambda \mu c} p'(z'), \quad t = \frac{t' c}{\lambda}, \quad Re = \frac{\rho c a_{20}}{\mu} \\ \delta = \frac{a_{20}}{\lambda}, \quad V_0 = \frac{V_0'}{c}, \quad r_1 = \frac{r_1'}{a_{20}} = \varepsilon, \quad r_2 = \frac{r_2'}{a_{20}} = 1 + \frac{\lambda k z}{a_{20}} + \phi \left(\frac{2\pi}{\lambda} (z - t) \right) \end{aligned} \quad (2.7)$$

$$\text{where } \phi(\text{amplitude ratio}) = \frac{b}{a_{20}} \leq 1$$

The equation of motion and boundary conditions in the dimensionless form becomes

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (2.8)$$

$$Re \delta^3 \left\{ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right\} = - \frac{\partial p}{\partial r} + \delta^2 \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(ru)}{\partial r} \right) + \delta^4 \frac{\partial^2 u}{\partial z^2} - \frac{\delta^2}{\gamma^2} \nabla^2 (\nabla^2 (u)) \quad (2.9)$$



$$\operatorname{Re} \delta \left\{ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right\} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (rw)}{\partial r} \right) + \delta^2 \frac{\partial^2 w}{\partial z^2} - \frac{1}{\gamma^2} \nabla^2 (\nabla^2 (w)) \quad (2.10)$$

Where $\gamma = \sqrt{\frac{\eta}{\mu a_{20}^2}}$ is the couple-stress fluid parameter

The boundary conditions are:

$$u=0 \quad w=V_0 \quad \nabla^2(u, w) \text{ finite at } r=r_1=\mathcal{E}, \quad (2.11a)$$

$$u = \frac{\partial r_2}{\partial y} \quad w=0 \quad \nabla^2(u, w)=0, \quad \text{at } r=r_2=1 + \frac{\lambda k z}{a_{20}} + \phi \sin[2\pi(z-t)] \quad (2.11b)$$

Using the long wavelength approximation and dropping terms of order δ it follows from equation (2.8-2.11) that the appropriate equation describing the flow in the laboratory frame of reference are

$$\frac{\partial p}{\partial r} = 0, \quad (2.12)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) - \frac{1}{\gamma^2} \nabla^2 (\nabla^2 (w)) \quad (2.13)$$

with dimensionless boundary condition

$$w=V_0 \quad \nabla^2(u, w) \text{ finite at } r=r_1=\mathcal{E},$$

$$w=0 \quad \nabla^2(u, w)=0 \quad \text{at } r=r_2=1 + \frac{\lambda k z}{a_{20}} + \phi \sin[2\pi(z-t)] \quad (2.14)$$

Integrating equation and using the boundary condition one finds the expression for the velocity profile as

$$w(z,t) = -\frac{1}{4} \left(\frac{\partial p}{\partial z} \right) \left[(r_2^2 - r_1^2) \left(\frac{\ln(r/r_1)}{\ln(r_2/r_1)} \right) - r^2 + r_1^2 \right] + \frac{1}{16\gamma^2} \left(\frac{\partial p}{\partial z} \right) \left\{ (r_2^2 - r_1^2)^2 \left(\frac{\ln(r/r_1)}{\ln(r_2/r_1)} \right)^2 \right. \\ \left. - (r^2 - r_1^2)^2 \right\} + \frac{V_0}{\ln(r_2/r_1)} \ln(r/r_2) \left\{ \frac{1}{4\gamma^2} \left\{ (r_2^2 - r_1^2) \left(\frac{\ln(r/r_1)}{\ln(r_2/r_1)} \right) \right. \right. \\ \left. \left. - r^2 + r_1^2 \right\} - 2 \frac{V_0}{\ln(r_2/r_1)} \ln(r/r_2) \right\} \quad (2.15)$$

The instantaneous volume flow rate $Q(z,t)$ is given by

$$Q(z,t) = \int_{r_1}^{r_2} 2\pi r w dr = -\frac{\pi \partial p}{8 \partial z} \left\{ (r_2^2 - r_1^2) \left[r_2^2 + r_1^2 - \left(\frac{r_2^2 - r_1^2}{\ln(r_2/r_1)} \right) \right] \right\} - \pi V_0 \left\{ \frac{r_1^2 - r_2^2}{\ln(r_2/r_1)} + r_1^2 \right\} + \\ \frac{\pi}{32\gamma^2} \left[(r_2^2 - r_1^2)^2 \left[(r_2^2 + r_1^2)^2 - \frac{(r_2^2 - r_1^2)^2}{\ln(r_2/r_1)^2} \right] \right] + \frac{\pi V_0}{8\gamma^2} \left\{ \frac{r_1^2 - r_2^2}{2\ln(r_2/r_1)} + r_1^2 \right\} \\ \left\{ (r_2^2 - r_1^2) \left[(r_2^2 + r_1^2) - \frac{(r_2^2 - r_1^2)}{\ln(r_2/r_1)} \right] \right\} \quad (2.16)$$

Or

$$\frac{\partial p}{\partial z} = -8 \left(\frac{Q/\pi \ln(r_2/r_1) + \frac{V_0}{2}(r_1^2 - r_2^2) + V_0 r_1^2 \ln(r_2/r_1) \left\{ 1 + \frac{1}{8\gamma^2} D \right\}}{(r_2^4 - r_1^4) \ln(r_2/r_1) - (r_2^2 - r_1^2)^2 \left\{ 1 + \frac{1}{4\gamma^2} D \right\}} \right) \quad (2.17)$$

Where



$$D = (r_2^4 - r_1^4) - \frac{(r_2^2 - r_1^2)^2}{\ln(r_2/r_1)}$$

The pressure rise $\Delta p_L(t)$ and friction force (at the wall) on the outer and the inner tubes $F_L^{(o)}(t)$ and $F_L^{(i)}(t)$ respectively, in a tube of length L, in their non-dimensional forms, are given by

$$\Delta p_L(t) = \int_0^A \frac{\partial p}{\partial z} dz \tag{2.18}$$

$$\Delta F_L^{(o)}(t) = \int_0^A r_2^2 \left(-\frac{\partial p}{\partial z}\right) dz, \tag{2.19}$$

$$\Delta F_L^{(i)}(t) = \int_0^A r_1^2 \left(-\frac{\partial p}{\partial z}\right) dz, \tag{2.20}$$

Where $A=L/\lambda$,

Substituting from equation (2.17) in equation (2.18-2.20) and with $r_1 = \varepsilon$ and $r_2(z,t) = 1 + \frac{\lambda kz}{a_{20}} + \phi \sin[2\pi(z-t)]$, we get

$$\begin{aligned} \Delta p_L(t) = & \int_0^A -8 \left\{ \frac{Q(z,t)}{\pi} \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \right\} + \frac{V_0}{2} \left[\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t) \right)^2 - \varepsilon^2 \right] \\ & + V_0 \varepsilon^2 \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \left\{ 1 + \frac{1}{8\gamma^2} X \right\} \left\{ 1 / \left[\left(\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t) \right)^4 - \varepsilon^4 \right) \right. \right. \\ & \left. \left. \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \left(1 + \frac{1}{4\gamma^2} X \right) \right] \right\} dz \end{aligned} \tag{2.21}$$

$$\begin{aligned} \Delta F_L^{(o)}(t) = & \int_0^A 8 \left\{ \left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t) \right)^2 \right\} \left\{ \frac{Q(z,t)}{\pi} \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \right\} \\ & + \frac{V_0}{2} \left[\varepsilon^2 - \left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t) \right)^2 \right] + V_0 \varepsilon^2 \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \\ & \left\{ 1 + \frac{1}{8\gamma^2} X \right\} \left\{ 1 / \left[\left(\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t) \right)^4 - \varepsilon^4 \right) \right. \right. \\ & \left. \left. \left(\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t) \right)^2 - \varepsilon^2 \right)^2 \right] \left[1 + \frac{1}{4\gamma^2} X \right] \right\} dz \end{aligned} \tag{2.22}$$

$$\begin{aligned} \Delta F_L^{(i)}(t) = & \int_0^A 8 \varepsilon^2 \left\{ \frac{Q(z,t)}{\pi} \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \right\} \\ & + \frac{V_0}{2} \left[\varepsilon^2 - \left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t) \right)^2 \right] + V_0 \varepsilon^2 \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \end{aligned}$$



$$\left\{1 + \frac{1}{8\gamma^2} X\right\} \left\{1 / \left[\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)\right)^4 - \varepsilon^4\right] - \left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)\right)^2 - \varepsilon\right\} \left\{ + \frac{1}{4\gamma^2} X \right. \\ \left. \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon} \right] \right\} \quad (2.23)$$

Where

$$X = \left\{ \left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)^2 - \varepsilon^2\right) \left[1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)^2 + \varepsilon^2\right] - \left(\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)^2 - \varepsilon^2}{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)} \right) \right\} \\ \ln \left[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)^2 - \varepsilon^2}{\varepsilon} \right]$$

The limiting of equation (2.15-2.17) as r_1 tends to zero gives the forms of the axial velocity and the pressure gradient for peristaltic flow in non uniform tube (without endoscope, $\varepsilon = 0$), these are

$$w(r, z, t) = -\frac{1}{4} \left(\frac{\partial p}{\partial z} \right) (r_2^2 - r^2) + \frac{1}{16} \left(\frac{\partial p}{\partial z} \right) (r_2^4 - r^4), \quad (2.24)$$

$$\frac{\partial p}{\partial z} = -\frac{8Q}{\pi r_2^4} + \frac{16Q}{\pi r_2^{16}}, \quad (2.25)$$

Hence the pressure rise and the outer friction force, in this case respectively, take the form

$$\Delta p_L(t) = -8 \int_0^A \frac{[Q(z, t) / \pi] dz}{\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)\right)^4} + 16 \int_0^A \frac{[Q(z, t) / \pi] dz}{\gamma^2 \left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)\right)^{16}} \quad (2.26)$$

$$\Delta F_L^{(o)}(t) = 8 \int_0^A \frac{[Q(z, t) / \pi] dz}{\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)\right)^2} + 16 \int_0^A \frac{[Q(z, t) / \pi] dz}{\gamma^2 \left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)\right)^4} \quad (2.27)$$

If $k=0$ in equations (2.26) and (2.27), we get expression for the pressure rise and friction force in a uniform tube. The analytical interpretation of our analysis with other theories are difficult to make at this stage, as the integrals in equation (2.21-2.23) and equation (2.26) and (2.27) are not integrable in closed form, neither for non-uniform nor uniform geometry ($k=0$). Thus further studies of our analysis are only possible after numerical evaluation of these integrals.

3. RESULT, DISCUSSION AND CONCLUSION

To discuss the results obtained above quantitatively assume the form of the instantaneous volume rate of the flow $Q(z, t)$, periodic in $(z-t)$ as [2.14-2.16]

$$\frac{Q(z, t)}{\pi} = \frac{\bar{Q}}{\pi} - \frac{\phi^2}{2} + 2\phi \sin(2\pi(z-t)) + \frac{2\lambda kz}{a_{20}} \phi \sin(2\pi(z-t)) + \phi^2 \sin^2(2\pi(z-t))$$

where \bar{Q} is the time average of the flow over one period of the wave. This form $Q(z, t)$ has been assumed in view of the fact that the constant value of $Q(z, t)$ gives $\Delta p_L(t)$ always negative, and hence will be no pumping action. Using this form of $Q(z, t)$, we shall now compute the dimensionless pressure rise $\Delta p_L(t)$, the inner friction force $F_L^{(i)}(t)$ (on the inner surface) and the outer friction force $F_L^{(o)}(t)$ (on the outer tube) over the tube length for various value of the dimensionless time t , dimensionless flow average \bar{Q} , amplitude ratio ϕ , radius ratio ε , couple stress parameter γ , and the velocity of the inner tube V_0 . The average rise in pressure $\Delta \bar{p}_L$, outer friction force $F_L^{-(o)}$ and the inner friction force $F_L^{-(i)}(t)$ are then evaluated by averaging $\Delta p_L(t)$, $F_L^{(o)}(t)$, and $F_L^{(i)}(t)$ over one period of the wave. As integrals

in equation (2.21-2.23) are not integrable in closed form, they are evaluated numerically using digital computer. Following Srivastava [15], we use the value of the various parameters in equation (2.21-2.23) as:

$$a_{20}=1.25\text{cm}, \quad L= \lambda =8.01\text{cm} \quad k = \frac{3a_{20}}{\lambda}.$$

Furthermore, since most routine upper gastrointestinal endoscopes are between 8-11 mm in diameter as reported Cotton and Williams [22] and the radius of the small intestine is 1.25 cm as reported in Srivastava [20] then the radius ratio ϵ , take the values 0.32, 0.38, and 0.44.

Figure (1) and (4) represent the variation of the dimensionless pressure with dimensionless time t for $\phi=0.4$, $V_0=0$, $\gamma=0.2$ and radius ratio $\epsilon=0.32, 0.38$, and 0.44 in the case of uniform and non uniform tube respectively. The difference of the pressure for different values of ϵ becomes smaller as the radius ratio increases, i.e as the inner radius of the tube increases. It can also be seen that the effect of increasing the flow rate is to reduce the pressure rise for various values of ϵ

Fig (2) and (3) represent the variation of the dimensionless pressure rise with dimensionless time t for $\phi=0.4$, $V_0=0$, $\gamma=0.2$ and velocity $V_0=-1, 0, 1$ for non uniform and uniform tube respectively. The result shows that the pressure rise increases as the inner tube velocity increases, i.e. pressure rise for the endoscope increases as the inner tube moves in the direction of the peristaltic waves.

Fig (6)-(7) and (8) shows the inner friction force (on the inner surface) and outer friction forces (on the outer surface) are plotted versus dimensionless time t for different values of $\epsilon=0.32, 0.38$, and 0.44 . It observed that as the radius ratio increases there is decrease in the inner friction force. It is noticed that the inner friction force behaves similar to the outer friction force for the same values of the parameter. Moreover, the outer friction force is greater than the inner friction force at the same values of the parameter.

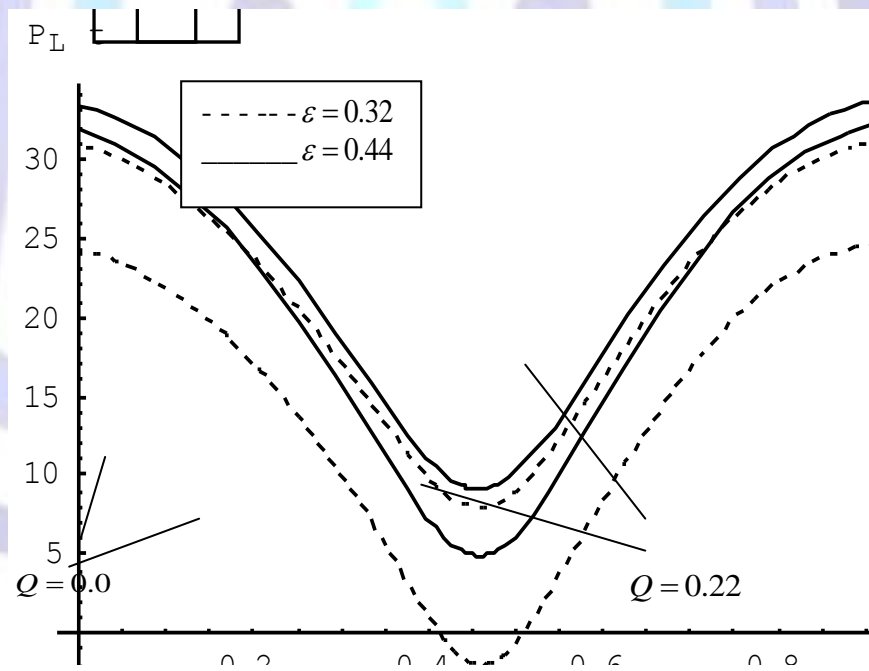


Fig (1) Variation of of pressure rise over the length of a uniform annulus at $\gamma=0.2$ $\phi=0.4$, $V_0=0$ and different values of ϵ

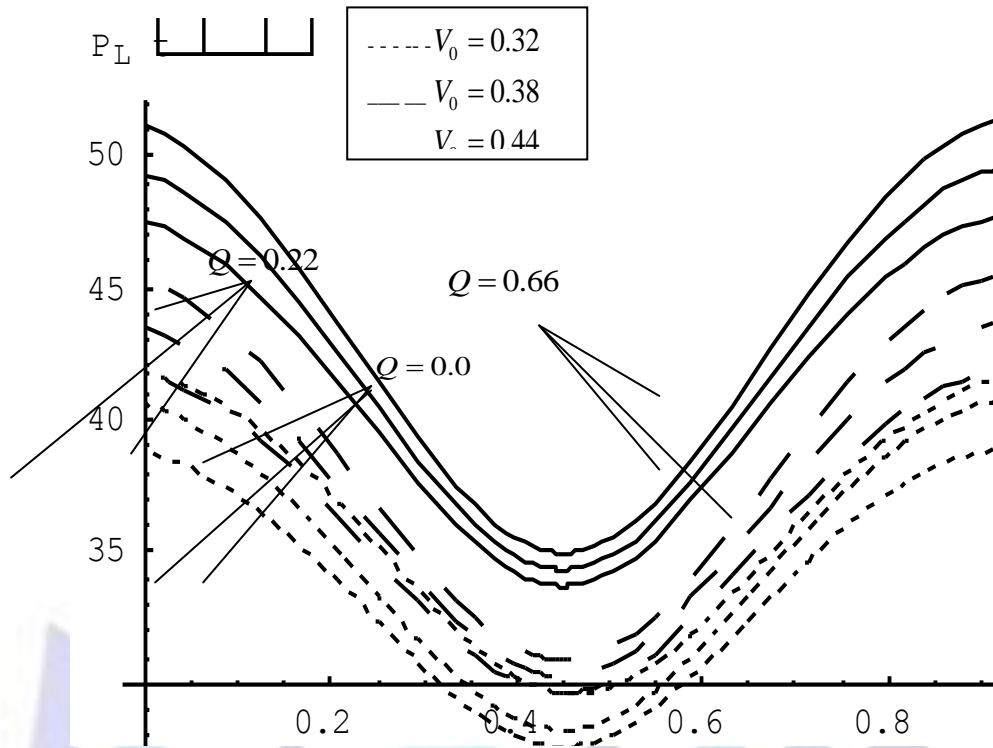


Fig (2) Variation of Pressure rise over the length of a non-uniform annulus at $\gamma = 0.2$ for different values of V_0 at $\phi = 0.4, \varepsilon = 0.38$

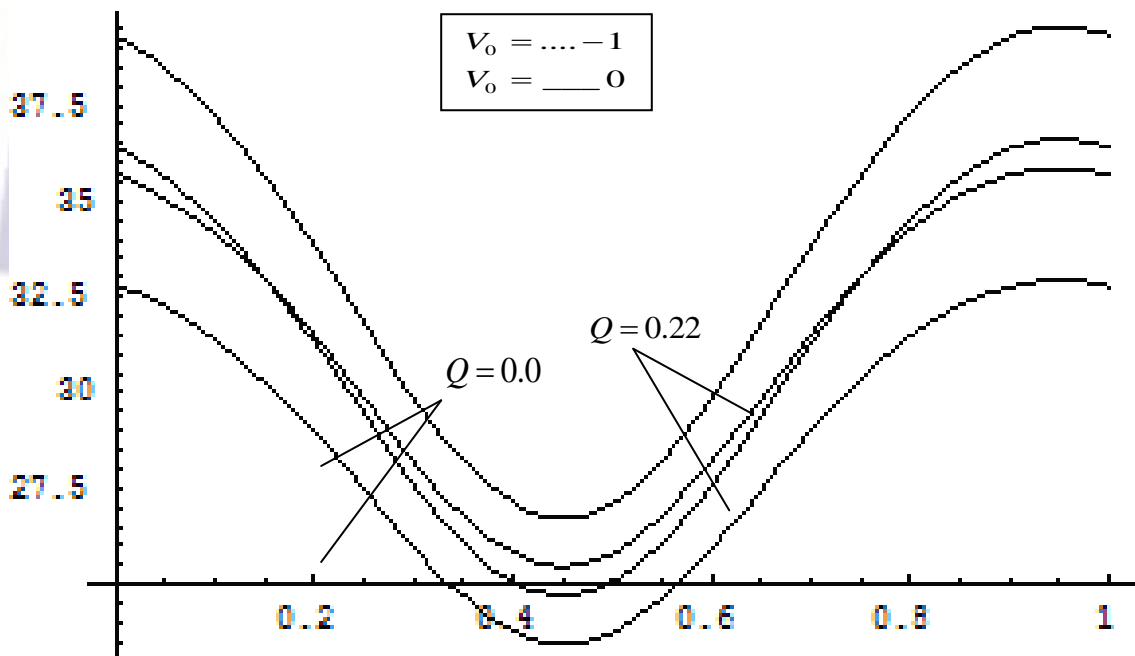


Fig.(3)Variation of pressure rise over the length of a uniform annulus at $\gamma = 0.2, \phi = 0.4, \varepsilon = 0.38$ for different values of V_0

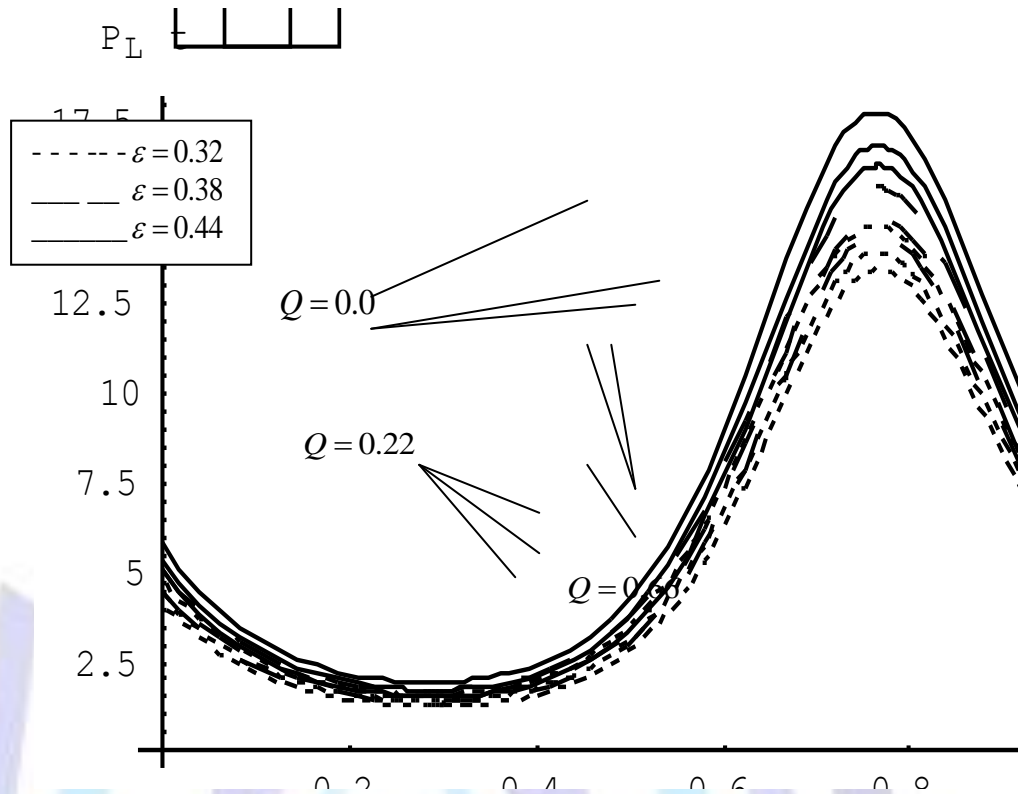
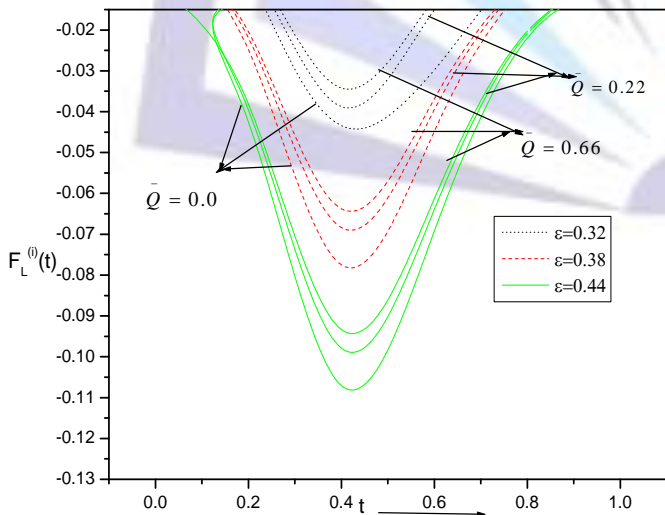
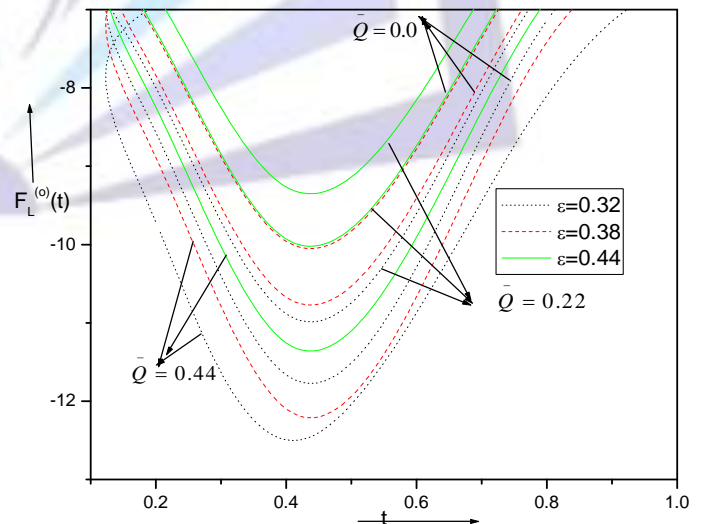


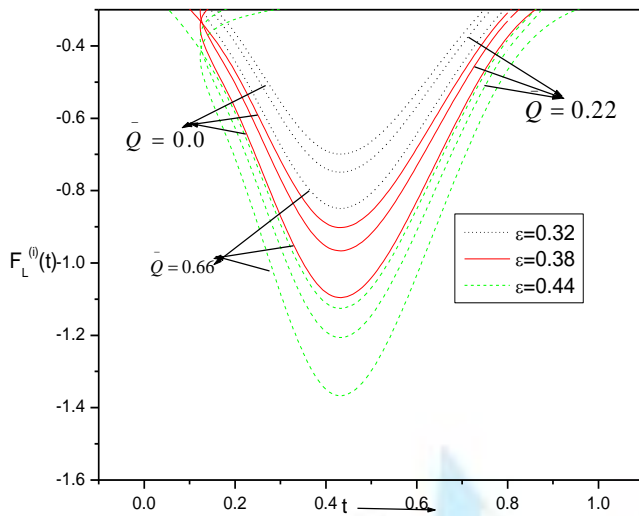
Fig (4) Variation of pressure rise over the length of non uniform at $\gamma = 0.2$ $\phi = 0.4$, $V_0 = 0$ and different values of ϵ



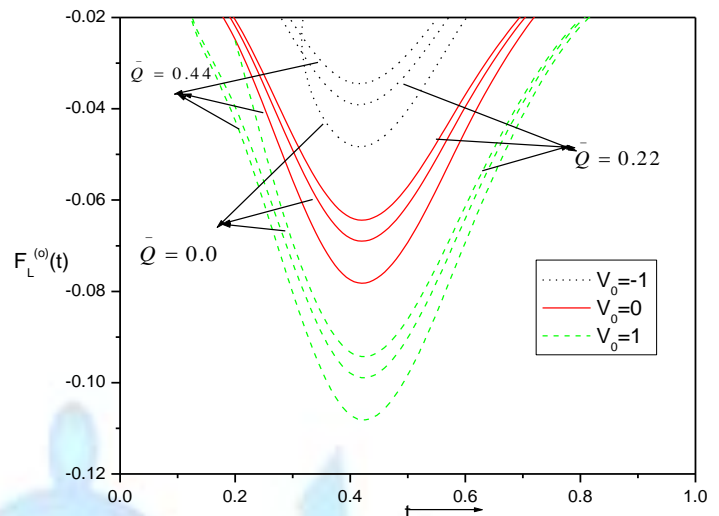
fig(5) Variation of inner friction force of non uniform annulus at $\gamma=0.2, \phi=0.4$, for different values of V_0 at $\epsilon=0.38$



fig(6) Variation of the outer force over the length of a non-uniform at $\gamma=0.2, \phi=0.4$, and different values of ϵ at $V_0=0$



fig(7) Variation of inner friction force over the length of non uniform annulus at $\gamma=0.2, \phi=0.4$, for different values at $V_0=0$



fig(8) Variation of the outer friction force over the length of non-uniform annulus at $\gamma=0.2, \phi=0.4$, and different values of V_0 at $\epsilon=0.38$

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