A Note On Soft Fuzzy Volterra Spaces

A.Haydar EŞ
Department of Mathematics Education, Başkent University, Bağlıca, 06490 Ankara, Turkey
haydares@baskent.edu.tr

ABSTRACT

In this paper, the concepts of soft fuzzy ε_r-Volterra spaces and soft fuzzy ε_p-Volterra spaces are introduced and studied. We will discuss several characterizations of those spaces.

Indexing terms/Keywords

Soft fuzzy topology; soft fuzzy Volterra spaces; soft fuzzy weakly Volterra spaces; soft fuzzy ε_r-Volterra spaces; soft fuzzy ε_p-Volterra spaces.

Academic Discipline And Sub-Disciplines

Mathematics; Topology.

SUBJECT CLASSIFICATION

Mathematics Subject Classification; 54A40, 03E72.

1. INTRODUCTION

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [12]. Chang in [1] introduced and developed the concept of fuzzy topological spaces. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concept of Volterra spaces have been studied extensively in classical topology in [3,4]. The concepts of fuzzy Volterra spaces, fuzzy weakly Volterra spaces and generalized fuzzy Volterra spaces in fuzzy topological spaces are introduced and studied by the authors in [5,6]. The concept of soft fuzzy topological space is introduced by I.U.Tiryaki [10]. The concept of almost P-spaces and almost GP-spaces in soft fuzzy setting was introduced by Es [2] . In this paper, in section 3, the concepts of soft fuzzy ε_r -Volterra spaces and soft fuzzy ε_p- Volterra spaces are introduced and studied.

2. PRELIMINARIES

We introduce some basic notions and results that are used in the sequel.

Definition 2.1. [8] Let (X,τ) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \wedge_{i=1}^{\infty} \mu_i$ where each μ_i is fuzzy open set. The complement of a fuzzy $G_\delta$set is fuzzy F_σ.

Definition 2.2. [10] Let X be a set, μ be a fuzzy subset of X and $M \subseteq X$. Then the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by $SF(X)$.

Proposition 2.3. [10] If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j) | j \in J\}$ has a meet, that is greatest lower bound, in $(SF(X), \sqsubseteq)$, denoted by $\prod_{j\in J}(\mu_j, M_j)$ such that $\prod_{j\in J}(\mu_j, M_j) = (\mu, M)$

where $\mu(x) = \wedge_{j\in J} \mu_j(x), \forall x \in X$,

$$M = \cap_{j\in J} M_j.$$

Definition 2.4. [10] Let X be a non-empty set and the soft fuzzy sets A and B in the form,

$$A = \{(\mu, M) | \mu(x) \epsilon I^X, \forall x \in X, M \subseteq X\}$$

$$B = \{(\lambda, N) | \lambda(x) \epsilon I^X, \forall x \in X, N \subseteq X\}$$

Then,

(i) $A \subseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N$.

(ii) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall x \in X, M = N$.

(iii) $A' \Leftrightarrow 1 - \mu(x), \forall x \in X, X \setminus M$.

(iv) $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall x \in X$ and $M \cap N$, for all $(\mu, M), (\lambda, N) \epsilon\, SF(X)$.

(v) $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X$ and $M \cup N$, for all $(\mu, M), (\lambda, N) \epsilon\, SF(X)$.

Definition 2.5. [10]

$(0, \emptyset) = \{(\lambda, N) | \lambda = 0, N = \emptyset\}$

$(1, X) = \{(\lambda, N) | \lambda = 1, N = X\}$

Definition 2.6. [11] For $(\mu, M) \in SF(X)$ the soft fuzzy set

$(\mu, M)' = (1 - \mu, X \setminus M)$ is called the complement of (μ, M).

Definition 2.7. [10] A subset $\tau \subseteq SF(X)$ is called an SF-topology on X if

(i) $(0, \emptyset)$ and $(1, X) \in \tau$

(ii) $(\mu_j, M_j) \in \tau, j = 1,2, \dots, n \Rightarrow \prod_{j=1}^{n}(\mu_j, M_j) \in \tau$

(iii) $(\mu_j, M_j), j \in J \Rightarrow \coprod_{j \in J}(\mu_j, M_j) \in \tau$. The elements of τ are called soft fuzzy open,

and those of $\tau' = \{(\mu, M) | (\mu, M)' \in \tau\}$ soft fuzzy closed.

If τ is SF-topology on X we call the pair (X, τ) SF-topological space (in short SFTS).

Definition 2.8. [10] The closure of a soft fuzzy set (μ, M) will be denoted by $\overline{(\mu, M)}$. It is given by

$$\overline{(\mu, M)} = \sqcap \{(\gamma, N) | (\mu, M) \sqsubseteq (\gamma, N), (\gamma, N) \in \tau'\}.$$

Likewise the interior is given by

$$(\mu, M)^{\circ} = \sqcup \{(\gamma, N) | (\gamma, N) \in \tau, (\gamma, N) \sqsubseteq (\mu, M)\}.$$

Note: $\overline{(\mu, M)} = cl(\mu, M)$ and $(\mu, M)^{\circ} = int(\mu, M)$.

Definition 2.9. [11] Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be a soft fuzzy set in (X, τ). Then

(i) (λ, N) is said to be soft fuzzy regular open if $(\lambda, N) = int(cl(\lambda, N))$.

(ii) (λ, N) is said to be soft fuzzy regular closed if $(\lambda, N) = cl(int(\lambda, N))$.

Definition 2.10. [7] A fuzzy topological space (X, τ) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X, τ) is fuzzy open. That is, every non-zero fuzzy G_{δ} set in (X, τ), is fuzzy open in (X, τ).

Definition 2.11. [8] A fuzzy topological space (X, τ) is called a fuzzy almost P-space if for every non-zero fuzzy G_{δ} set λ in (X, τ), $int(\lambda) \neq 0$ in (X, τ).

Definition 2.12. 8] A fuzzy topological space (X, τ) is called a weak fuzzy P-space if the countable intersection fuzzy regular open sets in (X, τ) is a fuzzy regular open set in (X, τ).

Definition 2.13. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy weak P-space if the countable intersection soft fuzzy regular open sets in (X, τ) is a soft fuzzy regular open set in (X, τ). That is, $\prod_{i=1}^{\infty}(\lambda_i, M_i)$ is a soft fuzzy regular open in (X, τ), where (λ_i, M_i)'s are soft fuzzy regular open sets in (X, τ).

Definition 2.14. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy P-space if countable intersection of soft fuzzy open sets in (X, τ) is soft fuzzy open. That is, every non-zero soft fuzzy G_{δ} set in (X, τ) is soft fuzzy open in (X, τ).

Definition 2.15. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy almost P-space if for every non-zero soft fuzzy G_{δ} set in (X, τ), $int(\lambda, M) \neq (0, \emptyset)$ in (X, τ).

Definition 2.16. [2]A soft fuzzy set (λ, M) in a soft fuzzy topological space (X, τ) is called a soft fuzzy nowhere dense if there exists no non-zero soft fuzzy open set (μ, N) in (X, τ) such that $(\mu, N) \sqsubseteq cl(\lambda, M)$. That is, $int(cl(\lambda, M)) = (0, \emptyset)$.

Definition 2.17. [2] A soft fuzzy set (λ, M) in a soft fuzzy topological space (X, τ) is called a soft fuzzy dense if there exists no soft fuzzy closed set (μ, N) in (X, τ) such that $(\lambda, M) \sqsubseteq (\mu, N) \sqsubseteq (1, X)$. That is, $cl(\lambda, M) = (1, X)$.

Definition 2.18. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy submaximal space if for each soft fuzzy set (λ, M) in (X, τ) such that $cl(\lambda, M) = (1, X)$, then (λ, M) in (X, τ).

Definition 2.19. [2] A soft fuzzy topological space (X,τ) is called a soft fuzzy almost GP-space if $int(\lambda,M) \neq (0,\emptyset)$, for each non- zero soft fuzzy dense and soft fuzzy $G_\delta\, set\ (\lambda,M)$ in (X,τ). That is, (X,τ) is a soft fuzzy almost GP-space if every non-zero soft fuzzy G_δ set in (X,τ) with $cl(\lambda,M) = (1,X)$, $int(\lambda,M) \neq (0,\emptyset)$.

Definition 2.20. [2] A soft fuzzy set (λ,M) in (X,τ) is called a soft fuzzy first category set if $(\lambda,M) = \sqcup_{i=1}^{\infty}(\lambda_i,M_i)$, where (λ_i,M_i)'s are soft fuzzy nowhere dense in (X,τ).

3. ON SOFT FUZZY VOLTERRA SPACES

Definition 3.1. Let (λ,M) be a soft fuzzy first category set in soft fuzzy topological space (X,τ). Then $(1,X)-(\lambda,M)$ is called a soft fuzzy residual set in (X,τ).

Definition 3.2. A SFTS (X,τ) is called a soft fuzzy ε_r -Volterra space if *cl* $(\prod_{i=1}^{n}(\lambda_i,M_i)) = (1,X)$, where (λ_i,M_i) 's are soft fuzzy dense and soft fuzzy residual sets in (X,τ).

Proposition 3.3. If the SFTS (X,τ) is a soft fuzzy ε_r -Volterra space, then

int $(\sqcup_{i=1}^{n}(\lambda_i,M_i)) = (0,\emptyset)$, where the soft fuzzy set (λ_i,M_i) 's are soft fuzzy first category sets such that *int* $(\lambda_i,M_i)=(0,\emptyset)$ in (X,τ).

Proof. Let (λ_i,M_i) 's (i=1,2,…,n) be soft fuzzy first category sets such that *int* $(\lambda_i,M_i) =(0,\emptyset)$ in (X,τ). Then $((1,X)-(\lambda_i,M_i))$'s are soft fuzzy residual sets such that *cl* $((1,X)-(\lambda_i,M_i))= (1,X)$ in (X,τ). Since (X,τ) is a soft fuzzy ε_r -Volterra space,

cl $(\prod_{i=1}^{n}((1,X)-(\lambda_i,M_i))) = (1,X)$. Then *cl* $((1,X) - \sqcup_{i=1}^{n}(\lambda_i,M_i)) = (1,X)$ and hence $(1,X)- int\left(\sqcup_{i=1}^{n}(\lambda_i,M_i)\right) = (1,X)$. Therefore, we have *int*$(\sqcup_{i=1}^{n}(\lambda_i,M_i)) = (0,\emptyset)$, where (λ_i,M_i) 's are soft fuzzy first category sets such that *int* $(\lambda_i,M_i)=(0,\emptyset)$.

Proposition 3.4. Let (X,τ) be a soft fuzzy ε_r -Volterra space. Then (X,τ) is a soft fuzzy Volterra space.

Proof. Let (λ_i,M_i) 's (i=1,2,…,n) be soft fuzzy dense and soft fuzzy G_δ sets in (X,τ). Then, (λ_i,M_i) 's are soft fuzzy residual sets in (X,τ).This implies that (λ_i,M_i) 's are soft fuzzy dense and soft fuzzy residual sets in (X,τ). Since (X,τ) is a soft fuzzy ε_r -Volterra space

cl $(\prod_{i=1}^{n}(\lambda_i,M_i)) = (1,X)$. Hence (X,τ) is a soft fuzzy Volterra space.

Proposition 3.5. If each soft fuzzy nowhere dense set is a soft fuzzy closed set in a soft fuzzy Volterra space (X,τ), then (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. Let (λ_i,M_i) 's (i=1,2,…,n) be soft fuzzy dense and soft fuzzy residual sets in (X,τ). Since (λ_i,M_i) 's are soft fuzzy residual sets, $((1,X)-(\lambda_i,M_i))$'s are soft fuzzy first category sets in (X,τ). Now $((1,X)-(\lambda_i,M_i)) = \sqcup_{j=1}^{\infty}(\mu_{ij},N_{ij}) \epsilon\, \tau$, where (μ_{ij},N_{ij}) 's are soft fuzzy nowhere dense sets in (X,τ) . From the hypothesis, the soft fuzzy nowhere dense sets (μ_{ij},N_{ij})'s are soft fuzzy closed sets and hence $((1,X)-(\lambda_i,M_i))$'s are soft fuzzy F_δ sets in (X,τ). Therefore (λ_i,M_i) 's are soft fuzzy G_δ sets in (X,τ). Since (X,τ) is a soft fuzzy Volterra space, *cl*$(\prod_{i=1}^{n}(\lambda_i,M_i)) = (1,X)$. Hence (X,τ) is a soft fuzzy ε_r -Volterra space.

Definition 3.6. A SFTS (X,τ) is called a soft fuzzy nodec space if every non-zero soft fuzzy nowhere dense set (λ,M) is soft fuzzy closed in (X,τ). That is, if (λ,M) is a soft fuzzy nowhere dense set in (X,τ), then $((1,X)-(\lambda,M))\epsilon\, \tau$.

Proposition 3.7. If the SFTS (X,τ) is a soft fuzzy Volterra space and soft fuzzy nodec space, then SFTS (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. Let (X,τ) be a soft fuzzy Volterra space and soft fuzzy nodec space and (λ_i,M_i) 's (i=1,2,…,n) be soft fuzzy dense and soft fuzzy residual sets in (X,τ). Since (λ_i,M_i) 's are soft fuzzy residual sets's are soft fuzzy first category sets in (X,τ). Now $(1,X)-(\lambda_i,M_i)=\sqcup_{j=1}^{\infty}(\mu_{ij},N_{ij})$, where (μ_{ij},N_{ij}) 's are soft fuzzy nowhere dense sets in (X,τ) .Since (X,τ) is a soft fuzzy nodec space, soft fuzzy nowhere dense sets (μ_{ij},N_{ij})'s are soft fuzzy closed in (X,τ). By the Proposition 3.5, (X,τ) is a soft fuzzy ε_r -Volterra space.

Proposition 3.8. If the SFTS (X,τ) is a soft fuzzy Volterra space and soft fuzzy submaximal space, then (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. Obvious.

Proposition 3.9. If the SFTS (X,τ) is a soft fuzzy ε_r -Volterra space and soft fuzzy D-Baire space, then *int*($\sqcup_{i=1}^{n}(\lambda_i, M_i)$) $= (0, \emptyset)$,where the (λ_i, M_i) 's are soft fuzzy first category sets in (X,τ).

Proof. Let (X,τ) be a soft fuzzy ε_r -Volterra space and soft fuzzy D-Baire space and (λ_i, M_i) 's (i=1,2,3,…,n) are soft fuzzy first category sets in (X,τ). Since (X,τ) is soft fuzzy D-Baire space, the soft fuzzy first category sets (λ_i, M_i) 's are soft fuzzy nowhere dense sets in (X,τ) and hence$((1,X) - (\lambda_i, M_i))$'s are soft fuzzy dense sets in (X,τ). Then $((1,X) - (\lambda_i, M_i))$'s are soft fuzzy dense and soft fuzzy residual sets in (X,τ). By the hypothesis, *cl* $(\prod_{i=1}^{n}((1,X) - (\lambda_i, M_i))) = (1,X)$. This implies that

cl $((1,X)$ - $\sqcup_{i=1}^{n}(\lambda_i, M_i)) = (1,X) - int\left(\sqcup_{i=1}^{n}(\lambda_i, M_i)\right) = (1,X)$. Therefore $int\left(\sqcup_{i=1}^{n}(\lambda_i, M_i)\right) = (0,\emptyset)$, where (λ_i, M_i) 's are soft fuzzy first category sets in (X,τ).

Definition 3.10. Let (X,τ) be a SFTS. Then (X,τ) is called a soft fuzzy Baire space if *int* $((\sqcup_{i=1}^{\infty}(\lambda_i, M_i)) = (0,\emptyset)$, where (λ_i, M_i) 's are soft fuzzy nowhere dense sets in (X,τ).

Proposition 3.11. If $\sqcup_{i=1}^{\infty}(\lambda_i, M_i)$, where the (λ_i, M_i) 's are soft fuzzy nowhere dense sets , is a soft fuzzy nowhere dense set in a soft fuzzy Baire space (X,τ), then (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. The proof is similar to Proposition 3.7.

Proposition 3.12. If each soft fuzzy first category set is a soft fuzzy closed set in a soft fuzzy Baire space (X,τ), then (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. Let (X,τ) be a soft fuzzy Baire space and (λ_i, M_i) 's (i=1,2,3,…,n) are soft fuzzy dense and soft fuzzy residual sets in (X,τ). Since (λ_i, M_i) 's are soft fuzzy residual sets, $((1,X) - (\lambda_i, M_i))$'s are soft fuzzy first category sets in (X,τ). By the hypothesis, the soft fuzzy first category sets $((1,X) - (\lambda_i, M_i))$'s are soft fuzzy closed sets in (X,τ) and hence (λ_i, M_i) 's are soft fuzzy open sets in (X,τ). Since (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy open sets in (X,τ), $((1,X) - (\lambda_i, M_i))$'s are soft fuzzy nowhere dense sets in (X,τ). Since (X,τ) is a soft fuzzy Baire space, $int(\sqcup_{i=1}^{n}((1,X) - (\lambda_i, M_i))) \sqsubseteq int(\sqcup_{i=1}^{\infty}((1,X) - (\lambda_i, M_i)))= (0,\emptyset)$. That is , $int(\sqcup_{i=1}^{n}((1,X) - (\lambda_i, M_i))) = (0,\emptyset)$. Hence *cl* $(\prod_{i=1}^{n}(\lambda_i, M_i)) = (1,X)$, where (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy residual sets in (X,τ). Therefore (X,τ) is a soft fuzzy ε_r -Volterra space.

Definition 3.13. A SFTS (X,τ) is called a soft fuzzy ε_p- Volterra space if *cl* $(\prod_{i=1}^{n}(\lambda_i, M_i)) = (1,X)$, where (λ_i, M_i) 's are soft fuzzy pre-open and soft fuzzy G_δ sets in (X,τ).

Proposition 3.14. If the SFTS (X,τ) is a soft fuzzy ε_p- Volterra space, then *Int* $((\sqcup_{i=1}^{n}(\lambda_i, M_i)) = (0,\emptyset))$, where (λ_i, M_i) 's are soft fuzzy pre-closed and a soft fuzzy F_6 sets in (X,τ).

Proof. Let (λ_i, M_i) 's (i=1,2,3,…,n) be soft fuzzy pre-closed and soft fuzzy F_6 sets in (X,τ). Then $((1,X) - (\lambda_i, M_i))$'s are soft fuzzy pre-open and soft fuzzy G_δ sets in (X,τ). By the hypothesis, *cl* $(\prod_{i=1}^{n}((1,X) - (\lambda_i, M_i))) = (1,X)$. Then *cl* $((1,X)$ - $\sqcup_{i=1}^{n}(\lambda_i, M_i)) = (1,X) - int\left(\sqcup_{i=1}^{n}(\lambda_i, M_i)\right) = (1,X)$. Therefore, we have *int*($\sqcup_{i=1}^{n}(\lambda_i, M_i)$) $= (0,\emptyset)$ where (λ_i, M_i) 's (i=1,2,3,…,n) be soft fuzzy pre-closed and soft fuzzy F_6 sets in (X,τ).

Proposition 3.15. If the SFTS (X,τ) is a soft fuzzy ε_p- Volterra space, then (X,τ) is a soft fuzzy Volterra space.

Proof. Let (λ_i, M_i) 's (i=1,2,3,…,n) be soft fuzzy dense and soft fuzzy G_δ sets in (X,τ). Since (λ_i, M_i) 's are soft fuzzy dense sets, *cl* (λ_i, M_i)= $(1,X)$. Now int(cl (λ_i, M_i)) =$(1,X)$. Then $(\lambda_i, M_i) \sqsubseteq$ int(cl(λ_i, M_i)). Since (X,τ) is a soft fuzzy ε_p-Volterra space and (λ_i, M_i) 's are soft fuzzy pre-open and soft fuzzy G_δ sets in (X,τ), *cl*$(\prod_{i=1}^{n}(\lambda_i, M_i)) = (1,X)$, where (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy G_δ sets in (X,τ). Hence (X,τ) is a soft fuzzy Volterra space.

Proposition 3.16. If *preint* $(\sqcup_{i=1}^{n}(\lambda_i, M_i)) = (0,\emptyset)$ where (λ_i, M_i) 's are soft fuzzy pre-closed sets in a soft fuzzy topological space (X,τ), then (X,τ) is a soft fuzzy ε_p- Volterra space.

Proof. Let (λ_i, M_i) 's (i=1,2,3,…,n) be soft fuzzy pre-open and soft fuzzy G_δ sets in (X,τ). Then $((1,X) - (\lambda_i, M_i))$'s are soft fuzzy pre-closed in (X,τ). By the hypothesis , preint($\sqcup_{i=1}^{n}((1,X) - (\lambda_i, M_i))$)= $(0,\emptyset)$. This implies that $(1,X) - precl\left(\sqcup_{i=1}^{n}(\lambda_i, M_i)\right) = (0,\emptyset)$ and hence $precl(\prod_{i=1}^{n}(\lambda_i, M_i)) = (1,X)$. Since $precl(\prod_{i=1}^{n}(\lambda_i, M_i)) \sqsubseteq cl(\prod_{i=1}^{n}(\lambda_i, M_i))$, we have $cl(\prod_{i=1}^{n}(\lambda_i, M_i))= (1,X)$.Therefore $cl(\prod_{i=1}^{n}(\lambda_i, M_i))= (1,X)$, where (λ_i, M_i) 's are soft fuzzy pre-open and soft fuzzy G_δ sets in (X,τ). Hence (X,τ) is a soft fuzzy ε_p- Volterra space.

Proposition 3.17. If *precl*($\prod_{i=1}^{n}(\lambda_i, M_i)$) $=(1, X)$, where(λ_i, M_i) 's are soft fuzzy pre-open sets in a soft fuzzy topological space (X,τ), then (X,τ) is a soft fuzzy ε_p- Volterra space.

Proof. The proof is similar to Proposition 3.16.

Proposition 3.18. If (λ, M) $=\sqcup_{i=1}^{n}(\lambda_i, M_i)$, where (λ_i, M_i) 's are soft fuzzy pre-closed sets, is a soft fuzzy nowhere dense set in a soft fuzzy topological space (X,τ), then (X,τ) is a soft fuzzy ε_p- Volterra space.

Proof. Suppose that (λ, M) $=\sqcup_{i=1}^{n}(\lambda_i, M_i)$, where (λ_i, M_i) *'s* are soft fuzzy pre-closed sets*,* and (λ, M) is a soft fuzzy nowhere dense sets in (X,τ). Then *preint*(*precl*(λ, M))= $(0, \emptyset)$. Since *preint*(λ, M) ⊑*preint*(*precl*(λ, M)), we have *preint*(λ, M)= $(0, \emptyset)$. Then *preint*($\sqcup_{i=1}^{n}(\lambda_i, M_i)$)= $(0, \emptyset)$, where (λ_i, M_i) 's are soft fuzzy pre-closed sets in (X,τ). Hence by Proposition 3.16, (X,τ) is a soft fuzzy ε_p- Volterra space.

Proposition 3.19. If each soft fuzzy pre-closed set is a soft fuzzy nowhere dense sets in soft fuzzy ε_r -Volterra space (X,τ), then (X,τ) is a soft fuzzy ε_p- Volterra space.

Proof. Obvious.

Proposition 3.20. If a soft fuzzy ε_p- Volterra space (X,τ) is a soft fuzzy submaximal space, then (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. Immediate from the definitions.

Proposition 3.21. If each soft fuzzy nowhere dense set is soft fuzzy closed set in a soft fuzzy ε_p- Volterra space (X,τ), then (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. It is clear from the definitions.

Proposition 3.22. If a soft fuzzy ε_p- Volterra space (X,τ) is a soft fuzzy nodec space, then (X,τ) is a soft fuzzy ε_r -Volterra space.

Proof. Let (X,τ) be a soft fuzzy ε_p- Volterra space and a soft fuzzy nodec space.Since (X,τ) is a soft fuzzy nodec space, each soft fuzzy nowhere dense set is a soft fuzzy closed set in (X,τ). By Proposition 3.21, (X,τ) is a soft fuzzy ε_r -Volterra space.

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