Minimal resistance of curves under the single impact assumption

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Abstract

We consider the hollow on the half-plane $\{(x,y) : y \le 0\} \subset \mathbb{R}^2$ defined by a function $u : (-1,\, 1) \to \mathbb{R}$, $u(x) < 0$ and a vertical flow of point particles incident on the hollow. It is assumed that u satisfies the so-called single impact condition (SIC): each incident particle is elastically reflected by graph(u) and goes away without hitting the graph of u anymore. We solve the problem: find the function u minimizing the force of resistance created by the flow. We show that the graph of the minimizer is formed by two arcs of parabolas symmetric to each other with respect to the y-axis. Assuming that the resistance of $u \equiv 0$ equals 1, we show that the minimal resistance equals $\pi/2 - 2\arctan(1/2) \approx 0.6435$. This result completes the previously obtained result [A. Plakhov. *The problem of minimal resistance for functions and domains.* SIAM J. Math. Anal. **46**, 27302742 (2014)] stating in particular that the minimal resistance of a hollow in higher dimensions equals 0.5.

We additionally consider a similar problem of minimal resistance, where the hollow in the half-space $\{(x_1,\dots,x_d,y) : y \le 0\} \subset \mathbb{R}^{d+1}$ is defined by a radial function U satisfying SIC, $U(x) = u(|x|)$, with $x = (x_1,\dots,x_d)$, $u(\xi) < 0$ for $0 \le \xi < 1$ and $u(\xi) = 0$ for $\xi \ge 1$, and the flow is parallel to the y-axis. The minimal resistance is greater than 0.5 (and coincides with 0.6435 when $d = 1$) and converges to 0.5 as $d \to \infty$.

Mathematics subject classifications: 49Q10, 49K30

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1 Introduction

Consider a function $u : \bar{\Omega} \to \mathbb{R}$, where $\Omega \subset \mathbb{R}^d$, $d \ge 1$ is an open connected bounded set. We assume that the gradient $\nabla u(x)$ exists and is continuous on an open full-measure subset of Ω, and

$$u(x) = 0 \;\text{ for }\; x \in \partial\Omega \quad\text{and}\quad u(x) \le 0 \;\text{ for }\; x \in \Omega. \tag{1}$$

Consider a parallel flow of point particles in $\mathbb{R}^{d+1}$ incident on the graph of u with the velocity $v = (0,\dots,0,\,-1)$. That is, the flow is parallel to the $(d+1)$th coordinate axis and is directed "downward".

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