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# Waves and Instabilities in $E \times B$ Dusty Plasma

*Sukhmander Singh*

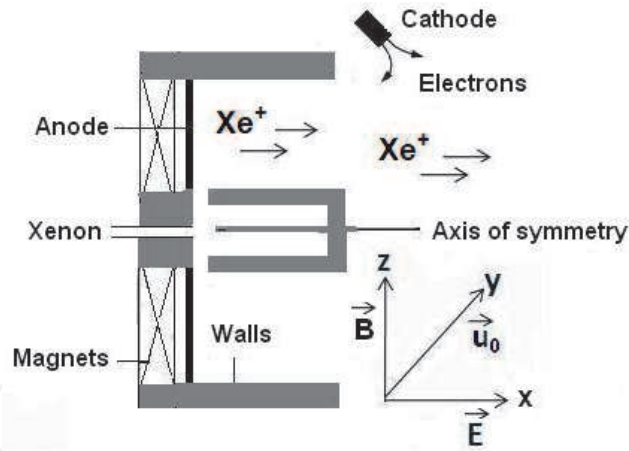
## Abstract

Hall thrusters are common examples of  $E \times B$  configuration, where electron trajectory gets trapped along the external magnetic field lines. This significantly increases the residence time of electrons in the plasma discharge channel. Hall thrusters are potential candidates for spacecraft station keeping, rephrasing and orbit topping applications because of its high thrust resolutions and efficiency. The goal of this chapter is to explain the working principle of Hall thrusters and to characterize the resistive instability in hot dusty plasma. The studies of these instabilities are useful to design efficient Hall thrusters and to understand the solar dusty plasma. The large amplitude of these oscillations has an adverse effect on the power processing unit of the devices. This reduces the efficiency and specific impulse and shortens the operating life of the Hall thruster. The theory of linearization of fluid equation for small oscillation has been given. The chapter also discusses the origin of plasma oscillation in a plasma discharge mechanics.

**Keywords:** plasma oscillations, instabilities, Hall thrusters, resistive plasma, growth rate

## 1. Introduction to $E \times B$ devices

There are some devices where plasma is weakly magnetized because of the larger Larmor radius of the massive ions against the length of the discharge channel. Electrons get trapped along the magnetic field lines in the channel. Hall thrusters and magnetrons are the common examples of  $E \times B$  devices. The  $E \times B$  configuration is used to confine electrons, increasing the electron residence time and allowing ionization and plasma sustainment. Hall thrusters have emerged as an integral part of propulsion technology. Unlike chemicals and electric rockets, in a Hall thruster, the propulsion thrust is achieved by gas which has high atomic number and low ionization potential. For this Xenon is mostly used. In the discharge channel, Xenon is ionized and then accelerated by electrostatic forces. Hall thrusters are versatile electric propulsion devices, where thrust efficiencies can exceed 50% and specific impulses are typically between 20 min and 1 h. The specific impulse has the dimension of time and is a measure for the effective lifetime of the thruster, when lifting its own propellant from the earth's surface. The specific impulse is defined as  $I_{sp} = T/\dot{m}_p g$ , where  $\dot{m}_p$  is the mass flow rate;  $T$  is the thrust, which is the total force undergone by the Hall Thruster in relation to the acceleration of the ions; and  $g$  is the acceleration due to gravity. If the propellant is fully ionized, the specific impulse is equal to the mean axial component of the final ion velocity divided by  $g$ .



**Figure 1.**  
Schematic diagram of a typical Hall plasma thruster [3].

It is interesting that these thrusters adjust their thrust and impulse by varying the acceleration voltage and the flow rate of the propellant. The fine tuning of the thrust correction can be done for the compensation of atmospheric drag for low-flying satellites using its high thrust resolution. The necessary thrust for such applications ranges from micro-Newtons to some Newtons with electric input powers of some 10 to some 10,000 Watts [1–4].

**Figure 1** shows the internal components of a Hall thruster which is generally made of an axis-symmetric cylindrical discharge chamber. A cathode is fixed outside to produce electrons to neutralize the outer surface of the device to overcome the space-charging problems. A high atomic weight number and low ionization potential gases are preferred propellant (Xenon, Argon) for Hall thrusters to get more thrust. The propellant enters from the left side of the channel via anode and gets ionized through the hollow cathode of the device. The electric field of strength  $\sim 1000$  V/m gets generated inside the discharge channel along the axial direction of the device. By using magnets around the annular channel and along the thruster centreline, a radial magnetic field of moderate strength ( $\sim 150$ – $200$  G) is created, which is strong enough for the electrons to get magnetized, i.e. they are able to gyrate within the discharge channel, but the ions remain unaffected due to their Larmor radius much larger than the dimension of the thruster [3].

We used a Cartesian coordinate system to understand the different forces on the particles inside the channel and let us suppose, the X-axis represents, the axis of the thruster. Generally, the applied electric and magnetic fields are in axial (along X axis) and radial (along Z axis) directions, respectively, of the device. Therefore because of the perpendicular electric and magnetic fields, the Lorentz forces act on the electrons along the Y axis ( $\vec{E} \times \vec{B}$  azimuthal direction). Since electrons have smaller Larmor radius than the length of the channel, therefore electrons rotate along the azimuthal direction under the influences of the Lorentz force [1–4].

## 2. Review on plasma instabilities in Hall thrusters

It is well known that plasma pressure drives the instabilities in plasma. Therefore the confined plasma is prone to non-equilibrium thermodynamic state. Therefore it must be important to know the consequences of these instabilities. It has been established that the amplitude and frequency of the oscillations in the Hall thrusters depend on mass flow rate, discharge voltage, geometry, magnetic field profile and

cathode operation mode. On the other hand, the plasma in a Hall thruster does not stay uniform, and an inhomogeneous plasma immersed in the external electric and magnetic fields is not in the thermodynamically equilibrium state; this deviation in general is a source of plasma instabilities.

In Hall thrusters, from low frequencies (few Hertz) to high frequencies (few GHz), oscillation spectra have been observed on theoretically as well as experimentally based studies. The oscillations in the range of 10–20 kHz are called as discharge oscillations, and oscillations in the range of 5–25 kHz are said to be ionization-driven oscillations. The drift instabilities and density gradient plasma are responsible to produce oscillations in the range of 20–60 kHz in a Hall thruster. The oscillations in the range of 70–500 kHz are also called transient time oscillations and are the order of ion residence time in the channel of the device. The oscillations associated with azimuthal waves are represented by high-frequency (0.5–5 MHz) oscillations [5]. Litvak and Fisch [6] have developed an analytical model for electrostatic and electromagnetic resistive instabilities in a Hall plasma for azimuthal disturbances. Singh and Malik investigated resistive instabilities for axial and azimuthal disturbances in a Hall thrusters [7, 8]. Fernandez et al. [9] did simulations for the growth of resistive instability. Litvak and Fisch [10] have analysed gradient-driven Rayleigh-type instabilities in a Hall thruster using two fluid hydrodynamic equations. Ducrocq et al. [11] have investigated high-frequency electron drift instability in the cross-field configuration of a Hall thruster. Barral and Ahedo [12] have developed a low-frequency model of breathing oscillations in Hall discharges, where they observed that unstable modes are strongly nonlinear and are characterized by frequencies obeying a scaling law different from that of linear modes. Chesta et al. [13] have developed a theoretical model to obtain the growth rate and frequencies of axial and azimuthally propagating plasma disturbances.

### 3. Studies of fine particles in plasma

The presence of heavy fine particles with a size of 1–50 microns and mass of orders  $10^{-10}$  to  $10^{-15}$  kg in a classical plasma acts as an external component in plasma. If the density of the dust particles is less than the plasma density, the system is called dusty plasma. These fine particles acquire some charges from the electrons to get charged. The magnitude of charge on dust grain is not constant. It depends on the type of dust grain, the surface properties of dust grain, the dust dynamics, the temperature, the density of plasma and the wave motion in the medium. The presence of fine particles in a plasma makes it more complex and these particles alter the dynamics of the plasma species which generate new propagating modes by exhibit their own dynamics. The dusty plasmas have an exciting property which has attracted researchers over the world in this area [14–22]. The presence of charged dust grains modifies the ion-acoustic waves, lower hybrid waves, ion-acoustic and introduces dust acoustic waves and dust ion acoustic waves [22]. Verma et al. have studied the electrostatic oscillation in the presence of grain charge perturbation in a dusty plasma [23]. They studied the property of electrostatic oscillation and instability phenomena taking into account the temporal evolution of the grain charge in an unmagnetized dusty plasma. Cui and Goree have studied the effect of fluctuations of the charge on a dust grain in plasma [24]. Sharma and Sugawa studied the effect of ion beam on dust charge fluctuations [25]. It is observed that growth rate of the instability increases with the relative density of negatively charged dust. If dust particle charge is  $eZ_d$ . The quasineutrality condition is given for dust particles by  $n_{e0} = n_{i0} + Z_d n_{d0}$ .

#### 4. Electron plasma discharge oscillation

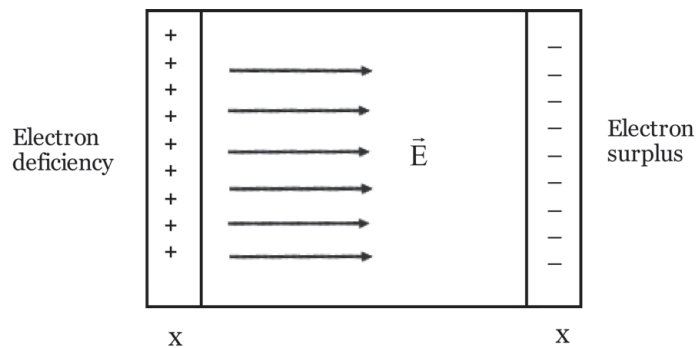
When electrons are displaced from the equilibrium position of the charged particles relative to the uniform background of the ions in plasma, an electric field is developed in such a direction that it tries to pull the electrons back to its equilibrium position to restore the neutrality. Because of the inertia effect, the electrons overshoot the equilibrium position, and now the electric field is developed in the opposite direction which again tries to pull back the electrons to their position of equilibrium. The massive ions are supposed to be fixed in the background and are not capable to respond the oscillating field generated by the oscillation of electrons. If  $n_0$  is the number of electrons per unit volume in infinite sheet plasma of thickness  $x$ , let the electron be displaced (as shown in **Figure 2**) to the right from their equilibrium position which results to generate surface charge density  $\sigma = en_0x$  on the left side of the sheet and equal and opposite surface charge density on the right side of the sheet. The generated electric field  $\vec{E} = en_0x/\epsilon_0$  tries to pull the electrons back, and thus oscillation takes place in plasma. From Newton's second law, we write  $m_e \frac{d^2x}{dt^2} = -e\vec{E} = -\frac{e^2n_0x}{\epsilon_0}$ . Thus the solution of the above second-order differential equation is given by  $x(t) = A \cos(\omega_{pe}t)$ , where  $\omega_{pe} = \sqrt{\frac{e^2n_0}{m_e\epsilon_0}}$  is the plasma frequency or character frequency at which imbalance charges oscillate. Using the values of various parameters, plasma frequency  $f$  (Hz) for electron is in the order  $\approx 9\sqrt{n_0}$  (Hz), when the electron density is taken per  $m^3$ . For example, for plasma having an electron density of  $10^{18}/m^3$ , we have  $f \approx 9$  GHz. Generally the plasma frequency lies in the microwave region. The above equation also shows that group velocity of these oscillations is zero, and hence there is no propagation of information. These waves are called stationary waves.

#### 5. Plasma oscillation when the motion of ions is also taken into account

Let  $x_1$  and  $x_2$  are the displacement from the equilibrium positions of electrons and ions, respectively, then the equation of motions is

$$m_e \frac{d^2x_1}{dt^2} = -\frac{e^2n_0(x_1 - x_2)}{\epsilon_0} \quad (1)$$

$$m_i \frac{d^2x_2}{dt^2} = \frac{e^2n_0(x_1 - x_2)}{\epsilon_0} \quad (2)$$



**Figure 2.** Schematic of electron displacement in plasma sheet of thickness  $x$ .

By combining the above equations, we obtain

$$\frac{d^2(x_1 - x_2)}{dt^2} = -\frac{e^2 n_0 (x_1 - x_2)}{\epsilon_0} \left( \frac{1}{m_e} + \frac{1}{m_i} \right) \quad (3)$$

or

$$\frac{d^2 X}{dt^2} = -\frac{e^2 n_0 X}{m_e \epsilon_0} \left( 1 + \frac{m_e}{m_i} \right) \quad (4)$$

where  $X = x_1 - x_2$ .

Therefore the frequency of oscillation is

$$\omega_{pie}^2 = \frac{e^2 n_0}{m_e \epsilon_0} \left( 1 + \frac{m_e}{m_i} \right) = \omega_{pe}^2 \left( 1 + \frac{m_e}{m_i} \right) \quad (5)$$

### 5.1 Frequency of oscillation for pair plasma

The pair plasma comprise of particles with opposite charge but equal mass, which gives plasma frequency  $\omega_{ie} = \sqrt{2}\omega_{pe}$ .

## 6. Concept of plasma resistivity

The equation of motion for electron in unmagnetized cold plasma can be given by the equation  $m \left( \frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \nabla) \vec{v}_e \right) = -e\vec{E} - mv\vec{v}_e$ . In a steady state, the collisional friction balances the electric acceleration, and the above equation results to  $\vec{E} = -\frac{mv\vec{v}_e}{e}$ . Since electrons move with respect to the ions, they carry the current density  $\vec{J} = -ne\vec{v}_e$ . Substituting the current density into electric field equation yields  $E = \eta J$ . Here we have defined the plasma resistivity  $\eta = \frac{mv}{ne^2}$ . Therefore plasma resistivity depends on the collision frequency between the neutral particles with plasma species.

## 7. Plasma model and basic equations

A Hall thruster with two-component plasma consisting of ions and electrons is considered in which only the electrons are magnetized and the ions are not. For the case of simplicity, the presence of dust particles has been ignored; otherwise the mathematical expression would become cumbersome. In order to realize the exact behaviour and the consequences of finite temperature on the thruster efficiency, it is of much importance to investigate the plasma disturbances in Hall thrusters by including the finite temperatures of the plasma species.

As discussed in Section 1, the electrons experience force along the azimuthal direction, and ions are accelerated along the exit side of the device to produce thrust by the external electric field. We use the common symbols to write the continuity and equation of motion for the ions and electrons under the thermal effects of ion and electron pressure gradient forces. The collision momentum transfer frequency ( $\nu$ ) between the electrons and neutral atoms is also taken into account to see the resistive effects in the plasma:

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot (\vec{v}_i n_i) = 0 \quad (6)$$

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \vec{\nabla}) \vec{v}_i = \frac{e\vec{E}}{M} - \frac{\vec{\nabla} p_i}{M n_i} \quad (7)$$

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot (\vec{v}_e n_e) = 0 \quad (8)$$

$$\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \vec{\nabla}) \vec{v}_e = -\frac{e}{m} (\vec{E} + \vec{v}_e \times \vec{B}) - v \vec{v}_e - \frac{\vec{\nabla} p_e}{m n_e} \quad (9)$$

## 8. Linearization of fluid equations

We consider the perturbed densities for ions and electrons as  $n_{i1}$  and  $n_{e1}$  velocities as  $\vec{v}_{i1}$  and  $\vec{v}_{e1}$  indicated by subscript 1 along with their unperturbed values as  $v_0$  and  $u_0$  in the X and Y directions respectively. The unperturbed part represents the state of the plasma in the absence of oscillations and is indicated by a subscript 0. The unperturbed density is taken as  $n_0$ , the electric field (magnetic field) as  $\vec{E}_0$  ( $\vec{B}_0$ ) and the perturbed value of the electric field (magnetic field) as  $\vec{E}_1$  ( $\vec{B}_1$ ). To linearize all the equations, let us write  $n_i = n_0 + n_{i1}$ ,  $\vec{v}_i = \vec{v}_{i1} + \vec{v}_0$ ,  $\vec{B} = \vec{B}_1 + \vec{B}_0$  and  $\vec{E} = \vec{E}_1 + \vec{E}_0$ . In view of small variations of both the density and magnetic field along the channel, the plasma inhomogeneities are neglected. The perturbations of the ion and electron densities are taken small enough ( $n_{i1}, n_{e1} \ll n_0$ ) so that the collisional effect due to the velocity perturbations dominate over the one due to the density perturbation. Since  $\vec{v}_0$  and  $u_0$  are constant, the terms  $(\vec{v}_0 \cdot \vec{\nabla}) n_0$ ,  $n_0 (\vec{\nabla} \cdot \vec{v}_0)$  and  $n_1 (\vec{\nabla} \cdot \vec{v}_0)$  are equal to zero. Further the terms  $(\vec{v}_1 \cdot \vec{\nabla}) n_1$  and  $n_1 (\vec{\nabla} \cdot \vec{v}_1)$  are neglected as they are quadratic in perturbation. The linearized form of the above equations thus reads

$$\frac{\partial n_{i1}}{\partial t} + v_0 \frac{\partial n_{i1}}{\partial x} + n_0 (\vec{\nabla} \cdot \vec{v}_{i1}) = 0 \quad (10)$$

$$\frac{\partial \vec{v}_{i1}}{\partial t} + v_0 \frac{\partial \vec{v}_{i1}}{\partial x} = \frac{e\vec{E}_1}{M} - \frac{\vec{\nabla} p_i}{M n_0} \quad (11)$$

$$\frac{\partial n_{e1}}{\partial t} + u_0 \frac{\partial n_{e1}}{\partial y} + n_0 (\vec{\nabla} \cdot \vec{v}_{e1}) = 0 \quad (12)$$

$$\frac{\partial \vec{v}_{e1}}{\partial t} + u_0 \frac{\partial \vec{v}_{e1}}{\partial y} = -\frac{e}{m} (\vec{E}_1 + \vec{v}_{e1} \times \vec{B}_0 + \vec{u}_0 \times \vec{B}_1) - v \vec{v}_{e1} - \frac{\vec{\nabla} p_e}{m n_0} \quad (13)$$

The initial drifts  $v_0$  and  $u_0$  of the ions and electrons in the channel are related to the electric and magnetic fields according to  $v_0 \frac{\partial v_0}{\partial x} = \frac{eE_0}{M}$  and  $u_0 = -\frac{E_0}{B_0}$  obtained from the unperturbed part of Eqs. (7) and (9). The electron pressure in Eqs. (9) and (13) is given by  $p_e = Y_e n_e T_e$  together with  $T_e$  as the electron temperature, which we consider to be constant, and  $Y_e$  as the ratio of specific heats.

**Normal mode analysis:** We seek the sinusoidal solution of the above equations; therefore the perturbed quantities are taken as  $f_1 \sim \exp(i\omega t - i\vec{k} \cdot \vec{r})$ . Then the

time derivative ( $\partial/\partial t$ ) can be replaced by  $i\omega$  and the gradient  $\vec{\nabla}$  by  $ik$ . Here  $f_1 \equiv n_{i1}, n_{e1}, \vec{v}_{i1}, \vec{v}_{e1}, \vec{E}_1$  and  $\vec{B}_1$  together with  $\omega$  as the frequency of oscillations and  $\vec{k}$  as the propagation vector.

## 9. Dispersion equation and growth rate of electrostatic oscillations

Since, we are only interested in electrostatic oscillations, and therefore in the meanwhile, the perturbed magnetic field can be ignored in Eq. (13). By using Fourier analysis in Eq. (10) and Eq. (13), the perturbed ion and electron densities are given as follows:

$$n_{i1} = \frac{n_0}{(\omega - k_x v_0)} (k_x v_{i1x} + k_y v_{i1y}) \quad (14)$$

$$n_{e1} = \frac{n_0}{(\omega - k_y u_0)} (k_x v_{e1x} + k_y v_{e1y}) \quad (15)$$

Using Eq. (11) into Eq. (14) gives

$$n_{i1} = \frac{ek^2 n_0 \varphi}{M(\omega - k_x v_0)^2} \left( 1 - \frac{k^2 V_{thI}^2}{(\omega - k_x v_0)^2} \right)^{-1} \quad (16)$$

where we used  $k^2 = k_x^2 + k_y^2$ .

The expression for the electron density  $n_{e1}$  contains the velocity components  $v_{e1x}$  and  $v_{e1y}$ , which are derived in terms of the potential  $\varphi$  under the assumption  $\Omega \gg \omega$ ,  $k_y u_0$  and  $v$  in view of the oscillations observed in Hall thrusters [13, 22, 23]. Letting  $\omega - k_y u_0 - iv \equiv \hat{\omega}$ ,  $\frac{eB_0}{m} \equiv \Omega$ ,  $\sqrt{\frac{Y_e T_e}{m}} \equiv V_{th}$ ,  $\sqrt{\frac{n_0 e^2}{m \epsilon_0}} \equiv \omega_e$  and  $\sqrt{\frac{n_0 e^2}{M \epsilon_0}} \equiv \omega_i$ .

we use Eq. (13) to write the velocity components

$$i(\omega - k_y u_0 - iv)v_{e1x} = \frac{e}{m} \frac{\partial \varphi}{\partial x} - \Omega v_{e1y} + \frac{ik_x n_{e1} V_{thE}^2}{n_0} \quad (17)$$

$$i(\omega - k_y u_0 - iv)v_{e1y} = \frac{e}{m} \frac{\partial \varphi}{\partial y} + \Omega v_{e1x} + \frac{ik_y n_{e1} V_{thE}^2}{n_0} \quad (18)$$

Further simplification gives

$$v_{e1x} = \frac{1}{m\Omega^2} \left( ik_y \Omega \varphi - \frac{im\hat{\omega}k_x V_{thE}^2 n_{e1}}{n_0} + k_x \hat{\omega} e \varphi - \frac{im\Omega k_y V_{thE}^2 n_{e1}}{n_0} \right) + \frac{i\hat{\omega}^2}{m\Omega^3} \left( ek_y \varphi - \frac{mk_y V_{thE}^2 n_{e1}}{n_0} \right) \quad (19)$$

$$v_{e1y} = \frac{1}{m\Omega^2} \left( imk_x V_{thE}^2 \Omega n_{e1} + k_y \hat{\omega} e \varphi - ik_x \Omega e \varphi - \frac{m\hat{\omega}k_y V_{thE}^2 n_{e1}}{n_0} \right) + \frac{i\hat{\omega}^2}{m\Omega^3} \left( \frac{mk_x V_{thE}^2 n_{e1}}{n_0} - k_x e \varphi \right) \quad (20)$$



With the above velocity components, the perturbed electron density  $n_{e1}$  can be expressed in terms of perturbed potential as follows:

$$n_{e1} = \frac{en_0\hat{\omega}k^2\varphi}{m\Omega^2(\omega - k_y u_0) + m\hat{\omega}k^2V_{thE}^2} \quad (21)$$

Finally, we use the expressions for the perturbed ion density  $n_{i1}$  and electron density  $n_{e1}$  in Poisson's equation  $\varepsilon_0\nabla^2\varphi = e(n_{e1} - n_{i1})$  in order to obtain

$$-k^2\varphi = \frac{\omega_e^2\hat{\omega}k^2\varphi}{\Omega^2(\omega - k_y u_0) + \hat{\omega}k^2V_{thE}^2} - \frac{\omega_i^2k^2\varphi}{(\omega - k_x v_0)^2 - k^2V_{thI}^2} \quad (22)$$

Since the perturbed potential is  $\varphi \neq 0$ , we have from Eq. (21)

$$\frac{\omega_e^2\hat{\omega}}{\Omega^2(\omega - k_y u_0) + \hat{\omega}k^2V_{thE}^2} + \frac{(\omega - k_x v_0)^2 - k^2V_{thI}^2 - \omega_i^2}{(\omega - k_x v_0)^2 - k^2V_{thI}^2} = 0 \quad (23)$$

This is the dispersion relation that governs the electrostatic waves in the Hall thruster's channel.

### 9.1 The limiting case

For smaller oscillations, that is,  $\omega \ll |k_y u_0|$ , the above relation yields

$$\frac{(\omega - k_x v_0)^2 = (k_y u_0 + iv) \left[ \frac{k^2 V_{thI}^2 \omega_e^2}{+k^2 V_{thE}^2 (\omega_i^2 + k^2 V_{thI}^2)} \right] + \Omega^2 k_y u_0 (\omega_i^2 + k^2 V_{thI}^2)}{(k_y u_0 + iv) (\omega_e^2 + k^2 V_{thE}^2) + \Omega^2 k_y u_0} \quad (24)$$

Now, using the conditions  $\Omega < \omega_e > \omega_i$  and  $V_{thI} \ll V_{thE}$  in the above equation and letting  $\omega_1 = \sqrt{\frac{(\Omega^2 \omega_i^2 + \omega_e^2 k^2 V_{thI}^2)}{(\Omega^2 + \omega_e^2)}}$ ,

$$(\omega - k_x v_0)^2 \approx \omega_1^2 \frac{\left[ 1 + \frac{iv\omega_i^2 k^2 V_{thE}^2}{k_y u_0 (\Omega^2 \omega_i^2 + \omega_e^2 k^2 V_{thI}^2)} \right]}{\left[ 1 + \frac{iv\omega_e^2}{k_y u_0 (\Omega^2 + \omega_e^2)} \right]} \quad (25)$$

Since the last terms in the second brackets of the numerator and denominator in the right-hand side of Eq. (25) are small, we obtain the following

$$\omega - k_x v_0 \approx \pm \omega_1 \left[ 1 + \frac{ivk^2 V_{thE}^2 \omega_i^2}{2k_y u_0 (\Omega^2 \omega_i^2 + \omega_e^2 k^2 V_{thI}^2)} \right] \left[ 1 - \frac{iv\omega_e^2}{2k_y u_0 (\Omega^2 + \omega_e^2)} \right] \quad (26)$$

## 10. Instability analysis

The roots  $\omega_j$  of the above polynomial may be real and/or complex number. For a real root, sinusoidal behaviour gives  $\exp(i\omega_j t - i\vec{k} \cdot \vec{r})$  showing an oscillatory

behaviour. For complex roots which always occur in complex conjugate, we write  $\omega_j = \omega_{rj} - i\gamma_j$ , where  $\omega_{rj}$  and  $\gamma_j$  are real numbers and the sinusoidal behaviour becomes  $e^{\gamma_j t} \exp(i\omega_{rj} t - i\vec{k} \cdot \vec{r})$  which is showing an exponentially growing wave for a positive value of  $\gamma_j$  or an exponentially damped wave for a negative value of  $\gamma_j$ . Since the complex roots always appear in a conjugate pair, i.e. we have both negative and positive values of  $\gamma_j$  simultaneously. Hence, for the complex roots, one of the waves is always unstable.

Finally, the growth rate  $\gamma$  of the resistive instability is calculated from Eq. (26) as below:

$$\gamma \approx \frac{v\omega_1}{2k_y u_0} \left[ \frac{\omega_e^2}{\Omega^2 + \omega_e^2} - \frac{\omega_i^2 k^2 V_{thE}^2}{\Omega^2 \omega_i^2 + \omega_e^2 k^2 V_{thI}^2} \right] \quad (27)$$

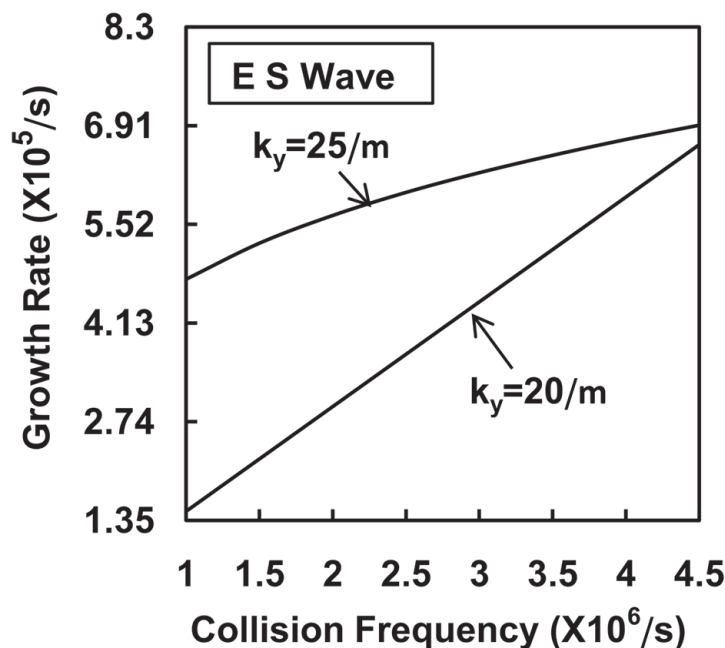
The corresponding real frequency is obtained as

$$\omega_r \approx k_x v_0 \pm \omega_1 \left[ 1 + \frac{v^2 k^2 V_{thE}^2 \omega_i^2 \omega_e^2}{4k_y^2 u_0^2 (\Omega^2 + \omega_e^2) (\Omega^2 \omega_i^2 + \omega_e^2 k^2 V_{thI}^2)} \right] \quad (28)$$

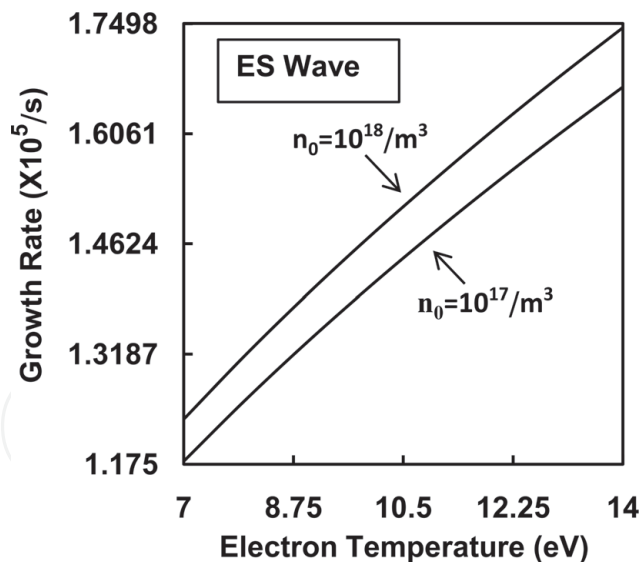
From Eq. (27), it is obvious that growth rate is directly proportional to the collisional (dissipative effects) frequency of the electrons which depends on various plasma parameters.

The results obtained in Eq. (27) matches with Litvak and Fisch [6] when the thermal effects become ignorable (i.e.  $T_i = T_e = 0$ ). Under this situation, the growth rate takes the form

$$\gamma \approx \frac{v}{2k_y u_0} \sqrt{\frac{\Omega^2 \omega_i^2}{(\Omega^2 + \omega_e^2)}} \left[ \frac{\omega_e^2}{(\Omega^2 + \omega_e^2)} \right]. \quad (29)$$



**Figure 3.** Variation of growth rate  $\gamma$  with collision frequency for different values of azimuthal wave number in a plasma having Xe ions ( $M = 131$  amu), when  $T_e = 10$  eV,  $n_0 = 10^{18}/m^3$ ,  $u_0 = 10^6$  m/s and  $B = 0.02$  T.



**Figure 4.**

Variation of growth rate  $\gamma$  with electron temperature for different values of plasma density in a plasma having Xe ions ( $M = 131$  amu), when  $v = 10^6/s$ ,  $k_y = 20/m$ ,  $u_0 = 10^6$  m/s and  $B = 0.02$  T.

In terms of lower hybrid frequency  $\omega_{LH} = \sqrt{\frac{\Omega^2 \omega_i^2}{\Omega^2 + \omega_e^2}}$  and in the limit  $\Omega \ll \omega_e$ , the growth rate can be written as  $\gamma \approx \pm \omega_{LH} \frac{v}{2k_y u_0}$ .

The above relation matches with Eq. (21) of [6].

Since it is not possible to find an analytical solution of the above equation, we look for the numerical solution along with typical values of  $B_0$ ,  $n_0$ ,  $T_e$ ,  $u_0$ ,  $k_y$ ,  $v$  and  $v_0$ . In Hall plasma thrusters, these parameters can have the values as thruster channel diameter = 4–10 cm,  $B_0 = 100 - 200$  G,  $n_0 = 5 \times 10^{17} - 10^{18}/m^3$ ,  $T_e = 10 - 15$  eV,  $u_0 \sim 10^6$  m/s,  $v \sim 10^6/s$  and  $v_0 = 2 \times 10^4 - 5 \times 10^4$  m/s [6–10]. With regard to the value of  $k_y$ , we constraint  $k_y = -m/r$  (where  $r$  is the radius of thruster channel) together with  $m = 1$  for the azimuthal mode propagation [6, 7]. Accordingly we set  $k_y = 20/m$ ; however, for higher mode ( $m > 1$ ) or larger value of  $k_y$ , the wavelength would be much smaller than the azimuthal dimension of the channel. The variations of the resistive growth rate are seen in the figures by solving Eq. (23) numerically. Some propagating modes and instabilities with smaller growth are also observed during the analysis of Eq. (23).

**Figure 3** confirms that the growth rate of the instability gets enhanced with the increase of collision frequency of the electrons due to the resistive coupling of the oscillations to the electron azimuthal drift. The growth rate also increases with the increase of the electron temperature (**Figure 4**), and it also increases with higher electron density of plasma. Therefore it can be concluded that the collisional effect is responsible to unstable the plasma system. The numerical value of the growth under the collision frequency is observed in the order of  $\sim 10^5/s$ .

## 11. Conclusions

In conclusions, we can say that the waves propagating in azimuthal and axial direction in a Hall thruster channel become unstable due to the resistive coupling to the electrons'  $\vec{E} \times \vec{B}$  flow in the presence of their collisions. By controlling the various parameters, the growth rate and the propagating frequency of the oscillation may be controlled to optimize the performance and lifetime of the device.

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