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# Contact Strength of Material

*Xiaofeng Qin*

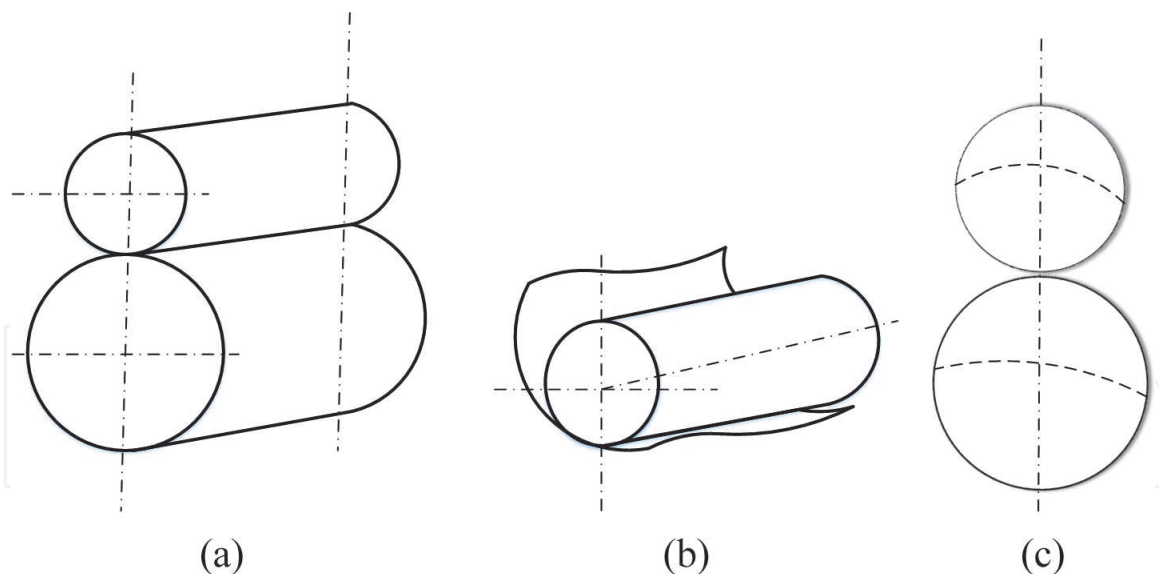
## Abstract

Contact is one of the common positions for relationship existing between surfaces of mechanical components with line or point shape, which lead to different stress condition between them. Classic Hertz theory solved the stress distribution of two components with different contact form. The different stress form of contact components will result in different failure modes, and different strength requirement is needed in design. For those works under conditions like classic Hertz contact, obvious plastic deformation should be avoided and the maximum stress should be smaller than the allowed limit stress which is usually equal to yield stress of material. While for the ones enduring load by extrusion of two contact surface, the crushing of two contact surface should be avoided and the maximum contact stress should be smaller than the ultimate strength of the material. But for those works under varying stresses, fatigue and wear are common failure modes. The typical failure form resulted from contact fatigue is pitting and spalling; the previous one usually results in small peeling of materials from surface and lots of small shallow pits will formed, while the spalling usually leads to lager part of material drop off from the surface and scrap of mechanical components. Wear is another failure form of components works under contact conditions, which is actually can be seen as the accumulation of pitting of asperities. Comparing the classic static strength theory and fatigue strength theory of material, the static contact strength is actually the limit condition of contact fatigue strength with the one circle stress loaded in the whole life. With the development of fatigue theory, lots of models were proposed to study the contact strength of material from the fracture mechanics view. The most popular ones are critical plane method and Dang Van multi-axial fatigue criterion, which are used to assess the crack initiation of materials works under contact load.

**Keywords:** mechanical component, static contact strength, contact fatigue strength, fracture mechanics, multi-axial fatigue

## 1. Introduction

Usually, strength reveals the mechanical properties of a kind of material or a component to resistant to fracture and over deformation. Based on the variation characteristics of stress endured by the material or components, the strength can be classified into static strength and variable stress intensity [1, 2]. In the real application, different materials are used for manufacturing mechanical components that work under contact load. According to the contact form and stress level induced by the load, the contact form can be classified into line contact as shown in



**Figure 1.**  
Diagram for different contact forms of mechanical parts.

**Figure 1(a)** and **(b)** and point contact as shown in **Figure 1(c)** [1–3]. The contact line or point will be transformed to load-bearing area [3, 4]. For actual application, the contact between gears, rolls in mill, rail/wheel, roller bearing and contact of bolt and holes are all under line contact form; while the ball and ring in bearing are under the point contact form [1–4].

As the strength is the capability of material enduring the load or stress, failure will occur if the load or stress is beyond the level that it can bear [4, 5]. The failure modes of material working under contact load show different forms with the different stress vibration characteristics such as plastic deformation, surface pitting, surface wear and crushing of contact surface. Typically, the plastic deformation and crushing of contact surface are both resulted from the stress which is exceeding the yield limit of the material. The surface pitting and wear usually are considered to be formed by the accumulation effect induced by varying stress. For ensuring the strength of material working under stress with different variation characteristics, different manner should be used to assess the safety of the material.

## 2. Static contact strength

### 2.1 Static contact stress

According to the classical elasticity theory, the deformation of material can be divided into three mechanical states: elastic state, plastic state and fracture. Some materials used for manufacturing the components works under static contact state, are not allowed have plastic deformation due to the requirement of working reliability and transmission accuracy. For the materials used in the manufacturing of the components working under contact state, most of them cannot work properly with obvious plastic deformation.

According to the different contact forms shown in **Figure 1**, the contact stress between two cylinders loaded by force  $F$  as shown in **Figure 2(a)** can be expressed as below [1, 3]:

$$\sigma_{H\max} = \frac{4 F}{\pi 2ab} = \sqrt{\frac{F \left( \frac{1}{\rho_1} \pm \frac{1}{\rho_2} \right)}{\pi b \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}} \quad (1)$$

The half-width  $a$  of contact area along the circumference is [3]:

$$a = \sqrt{\frac{4F \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}{\pi b \left( \frac{1}{\rho_1} \pm \frac{1}{\rho_2} \right)}} \quad (2)$$

where  $\rho_1$  and  $\rho_2$  are the curvature radius of contact area of two parts; the plus sign '+' means outer surface contact as shown in **Figure 1(a)**; the minus sign '-' means inner surface contact as shown in **Figure 1(b)**;  $E_1$  and  $E_2$  are the Young's modulus of materials of two components in contact, respectively;  $\mu_1$  and  $\mu_2$  are the Poisson's ratio of materials of two components in contact, respectively.

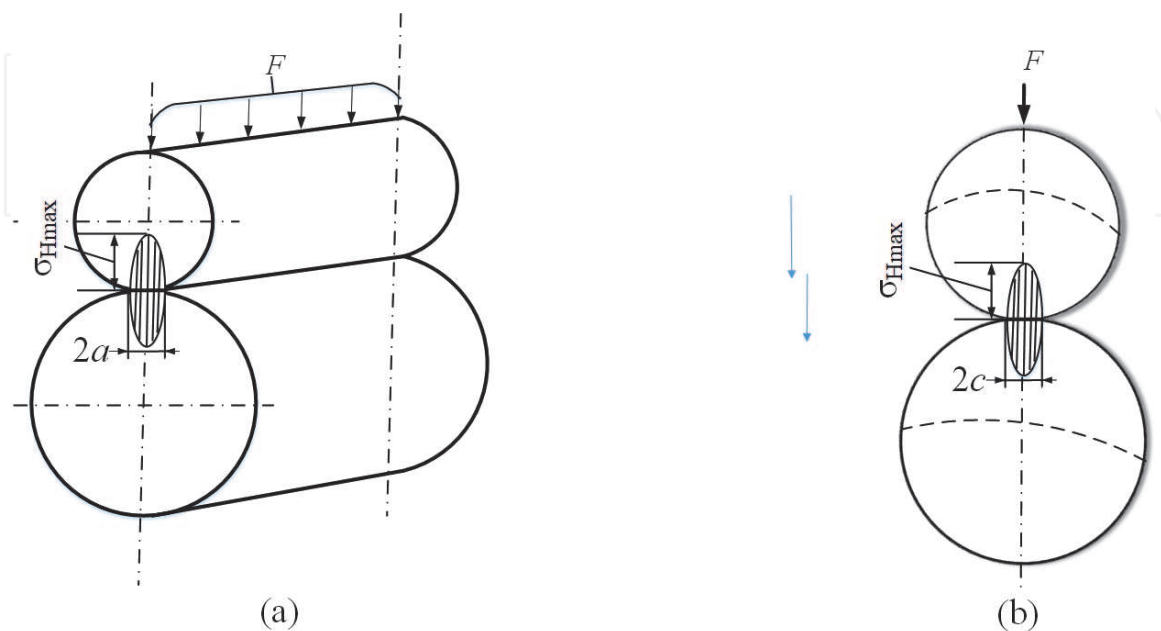
The contact stress between two balls loaded by force  $F$  shown in **Figure 2(b)** can be expressed as equation below [3]:

$$\sigma_{H\max} = \frac{3 F}{2 \pi c^2} = \frac{1}{\pi} \sqrt{6F \left( \frac{\left( \frac{1}{\rho_1} \pm \frac{1}{\rho_2} \right)}{\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}} \right)^2} \quad (3)$$

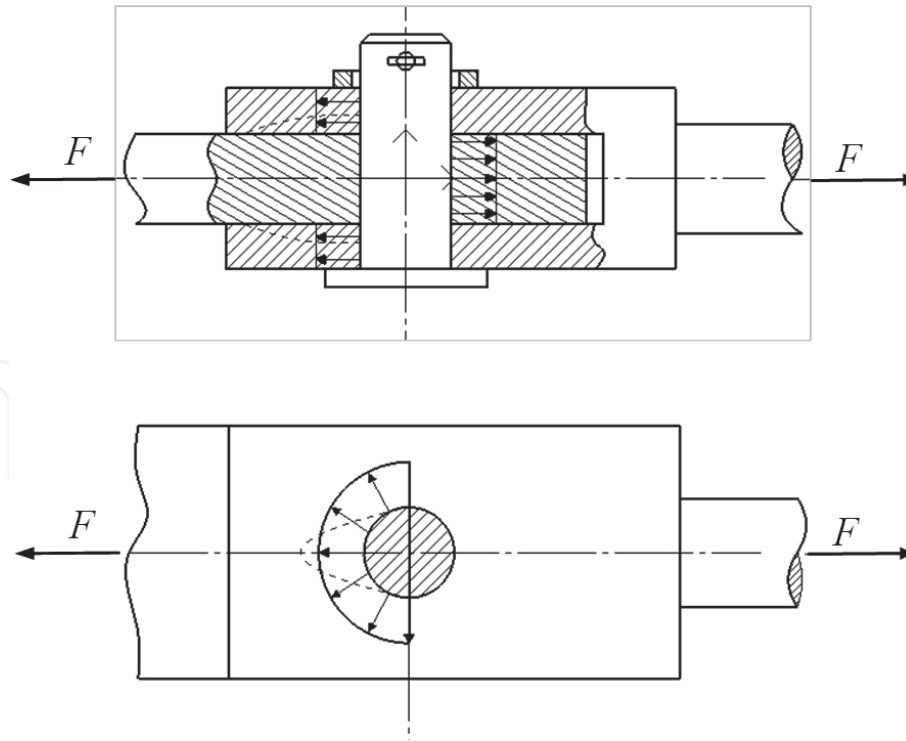
The radius  $c$  of contact area is [3]:

$$c = \left[ \frac{3F \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}{4 \left( \frac{1}{\rho_1} \pm \frac{1}{\rho_2} \right)} \right]^{1/3} \quad (4)$$

where  $c$  is the radius of contact area; the meaning of other symbols are same to that mentioned before.



**Figure 2.**  
 Diagram for stress distribution of different contact form.



**Figure 3.**  
Diagram of extrusion contact.

## 2.2 Static extrusion stress

For materials used in bolt connection, pin connection and key connection shown like in **Figure 3** mostly transfer the load by extrusion of match surface. Under contact form like the one shown in **Figure 3**, the main failure mode of the material located in the contact area is crushing. Based on the assumption of the stress distribution in the contact area, the stress between two match surfaces under extrusion load can be expressed as below:

$$\sigma_p = \frac{F}{A} \quad (5)$$

where  $F$  is the load acting on the two components and the acting direction of it is perpendicular to the bearing surface.  $A$  is the area size of match surface between two components when the bearing surface is flat or the projection area size of match surface between two components when the bearing surface is not flat.

## 2.3 Condition of static contact strength

For the linear state of stress, the condition of strength can be recorded as the following form:

$$\sigma \leq [\sigma] = \frac{\sigma_{\text{lim}}}{[S]} \quad (6)$$

where  $\sigma$  is the maximum stress active in a part or structure;  $[\sigma]$  is the admissible stress level which can be determined by  $\sigma_{\text{lim}}/[S]$ ;  $\sigma_{\text{lim}}$  is the ultimate stress of material; and the  $[S] > 0$  is termed the strength safety factor.

The materials for manufacturing the components' transmission load usually fails in the form of plastic deforms under static load. So the condition of materials under contact conditions, as shown in **Figure 1(a)** and **(b)** or **Figure 2**, is:

$$\sigma_{H \max} \leq [\sigma_H]_{\max} = \frac{\sigma_{\lim}}{[S]} \quad (7)$$

The condition of strength for the components enduring load by extrusion of match surface is:

$$\sigma_p \leq [\sigma_p] = \frac{\sigma_{\lim}}{[S]} \quad (8)$$

For different types of materials, the ultimate stress  $\sigma_{\lim}$  is different. If a material is not allowing the transition to the plastic state, then  $\sigma_{\lim} = \sigma_y$ ; where  $\sigma_y$  is the yield strength of material. If the material is of brittle type, then the  $\sigma_{\lim} = \sigma_b$ . Here,  $\sigma_b$  is the ultimate strength of the material, which can be obtained by standard tension test.

The safety factor [S] can be determined based on the safety requirement of components in the actual application conditions.

### 3. Contact fatigue strength of material

Any materials used in real application exist in the form of mechanical part. There is little mechanical part works under static stresses. For the time and space view, the amplitude of stresses may regularly or irregularly change with time. The fatigue is resulted from changes in the structure and properties of material led by the gradual accumulation of damage under the act of alternating stresses. Mechanically, the form of fatigue of material usually goes through the nucleation and growth of cracks, and ultimately leads to the volume fracture. This phenomenon occurring in the material of components working under contact load is often named pitting and wear.

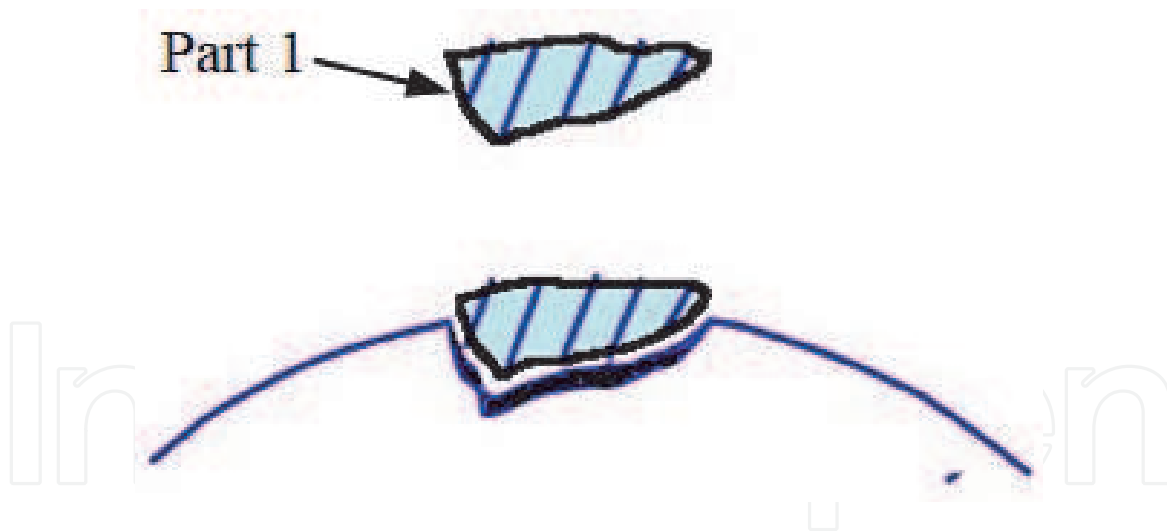
The detail of the contact fatigue process may vary with the difference of material and load conditions, but in almost all cases, the process can be manifested as the initiation and propagation of cracks. Based on the actual application of different materials under different contact conditions, the initiation of cracks led by the accumulation of contact fatigue damage is from the surface or near-surface layer. The final manifested form of cracks, initiated from different sites of contact components, after propagation until a piece of material detaches itself, will form a pit or spall; while from the point of micro view, the wear is the accumulation process of pit or spall of asperities of contact surface due to contact load.

#### 3.1 Pit and spall

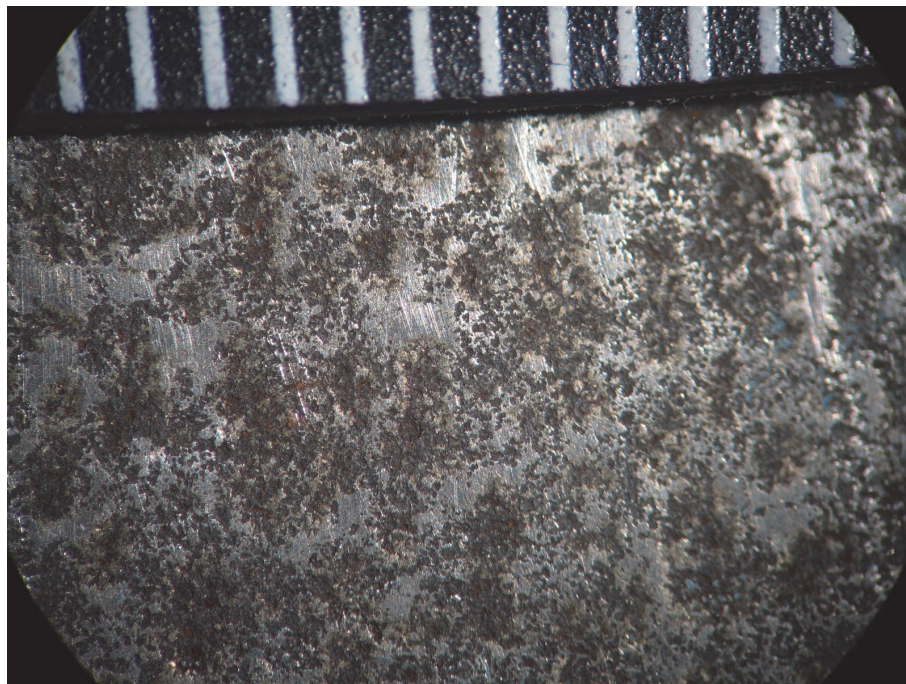
As mentioned before, pitting and spalling are the macro shown of material dropping from the contact surface due to damage accumulation with the cyclic of stresses.

Pitting is generally considered to be caused by the propagation of surface-initiated cracks. **Figure 4** shows the typical form of pitting of a contact surface. The process also includes three stages: crack initiate, crack propagate and material drop from the surface [6].

In the first stage, the cracks will initiate from the site where the damage accumulation reached the damage capacity limit of material. In the second stage, the cracks will propagate under the comprehensive act of tensile stress result from surface friction and traction and squeezing effect of the lubricant into the crack. In the last stage, the material will rupture from the surface if the remaining section between part 1 and the parent surface cannot endure the comprehensive act of



**Figure 4.**  
*Diagram for pitting form process.*

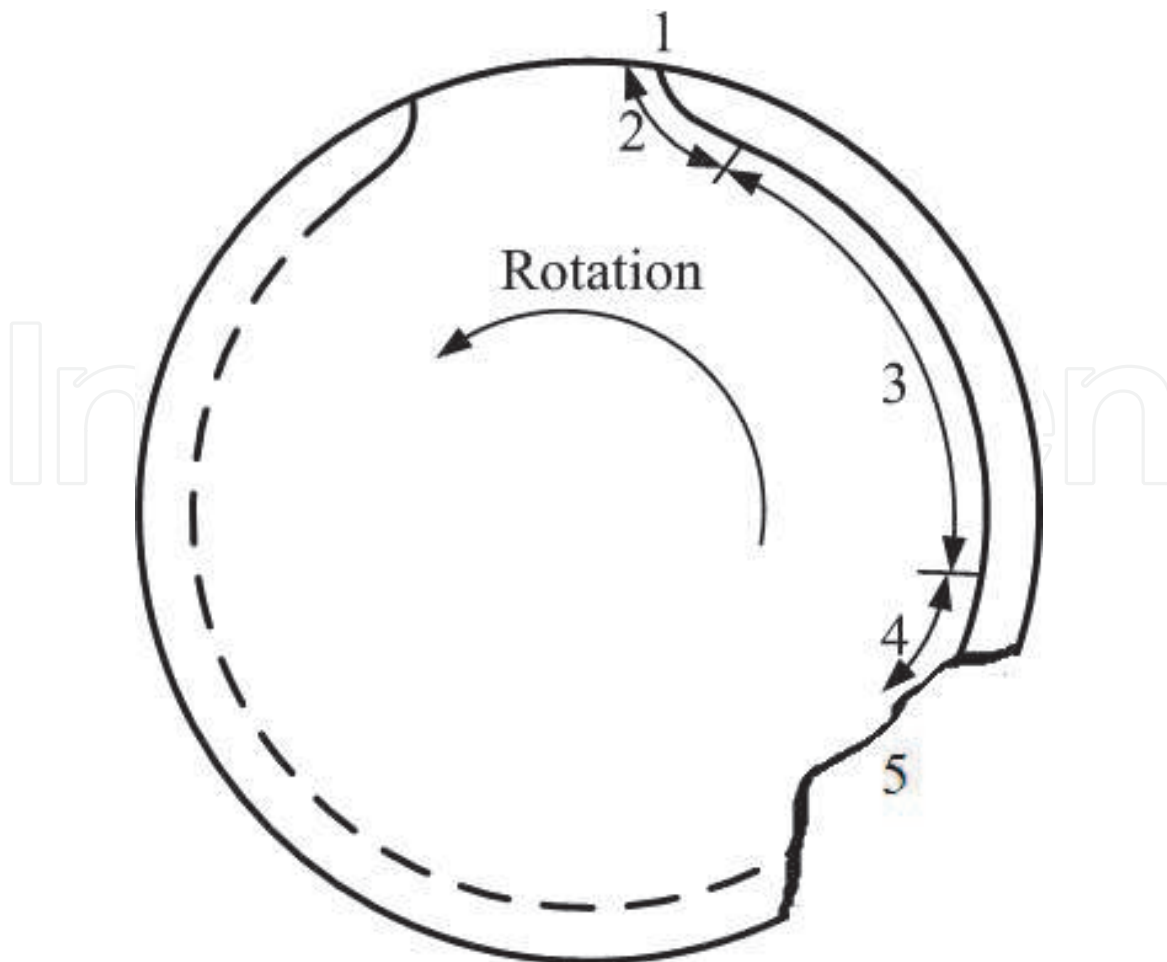


**Figure 5.**  
*Typical morphology of pitting.*

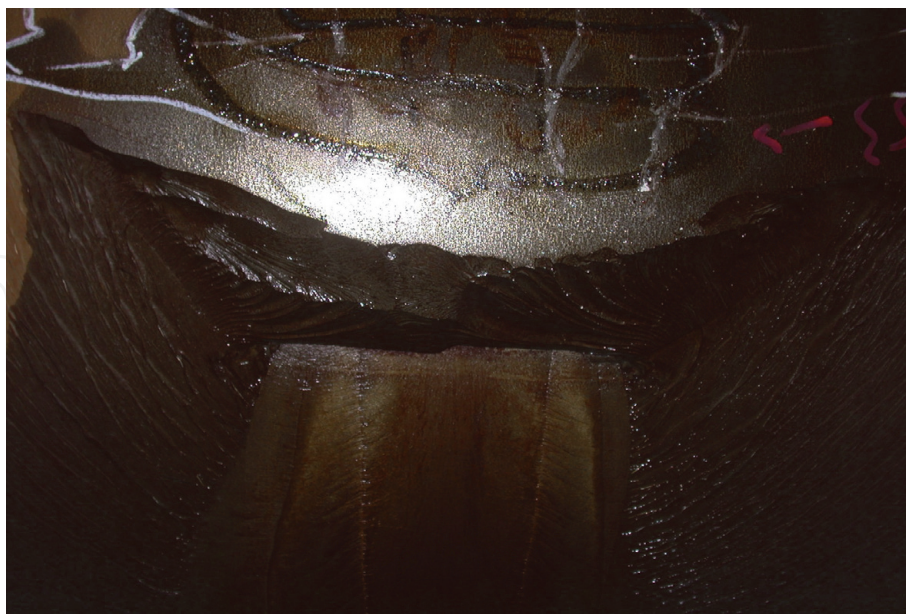
stress. Due to the edge of pitting, it will further result in the increase of stress by stress concentration effect; more cracks will initiate around the generated pitting and finally form a pitting surface as shown in **Figure 5**. Usually, the pitting caused by contact fatigue will not result in the loss of function of mechanical components.

Sometimes the cracks initiating from contact surface or subsurface will first propagate along the path which is at an acute angle to the surface. Then it is propagated along the path parallel to the surface which will let the crack to propagate along a relatively long path and eventually leading to large chunks peeled off from the surface. The whole process can be described by the sketch shown in **Figure 6** [7–9].

**Figure 7** shows a typical spalling morphology of backup roll used in steel production. Based on the morphology of spalling of roll shown in **Figure 7**, it can be seen that the crack propagates along the peripheral direction of roll and causes extensive surface peeling. Usually, the spalling can cause the complete loss of function of mechanical components.



**Figure 6.**  
*Spalling failure process of contact components [7].*

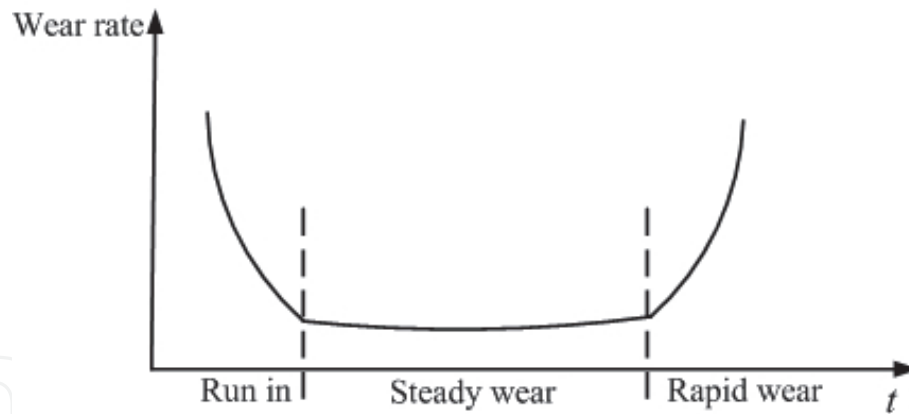


**Figure 7.**  
*Typical spalling of component result from contact fatigue.*

### 3.2 Wear

Wear is a common phenomenon for the mechanical components working under contact and having relative move. The wear resulted from contact fatigue is a more common phenomenon in components with good lubrication. As the machined





**Figure 8.**  
*Wear rate variation line in whole life.*

surface of components such as gear, ball and ring of bearing and wheel/rail consists of asperities, the load of two surface contact is supported by contact of lots of asperities and lubricates in micro level [10, 11]. Usually, we considered that the pitting occurs at the surface contact under macro level. If we observe two contact surfaces in micro level, the contact of separate asperities is equivalent to that contact of two surfaces in macro level. The only difference between them is the relative radius of curvature.

So if we considered the contact of asperities as two components contact under micro level, the formation of wear is actually the accumulation of micro pitting between the contacted asperities, and the variation process of wear during the whole life of mechanical components can be described by the formation mechanism of macro pitting. As it is known, the process of wear of mechanical component in its whole life can be divided into three stages including run-in process, steady wear and rapid wear process as shown in **Figure 8**.

In the run-in stage, the contact of two macro surfaces is supported by contact of asperities, which will result in the micro contact fatigue of asperities. The fatigue damage accumulation due to contact of asperities will result in the pitting which occurs on the surface of asperities. The pitting of asperities leading to the equivalent radius of curvature of asperities increase and contact stress between the two asperities decrease, which will decrease the damage during later contact and the wear process enter into steady wear process and last long time. With the further increase of using time, the damage of whole surface will increase and the whole strength of surface will degrade. Then the occurrence of pitting of asperities will speed up and wear rate of surface will sharply increase, which means the arrival of rapid wear.

#### 4. New trend of contact fatigue strength research

With the development of material process technology, the contact strength of it is increased gradually. But the failure caused by contact fatigue and wear cannot be avoided. Lots of models, established based on fracture mechanics and damage accumulation theory, are established to assess the contact fatigue process of material. In these models, the most popular ones are Dang Van criterion and the critical plane method.

##### 4.1 Critical plane model

The critical plane method considered that the crack initiation plane may occur on a plane near the maximum normal stress range for medium to high strength

steels, and the model used for assess on the occurs of critical plane can be expressed as [12, 13]

$$FP = \frac{\Delta\varepsilon}{2} \sigma_{\max} + J\Delta\gamma\Delta\tau \quad (9)$$

where  $\Delta\varepsilon$  is the normal strain range,  $\sigma_{\max}$  is the maximum normal stress,  $\Delta\gamma$  is the shear strain range,  $\Delta\tau$  is the shear stress range and  $J$  is material constant.

Based on the above equation, the relationship between the fatigue parameter and life is expressed as

$$(FP - FP_0)^m N_f = \text{constant} \quad (10)$$

where  $N_f$  is the fatigue life corresponding to fatigue parameter  $FP$ , and  $m$ ,  $FP_0$  and  $C$  are material fatigue properties determined from fatigue life experiments [13].

If the damage accumulates linearly, then the damage per loading cycle is [13]

$$\frac{dD_f}{dN} = \frac{1}{N_f} = \frac{(FP - FP_0)^m}{C} \quad (11)$$

where  $D_f$  is the fatigue damage equal or smaller than 1; and  $N$  represents the number of load cycles. If  $FP \leq FP_0$ , there is no damage resulted from the load cycle.

#### 4.2 Dang Van multi-axial fatigue criterion

Dang Van multi-axial fatigue criterion assumes there is elastic shakedown occurs before cracks initiation and considered two scales [14–16]. The first is that often used by engineers who used to analysis the fatigue of point that surrounded by an arbitrary elementary volume in macroscopic scale. The second one is used to subdivide the macroscopic scale element in mesoscopic scale. it thinks that macroscopic stress tension result in the mesoscopic one and the local inelastic deformation lead to the local residual stresses. Based on that assumptions, the model is expressed as an inequality of mesoscopic stresses at all instants  $t$  of the cycle to characterize the damage as below [14]:

$$\max_t [\tau(t) + ap(t)] \leq b \quad (12)$$

Where  $\tau(t)$  and  $p(t)$  are the instantaneous mesoscopic shear stress and hydrostatic stress,  $a$  and  $b$  are material constants which can be determined by classic bending and twisting fatigue test.

#### Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No. 51805355) and Natural Science Foundation for Young Scientists of Shanxi Province, China (Grant No. 201701D221136). The author is grateful to all the partners of College of Mechanical and Vehicle Engineering, Taiyuan University of Technology, China.

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