Chapter

Decision-Making in Fuzzy Environment: A Survey

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Abstract

Multi-criteria decision-making (MCDM) is a crucial process in many business and management applications. The final decision is based upon the relative weights to the decision-making team. The analytic hierarchy process (AHP) has found to be one of the most successful approaches for evaluations of the weights and the importance of the criteria. However, most of the evaluated values are not so precise due to the fuzziness of the evaluating environment. This chapter surveys essentially the basic analytic hierarchy process and the fuzzy analytic hierarchy process (FAHP). It depicts through an example the steps for using the original analytic hierarchy process for two levels of criteria. Then, it uses the same example to explain the fuzzy approach in the evaluation. Finally, it compares both approaches.

Keywords: analytic hierarchy process (AHP), fuzzy analytic hierarchy process (FAHP), multi-criteria decision-making (MCDM), Chang's extent analysis

1. Introduction

Multi-criteria decision-making (MCDM) is a discipline that interacts with decisions to select the most optimal alternative with respect to multiple criteria for a specific goal. MCDM is well known for impartially solving problems of decision-making and for comparing the alternative comparatively to deduce the relative priority of the alternatives. Based on the relative priority value, the optimal alternative is defined and selected as a choice that can achieves the decision target.

Different MCDM techniques are in recent times broadly applied and used to resolve various decisions and predictive problems. These techniques are such as weighted sum model (WSM), weighted product model (WPM), analytic hierarchy process (AHP), technique for order preference by similarly to ideal solution (TOPSIS) and fuzzy AHP which is the fuzzification version of the AHP. Among these techniques, we will discuss analytical hierarchy process (AHP) and fuzzy analytical hierarchy process (FAHP) in this chapter.

The main objective of this chapter is to introduce a comparative analysis of analytic hierarchy process (AHP) developed by Saaty [1] and fuzzy analytic hierarchy process (FAHP) developed by Chang [2]. Both techniques will be introduced using a simple example for decision-making.

Saaty introduced an example for determining the type of the job that would be best for the person upon getting his/her PhD. This example was selected to cope with the original work of Saaty about AHP.

In the flow of the chapter, first the classical AHP and fuzzy AHP methods are introduced, then the summary of calculations are presented for AHP and fuzzy AHP as the next section. Finally, the chapter ends with comparison results, findings and comments about these methods.

2. Analytic hierarchy process (AHP)

The analytic hierarchy process (AHP) is developed by Saaty [1] as a multi-criteria decision-making approach, which aids the decision maker to set relative priorities and to make the best decision. AHP has found to be one of the most successful approaches for evaluations the relative priorities of different criteria and for selection between alternatives. It gains recently high attention for many applications; see, for example, Ho and Ma [3]. AHP is especially suitable for complex decisions which involve the comparison of decision elements which are difficult to quantify. It is a technique for decision-making where there are a limited number of choices and these choices are characterized by a set of attributes (criteria). Each of these choices has different attributes' value.

To explain the core of AHP we consider the very simple example for building these relative priorities between three items, although later we will consider the development of priorities using AHP for two levels of criteria and in fuzzy environment. The AHP procedure can be described in an algorithmic way in five steps which give finally the relative priorities between criteria. These steps will be explained using the very simple example as follows:

Step 1: Define the problem: let us say we have three criteria A, B, C and we want to know the relative priorities (importance) between these criteria to achieve a specific goal.
Step 2: Construct a simple decision hierarchy structure to emphasize the goal and the criteria as shown in **Figure 1**. Although, the goals are generally selecting one of different alternatives, here the goal is the simplest one that is generating the relative priorities between the criteria A, B, and C.
Step 3: Construct a set of pairwise comparison methods to all criteria.

A pairwise comparison is a process used to compare the criteria in pairs to judge which criterion is more important than the others using Saaty's nine-point scale of pairwise comparison as shown in **Table 1**.

In general, consider a matrix **Z** with $n \times n$ matrix, where n is the number of evaluation criteria considered. Each entry z_{ij} of the matrix **Z** represents the importance of the ith criterion relative to the jth criterion. If $z_{ij} > 1$, then the ith criterion is more important than the jth criterion and in the otherwise, if $z_{ij} < 1$, then the ith criterion is less important than the jth criterion. If ith criterion and jth criterion have the same importance, then the entry z_{ij} is 1. The entries z_{ij} and z_{ji} satisfy the following constraint:

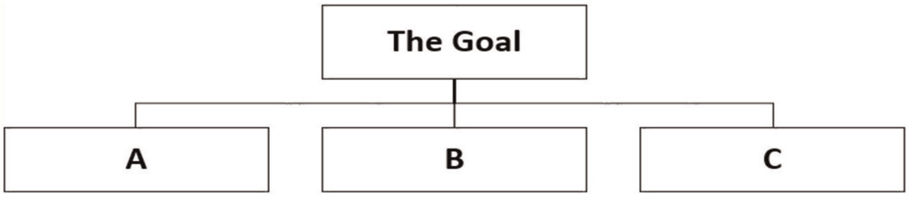

Figure 1.
Simple decision hierarchical structure.

Intensity of importance	Definition
1	Equally important
3	Moderately more important
5	Strongly important
7	Very strongly important
9	Extremely important
2, 4, 6, 8	Intermediate value between adjacent scales

Table 1.
Saaty's nine-point scale of pairwise comparison.

$$z_{ij} \,.\, z_{ji} = 1 \tag{1}$$

Assume that the comparisons between the criteria A, B, C are as follows: A is moderately more important than B, A is extremely important than C, and B is moderately more important than C. According to **Table 1**, we will have A = 3B, A = 9C, and B = 3C. This is also means that B = (1/3)A, C = (1/9)A, and C = (1/3)B. The result of these pairwise comparisons is traditionally related to what we call the pairwise comparison matrix as shown in **Table 2**.

Step 4: This is the normalization step which consist of two parts. In the first part, normalization is carried out for each column entries according to the following equation:

$$\overline{\mathrm{z}}_{\mathrm{ij}} = \frac{z_{ij}}{\sum_{i=1}^{n} z_{ij}} \tag{2}$$

The summation of the very simple example is shown in **Figure 2a** while the results of this normalization part are shown in **Figure 2b**.

	A	B	C
A	1	3	9
B	1/3	1	3
C	1/9	1/3	1

Table 2.
The pairwise comparison matrix for the considered three criteria.

	A	B	C
A	1	3	9
B	1/3	1	3
C	1/9	1/3	1
Sum	1.44	4.33	13

(a)

	A	B	C
A	0.693	0.693	0.693
B	0.231	0.231	0.231
C	0.077	0.077	0.077

(b)

Figure 2.
The normalization step for criteria: (a) summation of columns; and (b) dividing each cell by its columns' summation.

The second part, the weight w_i of the criterion i is calculated by taking the average of the entries on each row of matrix **Z**. This results in the weight vector **W** which in the very simple example becomes $\boldsymbol{W} = \begin{pmatrix} 0.693 \\ 0.231 \\ 0.077 \end{pmatrix}$

$$w_i = \frac{\sum_{j=1}^{n} \overline{z}_{ji}}{n} \tag{3}$$

Step 5: The final step is a test to check for the consistency associated with the comparison matrix to examine the extent of consistency by using consistency ratio (CR) using the formula:

$$\text{Consistency ratio } (CR) = \frac{CI}{RI} \tag{4}$$

If CR < 0.1, then the pairwise comparison matrix **Z** is reasonable consistence otherwise it is inconsistence. Here, RI is a random matrix consistency index obtained through experiments using samples with large quantities. Random index (RI) values for the matrix of the order n = [1, 10] are shown in **Table 3**.

The consistency index (CI) indicates whether a decision maker provides the comparison of consistent values in a set of evaluations. CI is calculated using the formula:

$$CI = \frac{\lambda_{max} - n}{(n-1)} \tag{5}$$

The calculation of the CI demands to compute the normalized eigenvector of the matrix and the principal eigenvalue $\lambda_{\max}$ of the matrix, which is obtained from summing the multiplication of the number of weights of all criteria in each column of the matrix with the eigenvector of the matrix.

$$\lambda_{max} = (1.44 \times 0.693) + (4.33 \times 0.231) + (13 \times 0.077) = 3 \tag{6}$$

For $\lambda_{\max}$ = 3 and n = 3, then the value of CI = 0. The consistency here is ideal due to the fact that there is full consistency between the three pairwise comparisons, A = 3B, A = 9C, and B = 3C, which means any of these three equations can be deduced from the other two equations.

As CI = 0, and RI for three elements = 0.58, the CR = 0 $<$ 0.1. This means that the evaluation of the matrix is consistent and all the comparisons of the elements are ideal (as CR = 0). This is the ideal case where the pairwise comparisons are perfect.

What happens if the ranking of the criteria is changed and the pairwise comparison matrix is reconstructed?

The new pairwise comparison matrix and the weight of each criterion are shown in **Figure 3a** and **b**, respectively.

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.2	1.32	1.41	1.45	1.49

Table 3.
Values of the random index (RI) for small problems.

	A	B	C
A	1	9	3
B	1/9	1	3
C	1/3	1/3	1

(a)

	A	B	C	Weight
A	1	9	3	0.665
B	1/9	1	3	0.201
C	1/3	1/3	1	0.134
Sum	1.44	10.33	7	1

(b)

Figure 3.
(a) The new pairwise comparison matrix, and (b) the weight of each criterion.

To check the consistency, the values of λ_{max}, CI, RI and CR which are:

$$\lambda_{\max} = 3.619, \text{CI} = 0.3095, \text{RI} = 0.58 \text{ and CR} = 0.534 > 0.1 \tag{7}$$

This means that the evaluation of the matrix is inconsistent and all the comparisons of the elements are needed to be reconsidered and the previous steps need to be repeated.

3. Fuzzy analytic hierarchy process (FAHP)

The conventional AHP is insufficient for dealing with fuzziness and uncertainty in multi-criteria decision-making (MCDM) because of inability of AHP to deal with the imprecision in the pairwise comparison process. Hence, the fuzzy AHP technique can be viewed as an advanced analytical method developed from the conventional AHP. The fuzzy AHP is proposed to find the uncertainty of AHP method. Different approaches are suggested as fuzzy AHP. The most two used methods for calculating the relative weights of the criteria are geometric means, which is proposed by Buckley [4], and the extent analysis methods which is proposed by Chang [2].

Fuzzy AHP (FAHP) has been shown successful in many applications [5–7]. The successfulness of FAHP attracts the researches to consider even different membership functions' form instead of using triangular membership functions to represents the fuzzy numbers which will be consider here after [8]. Also, in the following, we will consider only the extent analysis methods for calculating the relative weights of criteria.

The triangular number is denoted by three numbers A = (l, m, u) where "l" represents the lower value, "m" the medium value, and "u" the upper value, respectively ($l \le m \le u$). The reciprocal triangular number is denoted by A^{-1} and calculated as $A^{-1} = (1/u, 1/m, 1/l)$ as shown in **Table 4**.

The addition and the multiplications of two fuzzy numbers are explained by the following example:

Consider two triangular fuzzy numbers $A_1 = (l_1, m_1, u_1)$ and $A_2 = (l_2, m_2, u_{2.})$

The addition of two fuzzy numbers is defined by:

$$(l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \tag{8}$$

And the multiplication of two fuzzy numbers is defined by:

$$(l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 l_2, m_1 m_2, u_1 u_2) \tag{9}$$

Crisp importance value	Triangular fuzzy numbers	Linguistic scale for importance	Triangular fuzzy Reciprocal
1	(1, 1, 1)	Equally important	(1, 1, 1)
2	(1, 2, 3)	Intermediate value between 1 and 3	(1/3, 1/2, 1)
3	(2, 3, 4)	Moderately more important	(1/4, 1/3, 1/2)
4	(3, 4, 5)	Intermediate value between 3 and 5	(1/5, 1/4, 1/3)
5	(4, 5, 6)	Strongly important	(1/6, 1/5, 1/4)
6	(5, 6, 7)	Intermediate value between 5 and 7	(1/7, 1/6, 1/5)
7	(6, 7, 8)	Very strongly important	(1/8, 1/7, 1/6)
8	(7, 8, 9)	Intermediate value between 7 and 9	(1/9, 1/8, 1/7)
9	(9, 9, 9)	Extremely important	(1/9, 1/9, 1/9)

Table 4.
The scale of fuzzy AHP pairwise comparison.

We consider the same simple example to explain the core of fuzzy AHP for building these relative priorities. The following section outlines the Chang's extent analysis method on fuzzy AHP.

The fuzzy AHP procedure can be described in an algorithmic way in seven steps which give finally the relative priorities between criteria. These steps are:

Step 1: define the problem. This is the same example as mentioned before in AHP.
Step 2: develop the decision hierarchy like AHP step as mentioned before.
Step 3: construct the fuzzy pairwise comparison matrices to all criteria. Here, the general form of the fuzzy pairwise comparison will be as follows:

$$\mathbf{Z} = \left(\mathrm{z_{ij}}\right)_{\cdot n\times n} = \begin{bmatrix} (1,1,1) & (\mathrm{l_{12}}, \mathrm{m_{12}}, \mathrm{u_{12}}) & \dots & (\mathrm{l_{1n}}, \mathrm{m_{1n}}, \mathrm{u_{1n}}) \\ (\mathrm{l_{21}}, \mathrm{m_{21}}, \mathrm{u_{21}}) & (1,1,1) & \dots & (\mathrm{l_{2n}}, \mathrm{m_{2n}}, \mathrm{u_{2n}}) \\ . & . & . & . \\ . & . & . & . \\ (\mathrm{l_{n1}}, \mathrm{m_{n1}}, \mathrm{u_{n1}}) & (\mathrm{l_{n2}}, \mathrm{m_{n2}}, \mathrm{u_{n2}}) & \dots & (1,1,1) \end{bmatrix} \tag{10}$$

where $\mathrm{z_{ij}} = \left(l_{ij}, m_{ij}, u_{ij}\right)$, $\mathrm{z_{ji}} = \mathrm{z_{ij}}^{-1} = \left(1/u_{ij}, 1/m_{ij}, 1/l_{ij}\right)$ for $i, j = 1, \dots, \mathrm{n}$.

Using the linguistic scale for criteria and alternatives as shown in **Table 10** to compare the criteria in pairs to judge which criterion is more important than the others. As we use the same comparisons between the criteria A, B, C, the fuzzy pairwise comparisons between these criteria can be expressed in the matrix form as shown in **Table 5**.

Step 4: calculate the value of fuzzy synthetic extent $\mathrm{S_i}$ with respect to the ith criterion using the formula:

$$\mathrm{S_i} = \left(\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} u_{ij}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}}\right) \tag{11}$$

According to the previous example, the values of fuzzy synthetic extent are:

	A	B	C
A	(1, 1, 1)	(2, 3, 4)	(9, 9, 9)
B	(1/4, 1/3, 1/2)	(1, 1, 1)	(2, 3, 4)
C	(1/9, 1/9, 1/9)	(1/4, 1/3, 1/2)	(1, 1, 1)

Table 5.
The fuzzy pairwise comparison matrix for the considered three criteria.

$$S_A = (12, 13, 14) \otimes \left(\frac{1}{21.11}, \frac{1}{18.77}, \frac{1}{16.61}\right) = (0.568, 0.693, 0.842) \quad (12)$$

$$S_B = (3.25, 4.33, 5.5) \otimes \left(\frac{1}{21.11}, \frac{1}{18.77}, \frac{1}{16.61}\right) = (0.154, 0.231, 0.331) \quad (13)$$

$$S_C = (1.36, 1.44, 1.61) \otimes \left(\frac{1}{21.11}, \frac{1}{18.77}, \frac{1}{16.1}\right) = (0.064, 0.077, 0.097) \quad (14)$$

Step 5: compute the degree of possibility for each convex fuzzy number M_1 and M_2 that $M_1 \geq M_2$ which will be denoted by $V(M_1 \geq M_2)$ defined by the following definition.

$$V(M_1 \geq M_2) = \begin{cases} 1 \text{ if } m_1 \geq m_2 \\ 0 \text{ if } l_2 \geq u_1 \\ \dfrac{l_2 - u_1}{(m_1 - u_1) - (m_2 - l_2)} \text{ Otherwise} \end{cases} \quad (15)$$

To compare M_1 and M_2 both possibilities $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$ are needed. Considering **Figure 4** as an example, we have $(m_1 \geq m_2)$ which means that $V(M_1 \geq M_2) = 1$ and

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_1}(d) = D \quad (16)$$

where *hgt* is the highest intersection point, D is its value and d its ordinate as shown in **Figure 4**. Accordingly, D is given by:

$$D = V(M_2 \geq M_1) = \frac{u_2 - l_1}{(u_2 - m_2) + (m_1 - l_1)} \quad (17)$$

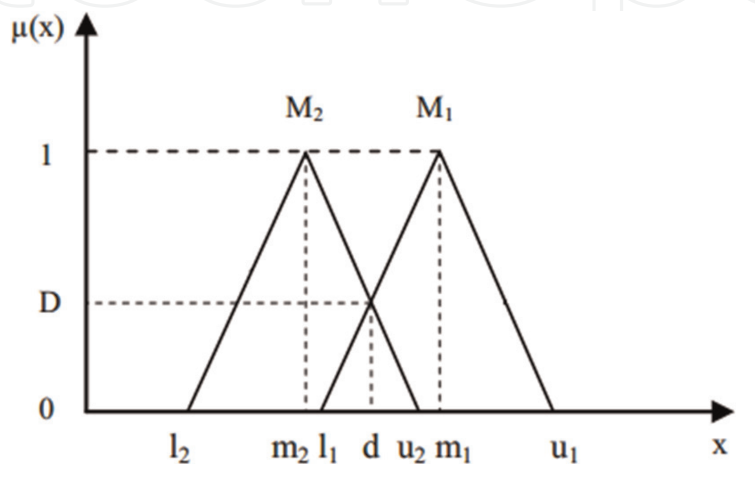

Figure 4.
Linguistic variables for the importance weight of each criterion.

For two fuzzy numbers only two values of possibilities are needed. As the considered fuzzy numbers increase, the numbers of the needed calculated possibilities are increased non-linearly. To compare n fuzzy numbers, we need $n(n-1)$ possible values. Consider the very simple example with the three criteria A, B, and C, the needed possibilities are:

$$V(M_A \geq M_B) = 1, V(M_A \geq M_C) = 1 \tag{18}$$

$$V(M_B \geq M_A) = 0, \quad V(M_B \geq M_C) = 1 \tag{19}$$

$$V(M_C \geq M_A) = 0, V(M_C \geq M_B) = 0 \tag{20}$$

Step 6: the degree of possibility for a convex fuzzy number to be greater than (k) convex fuzzy numbers $M_i(i = 1, 2, \ldots, k)$ can be defined by the following equation:

$$\begin{aligned} V(M_i \geq M_1, M_2, \ldots, M_k) &= V((M_i \geq M_1) \text{ and } (M_i \geq M_2) \text{ and} \ldots (M_i \geq M_k)) \\ &= \min V(M_i \geq M_k), (k = 1, 2, \ldots, n), (i = 1, 2, \ldots, n), k \neq i \end{aligned} \tag{21}$$

The minimum degrees of possibilities for criteria A, B, C are:

$$V(M_A \geq M_B, M_C) = \min (1, 1) = 1 \tag{22}$$

$$V(M_B \geq M_A, M_C) = \min (0, 1) = 0 \tag{23}$$

$$V(M_C \geq M_A, M_B) = \min (0, 0) = 0 \tag{24}$$

Step 7: the normalized weight vector $\boldsymbol{W} = (w_1, \ldots, w_2)^T$ of the fuzzy comparison matrix **Z** is:

Assuming $d'(z_i) = \min V((M_i \geq M_{k.}))$
For $(k = 1, 2, \ldots, n)$, $k \neq i$. Then the weight vector is given by:

$$\mathbf{W}` = (d'(z_1), d'(z_2), \ldots, d'(z_n))^T \tag{25}$$

Via normalization, the normalized weight vector is:

$$\mathbf{W} = (d(z_1), d(z_2), \ldots, d(z_n))^T \tag{26}$$

Therefore, the weight vector for A, B and C is:

$$\mathbf{W}` = (1, 0, 0) \tag{27}$$

And the normalized weight vector for A, B and C is:

$$\mathbf{W} = (1, 0, 0) \tag{28}$$

4. Examples of applications

The following example is proposed by Saaty about a simple decision for selecting a job [9] and it was selected to cope with the original work of Saaty about AHP. This example is a simple decision examined by someone to determine what kind of job would be best for him/her after getting his/her PhD. The goal is to determine the kind of job for which he/she is best suited as spelled out by the criteria. We will construct the pairwise comparison of criteria from the hierarchy structure shown in

Figure 5, apply AHP method and fuzzy AHP method and then compare the results between these two methods. As shown in **Figure 5**, the hierarchical structure consists of four levels. The first level (the top level) is the goal which is to determine the type of a suitable job, the second level is the criteria, the third level is the sub-criteria and the fourth level is the alternative (the lowest level) which the person will choose the kind of the job from these alternatives [10, 11].

According to **Figure 5**, 12 pairwise comparison matrices need to be stated: one for the criteria with respect to the goal, (flexibility, opportunity, security, reputation and salary), two for the sub-criteria which one of it is for the sub-criteria with respect to the flexibility (location, time and work), and the other is for the sub-criteria with respect to the opportunity (entrepreneurial, salary potential and top level position). Nine comparison matrices for the four alternatives with respect to the criteria and the sub-criteria "the covering criteria" connected to the alternatives (domestic company, international company, college and state university). The covering criteria are: the first six are sub-criteria in the third level and the last three are criteria from the second level. As Saaty listed only three pairwise comparison matrices of 12, we listed the rest of pairwise comparison matrices to emphasize the example and show the result. **Tables 6–8** indicate the pairwise comparison matrices for all criteria and sub-criteria.

Table 9 shows the calculation of the global weight for sub-criteria with respect to its criterion by multiplying weight of each criterion to the weights of sub-criteria that affect its criterion.

After computing the relative weights of criteria and sub-criteria, the next step is to compute relative weights of alternatives. **Tables 10–18** indicate the pairwise comparison matrices for alternatives with respect to the covering criteria.

Once the weight vector of covering criteria $\boldsymbol{W}$ and the weight vector of the alternative **S** have been computed, the AHP obtains a vector $\boldsymbol{V}$ of global scores by multiplying $\boldsymbol{S}$ and $\boldsymbol{W}$ as:

$$\boldsymbol{V} = \boldsymbol{S} \cdot \boldsymbol{W} \tag{29}$$

Finally, the alternative ranking is accomplished by ordering these global scores in a descending order. **Table 19** shows the final weights of the alternatives with

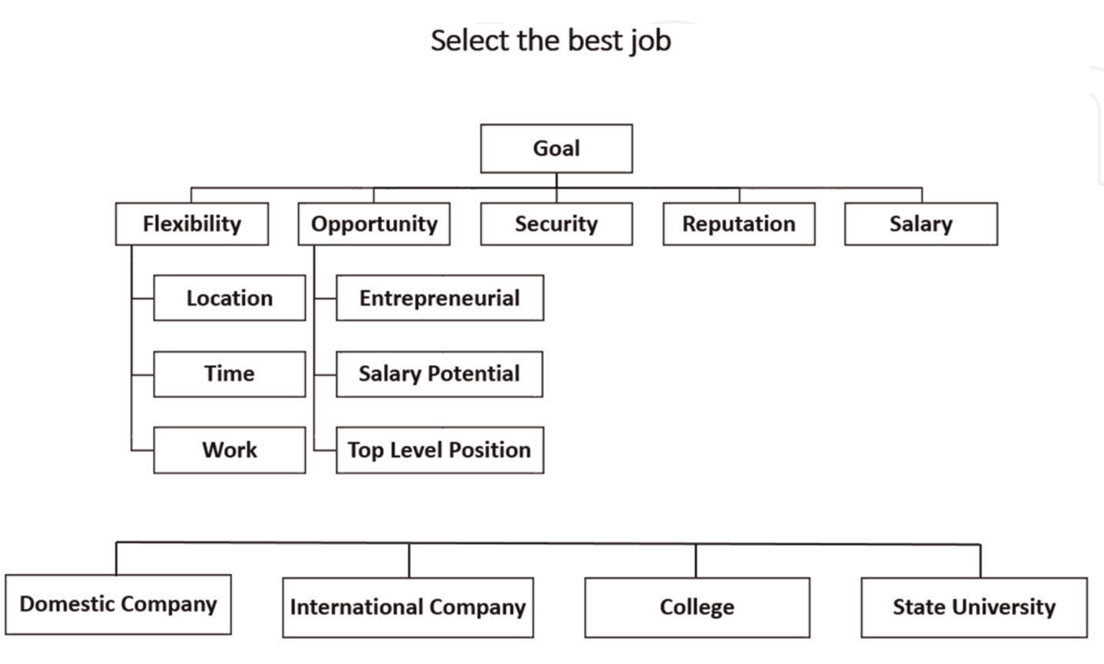

Figure 5.
Best job decision [9].

	Flexibility	Opportunities	Security	Reputation	Salary	Priorities
Flexibility	1	1/4	1/6	1/4	1/8	0.036
Opportunities	4	1	1/3	3	1/7	0.122
Security	6	3	1	4	1/2	0.262
Reputation	4	1/3	1/4	1	1/7	0.075
Salary	8	7	2	7	1	0.506

Table 6.
Pairwise comparison matrix of the main criteria with respect to the goal.

	Location	Time	Work	Priorities
Location	1	1/3	1/6	0.091
Time	3	1	1/4	0.218
Work	6	4	1	0.691

Table 7.
Pairwise comparison matrix for the sub-criteria with respect to flexibility.

	Ent	Sal-pot	Top level pos	Priorities
Ent	1	2	5	0.557
Sal-pot	1/2	1	1/4	0.158
Top level pos	1/5	4	1	0.283

Table 8.
Pairwise comparison matrix for the sub-criteria with respect to opportunity.

Criterion (C)	Local weight (CL)	Sub-criterion (SC)	Local weight (SCL)	Global weight (CL × SCL)
Flexibility	0.0367	Location	0.093	0.0034
		Time	0.221	0.0081
		Work	0.686	0.0251
Opportunity	0.123	Entrepreneurial	0.557	0.0685
		Salary potential	0.158	0.0194
		Top level position	0.283	0.0348

Table 9.
Local weight and global weight for criteria and sub-criteria.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	4	3	6	0.555
Int'l Co	1/4	1	3	5	0.258
College	1/3	1/3	1	2	0.124
State Univ.	1/6	1/5	1/2	1	0.064

Table 10.
Pairwise comparison matrix for the alternatives with respect to salary potential.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	1/5	4	1/3	0.192
Int'l Co	5	1	2	1/4	0.263
College	1/4	1/2	1	2	0.202
State Univ.	3	4	1/2	1	0.343

Table 11.
Pairwise comparison matrix for the alternatives with respect to location.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	1/3	3	6	0.32
Int'l Co	3	1	4	2	0.45
College	1/3	1/4	1	2	0.123
State Univ.	1/6	1/2	1/2	1	0.107

Table 12.
Pairwise comparison matrix for the alternatives with respect to work.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	2	1/3	1/4	0.133
Int'l Co	1/2	1	1/6	1/2	0.092
College	3	6	1	2	0.483
State Univ.	4	2	1/2	1	0.292

Table 13.
Pairwise comparison matrix for the alternatives with respect to time.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	2	4	6	0.502
Int'l Co	1/2	1	3	4	0.3
College	1/4	1/3	1	2	0.124
State Univ.	1/6	1/4	1/2	1	0.074

Table 14.
Pairwise comparison matrix for the alternatives with respect to entrepreneurial.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	1/5	1/3	1/2	0.072
Int'l Co	5	1	3	5	0.54
College	3	1/3	1	2	0.172
State Univ.	4	1/5	1/2	1	0.162

Table 15.
Pairwise comparison matrix for the alternatives with respect to top level position.

respect to the covering criteria. It is clear that the domestic company is the preferred candidate. The second candidate is the college, then the third candidate is the international company and the last candidate is the state university.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	3	2	5	0.47
Int'l Co	1/3	1	1/3	1/2	0.104
College	1/2	3	1	3	0.29
State Univ.	1/5	2	1/3	1	0.129

Table 16.
Pairwise comparison matrix for the alternatives with respect to security.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	4	1/2	3	0.33
Int'l Co	1/4	1	1/6	1/2	0.077
College	2	6	1	2	0.462
State Univ.	1/3	1/2	1/2	1	0.13

Table 17.
Pairwise comparison matrix for the alternatives with respect to salary.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	1	3	4	5	0.5
Int'l Co	1/3	1	1/2	5	0.197
College	1/4	2	1	2	0.19
State Univ.	1/5	1/5	1/2	1	0.072

Table 18.
Pairwise comparison matrix for the alternatives with respect to reputation.

Alternatives ↓ / Covering Criteria →	Location (0.0034)	Time (0.0081)	Work (0.0251)	Entrepreneurial (0.0685)	Salary potential (0.0194)	Top level position (0.0348)	Security (0.27)	Reputation (0.081)	Salary (0.49)	Overall weight
domestic company	0.192	0.133	0.32	0.575	0.587	0.072	0.47	0.5	0.33	**0.381**
international company	0.263	0.092	0.45	0.3	0.248	0.54	0.1	0.197	0.077	**0.138**
college	0.202	0.483	0.123	0.125	0.15	0.172	0.29	0.19	0.462	**0.345**
state university	0.343	0.292	0.107	0.074	0.076	0.162	0.129	0.072	0.13	**0.123**

Table 19.
Final weights of alternatives for AHP method.

As mentioned before, these results of a classical AHP are compared with the results of fuzzy AHP. Therefore, the evaluations are recalculated according to the fuzzy AHP on the same hierarchy structure. The 12 pairwise comparison matrices for all criteria, sub-criteria and alternatives are shown from **Tables 20–32**.

Table 23 shows the calculation of the global weight for sub-criteria with respect to its criterion by multiplying weight of each criterion to the weights of sub-criteria that affect its criterion.

	Flexibility	Opportunities	Security	Reputation	Salary	Priorities
Flexibility	(1, 1, 1)	(1/5, 1/4, 1/3)	(1/7, 1/6, 1/5)	(1/5, 1/4, 1/3)	(1/9, 1/8, 1/7)	0
Opportunities	(3, 4, 5)	(1, 1, 1)	(1/4, 1/3, 1/2)	(2, 3, 4)	(1/8, 1/7, 1/6)	0
Security	(5, 6, 7)	(2, 3, 4)	(1, 1, 1)	(3, 4, 5)	(1/3, 1/2, 1)	0.237
Reputation	(3, 4, 5)	(1/4, 1/3, 1/2)	(1/5, 1/4, 1/3)	(1, 1, 1)	(1/8, 1/7, 1/6)	0
Salary	(7, 8, 9)	(6, 7, 8)	(1, 2, 3)	(6, 7, 8)	(1, 1, 1)	0.763

Table 20.
Fuzzy pairwise comparison matrix of the main criteria with respect to the goal.

	Location	Time	Work	Priorities
Location	(1, 1, 1)	(1/4, 1/3, 1/2)	(1/7, 1/6, 1/5)	0
Time	(2, 3, 4)	(1, 1, 1)	(1/5, 1/4, 1/6)	0
Work	(5, 6, 7)	(3, 4, 5)	(1, 1, 1)	1

Table 21.
Fuzzy pairwise comparison matrix for the sub-criteria with respect to flexibility.

	Ent	Sal-pot	Top level pos	Priorities
Ent	(1, 1, 1)	(1, 2, 3)	(4, 5, 6)	0.65
Sal-pot	(1/3, 1/2, 1)	(1, 1, 1)	(1/5, 1/4, 1/3)	0
Top level pos	(1/6, 1/5, 1/4)	(3, 4, 5)	(1, 1, 1)	0.35

Table 22.
Fuzzy pairwise comparison matrix for the sub-criteria with respect to opportunity.

Criterion (C)	Local weight (CL)	Sub-criterion (SC)	Local weight (SCL)	Global weight (CL × SCL)
Flexibility	0	Location	**0**	0
		Time	**0**	0
		Work	**1**	0
Opportunity	0	Entrepreneurial	**0.65**	0
		Salary potential	**0**	0
		Top level position	**0.35**	0

Table 23.
Local weight and global weight for criteria and sub-criteria.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(3, 4, 5)	(2, 3, 4)	(5, 6, 7)	0.646
Int'l Co	(1/5, 1/4, 1/3)	(1, 1, 1)	(2, 3, 4)	(4, 5, 6)	0.354
College	(1/4, 1/3, 1/2)	(1/4, 1/3, 1/2)	(1, 1, 1)	(1, 2, 3)	0
State Univ.	(1/7, 1/6, 1/5)	(1/6, 1/5, 1/4)	(1/3, 1/2, 1)	(1, 1, 1)	0

Table 24.
Fuzzy pairwise comparison matrix for the alternatives with respect to salary potential.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(1/6, 1/5, 1/4)	(3, 4, 5)	(1/4, 1/3, 1/2)	0
Int'l Co	(4, 5, 6)	(1, 1, 1)	(1, 2, 3)	(1/5, 1/4, 1/3)	0
College	(1/5, 1/4, 1/3)	(1/3, 1/2, 1)	(1, 1, 1)	(1, 2, 3)	0.493
State Univ.	(2,3,4)	(3, 4, 5)	(1/3, 1/2, 1)	(1, 1, 1)	0.507

Table 25.
Fuzzy pairwise comparison matrix for the alternatives with respect to location.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(1/4, 1/3, 1/2)	(2, 3, 4)	(5, 6, 7)	0.508
Int'l Co	(2, 3, 4)	(1, 1, 1)	(3, 4, 5)	(1, 2, 3)	0.492
College	(1/4, 1/3, 1/2)	(1/5, 1/4, 1/3)	(1, 1, 1)	(1, 2, 3)	0
State Univ.	(1/7, 1/6, 1/5)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1, 1, 1)	0

Table 26.
Fuzzy pairwise comparison matrix for the alternatives with respect to work.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(1, 2, 3)	(1/4, 1/3, 1/2)	(1/5, 1/4, 1/3)	0
Int'l Co	(1/3, 1/2, 1)	(1, 1, 1)	(1/7, 1/6, 1/5)	(1/3, 1/2, 1)	0
College	(2, 3, 4)	(5, 6, 7)	(1, 1, 1)	(1, 2, 3)	0.626
State Univ.	(3, 4, 5)	(1, 2, 3)	(1/3, 1/2, 1)	(1, 1, 1)	0.374

Table 27.
Fuzzy pairwise comparison matrix for the alternatives with respect to time.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(1, 2, 3)	(3, 4, 5)	(5, 6, 7)	0.625
Int'l Co	(1/3, 1/2, 1)	(1, 1, 1)	(2, 3, 4)	(3, 4, 5)	0.375
College	(1/5, 1/4, 1/3)	(1/4, 1/3, 1/2)	(1, 1, 1)	(1, 2, 3)	0
State Univ.	(1/7, 1/6, 1/5)	(1/5, 1/4, 1/3)	(1/3, 1/2, 1)	(1, 1, 1)	0

Table 28.
Fuzzy pairwise comparison matrix for the alternatives with respect to entrepreneurial.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(1/6, 1/5, 1/4)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	0
Int'l Co	(4, 5, 6)	(1, 1, 1)	(2, 3, 4)	(4, 5, 6)	0.789
College	(2, 3, 4)	(1/4, 1/3, 1/2)	(1, 1, 1)	(1, 2, 3)	0.211
State Univ.	(1, 2, 3)	(1/6, 1/5, 1/4)	(1/3, 1/2, 1)	(1, 1, 1)	0

Table 29.
Fuzzy pairwise comparison matrix for the alternatives with respect to top level position.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(2, 3, 4)	(1, 2, 3)	(4, 5, 6)	0.573
Int'l Co	(1/4, 1/3, 1/2)	(1, 1, 1)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	0
College	(1/3, 1/2, 1)	(2, 3, 4)	(1, 1, 1)	(2, 3, 4)	0.394
State Univ.	(1/6, 1/5, 1/4)	(1, 2, 3)	(1/4, 1/3, 1/2)	(1, 1, 1)	0.033

Table 30.
Fuzzy pairwise comparison matrix for the alternatives with respect to security.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(3, 4, 5)	(1/3, 1/2, 1)	(2, 3, 4)	0.436
Int'l Co	(1/5, 1/4, 1/3)	(1, 1, 1)	(1/7, 1/6, 1/5)	(1/3, 1/2, 1)	0
College	(1, 2, 3)	(5, 6, 7)	(1, 1, 1)	(1, 2, 3)	0.564
State Univ.	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1, 1, 1)	0

Table 31.
Fuzzy pairwise comparison matrix for the alternatives with respect to salary.

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic Co	(1, 1, 1)	(2, 3, 4)	(3, 4, 5)	(4, 5, 6)	0.649
Int'l Co	(1/4, 1/3, 1/2)	(1, 1, 1)	(1/3, 1/2, 1)	(4, 5, 6)	0.228
College	(1/5, 1/4, 1/3)	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)	0.123
State Univ.	(1/6, 1/5, 1/4)	(1/6, 1/5, 1/4)	(1/3, 1/2, 1)	(1, 1, 1)	0

Table 32.
Fuzzy pairwise comparison matrix for the alternatives with respect to reputation.

Once the weight vector of covering criteria $\boldsymbol{W}$ and the weight vector of the alternative $\boldsymbol{S}$ have been computed, the fuzzy AHP obtains a vector $\boldsymbol{V}$ of global scores by multiplying $\boldsymbol{S}$ and $\boldsymbol{W}$ as:

$$\boldsymbol{V} = \boldsymbol{S} \cdot \boldsymbol{W} \tag{30}$$

Finally, the alternative ranking is accomplished by ordering these global scores in a descending order. **Table 33** shows the final weights of the alternatives with respect to the covering criteria. It is clear that the college is the preferred candidate. The second candidate is the domestic company, then the third candidate is the state university and the last candidate is international company.

Alternatives ↓ / Covering Criteria →	Local (0)	Time (0)	Work (0)	Entrepreneurial (0)	Salary potential (0)	Top level position (0)	Security (0.237)	Reputation (0)	Salary (0.763)	Overall weight
domestic company	0	0	0.508	0.625	0.646	0	0.573	0.649	0.436	0.469
international company	0	0	0.492	0.375	0.354	0.789	0	0.228	0	0
college	0.493	0.626	0	0	0	0.211	0.394	0.123	0.564	0.524
state university	0.507	0.374	0	0	0	0	0.033	0	0	0.007

Table 33.
Final weights of alternatives for fuzzy AHP method.

5. Conclusions

In this chapter, a comparative analysis of analytic hierarchy process and fuzzy analytic hierarchy process is presented using two levels of criteria example. The analytic hierarchy process method is mainly used in crisp values, the normalized weight of each alternative shows that domestic company has higher priority (0.381) than the other alternatives while the fuzzy analytic hierarchy process used in range values, the normalized weight of each alternative shows that college has higher priority (0.524) than the other alternatives.

The fuzzy analytic hierarchy process approach is preferred by decision makers than analytic hierarchy process approach because fuzzy analytic hierarchy process applies a range of values to incorporate the decision maker's uncertainly. It enhances the potential of the analytic hierarchy process for dealing with imprecise and uncertain human comparison judgments.

The example showed that weight values of some criteria, sub-criteria and alternatives in fuzzy analytic hierarchy process became zero, as shown in **Tables 20–22**, etc., which look odd as results, because normally all given criteria are used in pairwise comparisons and assumed to be evaluated to non-zero values. This is not a strange position because the decision makers may do not take into account one or more criteria for the evaluation even if these criteria are set in the hierarchy. Therefore, the fuzzy analytic hierarchy process approach provides to eliminate the unnecessary criterion or criteria if all of the decision makers assign "extremely important" value when compared with the other criteria and expresses the less important criteria.

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References

[1] Saaty TL. The Analytic Hierarchy Process. New York: McGraw-Hill; 1980. p. 324

[2] Chang DY. Applications of the extent analysis method on fuzzy AHP. European Journal of Operational Research. 1996;**95**(3):649-655

[3] Ho W, Ma X. The state-of-the-art integrations and applications of the analytic hierarchy process. European Journal of Operational Research. 2018; **267**(2):399-414

[4] Buckley JJ. Fuzzy hierarchical analysis. Fuzzy Sets and Systems. 1985; **17**(3):233-247

[5] Kubler S, Robert J, Derigent W, Voisin A, Le Traon Y. A state-of the-art survey & testbed of fuzzy AHP (FAHP) applications. Expert Systems with Applications. 2016;**65**:398-422

[6] Nguyen PT, Vu NB, Van Nguyen L, Le LP, Vo KD. The application of fuzzy analytic hierarchy process (F-AHP) in engineering project management. In 2018 IEEE 5th International Conference on Engineering Technologies and Applied Sciences (ICETAS). IEEE. November, 2018. pp. 1-4

[7] Ge Z, Liu Y. Analytic hierarchy process based fuzzy decision fusion system for model prioritization and process monitoring application. IEEE Transactions on Industrial Informatics. 2018;**15**(1):357-365

[8] Prascevic N, Prascevic Z. Application of fuzzy AHP for ranking and selection of alternatives in construction project management. Journal of Civil Engineering and Management. 2017;**23** (8):1123-1135

[9] Saaty TL. Decision making with the analytic hierarchy process. International Journal of Services Sciences. 2008;**1**(1): 83-98

[10] Saaty TL. How to make a decision: The analytic hierarchy process. European Journal of Operational Research. 1990;**48**(1):9-26

[11] Saaty TL, Vargas LG. Models, Methods, Concepts and Applications of the Analytic Hierarchy Process. Boston: Kluwer Academic Publishers; 2000