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Chapter

Discrete-Time Nonlinear Attitude Tracking Control of Spacecraft

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Abstract

Recent space programs require agile and large-angle attitude maneuvers for applications in various fields such as observational astronomy. To achieve agility and large-angle attitude maneuvers, it will be required to design an attitude control system that takes into account nonlinear motion because agile and large-angle rotational motion of a spacecraft in such missions represents a nonlinear system. Considerable research has been done about the nonlinear attitude tracking control of spacecraft, and these methods involve a continuous-time control framework. However, since a computer, which is a digital device, is employed as a spacecraft controller, the control method should have discrete-time control or sampled-data control framework. This chapter considers discrete-time nonlinear attitude tracking control problem of spacecraft. To this end, a Euler approximation system with respect to tracking error is first derived. Then, we design a discrete-time nonlinear attitude tracking controller so that the closed-loop system consisting of the Euler approximation system becomes input-to-state stable (ISS). Furthermore, the exact discrete-time system with a derived controller is indicated semiglobal practical asymptotic (SPA) stable. Finally, the effectiveness of proposed control method is verified by numerical simulations.

Keywords: spacecraft, attitude tracking control, discrete-time nonlinear control

1. Introduction

Recent space programs require agile and large-angle attitude maneuvers for applications in various fields such as observational astronomy [1–3]. To achieve agility and large-angle attitude maneuvers, it will be required to design an attitude control system that takes into account nonlinear motion because agile and large-angle rotational motion of a spacecraft in such missions represents a nonlinear system.

Considerable research has been done about the nonlinear attitude tracking control of spacecraft [4–12], and these methods involve a continuous-time control framework. However, since a computer, which is a digital device, is employed as a spacecraft controller, the control method should have discrete-time control or sampled-data control framework.

Although a sampled-data control method for nonlinear system did not advance because it is difficult to discretize a nonlinear system, a control method based on the Euler approximate model has been proposed in recent years [13, 14] and is applied to ship control [15]. Although our research group has proposed a sampled-data control method using backstepping [16] and a discrete-time control method based on sliding mode control [17] for spacecraft control problem, these methods are disadvantageous because control input amplitude depends on the sampling period T as the control law is of the form $u = a(x) + (b(x)/T)$.

For these facts, about the spacecraft attitude control problem that requires agile and large-angle attitude maneuvers, this chapter proposed a discrete-time nonlinear attitude tracking control in which the control input amplitude is independent of the sampling period T. The effectiveness of proposed control method is verified by numerical simulations.

The following notations are used throughout the chapter. Let R and N denote the real and the integer numbers. Rn and $R^{n\times m}$ are the sets of real vectors and matrices. For real vector $a \in R^n$, a^T is the vector transpose, $\|a\|$ denotes the Euclidean norm, and $a^\times \in R^{3\times 3}$ is the skew symmetric matrix

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

derived from vector $a \in R^3$. For real symmetric matrix A, $A > 0$ means the positive definite matrix. The identity matrix of size 3×3 is denoted by I_3. $\lambda_A^{\max} \in R$ and $\lambda_A^{\min} \in R$ are the maximal and the minimal eigenvalues of a matrix A, respectively.

2. Relative equation of motion and discrete-time model for spacecraft

In this chapter, as the kinematics represents the attitude of the spacecraft with respect to the inertia frame $\{i\}$, the modified Rodrigues parameters (MRPs) [5] are used. The rotational motion equations of the spacecraft's body-fixed frame $\{b\}$ are given by the following equations:

$$\dot{\sigma}(t) = G(\sigma(t))\omega(t), \tag{1}$$

$$G(\sigma(t)) = \frac{1}{2}\left\{\frac{1-\|\sigma(t)\|^2}{2}I_3 + \sigma(t)\sigma(t)^T + \sigma(t)^\times\right\},$$

$$\dot{\omega}(t) = J^{-1}\{-\omega(t)^\times J\omega(t) + u(t) + w(t)\}, \tag{2}$$

where Eq. (1) is the kinematics that represents the attitude of $\{b\}$ with respect to the $\{i\}$, Eq. (2) is the rotation dynamics, $\sigma(t) \in R^3$ [−] is the MRPs, $\omega(t) \in R^3$ [rad/s] is the angular velocity, $u(t) \in R^3$ [Nm] is the control torque (input), $w(t) \in R^3$ [Nm] is the disturbance input, and $J \in R^{3\times 3}$ [kg m^2] is the moment of inertia.

We consider a control problem in which a spacecraft tracks a desired attitude (MRPs) $\sigma_d(t) \in R^3$ and angular velocity $\omega_d(t) \in R^3$ in fixed frame $\{d\}$. The MRPs of the relative attitude $\sigma_e(t) \in R^3$ and the relative angular velocity $\omega_e(t) \in R^3$ in the frame $\{b\}$ are given by

$$\sigma_e(t) = \frac{N_e(t)}{1 + \|\sigma(t)\|^2\|\sigma_d(t)\|^2 + 2\sigma_d(t)^T\sigma(t)}, \tag{3}$$

$$N_e(t) = \left(1 - \|\sigma_d(t)\|^2\right)\sigma(t) - \left(1 - \|\sigma(t)\|^2\right)\sigma_d(t) + 2\sigma(t)^\times\sigma_d(t),$$

$$\omega_e(t) = \omega(t) - C(t)\omega_d(t), \tag{4}$$

where $C(t) \in \mathrm{R}^{3\times 3}$ is the direction cosine matrix from $\{b\}$ to $\{d\}$ that expresses the following Eq. [7]:

$$C(t) = I_3 + \frac{8\left(\sigma_e(t)^\times\right)^2 - 4\left(1 - \|\sigma_e(t)\|^2\right)\sigma_e(t)^\times}{\left(1 + \|\sigma_e(t)\|^2\right)^2}. \tag{5}$$

Substituting Eqs. (3) and (4) into Eqs. (1) and (2) using the identity $\dot{C}(t) = -\omega_e(t)^\times C(t)$ yields the following relative motion equations:

$$\dot{\sigma}_e(t) = G(\sigma_e(t))\omega_e(t), \tag{6}$$

$$\begin{aligned}\dot{\omega}_e(t) =& J^{-1}[-\{\omega_e(t) + C(t)\omega_d(t)\}^\times J\{\omega_e(t) + C(t)\omega_d(t)\} \\ &-J\{C(t)\dot{\omega}_d(t) - \omega_e(t)^\times C(t)\omega_d(t)\} + u(t) + w(t)]\end{aligned} \tag{7}$$

Hereafter, we assume that the variables of spacecraft $\sigma(t)$ and $\omega(t)$ are directly measurable and J is known. In addition, regarding the desired states $\sigma_d(t)$, $\omega_d(t)$, $\dot{\omega}_d(t)$, and the disturbance $w(t)$, the following assumption is made.

Assumption 1: the desired states $\sigma_d(t)$, $\omega_d(t)$, and $\dot{\omega}_d(t)$ are uniformly continuous and bounded $\forall t \in [0, \infty)$. The disturbance $w(t)$ is uniformly bounded $\forall t \in [0, \infty)$.

From Eqs. (A4) and (A5) in Appendix, the exact discrete-time model of relative motion equations is obtained as

$$\sigma_{e,k+1} = \sigma_{e,k} + \int_{kT}^{(k+1)T} G(\sigma_e(s))\omega_e(s)\, ds, \tag{8}$$

$$\begin{aligned}\omega_{e,k+1} =& \omega_{e,k} + \int_{kT}^{(k+1)T} [-\{\omega_e(s) + C(s)\omega_d(s)\}^\times J\{\omega_e(s) + C(s)\omega_d(s)\} \\ &-J\{C(s)\dot{\omega}_d(s) - \omega_e(s)^\times C(s)\omega_d(s)\} + u_k + w_k]ds\end{aligned} \tag{9}$$

and the Euler approximate model of relative motion equations are obtained as

$$\sigma_{e,k+1} = \sigma_{e,k} + TG(\sigma_{e,k})\omega_{e,k}, \tag{10}$$

$$\begin{aligned}\omega_{e,k+1} = \omega_{e,k} - TJ^{-1}[&-\{\omega_{e,k} + C_k\omega_{d,k}\}^\times J\{\omega_{e,k} + C_k\omega_{d,k}\} \\ &-J\{C_k\dot{\omega}_{d,k} - \omega_{e,k}{}^\times C_k\omega_{d,k}\} + u_k + w_k].\end{aligned} \tag{11}$$

3. Discrete-time nonlinear attitude tracking control

We derive a controller based on the backstepping approach that makes the closed-loop system consisting of the Euler approximate modes (10) and (11) become input-to-state stable (ISS), i.e., the state variable of closed-loop system $x_k = \left[\sigma_{e,k}^T \omega_{e,k}^T\right]^T$ satisfies the following equation:

$$\|x_{k+1}\| \le \rho(\|x_0\|, k) + \gamma(\|w_k\|), \quad \forall x_k \in \mathrm{R}^3, \quad \forall w_k \in \mathrm{R}^3,$$

where $\rho(\cdot)$ is the class KL function and $\gamma(\cdot)$ is the class K function. To this end, assume that $\omega_{e,k}$ is the virtual input to subsystem (10), and derive the stabilizing function α_k that $\sigma_{e,k}$ is asymptotic convergence to zero. Then, derive the control input u_k that closed-loop system becomes ISS. Here, regarding the variable $\sigma_{e,k}$, the following assumption is made.

Assumption 2: $\sigma_{e,k}$ lies in the region that satisfies the following equation:

$$0 \le \|\sigma_{e,k}\| \le 1, \quad \forall k.$$

Remark 1: from the relational expression

$$\sigma_{e,k} = \frac{\varepsilon_{e,k}}{1+\eta_{e,k}},$$

where $\varepsilon_{e,k} \in \mathrm{R}^3$ and $\eta_{e,k} \in \mathrm{R}$ are the quaternion $\left(\left\|\left[\varepsilon_{e,k}^T \eta_{e,k}\right]^T\right\| = 1, \|\varepsilon_{e,k}\| \le 1, \left|\eta_{e,k}\right| \le 1, \forall k\right)$. Assumption 2 is equivalent to $\eta_{e,k} \in [0,1]$.

In addition, Lemmas when using the derivation of the control law are shown below.

Lemma 1: for all $\sigma \in \mathrm{R}^3$, the following equations hold [5]:

$$\sigma^T G(\sigma) = b\sigma^T, \quad G(\sigma)^T G(\sigma) = b^2 I_3, \quad \left(b = \frac{1+\|\sigma\|^2}{4} > 0\right).$$

Lemma 2: when the quadratic equation

$$ax^2 + bx + c = 0 (a, b, c \in \mathrm{R})$$

has two distinct real roots $x = \alpha, \beta (\alpha < \beta)$, if $a > 0$, then the solution of the quadratic inequality

$$ax^2 + bx + c < 0$$

is $\alpha < x < \beta$.

3.1 Derivation of virtual input α_k

Assume that $\omega_{e,k}$ is the virtual input to subsystem (10), and define the stabilizing function such that

$$\omega_{e,k} = \alpha_k = -f_1 \sigma_{e,k}, \tag{12}$$

where $f_1 \in \mathrm{R}$ is the feedback gain. The candidate Lyapunov function for (10) is defined as

$$V_1(k) = \|\sigma_{e,k}\|^2. \tag{13}$$

From Lemma 1, the difference of Eq. (13) along the trajectories of the closed-loop system is given by

$$\Delta V_1(k) = V_1(k+1) - V_1(k) = \left\{ \left(Tf_1 b_k\right)^2 - 2Tf_1 b_k \right\} \|\sigma_{e,k}\|^2. \tag{14}$$

From Lemma 2, $\Delta V_1(k)$ becomes negative, i.e., the range of f_1 that holds the following equation

DOI: http://dx.doi.org/10.5772/intechopen.87191

$$\left(Tf_1 b_k\right)^2 - 2Tf_1 b_k < 0 \tag{15}$$

is obtained as

$$0 < f_1 < \frac{2}{Tb_k}. \tag{16}$$

In addition, since $2 \leq (1/b_k) \leq 4$ under Assumption 2, the range of f_1 that holds Eq. (15) is obtained as

$$0 < f_1 < \frac{4}{T}. \tag{17}$$

Therefore, if f_1 satisfies Eq. (17) and $\omega_{e,k} \to \alpha_k (k \to \infty)$, then $\sigma_{e,k} \to 0$.

3.2 Derivation of control input u_k

The error variable between the state $\omega_{e,k}$ and α_k is defined as

$$z_k := \omega_{e,k} - \alpha_k. \tag{18}$$

The control input u_k that makes the closed-loop system becomes ISS is derived. From Eq. (18), subsystem (10) becomes

$$\sigma_{e,k+1} = \sigma_{e,k} + TG(\sigma_{e,k})(z_k + \alpha_k). \tag{19}$$

From Eqs. (18) and (19) and the following equation

$$\alpha_k - \alpha_{k+1} = Tf_1\{G(\sigma_{e,k})z_k - f_1 b_k \sigma_{e,k}\},$$

the discrete-time equation with respect to z_k is

$$\begin{aligned} z_{k+1} = z_k &+ Tf_1\{G(\sigma_{e,k})z_k - f_1 b_k \sigma_{e,k}\} \\ &+ TJ^{-1}[-\{z_k + \alpha_{,k} + C_k\omega_{d,k}\} \times J\{z_k + \alpha_{,k} + C_k\omega_{d,k}\} \\ &- J\{C_k\dot{\omega}_{d,k} - (z_k + \alpha_{,k}) \times C_k\omega_{d,k}\} + u_k + w_k]. \end{aligned} \tag{20}$$

Now, by setting u_k to

$$\begin{aligned} u_k = &\{z_k + \alpha_{,k} + C_k\omega_{d,k}\} \times J\{z_k + \alpha_{,k} + C_k\omega_{d,k}\} \\ &+ J\{C_k\dot{\omega}_{d,k} - (z_k + \alpha_{,k}) \times C_k\omega_{d,k}\} \\ &- f_1 J\{G(\sigma_{e,k})z_k - f_1 b_k \sigma_{e,k}\} - f_2 J z_k, \end{aligned}$$

Eq. (20) becomes

$$z_{k+1} = (1 - Tf_2)z_k + TJ^{-1}w_k, \tag{21}$$

where $f_2 \in \mathrm{R}$ is the feedback gain. The candidate Lyapunov function for Eqs. (19) and (21) is defined as

$$V_2(k) = V_1(k) + \|z_k\|^2 = \|X_k\|^2, X_k = \left[\sigma_{e,k}^T z_k^T\right]^T. \tag{22}$$

As Eq. (14) is given by

$$\Delta V_1(k) = (Tb_k)^2\|z_k\|^2 + \left\{(Tf_1b_k)^2 - 2Tf_1b_k\right\}\|\sigma_{e,k}\|^2 + 2Tb_k(1 - Tf_1b_k)z_k^T\sigma_{e,k}^T$$

from Eq. (18), by using completing square, the difference of Eq. (22) along the trajectories of the closed-loop system is given by

$$\begin{aligned}\Delta V_2(k) &= \left(T^2f_2^2 - 2Tf_2 + T^2b_k^2\right)\|z_k\|^2 + \left\{(Tf_1b_k)^2 - 2Tf_1b_k\right\}\|\sigma_{e,k}\|^2 \\ &\quad + 2Tb_k(1 - Tf_1b_k)z_k^T\sigma_{e,k}^T + T^2w_k^TJ^{-2}w_k + 2T(1 - Tf_2)w_k^TJ^{-1}z_k \\ &\quad \le \left(2T^2f_2^2 - 4Tf_2 + T^2b_k^2 + 1\right)\|z_k\|^2 + \left\{(Tf_1b_k)^2 - 2Tf_1b_k\right\}\|\sigma_{e,k}\|^2 \\ &\quad + 2Tb_k(1 - Tf_1b_k)z_k^T\sigma_{e,k}^T + 2\left(\frac{T}{\lambda_J}\right)^2\|w_k\|^2 \\ &= X_k^TQ_kX_k + 2\left(\frac{T}{\lambda_J}\right)^2\|w_k\|^2,\end{aligned} \tag{23}$$

where

$$\lambda_J = \|J\|,\ Q_k = \begin{bmatrix} Q_{11,k} & Q_{12,k} \\ Q_{12,k}^T & Q_{22,k}\end{bmatrix},\ Q_{11,k} = \left\{(Tf_1b_k)^2 - 2Tf_1b_k\right\}I_3,$$

$$Q_{12,k} = Tb_k(1 - Tf_1b_k)I_3,\ Q_{22,k} = \left(2T^2f_2^2 - 4Tf_2 + T^2b_k^2 + 1\right)I_3.$$

In Eq. (23), if $Q_k < 0$, then

$$\Delta V_2(k) \le -\left|\lambda_{Q_k}^{\min}\right|\|X_k\|^2 + 2\left(\frac{T}{\lambda_J}\right)^2\|w_k\|^2,$$

where $\lambda_{Q_k}^{\min} < 0 \in \mathrm{R}$ is the minimum eigenvalue of Q_k and the condition of ISS holds [18]. Hereafter, conditions of f_1 and f_2 which the matrix Q_k holds $Q_k < 0$ are derived under Assumption 2.

From Schur complement, condition $Q_k < 0$ is equivalent to the following equations:

$$(Tf_1b_k)^2 - 2Tf_1b_k < 0, \tag{24}$$

$$2T^2f_2^2 - 4Tf_2 + c_k < 0 \quad \left(c_k = \frac{Tb_kf_1^2 - 2f_1 - Tb_k}{Tb_kf_1^2 - 2f_1}\right). \tag{25}$$

Condition (24) is the same as Eq. (15), and assume that Eq. (24) holds. From Lemma 2, the range of f_2 that holds for Eq. (25) is obtained as

$$\frac{2 - \sqrt{2(2 - c_k)}}{2T} < f_2 < \frac{2 + \sqrt{2(2 - c_k)}}{2T}, \tag{26}$$

and the following Eq.

$$2 - c_k > 0 \quad \Rightarrow \quad \frac{Tb_kf_1^2 - 2f_1 + Tb_k}{Tb_kf_1^2 - 2f_1} > 0 \tag{27}$$

must hold true in order to obtain a real number. As the denominator of Eq. (27) is the same as Eq. (24), the following equation must hold

$$Tb_k f_1^2 - 2f_1 + Tb_k < 0 \tag{28}$$

in order to hold Eq. (27). From Lemma 2, the range of f_1 that holds for Eq. (28) is obtained as

$$\frac{1-\sqrt{1-(Tb_k)^2}}{Tb_k} < f_1 < \frac{1+\sqrt{1-(Tb_k)^2}}{Tb_k}, \tag{29}$$

and the following Eq.

$$1-(Tb_k)^2 > 0 \quad \Rightarrow \quad 0 < T < \frac{1}{b_k} \tag{30}$$

must hold in order to have the real number. As $2 \le (1/b_k) \le 4$ under Assumption 2, T must satisfy the condition

$$0 < T < 2. \tag{31}$$

In addition, since

$$\max_{b_k} \frac{1-\sqrt{1-(Tb_k)^2}}{Tb_k} = \frac{2-\sqrt{4-T^2}}{T},$$
$$\min_{b_k} \frac{1+\sqrt{1-(Tb_k)^2}}{Tb_k} = \frac{2+\sqrt{4-T^2}}{T}$$

under Assumption 2, the condition (29) is given by

$$\frac{2-\sqrt{4-T^2}}{T} < f_1 < \frac{2+\sqrt{4-T^2}}{T} (0 < T < 2). \tag{32}$$

Therefore, if f_1 satisfies Eq. (32) under Assumption 2, Eqs. (27) and (28) hold. Furthermore, since

$$\max_{b_k} \frac{2-\sqrt{2(2-c_k)}}{2T} = \frac{1}{T} - \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2 f_1 (Tf_1 - 4)}},$$
$$\min_{b_k} \frac{2+\sqrt{2(2-c_k)}}{2T} = \frac{1}{T} + \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2 f_1 (Tf_1 - 4)}},$$

under Assumption 2, the condition (26) is given by.

$$\frac{1}{T} - \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2 f_1 (Tf_1 - 4)}} < f_2 < \frac{1}{T} + \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2 f_1 (Tf_1 - 4)}} (0 < T < 2). \tag{33}$$

Therefore, if f_1 and f_2 satisfy Eqs. (32) and (33) under Assumption 2, then $Q_k < 0$.

Summarizing the above, the following theorem can be obtained.

Theorem 1: if sampling period T and feedback gains f_1 and f_2 satisfy Eqs. (31), (32), and (33) under Assumption 2, then the closed-loop systems (10) and (11) with the following control law

$$\begin{aligned} u_k &= \{z_k + \alpha_{,k} + C_k\omega_{d,k}\}^{\times} J\{z_k + \alpha_{,k} + C_k\omega_{d,k}\} \\ &\quad + J\{C_k\dot{\omega}_{d,k} - (z_k + \alpha_{,k})^{\times} C_k\omega_{d,k}\} \\ &\quad - f_1 J\{G(\sigma_{e,k})z_k - f_1 b_k \sigma_{e,k}\} - f_2 J z_k \\ &= \omega_k{}^{\times} J\omega_k + J\{C_k\dot{\omega}_{d,k} - (z_k + \alpha_{,k})^{\times} C_k\omega_{d,k}\} \\ &\quad - f_1 f_2 J\sigma_{e,k} - J\{f_1 G(\sigma_{e,k}) + f_2 I_3\}\omega_{e,k} \end{aligned} \tag{34}$$

becomes ISS.

Then, we show that the pair $(u_k, V_2(k))$ is semiglobal practical asymptotic (SPA) stabilizing pair for the Euler approximate systems (10) and (11). Hereafter, suppose that sampling period T and feedback gains f_1 and f_2 satisfy Eqs. (31), (32), and (33) under Assumption 2. By using the following coordinate transformation

$$X_k = \begin{bmatrix} 1 & 0 \\ f_1 & 1 \end{bmatrix} \begin{bmatrix} \sigma_{e,k} \\ \omega_{e,k} \end{bmatrix} = Z\overline{X}_k,$$

Lyapunov function $V_2(k)$ and its difference $\Delta V_2(k)$ can be rewritten as

$$V_2(k) = \overline{X}_k^T Z^T Z \overline{X}_k = \overline{X}_k^T R \overline{X}_k,$$
$$\Delta V_2(k) = \overline{X}_k^T Z^T Q_k Z \overline{X}_k + 2\left(\frac{T}{\lambda_J}\right)^2 \|w_k\|^2 = \overline{X}_k^T \overline{Q}_k \overline{X}_k + 2\left(\frac{T}{\lambda_J}\right)^2 \|w_k\|^2.$$

Since $R > 0$ and $\overline{Q}_k < 0$, $V_2(k)$ and $\Delta V_2(k)$ satisfy following equations:

$$\lambda_R^{\min} \left\|\overline{X}_k\right\|^2 \le V_2(k) \le \lambda_R^{\max} \left\|\overline{X}_k\right\|^2, \tag{35}$$
$$\Delta V_2(k) \le -\left|\lambda_{\overline{Q}_k}^{\min}\right| \left\|\overline{X}_k\right\|^2 + 2\left(\frac{T}{\lambda_J}\right)^2 \|w_k\|^2. \tag{36}$$

In addition, $\overline{X}_k$ is bounded, and $V_2(k)$ is radially unbounded from Eqs. (35) and (36). Hence, the control input (34) satisfies the following equation under Assumption 1:

$$\|u_k\| \le M, \tag{37}$$

where M is a positive constant. Furthermore, $V_2(k)$ also satisfies the following equation for all $x, z \in \mathrm{R}^6$ with $\max\{\|x\|, \|z\|\} \le \Delta$:

$$\begin{aligned} |V_2(x) - V_2(z)| &= \left|x^T R x - z^T R z\right| = \left|(x+z)^T R (x-z)\right| \\ &= \lambda_R^{\max} \|x+z\| \|x-z\| \le 2\Delta \lambda_R^{\max} \|x-z\|, \end{aligned} \tag{38}$$

where Δ is a positive constant. Therefore, from Eqs. (35) to (38), Lyapunov function $V_2(k)$ and control input u_k satisfied Eqs. (A8)–(A11) in Definition 2 under Assumptions 1 and 2, and the pair $(u_k, V_2(k))$ becomes SPA stabilizing pair for the

Euler approximate systems (10) and (11). Then, the following theorem can be obtained by Theorem A.1 in Appendix.

Theorem 2: control input (34) is SPA stabilizing for exact discrete-time systems (8) and (9).

4. Numerical simulation

The properties of the proposed method are discussed in the numerical study. For this purpose, parameter setting of simulation is as follows:

$$J = \begin{bmatrix} 7050.0 & -0.536 & 43.9 \\ -0.536 & 2390 & 1640.0 \\ 43.9 & 1640.0 & 6130.0 \end{bmatrix} \text{kgm}^2, \sigma(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \omega(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{rad/s}$$

$$T = \begin{cases} 1.0 : \text{Case 1} \\ 0.5 : \text{Case 2}, f_1 = 0.6, f_2 = 0.8. \\ 0.1 : \text{Case 3} \end{cases}$$

The moment of inertia J is from [1]. The initial values $\sigma(0)$ correspond to Euler angles of 1–2-3 system of $\theta(0) = [\theta_1(0)\theta_2(0)\theta_3(0)]^T = [0\ \ 0\ \ 0]^T$ [deg]. The feedback gains f_1 and f_2 satisfy Eqs. (25) and (28) for all cases of T. The desired states $\sigma_d(t)$, $\omega_d(t)$, and $\dot{\omega}_d(t)$ in this simulation are the switching maneuver as shown in **Figure 1**.

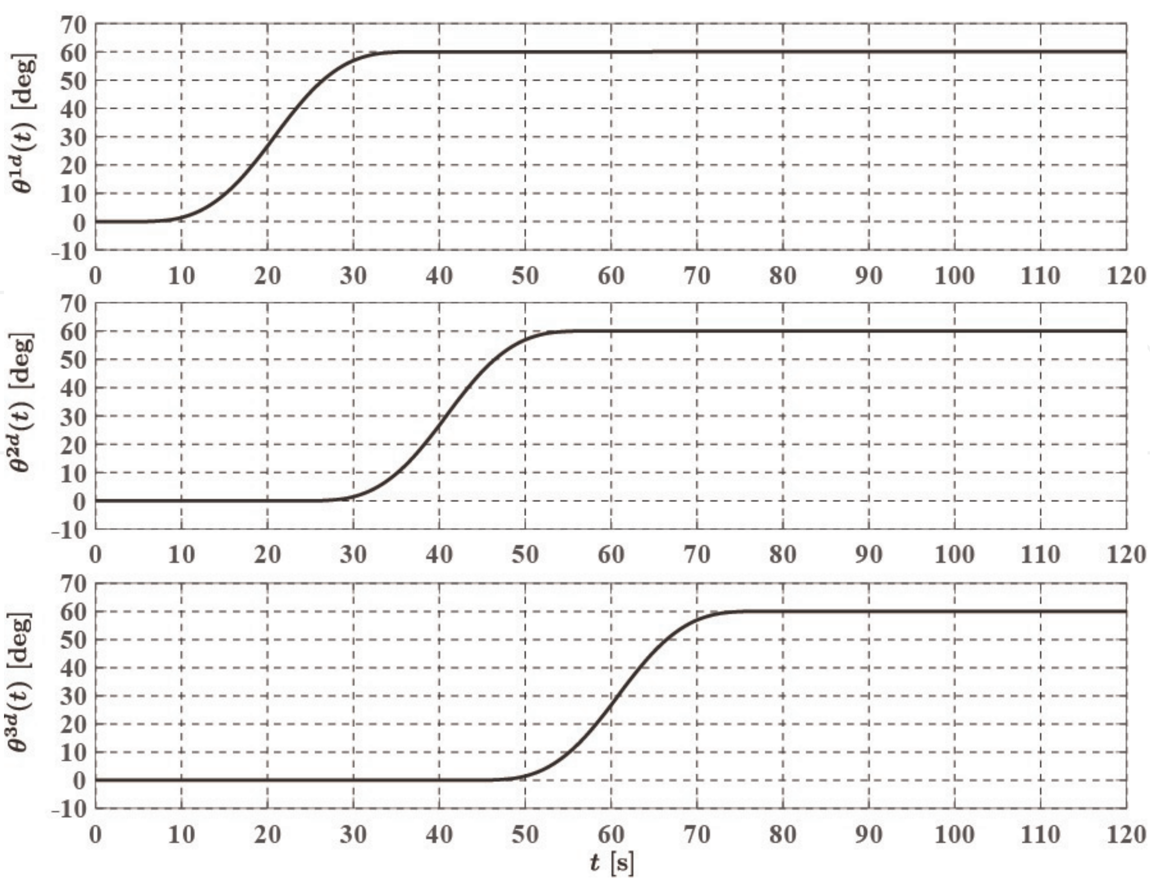

Figure 1.
Switching maneuver.

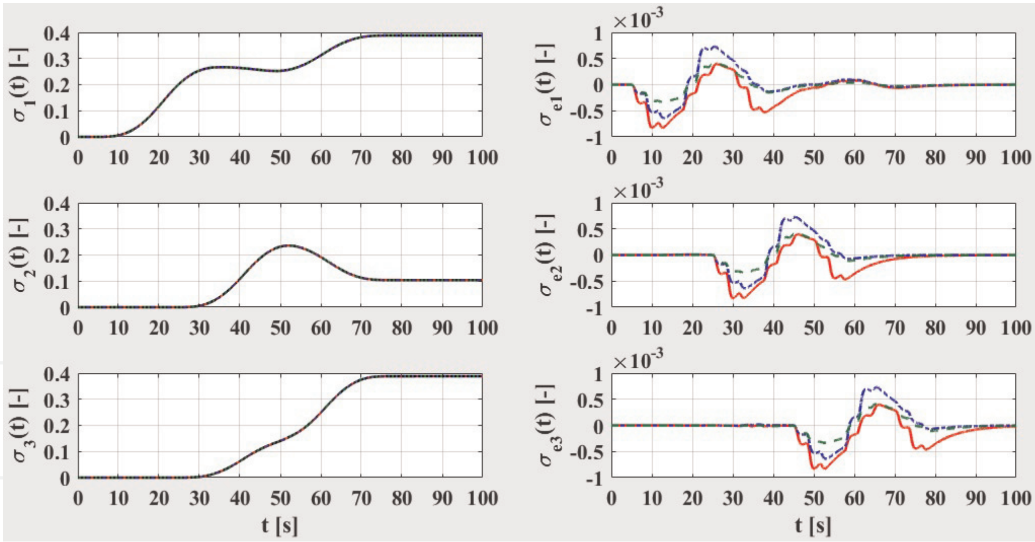

Figure 2.
Time histories of MRPs $\sigma(t)$ and $\sigma_e(t)$ (solid line, case 1; dashed-dotted line, case 2; dashed line, and case 3; dotted line, $\sigma_d(t)$).

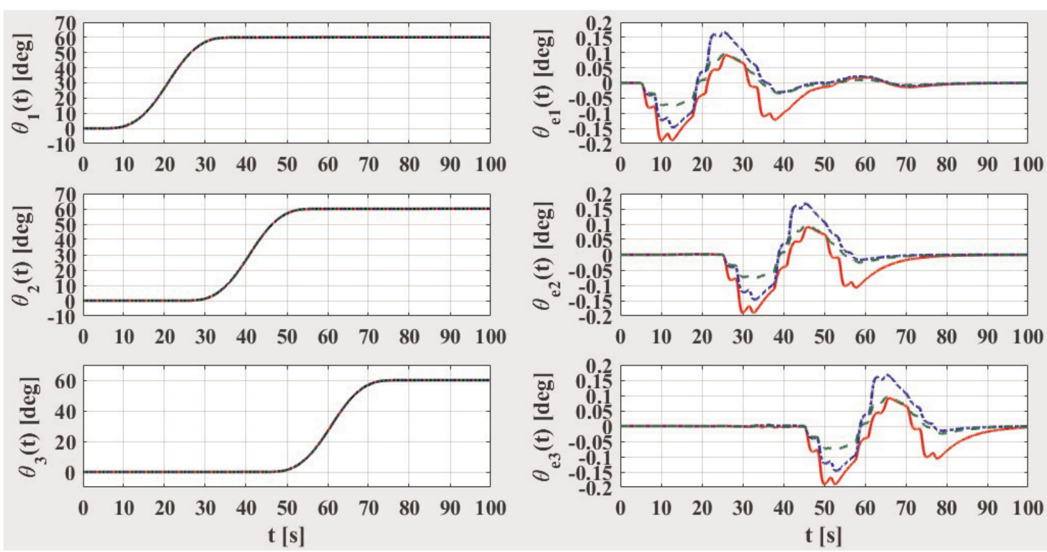

Figure 3.
Time histories of attitude angles $\theta(t)$ and $\theta_e(t)$ (solid line, case 1; dashed-dotted line, case 2; dashed line, and case 3; dotted line, $\theta_d(t)$).

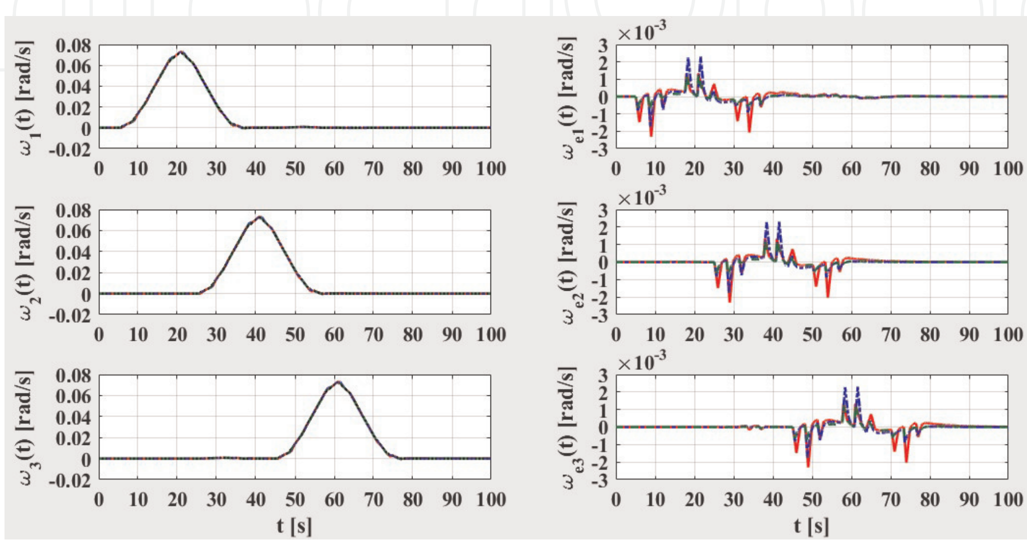

Figure 4.
Time histories of angular velocities $\omega(t)$ and $\omega_e(t)$ (solid line, case 1; dashed-dotted line, case 2; dashed line, and case 3; dotted line, $\omega_d(t)$).

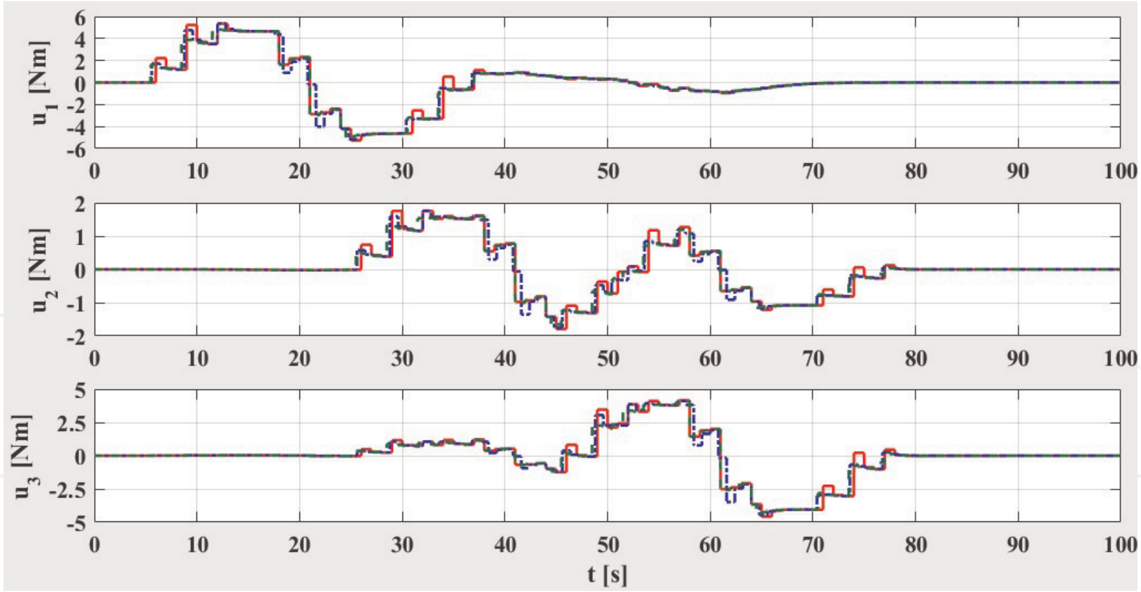

Figure 5.
Time histories of control input $u(t)$ (solid line, case 1; dashed-dotted line, case 2; and dashed line, case 3).

The results of the numerical simulation are shown in **Figures 2–5**. The relative attitude $\sigma_e(t)$ and relative angular velocity $\omega_e(t)$ converge to the neighborhood of $(\sigma_e(t), \omega_e(t)) = (0, 0)$, and the control input amplitude $u(t)$ does not depend on the sampling period T although there is a slight difference in the maximal value of $u(t)$.

5. Conclusion

This chapter considers the spacecraft attitude tracking control problem that requires agile and large-angle attitude maneuvers and proposed a discrete-time nonlinear attitude tracking control that the amplitude of the control input does not depend on the sampling period T. The effectiveness of proposed control method is verified by numerical simulations. Extension to the guarantee of stability as sampled-data control system will be subject to future work.

Appendix: sampled-data control of nonlinear system

This section shows preliminary results for nonlinear sampled-data control [13, 14, 19].

Let us consider the following nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)), x(0) = x_0, f(0, 0) = 0, \tag{A1}$$

where $x(t) \in \mathrm{R}^n$ is the state variable and $x(t) \in \mathrm{R}^m$ is the control input. The function $f(x(t), u(t))$ in Eq. (A1) is assumed to be such that, for each initial condition and each constant control input, there exists a unique solution defined on some intervals of $x[0, \tau)$.

The nonlinear system (A1) is assumed to be between a sampler (A/D converter) and zero-order hold (D/A converter), and the control signal is assumed to be piecewise constant, that is,

$$u(t) = u(kT) =: u(k), \forall t \in [kT, (k+1)T], k \in \{0\} \cup \mathrm{N}, \tag{A2}$$

where $T > 0$ is a sampling period. In addition, assume that the state variable

$$x(k) := x(kT) \tag{A3}$$

is measurable at each sampling instance. The exact discrete-time model and Euler approximate model of the nonlinear sampled-data systems (A1)–(A3) are expressed as follows, respectively:

$$x_{k+1} = x_k + \int_{kT}^{(k+1)T} f(x(s), u_k)\, ds =: F_T^e(x_k, u_k), \tag{A4}$$

$$x_{k+1} = x_k + Tf(x_k, u_k) =: F_T^{Euler}(x_k, u_k), \tag{A5}$$

where we abbreviate $x(k)$ and $u(k)$ to x_k and u_k. For the stability of the exact discrete-time model (A4) (F_T^e) and Euler approximate model (A5) (F_T^{Euler}), the following definitions are used [13, 14, 19].

Definition 1: consider the following discrete-time nonlinear system:

$$x_{k+1} = F_T(x_k, u_T(x_k)), \tag{A6}$$

where $x_k \in \mathrm{R}^n$ is the state variable and $u_T(x_k) \in \mathrm{R}^m$ is a control input. The family of controllers $u_T(x_k)$ SPA stabilizes the system (A6) if there exists a class KL function $\beta(\cdot)$ such that for any strictly positive real numbers (D, ν), there exists $T^* > 0$, and such that for all $T \in (0, T^*)$ and all initial state x_0 with $\|x_0\| \le D$, the solution of the system satisfies

$$\|x_k\| \le \beta(\|x_0\|, kT) + \nu, \ \forall k \in \{0\} \cup \mathrm{N}. \tag{A7}$$

Definition 2: let $\hat{T} > 0$ be given, and for each $T \in \left(0, \hat{T}\right)$, let functions $V_T : \mathrm{R}^n \to \mathrm{R}$ and $u_T : \mathrm{R}^n \to \mathrm{R}^m$ be defined. The pair of families (u_T, V_T) is a SPA stabilizing pair for the system (A7) if there exist a class K_∞ functions α_1, α_2, and α_3 such that for any pair of strictly positive real numbers (Δ, δ), there exists a triple of strictly positive real numbers $(T^*, L, M)\left(T^* \le \hat{T}\right)$ such that for all $x, z \in \mathrm{R}^n$ with $\max\{\|x\|, \|z\|\} \le \Delta$, and $T \in (0, T^*)$:

$$\alpha_1(\|x\|) \le V_T(x) \le \alpha_2(\|x\|), \tag{A8}$$

$$V_T(F_T(x, u_T(x))) - V_T(x) \le -\alpha_3(\|x\|) + T\delta, \tag{A9}$$

$$|V_T(x) - V_T(z)| \le L\|x - z\|, \tag{A10}$$

$$\|u_T(x)\| \le M. \tag{A11}$$

In addition, if there exists $T^{**} > 0$ such that Eqs. (A8)–(A11) with $\delta = 0$ hold for all $x \in \mathrm{R}^n$ and $T \in (0, T^{**})$, then the pair (u_T, V_T) is globally asymptotic (GA) stabilizing pair for the system (A6).

Using the above definitions, the following theorem is obtained by literatures [13, 14, 19].

Theorem A.1: if the pair (u_T, V_T) is SPA stabilizing for F_T^{Euler}, then u_T is SPA stabilizing for F_T^e .

Hence, if we can find a family of pairs of (u_T, V_T) that is a GA or SPA stabilizing pair for F_T^{Euler}, then the controller u_T will stabilize the exact model F_T^e.

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