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Chapter

Coherence Proprieties of Entangled Bi-Modal Field and Its Application in Holography and Communication

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Abstract

This chapter examines the coherence properties of two modes of entangled photons and its application quantum communication and holography. It is proposed novel two-photon entangled sources which take into account the coherence and collective phenomena between the photon belonging to two different modes obtained in two-photon cooperative emission or Raman or lasing. The generation of the correlated bimodal entangled field in two-photon emission or Raman Pump, Stokes and anti-Stokes modes is proposed in the free space and cavity-induced emission. The application two-photon and Raman bimodal coherent field in communication and holography are given in accordance with the definition of amplitude and phase of such entangled states of light. At first, this method does not appear to be essentially different in comparison with the classical coherent state of information processing, but if we send this information in dispersive media, which separates the anti-Stokes and Stokes photons from coherent entanglement fields, the information is drastically destroyed, due to the quantum distribution of photons in the big number modes may be realized in the situation in which the mean value of strength of bimodal field tends to zero. The possibility of restoration of the signal after the propagation of the bimodal field through different fibers, we may restore the common square amplitude and phase.

Keywords: quantum bimodal field, cooperative effects between blocks of photons, quantum communication, quantum holography

1. Introduction in the specific properties of correlated bimodal radiation field

Generated radiation in two- or multi-quantum processes opens new perspectives in studying new communication systems, holographic phase correlations, in the interaction of light with biomolecules and living systems. The specific attention is given to the new type of coherent emissions, which occurs not only between the quantum but between the photon groups generated in the non-linear interaction of the electromagnetic field (EMF) with emitters (atoms, molecules, biomolecules, etc.). This type of light generation supports the idea of coherent correlation that appears in the bi-modal field, in which it is generated the entangled photons.

A physical characteristic of field formed from the blocs of well-correlated bi-modes must be determined by the intensity of the electric field of each mode characteristic in such superposition. The applications of such a field characteristic can be fruitful both in quantum communication and holography. An attractive aspect of the problem consists in the selective two-quantum excitation of some atoms or molecules of the system, where it is necessary minimize the dipole active action of total photon flux over single-photon resonance of dipole-active transitions. The last idea can be applied in microbiology, where a selective dis-activation of some molecular structures (e.g. of viruses) in the tissue may become possible in two-quanta excitations. In this situation appears the necessity for a good description of both amplitude and phase of this new type of radiation formed from bimodal correlated photons.

The new concept of phase and amplitude correlations are important not only in interferometry but also in the holographic registration of information and are related to the conceptual aspects of physics, chemistry and microbiology for the recording of three and multi-dimensional images in cosmology [1–3]. According to the invention of Dennis Gabor [4] in 1947, the hologram is defined by the interference between two waves, the 'object wave' and the 'reference wave'. Like in laser experiments, this interference between the two waves requires to use the temporally and spatially consistent source, described by an intensity pattern, which represents the modulus squared of the sum of the two complex amplitudes. The reconstruction of the object field encoded within the hologram is based on the principle of light diffraction. This type of diffraction and interference can be keyed out by other coherent states, which can be an eigenstate of square parts of positive frequency strength of EMF. According to this description, the eigenvalue of vectors of square strength has the good amplitude and phase. For example, in the two photon cooperative emission by the pencil shape system of radiators (or by the cavity two-photon induced emission) the coherence is based between the photon pairs rather than between the individual photons. This effect is evident, when the pairs of photons are generated in the broadband spectral region of the EMF so, that the total energy of two photons in each pair is constant $2\hbar\omega_0 = \hbar\omega_{k1} + \hbar\omega_{k2} = Const.$ Considering that the frequencies of the photons in the pairs are aleatory distributed, $\omega_{ki} \neq \omega_{kj}$, we conclude that such systems generates the higher the second order coherence relative to first order one. In this context appear the problem of the application of such field in communication and holography, using its good amplitude and phase of squared strength, generated by the nominated sources. This chapter discusses the problem associated with the possibilities to divide the wave front of the photon-pairs into two wave fronts. Studying the interference between each part, "object bimodal waves" $\hat{E}^+(t+\tau)\hat{E}^+(t+\tau)$ and "reference bimodal waves" $\hat{E}^+(t)\hat{E}^+(t)$, we may create the hologram image consisted of the interference and diffraction fringes between the bi-photons belonging to wave fronts of square vectors of field consisted from the ensemble of bi-modal field, $\langle\hat{E}^+(t+\tau)\hat{E}^+(t+\tau)\hat{E}^-(t)\hat{E}^-(t)\rangle$.

To understood this type of coherence let us look at the light that consists of distinctive photons, which belong to broadband spectrum energy. Since the number of modes is relatively large, it is virtually impossible to find the two photons in the same mode and to create the coherent states from them, $\langle\hat{E}_k^-(t,z)\hat{E}_{k'}^+(t,z)\rangle \approx 0$, where $\hat{E}_k^-(t,z)$ and $\hat{E}_{k'}^+(t,z)$ is the Fourier transform the negative or positive defined EMF strength components of the radiation modes $k \neq k'$ obtained from the inverted atomic ensemble in z direction. Of course, the total intensity of such a light, obtained from individual sources (nuclei, atoms, molecules) becomes proportional to

the number of atomic sources, $I = \sum_j \langle \hat{\mathcal{E}}_j^-(t,z) \hat{\mathcal{E}}_j^+(t,z) \rangle \propto N$, because the correlation function between the strengths of different radiators is $\langle \hat{\mathcal{E}}_j^-(t,z) \hat{\mathcal{E}}_l^+(tz) \rangle|_{l \neq j} \approx 0$. Here $\hat{\mathcal{E}}_j^-(t,z)$ $(\hat{\mathcal{E}}_j^+(t,z))$ is the negative (positive) frequency component of the EMF through of each atom.

The creation of entangled photons in two- and multi-quantum processes opens the new possibilities in quantum communication and quantum holography. For example in the paper of prof. Teich et al. [5] it is proposed to make use of quantum entanglement for extracting holographic information about a remote $3 - D$ object in a confined space which light enters, but from which it cannot escape. Light scattered from the object is detected in this confined space entirely without the benefit of spatial resolution. Quantum holography offers this possibility by virtue of the fourth-order quantum coherence inherent in entangled beams. This new conception is based on the application of second ordered coherence function, proposed firstly by Glauber [6], and intensively developed in the last years. The possibility to use of the fourth-order quantum coherence of entangled beams was studied in Ref. [7]. Here it is proposed a two-photon analog of classical holography. Not so far the authors of Refs. [8, 9] used an innovative equipment registered the behavior of pairs of distinguishable and non-distinguishable photons entering a beam splitter. When the photons are distinguishable, their behavior at the beam splitter is random: one or both photons can be transmitted or reflected. Non-distinguishable photons exhibit quantum interference, which alters their behavior: they join into pairs and are always transmitted or reflected together. This is known as two-photon interference or the Hong-Ou-Mandel effect. The visibility of the hologram of a single photon fringe, V, is defined by a spectral mode overlap which can be high and stable for photons generated by different sources such as two independent spontaneous parametric down-conversion. As the authors mentioned, in the registration of single photon hologram [8], the quantum interference can be observed by registering pairs of photons. The experiment needs to be repeated several times using the two photon pairs with identical properties.

The authors of the Refs. [10-13] have proposed to investigate the coherence which appears between undistinguished photon pairs and the possibility to generate such a pair in the two-photon quantum generators. The increased interest not only to two-photon generation, but to induce Raman microscopy in special medicine and biology opens the new perspective the coherent proprieties of bimodal fields. Compared to spontaneous Raman scattering, coherent Raman scattering techniques can produce much stronger vibrational sensitive signals. This excitation needs a strong phase correlation between the pump, Stokes, and anti-Stokes components of the induced Raman process. These difficulties have been overcome by recent advances in coherent Raman scattering microscopy, which is based on either coherent anti-Stokes Raman scattering or stimulated Raman scattering [14, 15]. Appear a possibility to use this type of coherent states of bimodal field [12, 13], and to propose a new studies of vibrational aspects of molecules.

Following this idea let us discuss another effect related to the photon scattering processes into the pump, Stokes and anti-Stokes modes. Taking into consideration that in the Λ-type three-level system persists only pumping and Stokes modes when the atomic system is prepared in the ground state, or it may be reduced to the pump and anti-Stokes modes, when the atomic system is prepared in the excited state, we could reduce this cooperative scattering effect to the ensemble of correlated pairs of modes in the resonator. This is possible due to big detuning between the third level and pump modes when the system of atoms is prepared in the ground state. In this situation, it is possible to generate m pairs of correlated mods between

the pump and Stokes components, so that the frequency difference between the modes of each pair, $\omega_{kp_1} - \omega_{ks_1}$, $\omega_{ks_2} - \omega_{ks_2}$, $\omega_{kp_j} - \omega_{ksj}$, $\omega_{kp_m} - \omega_{ks_m}$ is equal to the transition energy between the ground and excited levels $2\omega_0$. In this description, the pump, and Stokes fields are considered to be incoherent due to the fact that the correlation functions between the pump and Stokes modes, belonging to different partitions gives zero contributions in the intensity correlation function $\langle \hat{E}_{k_{pj}}^{(-)}(t) \hat{E}_{k_{pl}}^{(+)}(t) \rangle_{j \neq l}$ and $\langle \hat{E}_{k_{sj}}^{(-)}(t) \hat{E}_{k_{sl}}^{(+)}(t) \rangle_{j \neq l}$. According to this only the diagonal elements belonging to the same modes remain non-zero so that the field intensity is proportional to this number "m". The resulting intensity of the pumping light is equal to the square strength of each pumping (or Stokes) modes $I_p = \sum_j \langle \hat{E}_{k_{pj}}^{(-)}(t) \hat{E}_{k_{pj}}^{(+)}(t) \rangle \sim m$ (or $I_s = \sum_j \langle \hat{E}_{k_{sj}}^{(-)}(t) \hat{E}_{k_{sj}}^{(+)}(t) \rangle \sim m$). The realization of such cooperative effects between the incoherent bi-modes can be obtained exactly as in the case of the two-photon generation in a wide spectrum at Raman emission (e.g., in the multimodal cavity or crossing the pumping pulse through a multimodal optical fiber). The total pumping field strength and Stokes is a multi-mode superposition for both pumping and Stokes mods, where the pumping field and Stokes are decomposed into the quantized states of the optical cavity.

The possibilities of correlations between the anti-Stokes, Stokes and pump modes have been overcome by recent advances in coherent Raman scattering microscopy, which is based on either coherent coherent anti-Stokes Raman scattering or coherent Raman scattering [14, 15]. In many cases, the phase correlations between these components become not so simple in the experimental realization. Appear the possibility to apply here the coherent states of bimodal field proposed in this chapter and a possibility to use holographic aspects of such bimodal field in biology and medicine where the phase and amplitude of Raman component are already correlated for coherent excitation of molecular vibrations $\hat{\Pi}^-(t) = \sum_j \hat{E}_{k_{sj}}^{(-)}(t) \hat{E}_{k_{pj}}^{(+)}(t)$. Here we notice that this new characteristic of the field in induced Raman process may have a good phase and amplitude as the traditional coherent field, $\Pi^-(t) = \Pi_0 \exp[-i\phi]$, the correlatives between the adjacent modes is proportional to the square number of adjacent bi-mods $\langle \hat{\Pi}^-(t) \hat{\Pi}^+(t) \rangle \sim m^2$. Phase $\phi = 2\omega_0 t - Kz$ contains a fixed frequency $2\omega_0 = \omega_{pki} - \omega_{ski}$ (here $i = 1, 2, ...m$) and a well-defined wave vector, $K = k_{pi} - k_{si}$ in the collinear cavity conversion of the photons from the pump field.

In the Section 2 we give the definition of bimodal coherent states in analogy with single photon coherent states. The definition of phase and amplitude of this bimodal field is also granted, taking into account the coherent states of bimodal superposition of entangled photon pairs and bimodal superposition of Stokes pump and anti-Stokes modes in the Raman scattering process. The lithographic proprieties of such bimodal field are given, taking into consideration multi-mode aspects of generation light.

The Section 3 is devoted to applications of coherent emission of two subgroups of photons, the total (or difference) energies of which can be reckoned as a constant, so that coherence appear between the vectors formed from the product of two electromagnetic field strengths. As it is shown that the coherence between such vectors is manifested if the emitted bi-photons belonging to broadband spectrum, hence that the coherence between individual photons can be neglected. The application of product strength amplitudes and phases in holographic registration is advised. The superposition of two vectors of bimodal field obtained in two-photon or Raman lasing effect is estimated for construction of the holographic image of the object.

2. Generation of biharmonic strength operators and their coherent proprieties

Let us consider two nonlinear processes of light generation in laser [16, 13] and collective decay phenomena [17, 18]. In the second order of interaction of light with matter, these processes strongly connect the quantum fluctuations of two waves. In the output detection region, this effect gives us the possibility to obtain the coherent effects between the bimodal fields as this is proposed in Section 1. In this nonlinear generation of light, the new signal at another frequency has a common coherent phase with impute mode in the nonlinear medium. We discuss the situation when the phases of the emitted waves are random relative to one another so that the total field average of EMF strength takes zero value $\langle \hat{E}(x,t) \rangle = 0$. In such a situation the emission is considered non-coherent. An opposite conception appears in quantum optics in which it is proposed a lot of effects connected with quantum entanglement and coherent proprieties of bimodal field [19]. Here as we discuss in Section 1 we have the possibility to introduce another type of coherence [12, 19, 20], which appear not between the photons of the same mode, but between the biphotons from the ensemble of pairs of modes or correlations of photons belonging to scattered modes (bi-modes) $\langle \hat{E}_r \hat{E}_s \rangle \neq 0$. When the number of bi-modes with the same energy of the photons in the pair increase [21, 22], the quantum fluctuations in each mode may achieve zero value $\langle \hat{E}_r \rangle = \langle \hat{E}_s \rangle = 0$. Following this conception, the similar coherence between the photons we introduced in the Raman emission processes [12, 13, 23]. In this case one photon from non-coherent driving field is absorbed (Stokes photon a_s) and other photon is generated (anti-Stokes b_a^+) $\langle \hat{E}_r \hat{E}_a \rangle \neq 0$, $\langle \hat{E}_r \hat{E}_s \rangle \neq 0$. Below we consider below two situations.

Case A. correspond to the generation of the coherent bi-photons along the axes of the pencil shape system of an inverted atomic system relative a dipole forbidden transition [19] together with two dipole active subsystems of radiators, S and R (see Ref. [23]). Here we propose another effect, in which the two-quantum cooperative emissions are ignited by single-photon decay process. As one-photon decay process of an exciting ensemble of atoms passes into Dicke super-fluorescence [24], we propose the situation in which this effect can be inhibited and the new cooperative interaction of this ensemble with dipole forbidden transitions of other atomic subsystems will stimulate another cooperative decay process, in which the coherence is established between the photon pairs. Indeed, if we consider an ensemble of excited atoms with non-equidistant transition energy, we may observe that in this situation the phase correlations between the atoms may be neglected. The non-equidistant dipole-active ensemble may be divided into two sub-ensembles of excited atoms, so that the pair of excited radiators from each sub-ensemble enters in resonance dipole-forbidden transition $(n + 1)S - nS$ of D of sub-ensemble D. In other words, we are interested in cooperative interaction between two dipole-active sub-ensembles and dipole-forbidden one as this is represented in **Figure 1**. Such super-radiance has the coherence between the photon-pairs and the coherence between the individual photons (first-order coherence) becomes inhibited.

Let us first discuss the three particle cooperative effects represented in **Figure 1** described in Refs. [17–19]. This interaction is focused on a new type of three particle collective spontaneous emission, in which the decay rate of three atomic subsystems is proportional to the product of the numbers of atoms in each subsystem, $N_s N_r N_d$ in the case all three sub-ensemble are equidistant. The quantum master equations take into consideration the correlations between three subsystems S, R and D in the single and two-photons cooperative exchanges between the atoms of each
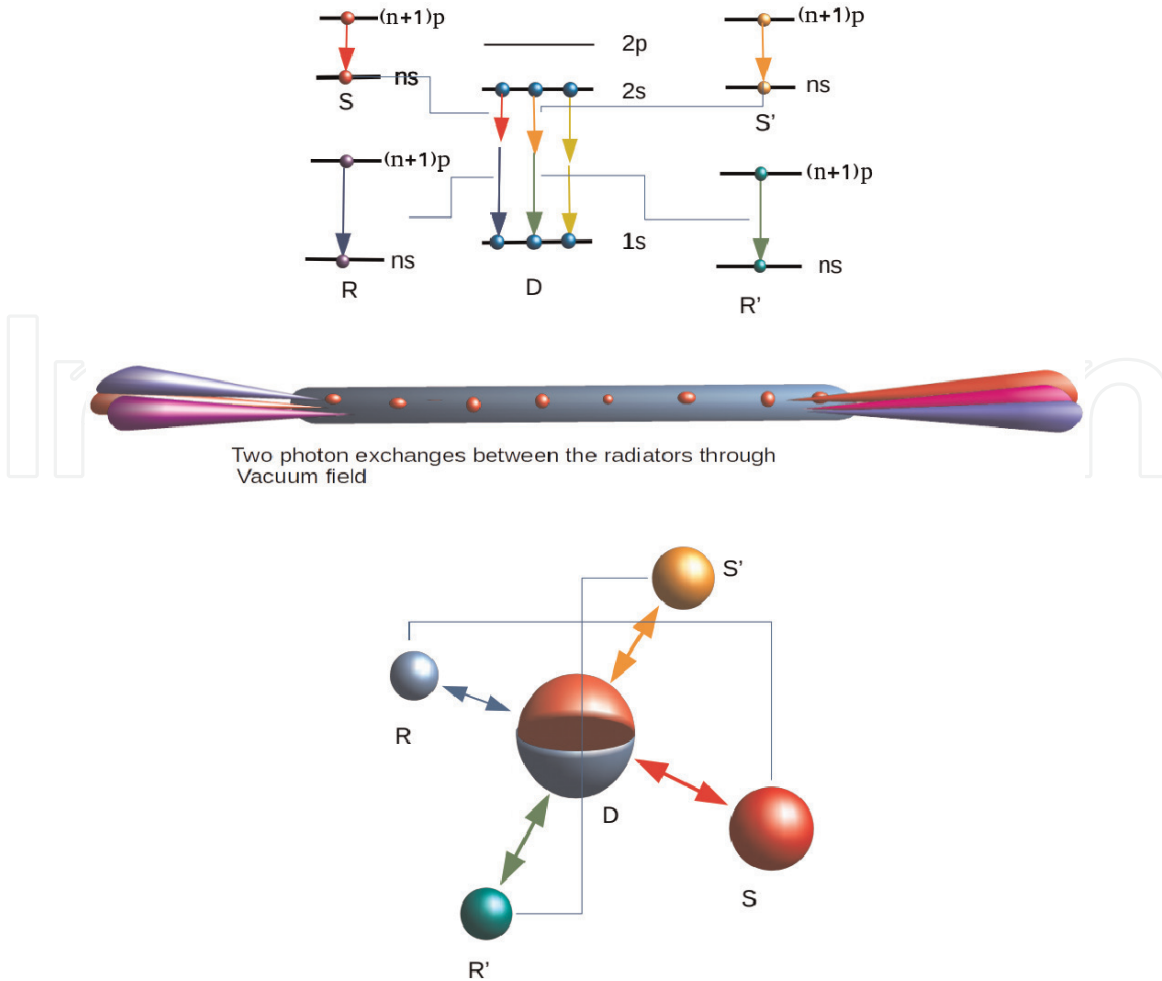
Figure 1.
The stimulation of two-photon cooperative effects of hydrogen-like (or helium-like) atomic transition $1S - 2S$ of D ensemble by dipole-active emission of S_j and R_j sub-ensembles in two-photon resonance described by the correlations (1a) and (1b). This two-photon cooperative decay can be prepared in pencil shape extended system of S, R and D radiators [19].

sub-ensemble (see Refs. [17–19]). The three-particles cooperative interaction through the vacuum of EMF is established taking into consideration the mutual influences between the single-photon polarization of S and R atomic subsystems and the non-linear polarization of the D atom (see Refs. [17–19]). To understand this effect it is necessary to examine the new correlation function which appears between the polarization of three different radiators from S, R and D subsystems: $\langle \hat{D}_n(t)\hat{R}_j^-(t)\hat{S}_l^-(t)\rangle$ and $\langle \hat{R}_l(t)\hat{S}_j(t)\hat{D}_m^-(t)\rangle$. The ignition role of S and R atoms is observed in the third-order terms, which contain two-photon resonances between three-particles represented in **Figure 1**, described by the above correlation functions. In order to experimentally observe these correlations, we must maximally destroy the single photon Dicke superradiance between atoms of S and R subsystems. This is possible if we choose the broadband sub-ensemble of excited dipole active atoms S and R. In this case, the single photon correlations between the atoms of this type become oscillatory with detuning frequency $\omega_{jl}^s(r)$ for j and l atoms. This corresponds to the situation when the correlations like $\langle \hat{S}_j(t)\hat{S}_l^-(t)\rangle$ (or $\langle \hat{R}_j(t)\hat{R}_l^-(t)\rangle$) become proportional to a $\exp[i\omega_{jl}^s(r)t]$ and rapidly oscillates during the cooperative decay process. But in this case in the sub-ensemble S, R we must have the big number of pairs S_j, R_j, $j = 1, 2, ..., N_p$ so that the established phase of each pair $\hat{S}_j^+$ and $\hat{R}_j^+$ will compensate the phase of $D_n^-(t)$ atom from equidistant D ensemble so that above defined three-particle correlators becomes smooth functions, giving the

maximal contribution in two-photon decay of the ensemble. But in this case, the sums of the three-particle correlators on the indexes: j, l and m, containing non-equidistant two-level atoms, S, and R, becomes proportional to the number of pairs of this type of atoms and number of D atoms, $N_p N_d$.

In this situation is respect all resonance conditions between the pairs and equidistant D- ensemble: $2\omega_0 = \omega_{r_j} + \omega_{s_l}$. This resonance between the two-photon transitions of D atomic subsystem and the pairs of the two dipole active atoms of R and S subsystems is represented in **Figure 1**. We can extend our attention to a big ensemble of three particles in such a cooperative process. Three atoms D, R and S are situated at relative small distances r_{ds}, r_{dr} and r_{rs} in comparison with emission wavelength. Such atomic may file up the volume with a dimension larger than the emission wavelengths. The exchange energies between the subsystems were analyzed in the literature and an attractive problem is connected with pencil shape atomic mixture described above. In this situation, the radiation can be observed along the pencil-shape atomic system (see **Figure 1**).

Following this conception, we observe that for the big ensemble of radiators the first order correlation function $G_I(t, t + \tau) = \langle \hat{E}^{(-)}(t) \hat{E}^{(+)}(t + \tau) \rangle$ becomes smaller than second order one. The second order correlation functions between the photons can be divided into two parts: $G_{II}^b(t, t + \tau) = \langle \hat{E}^{(-)}(t) \hat{E}^{(-)}(t) \hat{E}^{(+)}(t + \tau) \hat{E}^{(+)}(t + \tau) \rangle$ and $G_{II}^s(t, t + \tau) = \langle \hat{E}^{(+)}(t) \hat{E}^{(-)}(t) \hat{E}^{(+)}(t + \tau) \hat{E}^{(-)}(t + \tau) \rangle$. The first part describes the correlation between the photon pairs generated into the broadband interval and the second part describes the correlation between the bi-modes of scattered field. Here the positive and negative parts of the field strength $\hat{E}^{(+)}(t) = \sum_k g_k \hat{a}_k(t) \exp[i(\mathbf{k}, \mathbf{r})]$ and $\hat{E}^{(-)}(t) = \sum_k g_k \hat{a}_k^\dagger(t) \exp[-i(\mathbf{k}, \mathbf{r})]$ are expressed through the superposition of the annihilation $\hat{a}_k(t)$ and generation $\hat{a}_k^\dagger(t)$ field operators with wave vector k and polarization λ respectively. Using the method of elimination operator developed in Ref. [19], we demonstrated, that the second order correlation function $G_{II}^b(t, t + \tau)$ between the bi-photons is proportional not only to the correlation function between the atoms of ensemble D but consists from the sum of two types of correlations, which contain the intrinsic correlation of D ensemble like in the Dicke process [24] and correlations between the D ensemble and dipole active sub-ensemble R and S $G_{II}^b(t, t + \tau) = G_{II}^{bi}(t, t + \tau) + G_{II}^{b(sr)}(t, t + \tau)$ according to the Refs. [17, 18] the correlation between the atoms of D ensemble is proportional to the function

$$G_{II}^{b(rs)}(t, t + \tau) \sim \int_0^{k_0} dk(k - k_0)^2 k^2 \sum_{j, n} \frac{\sin k r_{jn}}{k r_{jn}} \frac{\sin (k_0 - k) r_{jn}}{(k_0 - k) r_{jn}} \quad (1a)$$

$$\times \left\langle \hat{D}_j^+(t) \hat{D}_n^-(t + \tau) \right\rangle,$$

and two photon cooperative ignition by the S ad R subensamble

$$G_{II}^{b(rs)}(t, t + \tau) \sim \left\langle \hat{R}_l^+(t) \hat{S}_j^+(t) \hat{D}_n^-(t + \tau) \right\rangle$$
$$\times \exp[i(\mathbf{k}_1 + \mathbf{k}, \mathbf{r}_n)] \exp[-i(\mathbf{k}', \mathbf{r}_j)] \exp[-i(\mathbf{k}', \mathbf{r}_l)]. \quad (1b)$$

Here we consider the sums on the repeated indexes. It is observed, that such a sum is proportional to the number dipole-active pairs N_p and number of D radiators. In the degenerate case when all three sub-ensemble are equidistant the number of term increase in the system [19], but in the system substantially increase first

order coherence between the same photons. In other words, the single photon process substantially ignites the generation of coherent photon pairs.

In the second order of interaction of light with matter, these processes strongly connect two waves in the output detection schemes and they give us the possibility to distinguish the coherent effects between entangled photons. For traditional single-mode coherence, it is well known the possible lithographic limits in measurements $\Delta \geq \lambda/2$. Taking into account the concept about the dropped lithographic limit in two-quantum coherent processes, the authors of Ref. [20] proposed new lithographic limit in the two-photon processes with a magnitude two times smaller than traditional $\Delta \geq \lambda/4$. This take place when frequencies of the signal and idler photons have the same value $\omega_s = \omega_i$. This propriety is also contained and in two-photon super-radiance [21] but here $\lambda = 2\lambda_{si}\lambda_{ri}/(\lambda_{si} + \lambda_{ri})$. Here λ_{ri} and λ_{si} are the emitted wavelengths by S ad R from the pair i, $i = 1, 2, ..N_p$. An interesting effect of two photon cooperative emission is possible in micro-cavities. In this case the mode structure of the cavity stimulates the two-photon decay effect in comparison with cascade effect [19, 25, 26] (see **Figure 2**).

The coherent properties and entanglement between the photons, emitted in two-quantum lasers and parametric down conversion has a great impact on application in quantum information and communication. The possibility of induced two-photon generation per atomic transition was suggested by Sorokin, Braslau and Prohorov [27, 28]. The scattering effects in two-photon amplifier attenuate the possibility to realize two-photon lasing. The first experiments demonstrated that two-photon amplification and lasing in the presence of external sources are possible [16, 29]. These ideas open the new conception about the coherence. Indeed, introducing the amplitude of two-quantum field encapsulated in two-photon lasers we can observe that the generation amplitude is described by the field product

$$\hat{P}^{(+)}(t,z) = \hat{E}_s^+(z,t)\hat{E}_r^+(z,t)$$
$$= G(k_s, k_r)\hat{a}_s\hat{a}_r \exp\left[2i\omega_0 t - i(k_s + k_r)z\right], \tag{2}$$

where $2\omega_0 = \omega_s + \omega_r = \omega_{21}$ is the total frequency of generated photons, $2k_0 = k_s + k_r$. In this case we can introduce the following operators of bi-bosin field $\hat{I}^+ = \hat{a}_s^\dagger\hat{a}_r^\dagger$ and $\hat{I}^- = \hat{a}_s\hat{a}_r$; $\hat{I}_z = \left(\hat{a}_s^\dagger\hat{a}_s + \hat{a}_r^\dagger\hat{a}_r\right)/2$, which satisfy the commutation relations $[\hat{I}^+, \hat{I}^-] = -2\hat{I}_z$, $[\hat{I}_z, \hat{I}^\pm] = \pm\hat{I}^\pm$. Such a generation possibilities was proposed in Refs. [17, 18]. According to the representation of these operators, we may introduce the following coherent states for this field $|\mu\rangle = \exp(\mu\hat{I}^+)|j,j\rangle/1 - |\mu|^{2j}$, which belong to the $su(1,1)$ symmetry described in Refs. [11, 19]. Here μ is the coherent displacement of bi-photon oscillator. Following this conception, the function $P^{(+)}(t,z)$ has the same behavior as the electrical component of single photon laser. For example, the mean value of this function on the coherent state can be represented through the harmonic functions with given phase and amplitude

$$\left\langle \hat{P}^{(+)}(t,z) \right\rangle = P_0 \exp\left[2i\omega_0 t - i(k_s + k_r)z + \varphi\right], \tag{3}$$

where $P_0 = G(k_s, k_r)|\langle\hat{a}_s\hat{a}_r\rangle|$ is the amplitude and $\varphi = Arg\langle\hat{a}_s\hat{a}_r\rangle$ is the phase of electrical field strengths of two fields a and b . In the detection scheme represented in **Figure 2B** it is observe delay time through z-dependence of such functions. The lithographic limit follows from the difference between the maximum and minimum of two sit experiment represented in **Figure 2**. According to the expression (3) and the distinguish distance Δ between the slits follows, that the second
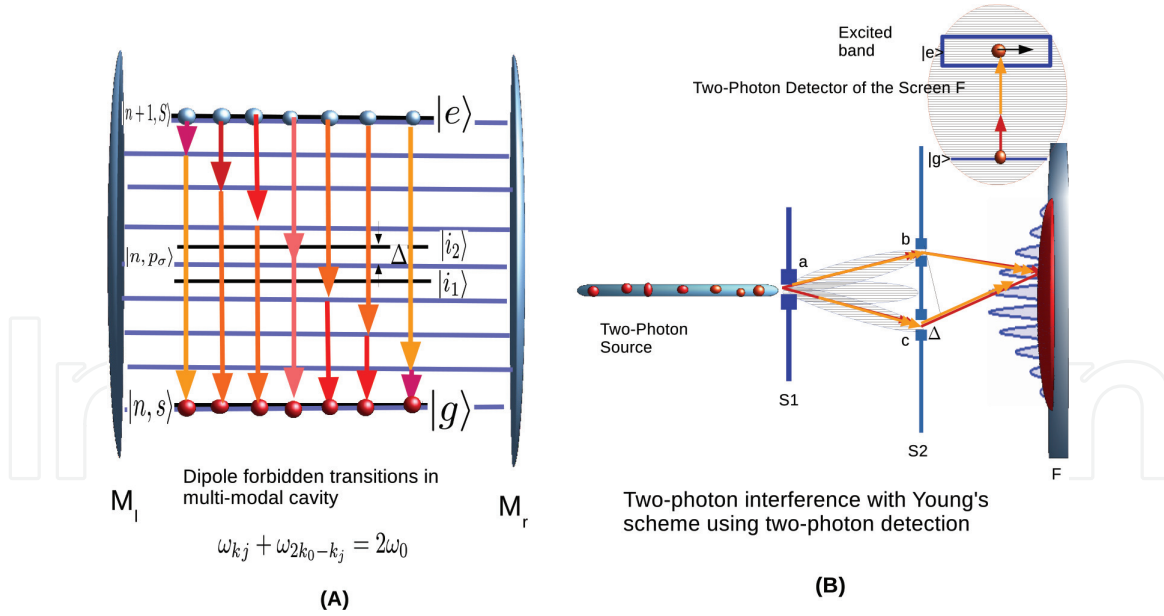
Figure 2.
(A) Sources of entangled photons in the two-photon bimodal processes. The horizontal blue lines represent the modes of the optical cavity modes formed between the vertical mirrors, M_l and M_r. (B) Two slits experiments with interference between pairs of modes which form the strengths product of EMF. The interference picture can be observed in tow-photon.

order correlation function $G_2(\Delta) = \langle \hat{P}^-(z)\hat{P}^+(z+\Delta)\rangle$ pass from maximal to minimal values for $\Delta(k_s + k_r)\sin\theta = \pi$. From this expression follows the some lithographic limit as in the Ref. [20].

To decorrelate the coherence between the photons of the same mode, in Ref. [10, 11] we proposed the cooperative multi-mode operators with similar commutation relations in the cavities. Mediating the amplitude of the bimodal fields we can introduce the collective modes field operators $\hat{I}^+ = \sum_{k \in (\omega_0, 0)} \hat{a}^\dagger_{2k_0-k}\hat{a}^\dagger_k$; $\hat{I}^- = \sum_{k \in (\omega_0, 0)} \hat{a}_{2k_0-k}\hat{a}_k$ and $\hat{I}_z = \sum_{k \in (\omega_0, 0)} (\hat{a}^\dagger_k\hat{a}_k + \hat{a}_{2k_0-k}\hat{a}^\dagger_{2k_0-k})/2$. As follows from above description the absolute value of conserved Casimir operator increase with increasing the number of bi-modes $\hat{I}^2 = (\hat{I}^z)^2 - 1/2(\hat{I}^+\hat{I}^- + \hat{I}^-\hat{I}^+)$. This effect is accompanied by the increasing of coherence between the bi-photons of each bi-modes relative the coherence which appears between the individual photons belonging to other modes. The similar coherent photon pairs may be generated in the broadband laser systems [11]. Following this idea the stationary solution of master equation for the bimodal cavity fled in the above multi-mode representation was obtained

$$\frac{\partial}{\partial t}\rho_m(t) = 2\kappa(m+1)(m+2j)\rho_{m+1}$$
$$- \frac{2(m+1)(m+2j)}{1+\beta(m+1)(m+2j)}\left(\alpha_1 + \alpha_2\frac{(m+1)(m+2j)}{1+\beta(m+1)(m+2j)}\right)\rho_m$$
$$+ \alpha_2\frac{2m^2(m+2j-1)^2}{[1+\beta m(m+2j-1)]^2}\rho_{m-1} - Ibid\{m \to m-1\},$$

where $\alpha_1 = \frac{2|g|^2 N\sigma_0\gamma}{(\omega-2\omega_0)^2+\gamma^2}$ represents the generation rate of photon pairs for full atomic inversion $N\sigma_0$, $\alpha_2 = \frac{T}{N\sigma_0}(\alpha_1^2 + \chi^2)$, $\beta = \frac{4|g|^2 T\gamma}{(\omega-2\omega_0)^2+\gamma^2}$, and $\chi = \frac{2|g|^2 N\sigma_0(\omega-2\omega_0)}{(\omega-2\omega_0)^2+\gamma^2}$. Here we used decomposition of density operator of bimodal field in the cavity $\rho(t) = \sum_{m=0}^\infty \rho_m|m,j\rangle\langle m,j|$. The evolution of correlation of the bi-photon intensity

$\langle \hat{I}^{+}(t)\hat{I}^{-}(t)\rangle$, and the sum of the photon correlation functions in each mode d $\langle : \hat{n}^2 : \rangle$ to lasing phase transition point for the following values of the j—collective parameter: $j = 0.5$ and $j = 10$. Here $\hat{n} = 2(\hat{I}_z - j)$. It is observed that with the increase of the number of the bi-modes the coherence between the bimodal field characteristic $\hat{P}^{-}(t+\tau,z)$ and $\hat{P}^{+}(t,z)$ increases: $\rangle\hat{P}^{-}(t+\tau,z)\hat{P}^{+}(t,z)$ $\langle \hat{I}^{+}(t)\hat{I}^{-}(t)\rangle \exp[2i\omega_0\tau - 2ik_0z]$. As it is observed from the behavior of parameter $\langle : n^2 : \rangle$, with increasing the number of modes, j, the coherence between the individual photons substantially decreases (see **Figure 3a** and **b**). This process of lasing stabilization is accompanied by the increasing the coherence between the photon pairs belonging to conjugate bi-modes and may be detected by the scheme represented in **Figure 2B**. The generation process of the coherent field in the some mode of the ensemble of the modes $2j$ is described by the sum of correlations $\sum_k\langle\hat{E}_k^{(-)}(t+\tau)\hat{E}_k^{(+)}(t)\rangle$, which become proportional to the sum of number of photons in each mode $\langle\hat{n}\rangle \exp[i\varphi(\tau)]$. As follows from **Figure 3** the amplitude of this function achieved the small all value with the increasing of the number of modes. In the single photon detection this correlations is described by the aleatory phase $\varphi(\tau)$ and may be represented by the smooth function (see "red" line) on the screen F of interference scheme **Figure 2B**.

Case B. Another possibility to create a coherent field for a big number of photons distributed in the broadband spectrum represents the bimodal spectrum of scattered photons. Indeed if we represent superposition between the photons obtained from A and S atoms as a combination $|\psi\rangle \sim |1\rangle_a|0\rangle_s + \exp i\phi|0\rangle_a|1\rangle_s/\sqrt{2}$ we may extrapolate such superposition for a big number of atoms from the dipole active sub-ensemble A and S belonging to $su(2)$ symmetry. Let us first discuss the three particle cooperative effects in the scattering interaction represented in **Figure 4** [17–19]. In the free space, such field may be generated with pencil shape process described by three ensembles of atoms D, S and A . This description is devoted to this a new type of three particle collective spontaneous emission, in which the decay rate of three atomic subsystems is proportional to the product of the numbers of atoms in each sub ensemble of equidistant atoms, $N_sN_rN_d$. In this situation only one possibility of resonance interaction between the dipole forbidden transition of D-Lambda atoms and ensemble of dipole active atoms S and A
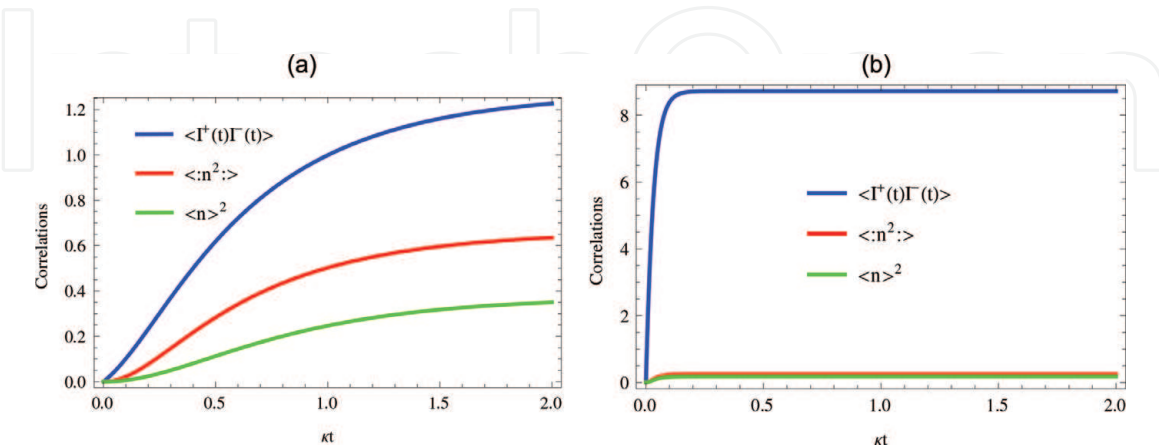

Figure 3.
The evolution of the photon correlations as function of the relative time $t\kappa$ to the phase transition for following parameters of the system: $\alpha_1/\kappa = 0.4$, $\alpha_1/\kappa = 0.01$ and $\beta/\kappa = 0.001$. Here it is represented: the square amplitudes of bimodal field $\langle\hat{I}^{+}\hat{I}^{-}\rangle$ (blue line), the correlation between the photons of each mode, $\langle : \hat{n}^2 : \rangle$ (red line) and the square of mean value of the photon number in each mode, $\langle\hat{n}\rangle^2$ (green line). Figure a corresponds to single mode two-photon emission, $j = 1/2$, and the numerical representation in figure b corresponds to the number of the bimodal cavity field $2j = 20$. As follows for the figures a and b the total photon correlations in each mode decreases with the increasing of the number of bi-modes (see the red and green lines).
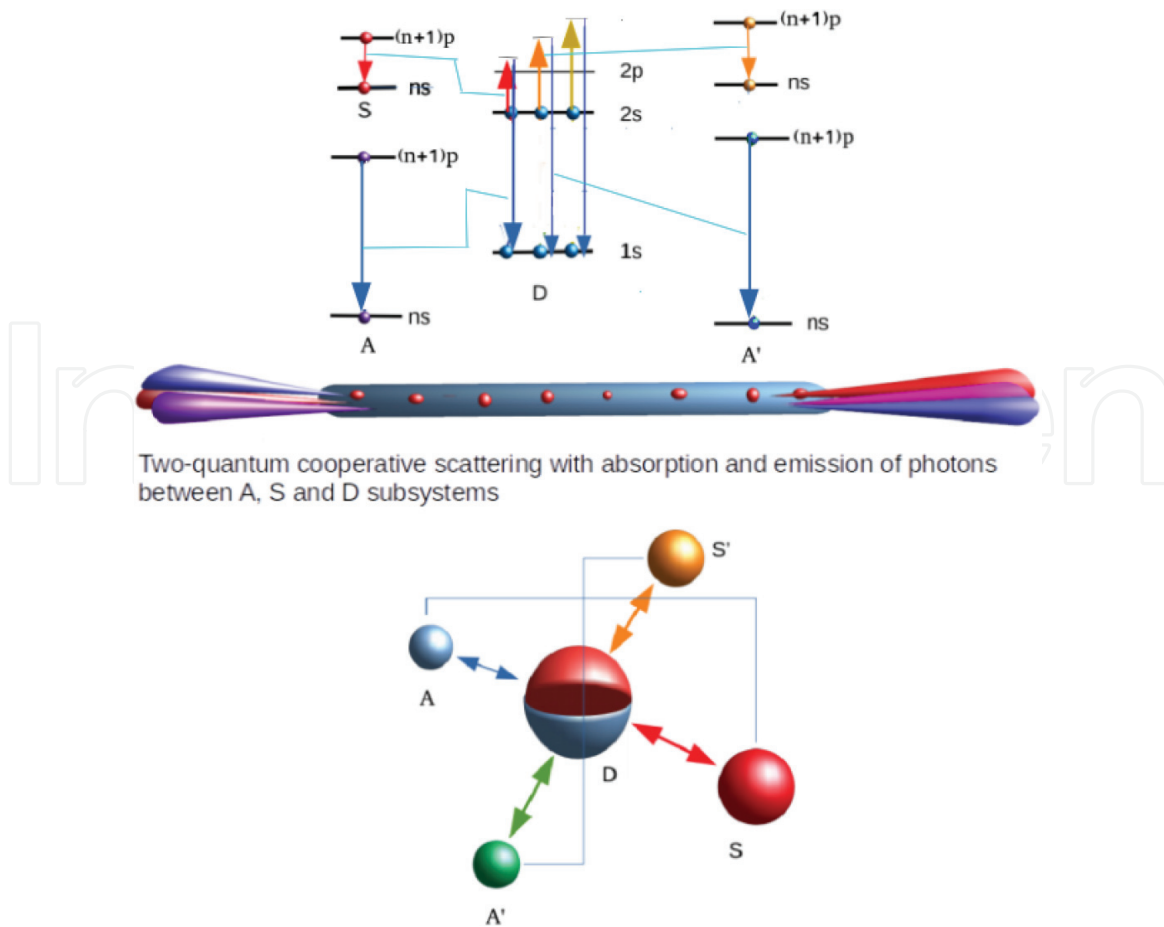
Figure 4.
The similar mutual transitions between dipole-active A_j and S_j atoms in scattering resonance with hydrogen-like (or helium like) ensemble of D atoms, described by the expression (4).

$2\omega_0 = \omega_a - \omega_s$, which correspond to scattering resonance between the three particles, respectively. This resonance situation for the decorrelated ensemble of dipole active atoms is represented in **Figure 4**.

We observe, that the Dicke cooperative effects between the sub-ensemble of atoms, S, and A can be neglected if the atoms in the sub-ensembles S and A are not equidistant relative their excited energy. In such a situation the Dicke cooperative effect in sub-systems of dipole-active atoms is negligible due to the consideration that the frequency width of broadband emission $\Delta\omega$ is large than the cooperative emission rate Γ_c. In this situation the cooperative correlations like $\langle \hat{A}_j^+(t)\hat{A}_l^-(t)\rangle$ and $\langle \hat{S}_j^+(t)\hat{S}_l^-(t)\rangle$ become proportional to the rapid oscillatory parts $\exp\left[i(\omega_{aj} - \omega_{al})t\right]$ and $\exp\left[i(\omega_{sj} - \omega_{sl})t\right]$, and vanishes after an average procedure on the time interval less than the decay time $1/\Gamma_c$. In such a situation, only the pairs of S, and A sub ensembles can excite the D-subsystem according to the third-order of perturbation decomposition [19]. It contains three particle scattering exchanges between the pairs of S, and A atoms and D-an ensemble of equidistant atomic represented by **Figure 4**. This exchange scheme of two S and A atomic pairs is described by the correlations between the pairs of A and S scattering resonance with D:
$\langle \hat{S}_j^+(t)\hat{D}_m^+(t)\hat{A}_l^-(t)\rangle$, $\langle \hat{A}_j^+(t)\hat{S}_l^-(t)\hat{D}_m^-(t)\rangle$ described by master equation in Ref. [23]. It corresponds to the scattering resonance between the pairs and D ensemble:
$\omega_{aj} - \omega_{sj} - 2\omega_0 = 0$. Here $i = 1, 2, N_p$, N_d is the number of atomic pairs of S and A sub-ensembles. In this situation, the second order coherence is also proportional to the product of two superposition of D-atoms and pairs of S_i and A_i atoms in the scattering resonance with D-equidistant ensemble as in the two-photon resonance

$$G_{II}^s(t, t+\tau) \sim \left\langle \hat{A}_l^+(t) \hat{S}_j^-(t+\tau) \hat{D}_n^-(t+\tau) \right\rangle$$
$$\times \exp\left[i(\mathbf{k}_1 + \mathbf{k}, \mathbf{r}_n)\right] \exp\left[-i(\mathbf{k}', \mathbf{r}_j)\right] \exp\left[-i(\mathbf{k}', \mathbf{r}_l)\right]. \tag{4}$$

As follows from the expression (4) and the scattering generation of correlation photons in the cavity **Figure 4** the scattered field into the blocs of two-modes ω_{ai}; ω_{si} and ω_{al}; ω_{sl} can form the coherent $su(2)$ state, which corresponds to the generators of the superposition of collective discrete bi-modes of EMF $\hat{J}^- = \sum_k \hat{a}_k^\dagger \hat{a}_{k+2k_0}$, $\hat{J}^+ = \sum_k \hat{a}_{k+2k_0}^\dagger \hat{a}_k$ and $\hat{J}_z = \sum_k \{\hat{a}_{k+2k_0}^\dagger \hat{a}_{k+2k_0} - \hat{a}_k^\dagger \hat{a}_k\}/2$ which satisfies the commutation relations for $su(2)$ algebra described in Section 2. In this cooperative effect, due to large number non-equidistant atoms in each ensemble A or S, the frequency differences between the scattered modes $\omega_{ai} - \omega_{si} = 2\omega_0$ and $\omega_{al} - \omega_{sl}$ have same wave vectors, $K_i = K_l$, where $\mathbf{K}_i = \mathbf{k}_{ai} - \mathbf{k}_{si}$ and $\mathbf{K}_l = \mathbf{k}_{al} - \mathbf{k}_{sl}$ and can be used in the coherent phenomena like holograms, or optical processing. In such coherence it is manifested the correlations between the ensemble of bi-modes generated by the pairs of atoms $\{S_l, A_l\}$, $l = 1, ... N_p$. These effects are accompanied with the interference between single- and two-quantum collective transitions of three inverted radiators from the ensemble. The three particle collective decay rate is defined in the description of the atomic correlation functions.

Let us study the interaction between the molecular systems and external Raman field prepared in the cooperative coherent process proposed in Refs. [12, 13, 30]. Following this Refs [19, 23], we can introduced the bimodal operators the product of which oscillates with the frequency $2\omega_0$ near the vibration frequency of the molecules (or bio-molecules) Ω

$$\hat{\Pi}^{(-)}(t, z) = \lambda \hat{E}_p^{(+)}(z, t) \hat{E}_a^{(-)}(z, t) + g \hat{E}_s^{(+)}(z, t) \hat{E}_p^{(-)}(z, t)$$

$$= G(k_p, k_a) \hat{b} \hat{a}^\dagger \exp\left[2i\omega_0 t - i(k_a - k_p)z\right] \tag{5}$$

$$+ G(k_s, k_p) \hat{s} \hat{b}^\dagger \exp\left[2i\omega_0 t - i(k_p - k_s)z\right].$$

Here the annihilation (creation) operators, $\hat{b}(\hat{b}^\dagger)$, $\hat{s}(\hat{s}^\dagger)$ and $\hat{a}(\hat{a}^\dagger)$, correspond to the pump, Stokes and anti-Stokes modes, respectively, which satisfy the Bose commutation rules: $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{j,i}$, and $[\hat{a}_i, \hat{a}_j] = 0, j \equiv a, b, s)$. The interaction Hamiltonian of molecules (bio-molecules) with bimodal field is described by the Hamiltonian $\hat{H}_I = -\hat{P}(t, z)\hat{\Pi}^-(t, z) + H.c.$, where the vector $\hat{P}$ is proportional to the displacement of the molecular oscillator $\hat{P}(t, z) \sim \hat{Q}(t, z) \sim |e\rangle\langle g| + |g\rangle\langle e|$. In the interaction with atomic sub-system (for example, four level system represented in **Figure 5**) in many situations $\lambda \sim \hbar/\Delta_a$ and $g \sim \hbar/\Delta_s$. From commutation of the bi-photon field operators between them $[g\hat{s}\hat{b}^\dagger + \lambda\hat{b}\hat{a}^\dagger, g\hat{b}\hat{s}^\dagger + \lambda\hat{a}\hat{b}^\dagger] = g^2(\hat{b}^\dagger\hat{b} - \hat{s}\hat{s}^\dagger + 1) + \lambda^2(\hat{a}^\dagger\hat{a} - \hat{b}\hat{b}^\dagger + 1)$, it is not difficult to observe that, when the interaction constant of atoms with Stokes and anti-Stokes modes coincide $g = \lambda$, the new operators, belonging to angular momentum $SU(2)$ symmetry, can be easily introduced: $\hat{L}_z = \hat{a}^\dagger\hat{a} - \hat{s}^\dagger\hat{s}$, $\hat{L}^- = \sqrt{2}(\hat{b}\hat{s}^\dagger + \hat{a}\hat{b}^\dagger)$, $\hat{L}^+ = \sqrt{2}(\hat{s}\hat{b}^\dagger + \hat{b}\hat{a}^\dagger)$. The similar commutation relation can be obtained in the case, when the relations $\lambda \gg g$ or $\lambda \ll g$ are satisfied. In the last two cases we may neglect the Stokes or anti-Stokes scattering process so, that the similar operators may be defined for this two special situations: (a) $\hat{J}^- = \hat{a}\hat{b}^\dagger$; $\hat{J}^+ = \hat{b}\hat{a}^\dagger$, $\hat{J}_z = (\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})/2$ for $\lambda \gg g$ and (b) $\hat{J}^- = \hat{b}\hat{s}^\dagger$, $\hat{J}^+ = \hat{s}\hat{b}^\dagger, \hat{J}_z = (\hat{b}^\dagger\hat{b} - \hat{s}^\dagger\hat{s})/2$ for $\lambda \ll g$. The commutation relations between these
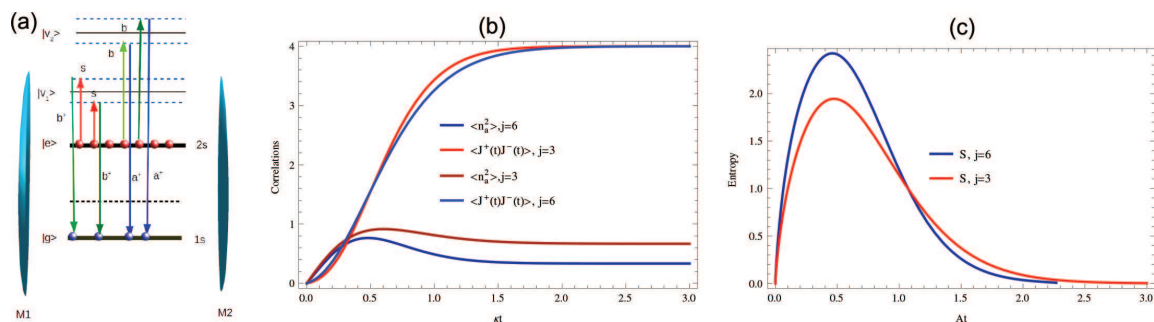
Figure 5.
(a) The Raman excitation of radiators (atoms, molecules) in the four-level energetic scheme with the conversion of the photons in stokes and anti-Stokes modes. Such atoms may fling through the cavity in **Figure 2A**. *(b) The time evolution of the relative correlations* $\langle \hat{J}^-(t)\hat{J}^-(t)\rangle/j^2$ *and* $\langle n^2 \rangle/j^2$ *for following parameters of the system* $\beta/A = 0.015$, $j = 3$ *and* $j = 6$. *(c) Evolution of von Neumann entropy for some parameters of the system. The substantial increase of the coherence between the bi-modal fields in comparison with coherence between the total numbers of photons belonging to each mode is observed. The entropy achieved minimal value in the lasing phase with increasing the number of uncorrelated modes in the pump and anti-Stokes field.*

operators are similar to the commutators of the collective atomic operators:
$[\hat{L}^+, \hat{L}^-] = 2\hat{L}_z$ and $[\hat{L}_z, \hat{L}^{\pm}] = \pm\hat{L}^{\pm}$. According to this description the new interaction Hamiltonian, $\hat{H}_I = \frac{\hbar g}{\sqrt{2}}\{|g\rangle\langle e|\hat{L}^{\dagger} + |e\rangle\langle g|\hat{L}^-\}$. Using the decomposition on the small operator of the system, $\hat{x}/(1+\hat{x})$, we may estimate the quantum fluctuations and the correlation functions as a function of evolution time and photon number in the applied field. Here $\hat{x} = 2\varepsilon\hat{L}^-\hat{L}^{\dagger}$ proportional to the small parameter $\varepsilon = g^2\gamma^{-2}$ described in Refs. [12, 31] ($\gamma^{-1} = \Lambda/v$ is main value of the flying time of atom, expressed through the atomic velocity, v, and cavity length, Λ). According to the projection operator method developed in Ref. [31] we start from the first order approximation of the master equation

$$\frac{d}{dt}\hat{W}(t) = -A\left[\hat{W}(t)\hat{L}^- \frac{1}{1+2\varepsilon\hat{L}^-\hat{L}^{\dagger}}, \hat{L}^{\dagger}\right] + H.c.,$$

where $A = Ng^2/2$ is the conversion rate, $\beta = 2A\varepsilon$ is the attenuation of the conversion rate, which increases with the increasing the mean value of the lifetime of the excited N-atoms flings through the cavity. The numerical solution of this equation is obtained, decomposing the density matrix on the angular momentum states, described by the eigenstates of the operator $\hat{L}^2$, $\hat{W} = \sum_{m=-j}^{j}P_m(t)|j,m\rangle\langle m,j|$. Here the Hilbert vectors $|j,m\rangle$ belong to the three mode states in the resonator (Pump, Stokes and anti-Stokes), $P_m(t)$ is the population probability of the $|j,m\rangle$ state. As in the two-photon emission (Case A) we are interested in the developing of the quantum between the photons belonging to scattered bi-modes. As a simple representation, we consider the situation when the non-correlated photons from the pump mode "b" is converted into the anti-Stokes mode "b". This process is possible for big detuning from resonance $\Delta_1 \gg \Delta_2$ (see **Figure 5a**). As follows from the interaction Hamiltonian in this situation the coherent function $G_2(\tau) = \langle \Pi^-(t,z)\Pi^+(t+\tau,z)\rangle$ becomes proportional to the expression $\langle \hat{J}^-(t)\hat{J}^-(t)\rangle \exp[i2\omega_0\tau]$. From **Figure 5** follows that in the process of conversion of the un-correlated pump photons into the anti-Stokes one $n_a = j + \langle \hat{J}_z(t)\rangle$ the cooperative phase of these two modes is established. The process achieved the saturation phase like in the single photon lasers. With the increasing the number of uncorrelated pump photons in broadband of the modes, this process is accompanied with the decreasing of relative coherence of the photons in each mode, so that the sum of total converted photons in anti-Stokes modes remain smaller than second order coherent function $G_2(0)$

(see **Figure 5**). The von Neumann entropy of the system, obtained from the representation $S_t = \sum_{m=-j}^{j} P_m \log[P_m]$ achieves the maximal value at the initial stage of conversion after that when it is established the coherence between the pump photons and converted one like in a similar way like in the super-radiance. After that, it decreases correspond to the established a new coherent phase described above.

Let us find the coherent phenomena which appears between two fields in Raman processes. If we study generation of Stokes light under the non-coherent pumping with anti-Stokes field, we can introduce the following representation of the bi-modal field

$$\hat{\Pi}^{-}(t,z) = \hat{E}_s^{(+)}(z,t)\hat{E}_a^{-}(z,t) = G(k_s,k_a)\hat{a}\hat{b}^{\dagger} \exp[2i\omega_0 t - i(k_a - k_s)z], \qquad (6)$$

where $\hat{E}_s^{(+)}(z,t)$ and $\hat{E}_a^{-}(z,t)$ are positive and negative defined strength of Stokes and anti-Stokes field (see **Figure 5a**), $\hat{a}_s^{\dagger}$, $\hat{b}_a^{\dagger}$ and $\hat{a}_s$, $\hat{b}_a$ are the annihilation and creation operators of electromagnetic field at Stokes, ω_s, and anti-Stokes, ω_a, frequencies respectively; $\omega_a - \omega_s = \omega_0$ is the fixed frequency of bi-modal field according to transition diagram represented in **Figure 5**. Following this definition one can introduced the new bi-quantum operators $\hat{J}^{-} = \hat{b}^{\dagger}\hat{a}$; $\hat{J}^{+} = \hat{a}^{\dagger}\hat{b}$; and $\hat{J}_z = (\hat{a}^{+}\hat{a} - \hat{b}^{\dagger}\hat{b})/2$. In this case for constant number of photons in resonator the conservation of Kasimir vector is possible, $J^2 = \hat{J}_z^2 + \hat{J}_x^2 + \hat{J}_y^2$, where $\hat{J}_x = (\hat{J}^{+} + \hat{J}^{-})/2$, $\hat{J}_y = (\hat{J}^{+} - \hat{J}^{-})/2i$. Considering that initially the photons are prepared in anti-Stokes mode of cavity $N = 2j$, one can describe the two photon scattering lasing processes by coherent state for this bi-boson field, belonging to $su(2)$ algebra.

$$|\alpha\rangle = \exp\left\{\alpha\hat{J}^{+}\right\}|-j,j\rangle\left\{1 + |\alpha|^2\right\}^{-j}, \qquad (7)$$

where $\alpha = \tan(\theta/2)$ is the amplitude of this bi-boson field obtained in the Raman lasing processes. Taking into account the coherent state (7) one can found the mean value of strength product $\langle\hat{\Pi}(t,z)\rangle = [\langle\hat{\Pi}^{-}(t,z)\rangle + \langle\hat{\Pi}^{+}(t,z)\rangle]/2$

$$\langle\hat{\Pi}(t,z)\rangle = \Pi_0 \cos[\omega_0 t - (k_a - k_s)z + \varphi], \qquad (8)$$

where $\Pi_0 = G(k_s,k_a)|\langle\hat{a}_s\hat{b}_a^{+}\rangle|$ and $\varphi = \arg\langle\hat{a}_s\hat{b}_a^{+}\rangle$ are the amplitude and phase of bimodal field formed from Stokes and anti-Stokes photons.

The lithographic limit between maximal and minimal values of amplitude of correlation function $G_2(\Delta) = \langle\Pi^{-}(z)\Pi^{+}(z + \Delta)\rangle$ in the two-slit experiments observed with two-photon detectors corresponds to the lithographic limit of this conjugate entangled bimodal field (see **Figure 6**). In this case, this limit is larger than in two-photon coherent emission $\Delta \geq \lambda_p \lambda_s / [2(\lambda_s - \lambda_p)]$. The frequency of this coherent field achieves the frequency of the vibration states of the molecule (bio-molecules) when the difference of wavelengths between the Stokes, pump, and anti-Stokes have the same magnitude. This coherent phenomenon between the Stokes and anti-Stokes fields can be used in Holographic representation of molecular vibrations and other coherent processes with phase memory. For holography, we propose the generation of new coherent states between Stokes pump and anti-Stokes field using nano-fiber systems [32]. In comparison with the cavity field, this type of generation permits to use the correlated bi modes out-site of generation
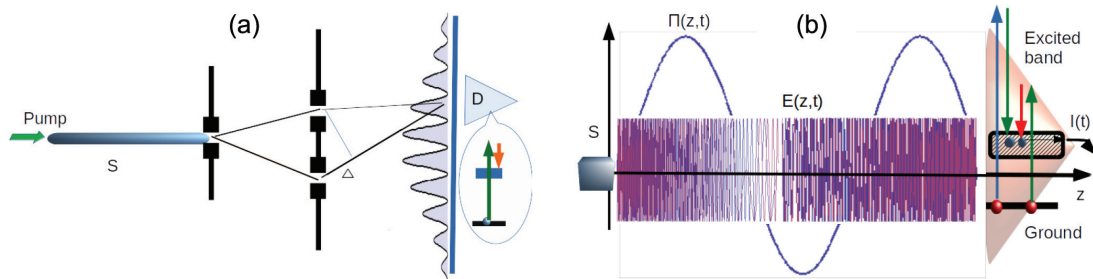
Figure 6.
*(a) The non-coherent pump of the bimodal field in the cavity (see **Figure 5a**) and scattering double slit interference on the frequency, $2\omega_0 = \omega_p - \omega_s$. (b) The time dependence of the mean value of vector $\langle \hat{\Pi}(z,t) \rangle$ (black line) and phase aliasing distribution in Raman components of the field $E(z,t)$. For the detection of the coherent properties (6)-(8) of $\langle \hat{\Pi}(z,t) \rangle$, it is possible to use a two-photon detection scheme.*

scheme represented in **Figure 9**. The atoms situated in the evanescent zone of nano-fiber stimulate the cooperative conversion of the photons from anti-Stokes pulse into the pump and Stokes pulses.

3. Quantum communication and holographic proprieties of bi-boson coherent field

The main differences between this bimodal field and the classical coherent field consists in the aleatory distribution of energies and phases between the photons of each pair, which enter in the coherent ensemble of bi-photons. Passing through the dispersion media's the common phrase of the ensemble may be drastically destroyed so, that the problem which appear consist in the restoration of common phase of the ensemble of photon pairs generated by the quantum sources. These phenomena of restoration of common phase of the ensemble have a quantum aspects and can be used in quantum communication and quantum holography.

In the case A we proposed the new possibilities in decreasing of coherent proprieties between the photon pairs of two-photon beam. The application of coherent effect of the bimodal field of communication and holography opens the new perspectives in the transmission of information not only through entangled state of photons but also through the second order coherence. At the first glance one observes that such coherent registration of information may have nothing to do with the traditional method. But looking to the scheme of **Figure 7** we observe that when the photon-pair pulses pass through a dispersive medium, the idler photons from the pair change their directions relative to signal photons. Focusing the signal and idler photons into different optical fibers, we are totally dropping the coherence among the photons. However, after a certain time interval, the idler and signal photons from the pairs could be mixed again (see **Figure 7**) and the coherence may be restored. The coherent state obtained in two-photon cooperative or laser emission takes into account not only entanglement between the pairs of photons, but the coherence between the bi-photons too, and can be used in mixed processing problems in which the quantum entanglement between the photon of each pair of photons is used simultaneously with classical coherence between the pairs.

Below we discuss how hologram can be constructed using the recording phase information of bimodal field on a medium sensitive to this phase, using two separate beams of bimodal field (one is the "usual" beam associated with the image to be recorded and the other is a known as the reference beam). Exploiting the interference pattern between these bi-boson fields described in the last section in principle this is possible. For example the Stokes and anti-Stokes fields can be regarded as a field with electromagnetic strength product (6), so that the common phases $\phi = 2\omega_0 t - k_0 z$ of
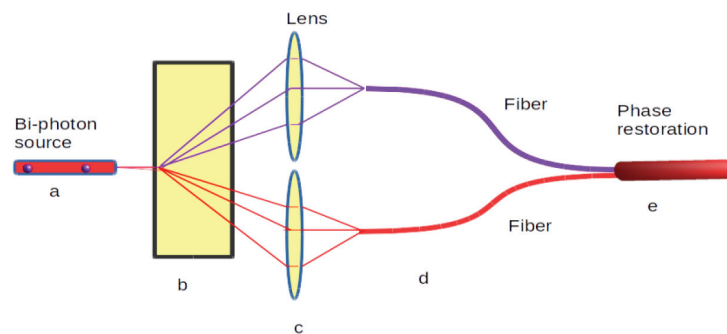
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Figure 7.
The two-quanta coherence and its possible experimental observations: a, two photon coherent generator; b, dispersive media; c, lenses; d, fibers; e, signal restoration.

these two fields amplitude a_s and b_a^+ has similar behavior as the phase of single mode coherent field, here $2\omega_0 = \omega_a - \omega_s$ and $k_0 = k_a - k_s$ are the frequencies and wave vector difference between the Stokes and anti-Stokes fields respectively. The coherent propriety of this product of the electric field components is proposed to apply in possibilities to construct the time, space holograms of real objects, taking in to account the conservation of phase of amplitude product in the propagating and interference processes. The quantum phase between the radiators can also be used in holography.

Presently exist a lot of proposals in which is manifested holographic principals of processing of quantum information [5, 8, 9]. One of them is the model of Prof. Teich with co-authors [5]. According to this model the correlations between the entangled photons, obtained in parametric down conversion, can be used in quantum holography. The hologram in parametrical down conversion is realized in terms of the correlations between the entangled photon in the single pair. The coherence between the pairs is not taken into consideration.

Following the idea of classical holograms, we changed the conception of two-photon holograms using the second order interference described in Refs. [19, 30]. This new type of hologram registration is based on the coherent proprieties (3) and (8). As well known the holographic code in single photon coherent effects appears on mixing the original wave (hereafter called the "object wave") I_0 with a known "reference wave" I_r and recording their interference pattern in the $z = 0$ plane. According to transmittance conception T of single-mode holograms, the correlations are proposed in the strength product of "object wave" and the "reference wave" waves $\langle \hat{E}_O(z,t)\hat{E}_r(z,t+\tau)\rangle$, where $\hat{E}_O(z,t) = (\hat{E}_O^+(z,t) + \hat{E}_O^-(z,t))/\sqrt{2}$ and $\hat{E}_r(z,t+\tau) = (\hat{E}_r^+(z,t+\tau) + \hat{E}_r^-(z,t+\tau))/\sqrt{2}$ the scheme of interference pattern is represented in **Figure 8** and has many analogies with classical holograms. The transmittance is given by interference between the original and reference bimodal waves at t (see for example [33]). Extending this conception we construct such a hologram, replacing the EMF strength the two-photon coherence using the field vector (2)-(3). According to the classical definition one can represent correlations between the original bimodal field through $\hat{P}_O(z,t) = (\hat{P}_O^+(z,t) + \hat{P}_O^-(z,t))/\sqrt{2}$ and reference bimodal wave vector, described by the expression $\hat{P}_r(z,t+\tau) = (\hat{P}_r^+(z,t+\tau) + \hat{P}_r^-(z,t+\tau))/\sqrt{2}$. In this case, the points on the plan of hologram, $z = 0$, we have the transmittance

$$T_b = \left\langle : \left(\hat{P}_O(z,t) + \hat{P}_r(z,t+\tau)\right)^2 : \right\rangle$$

$$= \left\langle \hat{P}_O^-(0,t)\hat{P}_O^+(0,t) \right\rangle + \left\langle \hat{P}_r^-(0,t+\tau)(\hat{P}_r^+(0,t+\tau) \right\rangle \qquad (9)$$

$$+ \left\langle \hat{P}_O^-(0,t)\hat{P}_r^+(0,t+\tau) \right\rangle + \left\langle \hat{P}_r^-(0,t+\tau)\hat{P}_O^+(0,t) \right\rangle.$$

In the expression (9) the function $G_{O2} = \langle \hat{P}_O^-(0,t)\hat{P}_O^+(0,t)\rangle$ represents the intensity of detecting bi-photons from the original bimodal field; $G_{r2} = \langle \hat{P}_r^-(0,t+\tau)\hat{P}_r^+(0,t+\tau)\rangle$ is the intensity of detecting bi-photons from reference bimodal wave from the object. The phase dependence of the image can be described by argument of complex number $\langle P_O^-(0,t)P_r^+(0,t+\tau)\rangle$ or $\arg(\hat{P}_O^+(0,t)) - \arg(\hat{P}_r^+(r,t+\tau))$. The propriety of two-photon bimodal field is described by the expressions (2)–(3) of the last Section 2. The detection scheme on the plan $z = 0$ is described in **Figure 8** and as in the scheme 7, we may take into consideration the fact, that the coherent blocks of bi-photons can be separated into idler and signal photons. This entangled effect may be registered by two separate detector screens represented in the **Figure 8**. For example the A screen, it is used for the registration of 'idler' photons while the screen B can be used for the registration of amplitude an fluctuations phase of 'signal' photons. The problem consists in the restoration of this bimodal field with coherent proprieties between the bi-photons. The transmittance can be detected by two photon detectors on the plane (x,y) and the interpretation of image can be expressed in classical terms

$$T_b = G_{O2} + G_{r2} + 2\sqrt{G_{r2}G_{O2}} \cos\left[\arg\left(P_O^+(0,t)\right) - \arg\left(P_r^+(r,t+\tau)\right)\right] \qquad (10)$$

The same behavior has the bimodal field formed from Stokes Pump and anti-Stokes photons. In the case of scattering bimodal field the coherent proprieties of the vector $\Pi_O(z,t) = \left(\Pi_O^+(z,t) + \Pi_O^-(z,t)\right)/\sqrt{2}$ can be found from the expressions (5)–(8) so that the transmittance

$$T_s = \left\langle : \left(\Pi_O(z,t) + \Pi_r(z,t+\tau)\right)^2 : \right\rangle$$

$$= \langle \Pi_O^-(0,t)\Pi_O^+(0,t)\rangle + \langle \Pi_r^-(0,t+\tau)\Pi_r^+(0,t+\tau)\rangle \qquad (11)$$

$$+ \langle \Pi_O^-(0,t)\Pi_r^+(0,t+\tau)\rangle + \langle \Pi_r^-(0,t+\tau)\Pi_O^+(0,t)\rangle.$$
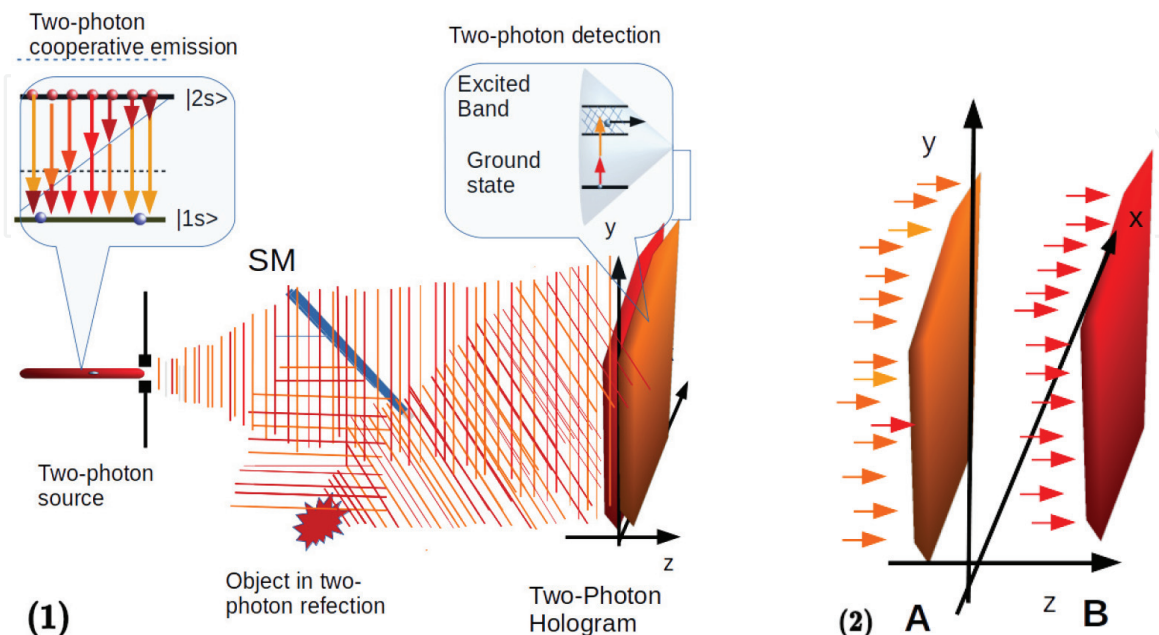

Figure 8.
(1) Two-photon coherent light described in Section 2 and the registration of hologram taking into consideration the phase and amplitude of bi-photon field of the "object" and "reference" waves. (2) The possibilities to detect the "signal" ($\omega_i \in (\omega_0, 2\omega_0)$) and "idler" ($\omega_i \in (0, \omega_0)$) photons on separate screens A and B.

This type of Holography takes into consideration the coherent process at low frequency $\omega_p - \omega_s$ (or $\omega_a - \omega_p$ which may coincide with the vibration frequencies of biomolecules. The popularity of coherent Raman scattering techniques in optical microscopy increases and it may be developed using another type of coherence described in the section. The holography developed on the bases of coherence proprieties between the two- (or three) conjugate modes of the scattering field opens this possibility not only for the description of the spectral diapason and time dependence of scattered field intensity, but the topological aspects of the molecular structures manifested in holographic representations of the vibrational modes of molecules. The coherence proposed in the Section 2 *B* needs the low intensity of each mode component in comparison with traditional Raman diagnostic proposed in Refs. [14, 15]. Using the coherent proprieties, described at the point *B* of the last section, we can estimate a lot of peculiarities connected with geometrical structures of biomolecules for lower intensities of each mode component of Raman process described by Refs. [14, 15]. In this case the transmission can be detected by the scheme proposed in **Figure 9** on the plan (x, y), where the interpretation of Hologram imaging can be expressed in classical terms.

$$T_s = G_{O2} + G_{r2} + 2\sqrt{G_{r2}G_{O2}} \cos\left[\arg\left(\Pi_O^+(0,t)\right) - \arg\left(\Pi_r^+(r,t+\tau)\right)\right] \qquad (12)$$

The entanglement between each mode of the field can be detected by two-photon detector schemes, placed in the plan of hologram represented in **Figure 9**. This procedure may be in tangency with proposed experimental detections of vibration modes of biomolecules [8, 9].

In comparison with the spontaneous parametric down-conversion the super-radiance [21] or cooperative scattering processes [12, 13] represented generators of non-classical light source—the two-photon quantum entangled state with the coherent aspects between the two conjugate modes. Two-modes from such processes may become incoherent, but the coherence can be revived in the two-photon excitations of the detector which represents the photon pairs from adjacent modes. The two-photon detection scheme an interference connected to it is shown in **Figure 6**. The similar effect appears between stokes, pump, and anti-Stokes photon in induced scattering. In the pioneer theoretical work of two-photon optics, Belinskii and Klyshko [7] predicted three spooky schemes: two-photon diffraction, two-photon holography, and two-photon transformation of two-dimensional images. The first and last schemes have been demonstrated in the experiments,
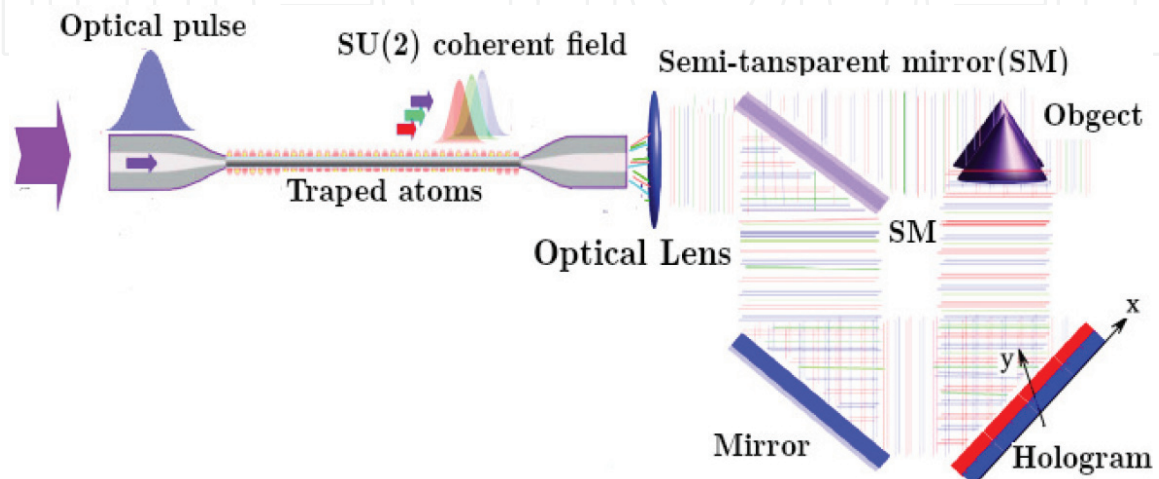

Figure 9.
Two-photon coherent light and principle of hologram registration taking into consideration the phase and amplitude of the three modes of Raman scattered field Stokes, pump and anti-Stokes modes.

known as ghost interference [34] and ghost imaging [35], respectively. These experiments are connected to the original gedankenexperiment of EPR paradox and open the way to the detection of two-photon holography [8, 9]. In such holograms the signal photons play both roles of "object wave" and "reference wave" in holography, but are recorded by a point detector providing only encoding information, while the "idler" photons travel freely and are locally manipulated with spatial resolution along the fibers becomes possible.

4. Conclusions

The encrypted information, using the coherence of multi-mode bimodal field in quantum holography, opens the new perspective, in which the coherence proprieties between bi-photons are used together with non-local states of entangled photon pairs. The possibilities to use this coherence in the quantum communication and holographic registration of objects is described by the expressions (9) and (10) and is proposed for future developments. The main distinguish between the traditional holograms and such a hologram registration becomes attractive from physical points of view because it must take into consideration the common phase of two light modes described by the expressions (9)-(12). It also discusses the cooperative behavior of three cavity modes which corresponds to pump, Stokes and anti-Stokes photons stimulated by the atomic inversion. A new type of cooperative generation described by the correlations of the expressions (1) and (4) may be used in quantum nucleonics [36] as an ignition mechanism of coherence generation gamma photons by long-lived nuclear isomers in the single and two-quantum interaction with other species of excited radiators.

This method of recording of information affords the new perspectives in quantum cryptography and quantum information and has the tendency to improve the conception about quantum holograms observed in in literature [5, 7–9]. All these methods open new possibilities in the coding and decoding of data.

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