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Analysis of Wavelet Transform Design via Filter Bank Technique

Peter Yusuf Dibal, Elizabeth Onwuka, James Agajo and Caroline Alenoghena

Abstract

The technique of filter banks has been extensively applied in signal processing in the last three decades. It provides a very efficient way of signal decomposition, characterization, and analysis. It is also the main driving idea in almost all frequency division multiplexing technologies. With the advent of wavelets and subsequent realization of its wide area of application, filter banks became even more important as it has been proven to be the most efficient way a wavelet system can be implemented. In this chapter, we present an analysis of the design of a wavelet transform using the filter bank technique. The analysis covers the different sections which make up a filter bank, i.e., analysis filters and synthesis filters, and also the upsamplers and downsamplers. We also investigate the mathematical properties of wavelets, which make them particularly suitable in the design of wavelets. The chapter then focuses attention to the particular role the analysis and the synthesis filters play in the design of a wavelet transform using filter banks. The precise procedure by which the design of a wavelet using filter banks can be achieved is presented in the last section of this chapter, and it includes the mathematical techniques involved in the design of wavelets.

Keywords: wavelet, filter bank, perfect reconstruction, orthogonality, paraunitary condition

1. Introduction

Filter banks can be defined as the cascaded arrangement of filters, i.e., low-pass, high-pass, and band-pass filters connected by sampling operators in such a manner as to achieve the decomposition and recomposition of a signal from a spectrum perspective. The sampling operators could either be downsamplers or upsamplers. The downsamplers are called decimators while the upsamplers are called expanders. The technique of filter banks plays an important role in most digital systems that rely on signal processing for their operations. Using this technique, any signal feature can be reliably extracted and analyzed; hence filter banks have wide applications in digital signal processing systems. A filter bank as shown in **Figure 1** [1, 2] consists of different parts, which collectively execute a desired function.

As can be seen in **Figure 1**, the filter bank is made of two sections: the analysis filter bank section (composed of analysis filters and downsamplers), and the synthesis filter bank section (composed of upsamplers and synthesis filters). In this chapter, we will discuss the analysis and synthesis filter bank sections, their
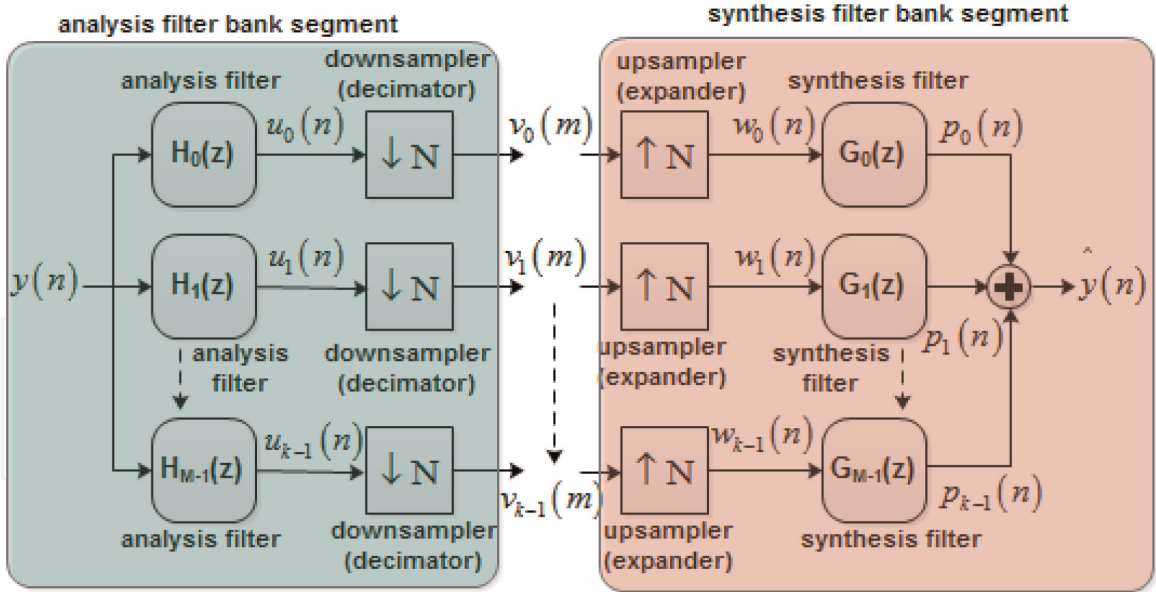
Figure 1.
k-Channel filter bank [1, 2].

responses to incoming signals, and how they work together in the derivation of a wavelet transform function.
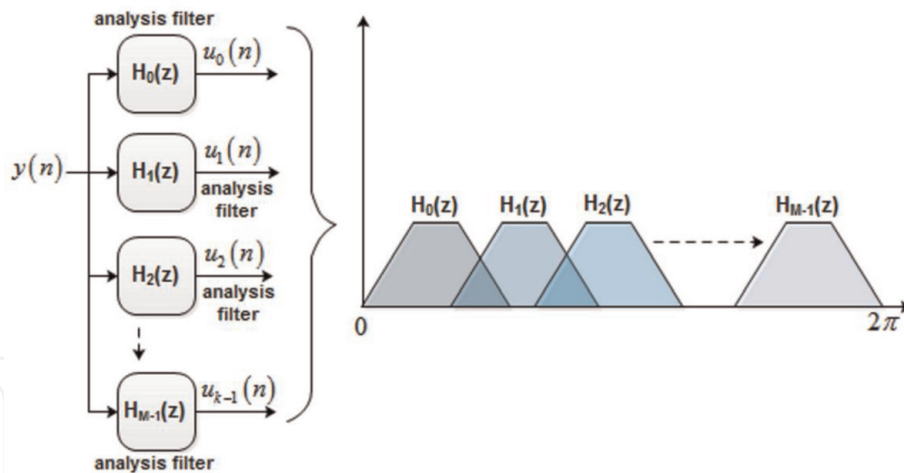
2. Analysis filter bank section

The analysis filter bank section is made up of the analysis filter banks, and downsamplers or decimators which together act on an input signal to perform a desired function through decomposition of the signal. In this section, we will analyze the mathematical relationship that exists between these two components. To have a thorough understanding of this relationship, it is important to briefly discuss these components separately.

2.1 Analysis filter bank

The filters that make up the analysis filter banks could either be low-pass filters, or high-pass filters. Each of these filters, as shown in **Figure 2**, allows the passage of only a particular frequency component of the input signal $y(n)$. Thus, specific features of the input signal embedded at different frequencies can be individually extracted and investigated using the analysis filter bank [3, 4]. The k-channel filter bank in **Figure 2** separates the frequencies of the input signal in the manner presented.

It can be seen from the frequency responses that the output of the filters overlap each other. This is because in practice, the filters are not ideal. However, the overlapping condition can be improved through an optimized design of the filters. Mathematically, the effect of each of the filters in the filter bank on the input signal $y(n)$ can be stated as follows:

$$U_0(Z) = Y(Z)H_0(Z)$$
$$U_1(Z) = Y(Z)H_1(Z)$$
$$U_2(Z) = Y(Z)H_2(Z)$$
$$U_{k-1}(Z) = Y(Z)H_{M-1}(Z)$$

$$(1)$$

Figure 2.
Separation of input signals into sub-band frequencies by analysis filter bank.

where $U_i(z)$ is the z-transform of the result from the convolution operation between the z-transform of the input signal $Y(Z)$ and the z-transform of the filter $H_i(Z)$. The output $U_i(z)$ in **Figure 2** is fed into the corresponding downsampler of **Figure 1**. In the next section, we will analyze the downsampler and state the mathematical operation it performs on a given signal.

2.2 Downsampler/decimator

The downsampler shown in **Figure 1** downsamples an input signal by a factor of N. This implies that it only retains all the Nth samples in a given sequence. For example, if $N = 2$, then the downsampler will retain all even samples in a given sequence. Given an input signal $x(n)$, the downsampler with a factor of 2 will downsample the signal as:

$$\hat{x}(n) = x(2n), \forall n \in \mathbb{Z} \qquad (2)$$

Figure 3 shows the conceptual depiction of the relationship in Eq. (2).

Mathematically, the output of the decimator in **Figure 1** can be expressed as a product of the input sequence $u_i(n)$ and the sequence of unit impulses which are N samples apart, i.e.,

$$v_i(n) = \sum_{k \in \mathbb{Z}} u_i(n)\delta(n - kN), \forall k \in \mathbb{Z} \qquad (3)$$

The relationship in Eq. (3) will only select the kN^{th} sample of $u_i(n)$, and the Fourier series expansion of the impulse series can be expressed as [5]:
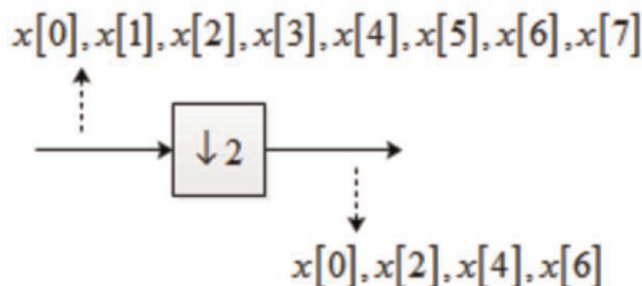
Figure 3.
Decimation by a factor of 2.

$$\sum_{k \in \mathbb{Z}} \delta(n - kN) = \frac{1}{N} \sum_{k=0}^{N-1} e^{-j2\pi kn/N} \tag{4}$$

Setting $W_N = e^{-j2\pi/N}$ and $n = 1$, the relationship in Eq. (4) becomes:

$$\sum_{k \in \mathbb{Z}} \delta(n - kN) = \frac{1}{N} \sum_{k=0}^{N-1} W_N^{-k} \tag{5}$$

Substituting Eq. (5) into (3) yields:

$$v_i(n) = \frac{1}{N} \sum_{k=0}^{N-1} u_i(n) W_N^{-k} \tag{6}$$

In terms of z-transformation, the relationship in Eq. (6) can be expressed as:

$$V_i(Z) = \frac{1}{N} \sum_{k=0}^{N-1} U_i \left(Z^{\frac{1}{N}} W_N^{-k} \right) \tag{7}$$

Having looked at the analysis filters and downsamplers, we will now turn our attention to synthesis filter bank section of **Figure 1**.

3. Synthesis filter bank section

The synthesis filter bank section is made of the upsamplers and synthesis filter banks. These components work together to perform the opposite operation performed by the analysis filter bank section shown in **Figure 1**. In this section, we will make an analysis of the mathematical relationship that governs the operation of the synthesis filters and upsamplers.

3.1 Synthesis filter bank

Similar to the analysis filter bank, the synthesis filter bank is made of low-pass and high-pass filters. The output of these filters as shown in **Figure 1**, are summed to a common output. In typical filter bank applications, the frequency responses of these filters are typically matched to those of the analysis filters shown in **Figure 2**. The mathematical expression for the effect each of these filters has on the corresponding input signal $w_i(n)$ is as stated below [6]:

$$
\begin{aligned}
P_0(Z) &= W_0(Z)G_0(Z) \\
P_1(Z) &= W_1(Z)G_1(Z) \\
P_2(Z) &= W_2(Z)G_2(Z) \\
P_{k-1}(Z) &= W_{k-1}(Z)G_{M-1}(Z)
\end{aligned} \tag{8}
$$

In **Figure 2**, the input to the synthesis filter bank is upsamplers or expanders. The next section gives a brief review of the upsamplers.

3.2 Upsampler/expander

The upsampler expands an input signal by a factor N. It does this by inserting zeros at every nth position in the sequence of the input signal. For example, if

$$x[0], x[1], x[2], x[3], x[4]$$

$$\boxed{\downarrow 2}$$

$$x[0], 0, x[1], 0, x[2], 0, x[3], 0, x[4]$$
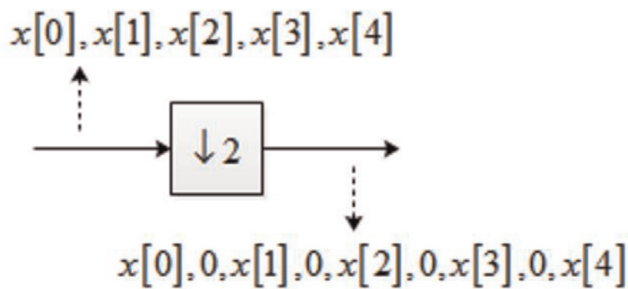
Figure 4.
Upsampling by a factor of 2.

$N = 2$, then the upsampler will insert a zero between every two adjacent samples in a given sequence as shown in **Figure 4**.

Given an input signal $v_i(n)$ in **Figure 1**, an upsampler with a factor of 2 will upsample the signal using the relationship [7]:

$$w_i(n) = \sum_{k \in \mathbb{Z}} v_i(n)\delta(n - kN), \forall k \in \mathbb{Z} \tag{9}$$

Similar to the expression in Eq. (3), the z-transform of the expression in Eq. (9) which is an upsampler is stated as follows [8]:

$$W_i(Z) = \frac{1}{N} \sum_{k=0}^{N-1} V_i\left(Z^N W_N^{-k}\right) \tag{10}$$

To be useful in wavelet designs, filter banks must be designed to have certain characteristics which guarantee that a signal at the input of a filter bank will be received accurately at the output of the filter bank. In the next section, we will examine the properties of filter banks and how these properties influence the design of wavelets.

4. Properties of filter banks for wavelet design

In wavelet designs, filter banks are required to possess three important properties which are fundamental to the realization of a wavelet function. These properties include: perfect reconstruction, orthogonality, and paraunitary condition.

4.1 Perfect reconstruction

This property guarantees that the signal at the output of a given filter bank is a delayed version of the signal at the input of the filter bank. Perfect reconstruction is an important property of a filter bank because it cancels the effect of aliasing of the input signal at the output, caused by the downsamplers and upsamplers. To understand this point, consider a two-channel finite impulse response FIR filter bank shown in **Figure 5**.
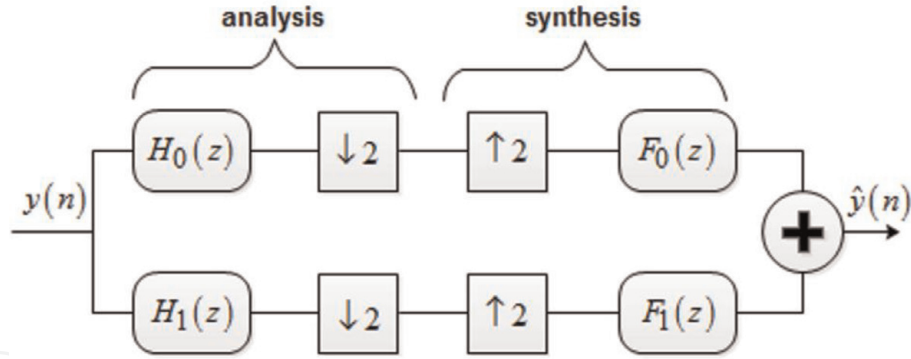
The output $\hat{y}(n)$ is derived using Eqs. (6) and (10) as follows in terms of the signal component and aliasing component as:

$$\hat{Y}(z) = \text{signal\_component} + \text{aliasing\_component} \tag{11}$$

where the signal_component and aliasing_component are defined as:

$$\left.\begin{aligned}
\text{signal\_component} &= \frac{1}{2}[F_0(z)H_0(z) + F_1(z)H_1(z)]X(z) \\
\text{aliasing\_component} &= \frac{1}{2}[F_0(z)H_0(-z) + F_1(z)H_1(-z)]X(-z)
\end{aligned}\right\} \tag{12}$$

Figure 5.
Two-channel FIR filter bank.

To achieve perfect reconstruction, the following condition must be satisfied [1]:

$$\left.\begin{array}{c} F_0(z)H_0(z) + F_1(z)H_1(z) = 2z^{-1} \\ F_0(z)H_0(-z) + F_1(z)H_1(-z) = 0 \end{array}\right\} \qquad (13)$$

The relationships in Eqs. (11) and (13) are possible when the filter bank is constructed as a QMF (quadrature mirror filter) filter bank or CQF (conjugate quadrature filter) filter bank. Both QMF and CQF banks provide a mechanism by which complete cancellation of the aliasing component in Eq. (11) can be accomplished. Using QMF, aliasing cancellation can be achieved by constructing the filters in **Figure 5** based on the following relationships [4, 5]:

$$\left.\begin{array}{c} F_0(z) = H_0(z) \\ H_1(z) = H_0(-z) \\ F_1(z) = -H_1(z) \end{array}\right\} \qquad (14)$$

In Eq. (14), the synthesis filter $F_0(z)$ has the same coefficients as the analysis filter $H_0(z)$; the analysis filter $H_1(z)$ has the same coefficients as the analysis filter $H_0(z)$, but every other value is negated; the synthesis filter $F_1(z)$ is a negative copy of the analysis filter $H_1(z)$. For example, if the analysis filter $H_0(z)$ has coefficientsp, q, r, s, then the filter bank in **Figure 5** will assume the structure shown in **Figure 6**.

For the CQF bank, the coefficients of the analysis filter $H_1(z)$ are a reversed version of the analysis filter $H_0(z)$ with every other value negated. The synthesis filters $F_0(z)$ and $F_1(z)$ are a reversed versions of the analysis filters $H_0(z)$ and $H_1(z)$, respectively. These relationships can be stated mathematically as follows [10]:
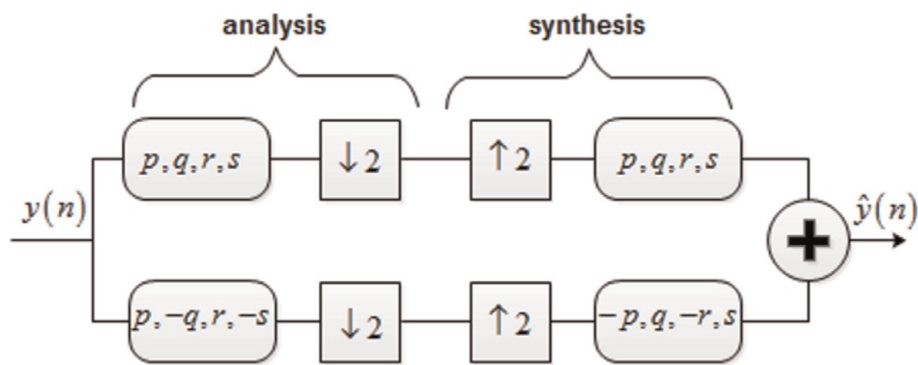
$$\left.\begin{array}{c} H_1(z) = z^{-1}H_0(-z^{-1}) \\ F_0(z) = H_1(-z) \\ F_1(z) = -H_0(-z) \end{array}\right\} \qquad (15)$$

Based on the relationship in Eq. (15), the filter bank shown in **Figure 6** for CQF will assume the structure shown in **Figure 7**.
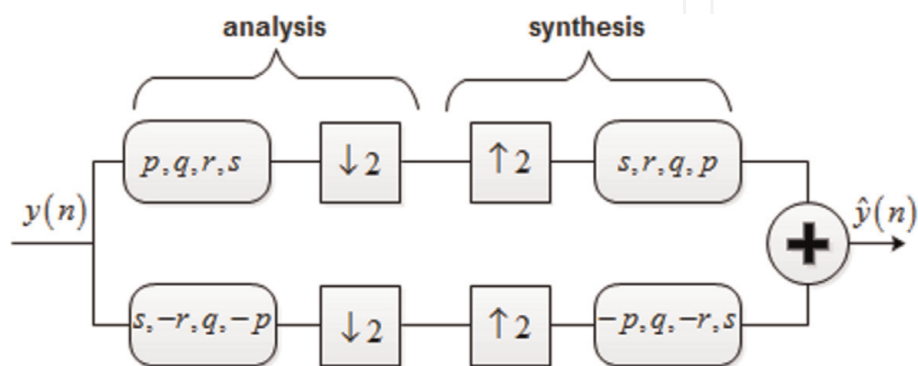
Based on the structure of **Figures 6** or **7**, the output signal $\hat{y}[n]$ is related to the input signal $y[n]$ by the expression:

$$\hat{y}[n] = (pp + qq + rr + ss)y[n - 3] \qquad (16)$$

If we impose the condition that $pp + qq + rr + ss = 1$, then Eq. (16) becomes:

Figure 6.
QMF two-channel FIR filter bank.

Figure 7.
CQF two-channel FIR filter bank.

$$\hat{y}[n] = y[n-3] \tag{17}$$

The relationship in Eq. (17) states that the output signal $\hat{y}[n]$ is delayed version of the input signal $y[n]$ by three samples. We leave the verification of Eq. (16) as an exercise for the reader.

Having looked at perfect reconstruction as a necessary property for a filter bank in wavelet design, we now look at orthogonality as also an essential property for a filter bank in the design of wavelets.

4.2 Orthogonality

Orthogonality in a filter bank is a situation in which the synthesis filter bank is a transpose of the analysis filter bank. This is a useful property in the sense that it allows for the energy preservation of the signal being processed. This important property is achieved through the imposition of the orthogonality condition on both the analysis and filter bank sections while at the same time preserving the perfect reconstruction condition of the filter bank. The imposition of the orthogonality condition in a filter bank (see **Figure 5**) occurs when the following relationships are satisfied [11]:

$$\langle \tilde{f}_0(n-2k), h_1(n-2l) \rangle = 0 \; \langle \tilde{f}_1(n-2k), h_0(n-2l) \rangle = 0 \tag{18}$$

where

$$\tilde{g}_i(n) = g_i(-n)$$

and

$$\langle \tilde{g}_0(n-2k), h_0(n)\rangle = \delta_k \ \langle \tilde{g}_1(n-2k), h_1(n)\rangle = \delta_k \tag{19}$$

In Eq. (18), the inner product of the coefficients of the synthesis filter $F_0(z)$ and the analysis filter $H_1(z)$ must be zero and the inner product of the coefficients of the synthesis filter $F_1(z)$ and the analysis filter $H_0(z)$ must also be zero for the orthogonality condition to hold.

Also, the low-pass analysis filter $H_0(z)$ is related to the other three filters through the following expressions [12]:

$$\left.\begin{array}{l} H_1(z) = cz^{-(L-1)}\tilde{H}_0(-z) \\ F_0(z) = z^{-(L-1)}\tilde{H}_0(z) \\ F_1(z) = z^{-(L-1)}\tilde{H}_1(z) \end{array}\right\} \tag{20}$$

where L denotes the length of the filter which must be even, and c is a constant with $|c| = 1$; $\tilde{H}_0(-z)$ is the flipped and conjugated version of $H_0(z)$, $\tilde{H}_0(z)$ is the conjugated version of $H_0(z)$, and $\tilde{H}_1(z)$ is the conjugated version of $H_1(z)$.

The condition in Eq. (20) also describe the necessary requirement for a filter bank to be paraunitary (which we shall examine in the next section), i.e., the low-pass filter $H_0(z)$ satisfy the following power symmetry of halfband condition [8, 9]:

$$P(z) + P(-z) = 2 \tag{21}$$

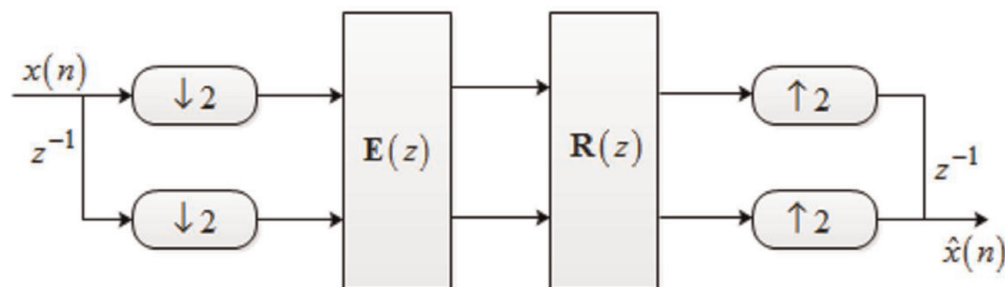where $P(z) = H_0(z)\hat{H}_0(z)$. If the low-pass filter $H_0(z)$ satisfies the required symmetry condition:

$$H_0(z) = z^{-(L-1)}H_0(z^{-1}) \tag{22}$$

then $P(z)$ is said to be a real filter. The implication of the constraint in Eq. (21) is that $H_1(z)$ and $F_1(z)$ be antisymmetric filters, and $F_0(z)$ is a symmetric filter. The relationships in Eqs. (20)–(22) give the necessary and sufficient condition for the characterization of a filter bank with orthogonality and symmetry.

The orthogonality condition for a filter bank can also be examined from a polyphase perspective. Consider the polyphase representation of the filter bank in **Figure 5** as illustrated in **Figure 8** [13].

If $\mathbf{E}(z)$ in **Figure 8** is type-I analysis polyphase matrix, and $\mathbf{R}(z)$ is type-II synthesis polyphase matrix, then [13]:

$$[H_0(z) \ H_1(z)]^T = \mathbf{E}(z^2)\begin{bmatrix}1 & z^{-1}\end{bmatrix}^T \\ [F_0(z) \ F_1(z)] = \begin{bmatrix}z^{-1} & 1\end{bmatrix}\mathbf{R}(z^2) \tag{23}$$

Figure 8.
Polyphase implementation of filter bank.

8

The conditions in Eqs. (20)–(22) hold true iff $\mathbf{E}(z)$ and $\mathbf{R}(z)$ satisfy the following:

$$\left.\begin{array}{r}\hat{\mathbf{E}}(z)\mathbf{E}(z) = \mathbf{I} \\ \mathbf{R}(z) = z^{-(k-1)}\mathbf{E}(z) \\ \mathbf{E}(z) = z^{-(k-1)}\text{diag}(1, -1)\mathbf{E}(z^{-1})\mathbf{J}\end{array}\right\} \quad (24)$$

where $k = L/2$, with the first and second condition in Eq. (24) relating to the filter bank orthogonality condition, and the last represents the filter bank symmetry.

We now look at the paraunitary condition of a filter bank, which is also a necessary property in filter bank implementation of wavelets.

4.3 Paraunitary condition

In the filter bank implementation of a wavelet transform, the paraunitary condition plays the critical role of guaranteeing the generation of orthonormal wavelets, and also perfect recovery of a decomposed signal. The paraunitary condition guarantees that recovered signal will suffer no phase or aliasing effect if a filter bank satisfies the paraunitary condition [14].

Given a polyphase transfer function matrix $\mathbf{E}(z)$, the paraunitary condition is established by the matrix iff [15]:

$$\mathbf{E}^H(z^{-1})\mathbf{E}(z) = \mathbf{I} \quad (25)$$

where the H superscript denotes the conjugated transpose, and $\mathbf{I}$ denotes the identity matrix. Paraunitary filter banks also have an attractive property of losslessness, which implies that for every frequency, the total signal power is conserved [16]. From this property [17], any $M \times M$ real-coefficient lossless matrix with $N - 1$ degree can be realized using the structure shown in **Figure 9** [18].

If the real-coefficient lossless matrix is denoted by $\mathbf{E}(z)$; then the matrix is said to have a special case of lossless degree of one iff it can be characterized by the relationship [18]:

$$\mathbf{E}(z) = \left[\mathbf{I} - \mathbf{v}\mathbf{v}^+ + z^{-1}\mathbf{v}\mathbf{v}^+\right]\mathbf{R} \quad (26)$$

where $\mathbf{R}$ is an arbitrary $M \times M$ unitary matrix and $\mathbf{v}$ is an $M \times 1$ column vector with unit norm. From Eq. (26), the paraunitary condition for a filter bank is obtained as follows [18]:

$$\left[\mathbf{I} - \mathbf{v}_k\mathbf{v}_k^+ + \mathbf{v}_k\mathbf{v}_k^+ z\right]\mathbf{E}_k(z) = \mathbf{E}_{k-1}(z) \quad (27)$$
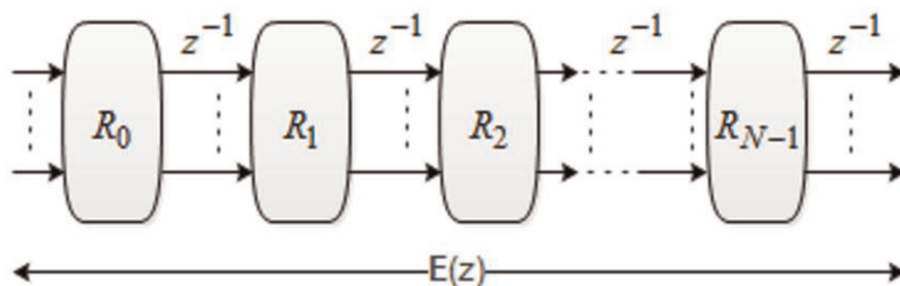
Figure 9.
Cascade implementation of $\mathbf{E}(z)$ as FIR lossless unitary matrices separated by delays.

Having looked at the filter bank and its three important properties for the design of a wavelet, we will in the next section examine the application of these properties in the design of a wavelet.

5. Filter bank design of a wavelet transform

The filter bank design of a wavelet transform is usually implemented from the analysis filter bank segment to the synthesis filter bank segment.

5.1 Analysis filter bank in wavelet transform design

Given that the expression for a scaling function $\varphi[n]$ is the series sum of the shifted versions of $\varphi[2n]$, then according to [15, 16], $\varphi[n]$ can be represented as:

$$\varphi[n] = \sum_k h[k]\sqrt{2}\varphi(2n - k), \forall k \in \mathbb{Z} \tag{28}$$

where $h[k]$ denotes the scaling coefficients. If n is transformed such that $n \to 2^\alpha n - \beta$, then the relationship in Eq. (28) becomes [14]:

$$\varphi[2^\alpha n - \beta] = \sum_k h[k]\sqrt{2}\varphi[2(2^\alpha n - \beta) - k] \tag{29}$$

which translates into:

$$\varphi[2^\alpha n - \beta] = \sum_{m=2\beta+k} h[m - 2\beta]\sqrt{2}\varphi[2^{\alpha+1}n - m] \tag{30}$$

when $k = m - 2\beta$.

In a similar consideration to Eq. (28), the wavelet function $\psi[n]$ can be represented as [19]:

$$\psi[n] = \sum_k g[k]\sqrt{2}\varphi(2n - k), \forall k \in \mathbb{Z} \tag{31}$$

where $g[k]$ denotes the wavelet coefficients. Also, if n is transformed such that $n \to 2^\alpha n - \beta$, then the relationship in Eq. (31) becomes [14]:

$$\varphi[2^\alpha n - \beta] = \sum_k g[k]\sqrt{2}\varphi[2(2^\alpha n - \beta) - k] \tag{32}$$

which translates into:

$$\psi[2^\alpha n - \beta] = \sum_{m=2\beta+k} g[m - 2\beta]\sqrt{2}\varphi[2^{\alpha+1}n - m] \tag{33}$$

when $k = m - 2\beta$.

5.2 Synthesis filter bank in wavelet transform design

In the synthesis filter bank, the reconstruction of the original coefficients of a signal can be achieved through the combination of the scaling and wavelet function coefficients at a coarse level of resolution. Given a signal at $\alpha + 1$ scaling space $f[n] \in V_{\alpha+1}$, then according to [16, 17], the reconstruction is derived as follows:
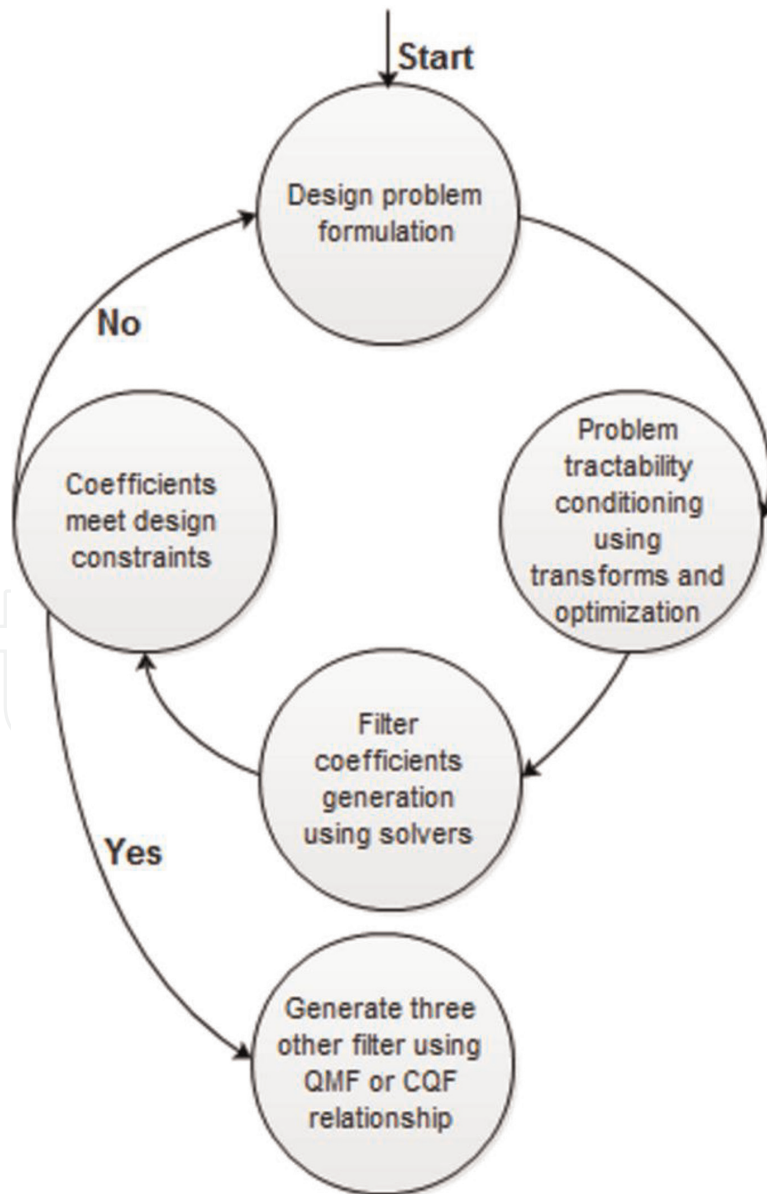
$$f[n] = \frac{1}{\sqrt{M}} \left(\sum_{\beta=-\infty}^{\infty} \lambda_{\alpha+1,\beta} \varphi_{\alpha+1,\beta}[n] \right) = \frac{1}{\sqrt{M}} \left(\sum_{\beta=-\infty}^{\infty} \lambda_{\alpha+1,\beta} \sqrt{2^{\alpha+1}} \psi[2^{\alpha+1}n - \beta] \right)$$

(34)

For the next scale, Eq. (34) becomes:

$$f[n] = \frac{1}{\sqrt{M}} \left(\sum_{\beta} \lambda_{\alpha,\beta} 2^{\alpha/2} \varphi[2^{\alpha}n - \beta] + \sum_{\beta} \gamma_{\alpha,\beta} 2^{\alpha/2} \psi[2^{\alpha}n - \beta] \right)$$

(35)

Substituting Eqs. (28) and (31) into Eq. (35) and after algebraic manipulations yields [14]:

$$\lambda_{\alpha+1,\beta} = \sum_{m} \lambda_{\alpha,\beta} h[\beta - 2m] + \sum_{m} \gamma_{\alpha,\beta} g[\beta - 2m]$$

(36)

Figure 10.
State chart for wavelet design procedure.

6. Wavelet transform design procedure using filter banks

In the design of a wavelet system using filter banks, it is of utmost importance that the filters which will execute the filter bank system as shown in **Figure 1**, possess the properties discussed in Section 4. Owing to the fact that in a filter bank, all the filters can be derived from an initial filter H_0 as described in Eq. (13), then this initial filter must be designed in such a manner that the relationships in Sections 5.1 and 5.2 are realized. To this end, the following steps as shown in the state diagram in **Figure 10** are necessary.

In the first state in **Figure 10**, the design problem formulation which can be achieved using trigonometric polynomial, takes the following into consideration [14]:

i. Compact support which guarantees that the wavelet is characterized by finite non-zero coefficient.

ii. Paraunitary condition which guarantees the generation of orthonormal wavelets.

iii. Flatness/k-regularity which guarantees the smoothness of the wavelet in both time and frequency domains.

The second state which involves conditioning the problem as a tractable problem involves, if necessary, transforming a non-linear formulation of the problem to a linear formulation, and then optimizing the problem using techniques like convex optimization. The generation of the filter coefficients using solvers in the third state of the machine involves techniques like spectral factorization. Through simulation in the fourth state of the chart, the generated coefficients can be verified whether or not they meet the design constraints. Using the QMF or CQF relationships in Eqs. (13) and (14), the other filters in the filter bank are generated in the fifth state of the chart.

7. Conclusion

In this chapter, we have presented an analysis of the design of wavelets using filter bank technique. The chapter looked at the two major components of a filter bank which the analysis and the synthesis components. The properties of filter banks which are desirable in the design of wavelets were also investigated, alongside the mathematical description of these properties. The chapter also gave a brief mathematical description of the role the analysis and the synthesis filter banks play in the design of wavelets. Finally, the required general procedure for the design of wavelets was presented, showing the necessary steps to take in order to achieve an effective design.

The major contribution of this chapter is the provision of a step by step analysis and procedure for the design of filter banks in a precise and concise manner.

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