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Review Heat Transfer of Non-Newtonian Fluids in Agitated Tanks

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Abstract

The heating and cooling of non-Newtonian liquids in tanks with mechanical impellers are operations commonly employed as chemical reactors, heat exchangers, distillers, extractors, thinners and decanters. In particular, the design of heat exchangers (jackets, helical coils, spiral coils and vertical tubular baffles) in tanks requires the prior knowledge of the rheology of the liquid for the calculation of the convection coefficients and the Reynolds number, in order to obtain the area thermal exchange. This chapter aimed to present the basic concepts of tanks with agitation, non-Newtonian liquids, hydrodynamics, heat transfer and, finally, with a practical design example for engineers and undergraduate students.

Keywords: non-Newtonian liquid, tank agitation, heat transfer, rheology

1. Introduction

The agitation and mixing of liquids are an operation commonly used in chemical, petrochemical, food and pharmaceutical processes. In general, the aforementioned operation is performed in tanks (usually cylindrical shape) with mechanical impellers [1]. Generally, agitation refers to forcing a fluid by mechanical means to flow in circulatory or similar pattern inside a vessel [2]. Mixing is the random distribution into and through one another, of two or more initially separate phases [3]. Mixing is achieved by moving material from one region to another [4].

The quality of the mixture is the main parameter that is associated with the efficiency of the heat transfer and mass transfer operations occurring in tanks with agitation and mixing.

Most of the liquids stirred and mixed in tanks, are polymer solutions and paint mixtures, as well as mineral pulps and food pastes, which exhibit a non-Newtonian rheology [5].

Thus, this article will address a chapter with the main concepts on the hydrodynamics of agitation, and, subsequently, the basic parameters of heat transfer in the agitation of non-Newtonian liquids independent of time are detailed, because they represent about 85% of all liquids occurring in industrial processes. At the end of the article, an example of the design of a tank with heating and agitation of a pseudoplastic liquid will be presented, applying the equations that will be presented in the course of this text.

2. Non-Newtonian liquids

Non-Newtonian liquids have a non-linear relationship between shear stress and shear rate, so that the apparent viscosity of this liquid is a function of the rate applied by the source promoter of the amount of movement.

Rheology classifies non-Newtonian liquids into three large groups: (a) time-independent, (b) time-dependent and (c) viscoelastic [6].

Liquids independent of time have their apparent viscosity varying depending on the shear rate and the temperature (**Figure 1**).

The rheological model of Ostwald de Waele [7] or model of the law of the powers, as presented in Eq. (1), has a good adjustment to the data obtained experimentally in rheometers and viscometer, in the shear rate range between 10 and 1000 s^{-1}.

$$\tau = k\left(\nabla\vec{v}\right)^{n} \tag{1}$$

The constant k (consistency factor) and the exponent N (consistency index) are obtained experimentally in viscometers and rheometers. When index n is less than 1, the liquid is pseudoplastic with decrease in apparent viscosity according to the variation in shear rate, such as the food liquids and polymer solutions. With index n equals 1 the liquid is Newtonian as water and hydrocarbons. In case of index n greater than 1, the liquid is dilating with increase in apparent viscosity as the increase in shear rate, such as the starch suspensions and some mineral sludge.

Viscoplastic liquids, which require a yield stress, are adjusted satisfactorily by the Herschel-Bulkley model (Eq. (2)).

$$\tau - \tau_0 = k\left(\nabla\vec{v}\right)^{n} \tag{2}$$

The yield stress (τ_0) is obtained experimentally being a constant parameter of each type of liquid.

The apparent viscosity can be calculated from Eqs. (1) and (2) replacing the shear stress term by the following definition (Eq. (3)):
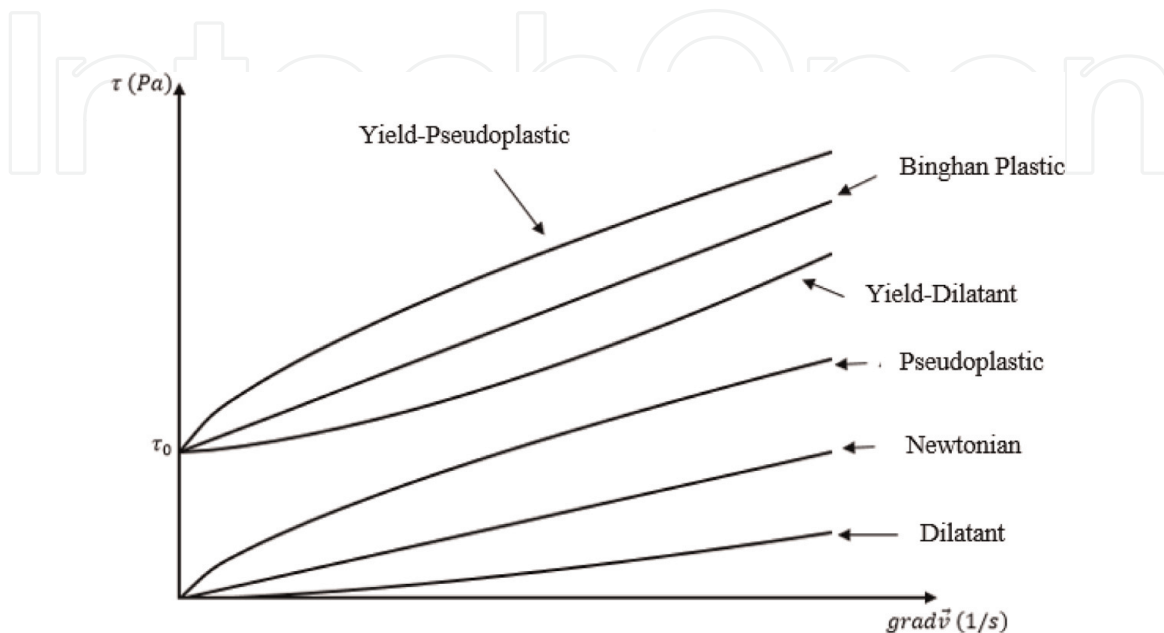
Figure 1.
Classification of time-independent liquids.

$$\eta = \tau / \nabla \left(\vec{v} \right)^{n} \tag{3}$$

There are other models (Casson, Ellis, and Carreau) that have a more accurate adjustment of the apparent viscosity variation with the shear rate, when compared with the model of the law of the powers. However, these models have several constants that must be experimentally determined.

Time-dependent liquids (**Figure 2**) are classified as thixotropics with decreased apparent viscosity with time and reoptical with increased apparent viscosity over time [8].

The thixotropic fluid has an ascending curve similar to pseudoplastics, however, due to hysteresis (phenomenon caused by the variation of apparent viscosity with time), the relaxation of the liquid occurs by a downward curve that differs from the curve up. In the case of the reopetic liquid, the inverse occurs, and the ascending curve is similar to the dilating fluid.

The influence of time on apparent viscosity variation is difficult to achieve and with little accuracy by theoretical models, however, liquids such as food pastes have a great variation in hysteresis, which should be considered in the design of a tank with agitation. The most practical form is from tests in a rheometer in the so-called round-trip test. The sample is subjected to countless cycles of ascending and descending in the shear rate, in order to raise curves similar to those shown in **Figure 2**.

In the design of tanks with mechanical impellers and even other unitary operations (such as pumping and heat exchangers), the variation of hysteresis is considered almost negligible in these liquids, so that the model of the law of the powers satisfies design requirements perfectly.

Finally, the last class of non-Newtonian liquids is called viscoelastic. These liquids have at low shear rates a behavior tending to the solid state, and parameters such as modulus of elasticity are obtained experimentally in vibratory rheometers. In the case of submission to high shear rates, these liquids have similar behavior to
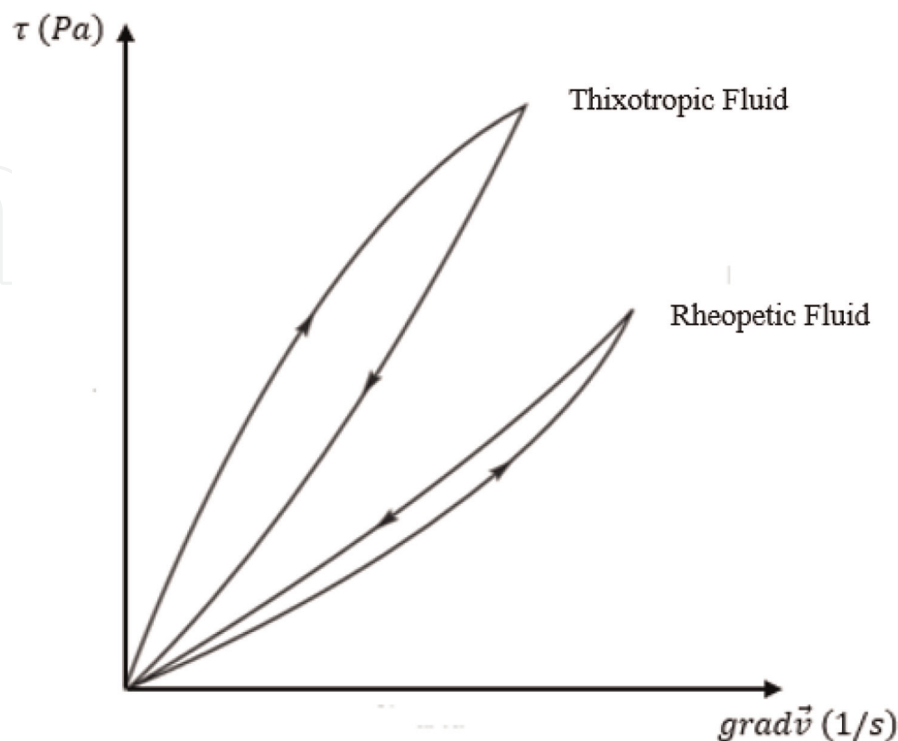

Figure 2.
Time-dependent liquids.

the liquid phase as the anionic polyacrylamide, which is used as an agent for flocculant water treatment plants.

A curious effect of viscoelastic liquids during a stirring and blending operation is the so-called effect of Weissenberg, which is characterized by the liquid being fixed in the impeller during its rotation and climbing through the shaft. Therefore, when this type of liquid is agitated, the rotations employed must be low (lower than 50 rpm) and it is recommended to use a helical impeller of double tape.

The heat transfer in the agitation of Newtonian liquids and non-Newtonian liquids is intimately linked to hydrodynamics, because the flow occurring in the fluid and in the peripheries of the tank influences significantly how the heat is transmitted between the promoter source for all the liquid that will be warmed or cooled.

3. Hydrodynamics of agitation and mixing of non-Newtonian liquids

The flow in stirring and mixing units is a function of the type of mechanical impeller used, which can be of axial type and radial type.

The axial type impeller promotes a flow predominantly parallel to the impeller shaft, directing the liquid to the base of the tank. The most common axial impeller used in industrial processes is the pitched blade turbine (PBT), which can have from two to four paddles and angles between 30 and 60°, the conventional one, a PBT with four paddles inclined to 45° (**Figure 3A**). They are recommended for mixing suspended solids and in processes requiring low to moderate turbulence (with Reynolds number between 4000 and 40,000). There are other types of axial impellers such as the helical tape (for liquids with high viscosities), the naval propeller (for mixtures of immiscible liquids with moderate turbulence) and the propellants with teeth (for high shear in the mixture of pigments) [9, 10].

Radial type impellers promote fluid flow, in the output of the same and in the direction of the tank wall, and therefore, the power consumption by the electric motor is higher when compared to an axial impeller. The most common radial impeller used in industrial processes is the Rushton turbine (RT) with six flat blades or flat Six Blade Impeller (**Figure 3B**). There are turbines of four paddles to eight paddles, but they are poorly used and, in the case of liquids with high viscosity, it is recommended to use radial impeller type anchor [11].
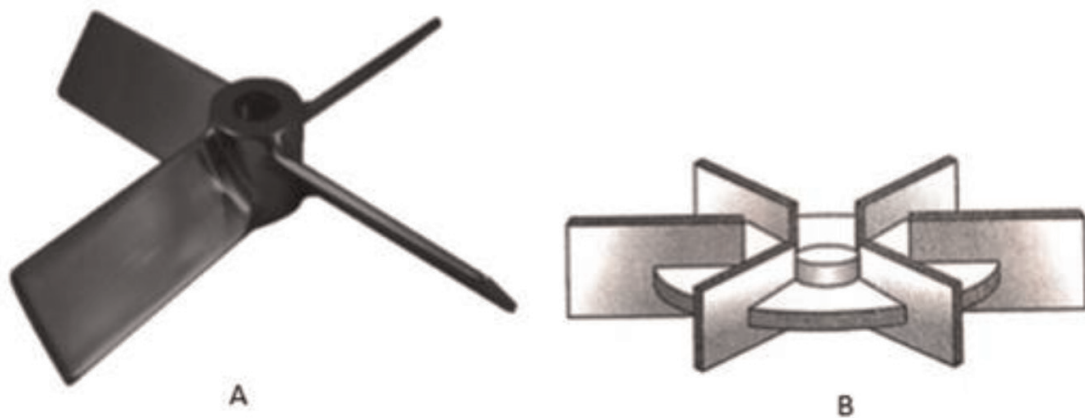

Figure 3.
(A) Pitched blade turbine and (B) Rushton turbine.

The radial impeller is used for mixtures of mixed and immiscible liquids and gases with liquids, usually is situations that require major turbulence, aiming at rapid mixing times (in the range of 9–14 s).

A big problem with the processing and blending in tanks was the standardization of the dimensions that the tank must possess in relation to its peripherals (impeller, impeller position in the tank, baffles and draft tubes). Rushton et al. [12] presented several relationships based on experimental results, in which if followed, the quality of the mixture is potentiated, the amount of plates of the tank is minimized (diameter equal to the height to tank with lid) and the power consumption is close to the smallest possible, which in terms of engineering is excellent.

Figure 4 presents a scheme of the standard dimensions proposed by the aforementioned researchers. When a tank is designed out of these specifications, this tank is called non-standard.

The main design parameters of a tank with stirring and mixing (regardless of the operation in which the equipment is intended) are the power consumption, the mixing time, the thermal efficiency and the efficiency of the mass transfer.

Non-Newtonian liquids have high apparent viscosities, usually in the range of 500–2000 cP so that the mechanical impeller when stirring this liquid will cause a viscous energy dissipation, which is directly proportional to the consumption of useful power by the impeller. In the project, the introduction of energy into the system and calculation of its contribution in the source of heating or cooling in the tank should be taken into account.

The viscous dissipation function for a stirring system is written according to Eq. (4) [14].

$$\phi_V = \left(2\tau_{rr}{}^2 + 2\tau_{\theta\theta}{}^2 + 2\tau_{ZZ}{}^2 + \tau_{rZ}{}^2 + \tau_{r\theta}{}^2 + \tau_{Z\theta}{}^2\right)/\eta^2 \tag{4}$$

Eq. (4) is considered the components of normal stresses in the three directions in cylindrical coordinates and the shear components in the radial direction with the axial and tangential planes and in the axial direction with the tangential plane. The shear stress components are suitable for a rheological model appropriate to the type of liquid that will be agitated and heated or cooled.
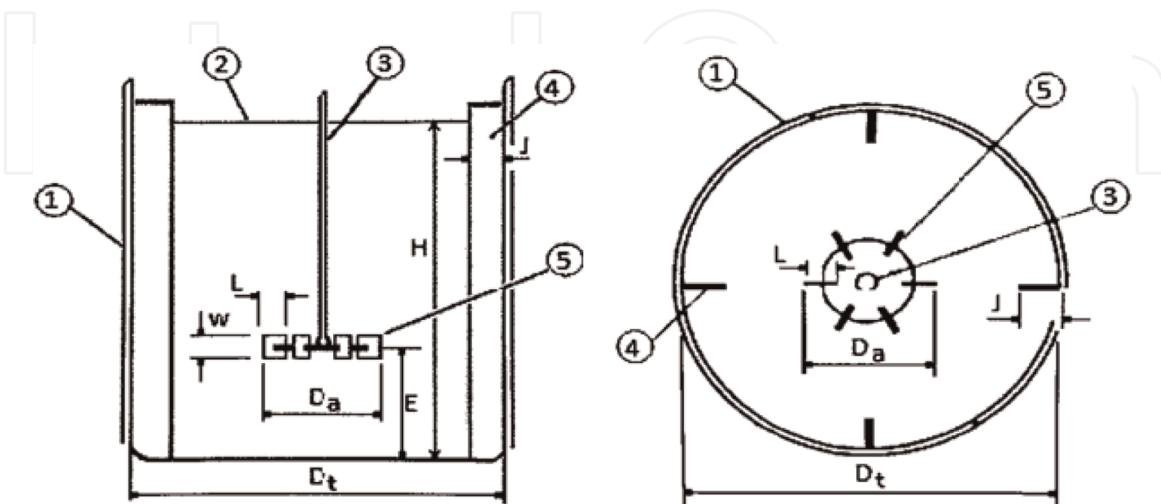

Figure 4.
Geometric relations fixed by Rushton, Costich and Everett 7: 1) tank wall; 2) height of liquid level; 3) mechanical impeller shaft; 4) baffle; 5) mechanical impeller. With the following relationships:
$S_1 = D_t/D_a = 3; S_2 = E/D_a = 1; S_3 = L/D_a = 1/4; S_4 = W/D_a = 1/5; S_5 = J/D_t = 0.1$ and $S_6 = H/D_t = 1$
Moraes Júnior e Moraes [13].

Considering a pseudoplastic liquid, shear stresses can be calculated by the model of the Power Law (Eq. (1)). Finally, the power consumed by the mechanical impeller is given by Eq. (5).

$$P = \int \eta \phi_V r \, dr \, d\theta \, dz \tag{5}$$

The analytical solution of Eq. (5) is not feasible due to the complexity of the viscous dissipation function by involving nonhomogeneous partial derivatives of high order.

Another alternative is the solution by semi-empirical models obtained by Buckingham's Pi theory, which relates the response variable (in this case the power consumption) with the independent variables through the fundamental quantities.

In Eq. (6), the empirical model is presented for calculating power consumption as a function of adimensional numbers.

$$\frac{P}{\rho N^3 D_a^5} = K' \left(\frac{N D_a^2 \rho}{\eta} \right)^{a'} \left(\frac{D_a N^2}{g} \right)^{b'} (geometry)^{c'} \tag{6}$$

The first term to the left in Eq. (6) is the number of power (N_P); on the right limb evaluating from left to right has itself the Reynolds number, the number of Froude, and the geometric relations. It is noteworthy that the constant K' and the exponents a', b' and c' are obtained experimentally and are functions of the type of impeller, the type of tank, the presence of baffles and the rheology of the fluid in agitation.

In most agitation systems, the same contains baffles, so the number of Froude becomes negligible in relation to the Reynolds number. Eq. (6) can be rewritten for a given geometry simplified in Eq. (7).

$$N_p = K'(Re)^{a'} \tag{7}$$

It can be noted that the Reynolds number (flow parameter) has a significant effect on power consumption, as well as in heat transfer, as discussed in topic 3.

In the agitation and mixing of Newtonian liquids, the calculation of the Reynolds number is simple, because the apparent viscosity reduces the dynamic viscosity, which is tabulated as a function of the temperature for several liquids. In the case of non-Newtonian liquids the calculation is no longer simple and becomes a problem of closure, because a rheological model needs to be chosen and its parameters determined experimentally.

Considering that the liquid follows the model of the power law (valid for pseudoplastics and dilatants), the apparent viscosity is calculated by Eq. (8).

$$\eta = k \left(\nabla \vec{v} \right)^{n-1} \tag{8}$$

Thus, the Reynolds number is written as (Eq. (9)):

$$Re = \frac{N D_a^2 \rho}{k \left(\nabla \vec{v} \right)^{n-1}} \tag{9}$$

The shear rate required for calculating the Reynolds number can be calculated by solving the equation of the amount of motion applied to a control volume (in this case the tank with agitation and mixing), as shown in Eq. (10).

$$\rho Dv/D\theta = -\nabla p + \nabla(\eta \nabla v) + \rho g \qquad (10)$$

As in Eq. (5), the solution of Eq. (10) is unfeasible, since the velocity components in the three directions and the temporal component are relevant in agitation and mixing systems. Besides that, the internal geometry of the tank (through the presence of the mechanical impeller and the baffles) makes the analytical solution impracticable. An alternative to numerical solution is the determination of some function that describes the variation of shear rate in relation to rotation of the mechanical impeller, thus, it is not necessary to directly determine the shear rate, which would enable the calculation of the Reynolds number.

Metzner and Otto [15] performed a pioneering work in the agitation and mixing of non-Newtonian liquids in the search for this function that relates the shear rate and the rotation of the mechanical impeller. The researchers worked with tanks in the range of 6–22 inches stirring carboxymethylcellulose (CMC) solutions, Carbopol and the Attasol. All these substances are polymers widely used in industrial processes and it was found that in aqueous solutions, these polymers follow the model of the law of the powers.

By following the model of the law of the powers, these polymers in aqueous solution have their apparent viscosity as a function only of temperature and variation of the shear rate, which decreases the complexity of the problem.

The researchers presented a method to relate, in an experimental way, the shear rate with the rotation of the mechanical impeller, based on the following assumptions: (a) the consistency index (n) of the power law was adopted as constant, despite there is a slight variation of this parameter with the shear rate, but in terms of design, this variation is negligible; (b) the flow of non-Newtonian liquids occurs preferentially in the laminar regime so that there is no detachment of the boundary layer which is the surface of the mechanical impeller and (c) the variation of the shear rate occurs exclusively due to the rotation of the mechanical impeller and not in relation to the rheology of the liquid.

With the last premise, it was assumed that there is a mean shear rate that varies only in function of rotation so that this parameter represents all the fluctuations of shear rate occurring during the agitation of the liquid. In Eq. (11), the definition of this mean parameter is presented:

$$\overline{\nabla \vec{v}} = k_s N \qquad (11)$$

The constant k_s is determined experimentally according to the type of mechanical impeller and its geometry and the tank under analysis. **Table 1** shows the values of the constant k_s for some types of mechanical impellers. It is noteworthy that Eq. (11) is valid only for non-Newtonian liquids independent of the pseudoplastics-type time (n < 1).

Impeller	Number of baffles	D_a (m)	D_t/D_a	$k_s(n < 1)$
Radial with 6 flat blades	0	0.051–0.20	1.3–5.5	11.5 ± 1.5
Radial with 6 flat blades	4	0.051–0.20	1.3–5.5	11.5 ± 1.5
Axial with 4 paddles	4	0.12	2.13	10.0
Anchor	0	0.28	1.02	11.0 ± 1.5

Table 1.
Values of k_s for various types of mechanical impellers [5].

In Eq. (12), the Reynolds of Metzner and Otto is presented, and the use of it depends on the knowledge of the rheological parameters of the model of the power law (k and n) and the constant k_s.

$$Re_{MO} = \frac{N^{2-n} . D_a{}^2 \rho}{k . k_s{}^{n-1}} \qquad (12)$$

Calderbank and Moo-Young [16] disagreed with Metzner and Otto [15] in determining the function that relates the shear rate with the rotation only in one of the premises; for them the variation of the shear rate besides relying on the rotation of the mechanical impeller, is also a function of the rheology of the liquid.

Thus, the researchers used a similarity approach with the Reynolds of Metzner and Reed [17] (Eq. (13)) modified for a stirring system.

$$Re_{MR} = \frac{D_a{}^2 N \rho (8N)^{1-n}}{k} \left(\frac{4n}{3n+1} \right)^n \qquad (13)$$

The constant 8 that appears in Eq. (13) is obtained from the original definition of Reynolds by Metzner and Reed, valid for the flow of non-Newtonian liquids inside pipes. Thus, Eq. (13) was generalized replacing the constant 8 with a constant B, which is the function of rheology and the shear rate of the fluid, as presented in Eq. (14) (Reynolds of Calderbank and Moo-Young [16]):

$$Re_{CM} = \frac{D_a{}^2 N \rho (BN)^{1-n}}{k} \left(\frac{4n}{3n+1} \right)^n \qquad (14)$$

The value of parameter B is calculated using Eq. (15) for pseudoplastic liquids, with an error of approximately 10%:

$$B = (D_t/D_a)^2 / \left[(D_t/D_a)^2 - 1 \right] \qquad (15)$$

If the tank follows the standard geometry model proposed by Rushton, Costich and Everett [10], the value of parameter B is 11.

Tanguy et al. [18] presented a more reticent view as to the observation of Calderbank and Moo-Young [16] in relation to the dependence of the shear rate with the rheology of the liquid.

The researchers investigated with various types of impellers such as anchor-type and observed that variations in the behavior index (n) in the range 0.3–0.95 did not provide a significant variation of the constant k_s and parameter B. Thus, they concluded that the Metzner and Otto model [15] for being simpler should be applied.

However, the concept presented by the researchers can be extended to the axial impellers with four paddles and radial turbine type. For example, the value of the constant k_s for the radial impeller turbine is 11.5, while the value of B is 11, presenting a deviation of 4.3%, which in terms of engineering design is little significant.

Soon it is recommended in tank projects with agitation, mixing and heat transfer of non-Newtonian liquids, the use of Reynolds by Metzner and Otto [15] is recommended for being simpler to calculate and with excellent accuracy.

The researchers previously mentioned found a relation to the shear rate with the rotation of the mechanical impeller through an empirical model based on data obtained in an experimental way. However, in the last decades, the numerical solution by computational fluid dynamics (CFD) has been employed to solve the equation of the amount of movement (Eq. (10)) for complex geometries, which

enables the determination of the field of speeds and thus the shear stresses at any point in the domain.

Ameur and Bouzit [14] studied the power consumed by a two-blade radial mechanical impeller in a tank without baffles in the agitation of a pseudoplastic liquid. The authors used the model of the law of the powers (Eq. (8)) for the prediction of the apparent viscosity and varied the index of behavior of the liquid between 0.7 and 1.0.

With the results obtained in the simulation, a theoretical model was determined to predict the number of power (Eq. (16)) according to a generalized Reynolds number (Eq. (17)), the height of the liquid level (H), diameter impeller (D_a), tank diameter (D_t) and rheology of the liquid:

$$N_p = \frac{0.000209}{Re_o}\left(114.1 + 0.56e^{\frac{n}{3}}\right)\left[170 - 166e^{\left(-\frac{H}{1.2D_t}\right)}\right]\left[6.2e^{\left(\frac{D_a}{0.23D_t}\right)}\right] \qquad (16)$$

$$Re_0 = \frac{\rho N^{2-n}D_a^2}{k} \qquad (17)$$

Eq. (16) has validity for agitation of pseudoplastic liquids in tanks without chicane with Reynolds between 0.1 and 10. The results obtained in the simulation were validated with the experimental work of Bertrand and Couderc [19] so that there was an excellent adherence of the experimental data with Eq. (16).

In terms of design, this work contemplate very low Reynolds number. In most cases found industrially, the values for the Reynolds are in the range of 200 to 800, so that the results obtained in the simulation should be uses with caution.

A phenomenon of difficult prediction in the agitation of Newtonian and non-Newtonian liquids is turbulence, which can be defined in a few words such as a transient, rotational event and fluctuations in velocity components. These variables make the analysis of the equation of the movement very complex, originating terms, such as the Reynolds tensor, which requires experimental parameters to give a closure to the equation.

Wu [20] studied the application of six turbulence models, such as the "Reynolds stress model", in the agitation of non-Newtonian fluid in a tank with mechanical impeller. The tank under study is an anaerobic reactor and non-Newtonian fluid is a mud characterized as pseudoplastic following the model of the law of powers with consistency index (n) ranging from 0367 to 1000.

Turbulence models are applied with the agitation from an axial impeller with four flat blades at 45° (PBT) and a modification of the same with curved blades. The author presents with detail the mathematical treatment of the constitutive equations and the difficulty of obtaining the results in the simulation by CFD, because in this case, he considered the transient process (which exponentially increases computational time). Finally, the results were validated experimentally and there was a distancing of 30% of the results.

Taking into account the complexity from the analysis of turbulence in tanks with agitation, the variation of rheological properties and errors of the numerical solution, the deviation presented is acceptable and the study becomes very relevant for the theme in question.

Sossa-Echeverria and Taghipour [21] evaluated the use of axial-type impellers placed on the lateral side of the tank aiming to stir pseudoplastic liquids. The CFD tool was used for the analysis of the vector field formed by the impellers, and the study was conducted in the laminar region, with Reynolds between 10 and 200.

The results were experimentally validated with the agitation of Carbopol solutions and the field of velocities was obtained by the particle image velocimetry (PIV) technique. In this technique, spherical particles of a reflective material are

placed in the tank and a laser positioned orthogonal to the tank wall, which emits radiation and the same is reflected in the moving particles, thus enabling in real time the determination of vectors. There was an excellent adherence between the experimental data and those predicted by the simulation, so the authors provided a diagram of the number of power according to the Reynolds number.

In general, in terms of design, CFD simulation has been quite useful in estimating parameters during agitation without the need to perform a large amount of experiments, however, it should not be forgotten that the experimental results are validation of the models obtained in the simulations.

4. Heat transfer

4.1 Calculation of the thermal exchange area

Heat exchangers are chemical and biochemical reactors and in tanks with mechanical impellers are the jackets, the helical coils, the spiral coils and the vertical tube baffles [22]. The vertical tube baffles in addition to exchanging heat with the stirring fluid avoid the formation of vortices that can be harmful to the quality efficiency of the mixture [23].

Optimally, the heat exchange system of a tank is projected on a permanent basis through the classical design equation (Eq. (18)):

$$A = Q/U.LMTD \qquad (18)$$

The coefficient U usually called the overall heat transfer coefficient or the global coefficient of performance of the heat exchanger is the parameter that relates the mechanisms of thermal exchange (conduction, convection and radiation), the flow, the properties of the liquid and the geometry of the control volume with the thermal exchange area.

In Eq. (18), the coefficient U is assumed to be constant independent of the temperature variation and liquid viscosity, besides being invariant with the process time. In the case of liquids with high viscosities (such as heavy oils), the coefficient U may have variations during the process due to the viscosity gradient that occurs as a function of temperature, in which case this problem is eliminated with the introduction of caloric temperature (valid for Newtonian liquids).

However, in the case of non-Newtonian liquids, the approach of the caloric temperature is not adequate due to the variation of the apparent viscosity with the shear rates caused by the rotation of the mechanical impeller. In any case, the transient regime is very prominent in the heating or cooling of non-Newtonian liquids, which makes it impossible to use Eq. (18) for calculating the thermal exchange area [24].

Rosa and Moraes [1] presented a deduction from the first law of thermodynamics applied to a tank in Newtonian fluid agitation for transient regime operations; however, at that moment the effect of viscous dissipation in the process as described in Item 2 of this text was not considered, which for non-Newtonian liquids is indispensable.

In order to contemplate the effect of viscous dissipation, consider a tank perfectly insulated in which a non-Newtonian liquid is placed and a mechanical impeller. This liquid will be heated by another liquid through a heat exchanger (which can be a jacket or serpentine or vertical tubular chicane). The operation will occur in batch. Eq. (19) illustrates the first law of thermodynamics in terms of rate:

$$Q_{vc} - W_{vc} + w_e(h_e + v_e^2/2 + gz_e) - w_s(h_s + v_s^2/2 + gz_s) = dE_{vc}/d\theta \tag{19}$$

Consider the following hypotheses for design: (1) As the tank is perfectly insulated, there are no liquid flows of heat between the tank and the external environment, (2) the kinetic and potential energy variations are negligible in relation to the enthalpies and (3) perfectly agitated tank. The term regarding to the work is related to the contribution of the viscous dissipation generated by the rotation of the impeller, which will be considered.

The derivative present in Eq. (19) is referent to the total energy variation of the control volume. With the hypotheses mentioned above, this derivative summarizes the variation of internal energy so that it is equal to the variation of enthalpy for incompressible liquids. Finally, the enthalpy can be written in function of the specific heat as the constant pressure and the derivative is summarized as the "bulk" temperature variation (perfectly agitated tank condition).

Eq. (20) is a presented Eq. (18) after the hypotheses mentioned:

$$w_h c_{ph}(T_1 - T) - W_{vc} = M c_{pc} \frac{dt_b}{d\theta} \tag{20}$$

The left-hand side of Eq. (18) is a source of thermal exchange, so that it can be written according to coefficient U (Eq. (21)).

$$w_h c_{ph}(T_1 - T) = UALMTD \tag{21}$$

Replacing Eq. (21) in Eq. (20), you have:

$$UA\{(T_1 - T)/[ln(T_1 - t_b/T - t_b)]\} - W_{vc} = M c_{pc} \frac{dt_b}{d\theta} \tag{22}$$

The integration of Eq. (22) depends on two analyses: (1) The coefficient U must be constant during the process; otherwise, it is necessary to know its variation according to the apparent viscosity and time. (2) It is necessary to determine an expression for the work (viscous dissipation).

The work provided by the impeller to the liquid is equivalent to the power consumption, so the same can be calculated by the power number, according to Eq. (23).

$$W_{vc} = N_p \rho N^3 D_a^5 \tag{23}$$

According Rosa [24], the heating of non-Newtonian liquids of the pseudoplastic type occurs with the constant U coefficient. Therefore, replacing Eq. (23) in Eq. (22), considering the constant U coefficient and integrating, you have:

$$ln\left(\frac{K_1 w_h c_{ph}(T_1 - T_b) - W_{vc}}{K_1 w_h c_{ph}(T_1 - T_{b_0}) - W_{vc}}\right) = -\frac{K_1 w_h c_{ph}}{M c_{pc}}\theta \tag{24}$$

Eq. (24) should be solved by trial and error for the constant K_1 until the convergence between the right and left limbs occurs. The thermal exchange area is calculated from Eq. (25).

$$A = \frac{w_h c_{ph}}{U} ln\left(\frac{1}{1 - K_1}\right) \tag{25}$$

The solution of Eq. (25) implies the determination of coefficient U, which will be detailed in Section 4.2.

4.2 Overall heat transfer coefficient

Mathematically, the coefficient U is written according to the thermal exchange mechanisms however, in the heating and cooling of Newtonian and non-Newtonian liquids in tanks, convection is the predominant mechanism, so that the conduction and radiation can be despising, according to Eq. (26).

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} \tag{26}$$

The internal n coefficient convection (h_i) is referring to the liquid traversing the interior of the heat exchanger, in this case the jacket or the serpentine or the vertical tubular baffle. Usually these liquids are Newtonians, so that the expressions for the calculation of the internal coefficient of convection are known in the literature.

Rosa and Moraes [1] discussed in detail the main equations used for the coefficient h_i in tanks with agitation, so in this topic, the main discussion will be in relation to the external convection coefficient (h_o).

The coefficient h_o represents the mechanism of forced convection occurring in the agitation of the non-Newtonian liquid, which depends on the geometry of the tank and the mechanical impeller, the flow and the physical properties of the liquid. The convection coefficient is calculated on the external surface of the heat exchanger based on the condition of equal thermal flow between the liquid traversing the interior of the surface and the liquid being agitated (Eq. (27)).

$$h_o = -\frac{k'(\partial T/\partial r)_{r=R}}{(T_s - T_{00})} \tag{27}$$

The solution of Eq. (27) depends on the determination of the temperature profile of the non-Newtonian liquid in agitation. The temperature profile can be calculated from the application of the energy equation (Eq. (28)) to the control volume (tank with agitation).

$$\rho c_p DT/D\theta = k'\nabla^2 T + \eta\Phi' + \beta' T Dp/D\theta + q''' \tag{28}$$

It is not possible to calculate the convection coefficient in an analytical way that the solution of Eq. (28) depends on the previous solution of the equation of the movement quantity (Eq. (10)) and as previously seen is not feasible due to the complex geometry of the tank.

As an alternative, as in power consumption, the convection coefficient is obtained by an empirical model, obtained through the Buckingham's Pi theory (Eq. (29)):

$$\frac{h_o D_t}{k'} = K\left(\frac{ND_a{}^2\rho}{\eta}\right)^a \left(\frac{C_p\eta}{k'}\right)^b \left(\frac{\eta}{\eta_w}\right)^c \tag{29}$$

In Eq. (29), the coefficient h_o is calculated from the number of the Nusselt, being a function of the number of Reynolds, Prandtl number and relation between the apparent viscosities calculated at the "bulk" temperature of the liquid in agitation and the viscosity calculated at the temperature of the external surface of the heat exchanger. The constant and exponents of Eq. (29) are experimentally determined according to the type of tank, the type of mechanical impeller, the type of heat exchanger and whether the tank contains baffles.

However, the use of Eq. (29) depends on two parameters: (1) The choice of a rheological model suitable for non-Newtonian fluid in agitation and (2) The model that will relate the shear rate occurring with the rotation of the mechanical impeller.

As quoted, most of the non-Newtonian liquids are pseudoplastics and follow the model of the law of the powers, so that the shear rates and the rotation of the mechanical impeller are mainly related by the models of Metzner and Otto [15] and Calderbank and Moo-Young [16]. But as if seen ahead, several researchers have used other ways to relate the shear rate with the rotation of the mechanical impeller.

Carreau et al. [25] conducted a study in the heating and cooling of pseudoplastic fluids (aqueous solutions of carboxymethylcellulose and Carbopol 934) in a tank with an internal diameter of 0.76 m equipped with an axial impeller with four paddles inclined to 45° and a simple jacket. The study was carried out in transient regimen and the authors neglected the effect of viscous dissipation caused by mechanical impeller to agitation fluid.

The authors used the method of Wilson similar to using by Chilton et al. [26] to obtain the parameters of Eq. (29); however, they noticed a convergence problem in the method in relation to obtaining the exponent of the number of Prandtl, due to the variation of the apparent viscosity with the rotation of the mechanical impeller.

In this way, the authors introduced a concept called differential viscosity (μ_d) that is nothing more than the relationship between the derivative of shear stress by the shear rate; however, for values of shear rates above 500 s^{-1}, the differential viscosity remains constant, as shown in Eq. (30). This is a reasonable consideration because at high rotations of the mechanical impeller, the apparent viscosity of pseudoplastic fluids tends to remain constant:

$$\mu_d = \left(d\tau / d\nabla \vec{v} \right)_{\nabla \vec{v} \to 00} \tag{30}$$

The authors presented a variation of the Reynolds number based on the concept of Metzner and Otto [15] and Calderbank and Moo-Young [16] as shown in Eq. (31).

$$Re_{Carreau} = \frac{N^{2-n} D_a{}^2 \rho}{(k/8)\left(\frac{6n+2}{n}\right)^n} \tag{31}$$

The authors proposed two correlations, one for heating and one for cooling, according to Eqs. (32) and (33), respectively:

$$Nu = 3.41 (Re_{Carreau})^{0.67} (Pr_{Carreau})^{0.33} \tag{32}$$

$$Nu = 1.43 (Re_{Carreau})^{0.67} (Pr_{Carreau})^{0.33} \tag{33}$$

In Eqs. (32) and (33), the absence of the relative term of viscosities in temperature "*Bulk*" by the temperature of the wall is noted, which caused a great difference between the proportionality constants, 3.41 for the heating and 1.43 for the cooling, although the inclinations are the same, based on the equality of the exponents of the Reynolds and Prandtl numbers. Eqs. (32) and (33) have an error of 11.8 and 14.0%, respectively.

In order to engage the heating and cooling phenomena in a single equation, the authors used the differential viscosity as a function of a modified exponent, as shown in Eq. (34).

$$Nu = 1.474 (Re_{Carreau})^{0.70} (Pr_{Carreau})^{0.33} \left(\frac{\mu}{\mu_{dw}} \right)^{0.24/n} \tag{34}$$

The error of Eq. (34) is 19.3%, slightly higher than the error of Eqs. (30) and (31). In terms of engineering design, the Nusselt equation, if possible, should be able to predict heating and cooling in the same system of agitation, aiming at ease for the engineer, but without losing the predictability of the phenomenon. Eq. (34) has validity for Reynolds between 100 and 5000, Prandtl in the range of 100–800, and consistency index between 0.343 and 0.633.

Hagedorn and Salamone [27] carried out a study of heating pseudoplastic fluids in a tank operating in a batch with a jacket, aiming to obtain an expression that would allow calculating the convection coefficient with axial, radial and anchor impellers. The authors applied the constitutive equations of continuity, amount of movement and energy in cylindrical coordinates and resolved them with dimensional analysis, as presented in Eq. (35). In the dimensional analysis, the authors used the concept of Metzner and Otto [15] for the average shear rate:

$$Nu = K Re_{MO}^{\left[\frac{a}{n+1}+b\right]} Pr_{MO}^{c} (Vi_{MO})^{d} \left(\frac{D_t}{D_a}\right)^{e} \left(\frac{W}{D_a}\right)^{f} n^{g} \qquad (35)$$

Eq. (35) is valid for Reynolds between 35 and 680,000, Prandtl in the range of 2–23,600 and consistency index between 0.36 and 1.0. A large variation of the Reynolds number is observed, which is explained using water (low viscosity) and high rotations. **Table 2** shows the exponents of Eq. (35) for the impellers used in the work.

Sandall and Patel [28] analyzed the heating of pseudoplastic fluids in tank equipped with Jacket, using two types of mechanical impellers, radial turbine with six flat blades and an anchor-type impeller. The fluids used were aqueous solutions of Carbopol and also two Newtonian fluids (water and glycerin), aiming to increase the number of Reynolds as it was done in the work of Hagedorn and Salamone [27].

The tank has an internal diameter of 0.18 m and baffles were used with the radial impeller and, with the anchor-type impeller, the tank was without baffles. The average shear rate required for calculating the adimensionals of Eq. (29) was based on the concept of Calderbank and Moo-Young [16]. Eqs. (36) and (37) present the Nusselt expressions for the radial impeller and anchor-type, respectively:

$$Nu = 0.315 (Re_{CM})^{0.67} (Pr_{CM})^{0.33} (Vi_{CM})^{0.12} \qquad (36)$$

$$Nu = 0.482 (Re_{CM})^{0.67} (Pr_{CM})^{0.33} (Vi_{CM})^{0.12} \qquad (37)$$

Eq. (36) has validity for Reynolds between 80 and 93,000, Prandtl in the range of 2.1–644 and consistency index between 0.35 and 1.0. Similarly, Eq. (37) is valid for Reynolds in the range 320–89,600, Prandtl between 2.1 and 644 and consistency index in the range from 0.35 to 1.0.

Comparing the error of the Nusselt equation to the turbine-type impeller (Eq. (36)), around 18%, with the error of the equation proposed by Hagedorn and

Impeller	K	a	b	c	d	e	f	g
Anchor	0.56	1.43	0	0.30	0.34	—	—	0.54
Paddle	2.51	0.96	0.15	0.26	0.31	−0.46	0.46	0.56
Axial	0.55	1.28	0	0.30	0.32	−0.40	—	1.32
Radial	3.57	1.25	0	0.24	0.30	—	0	0.78

Table 2.
Exponents of Eq. (35). Pseudoplastic fluids in tank with jacket.

Salamone [27], about 26.8%, it is noted that the model of Sandall and Patel [28] has a better fit to the phenomenon observed.

In 1971, Martone and Sandall [29] did a study of heat transfer in the agitation of sludge composed of calcium carbonate in volumetric concentrations of 12%, 15%, 23% and 26% and also water and glycerin to obtain a wide range of the Reynolds number. Rheology of these slurries follows the rheological model of Binghan (Eq. (38)), which has a yield stress.

$$\tau = \tau_0 + \mu_B \nabla \vec{v} \tag{38}$$

In this way, the number of Reynolds and, Prandtl and viscous ratio of Eq. (29) should be modified according to the initial voltage. Calcium carbonate slurries are heterogeneous suspensions in such a way that the influence of the volumetric fraction of the solid in the Nusselt number should be accounted for. Therefore, Eq. (29) is rewritten as presented in Eq. (39).

$$\frac{Nu}{(\varphi/\varphi - 1)^a} = K\left(\frac{D_a{}^2 N\rho}{\mu_b + (\tau_0/BN)}\right)^a \left(\frac{\mu_{ba} + (\tau_0/BN)}{k}\right)^b \left(\frac{\mu_b + (\tau_0/BN)}{(\mu_a + (\tau_0/BN))_w}\right)^c \tag{39}$$

Table 3 presents the constant and exponents of Eq. (39) for the radial impeller turbine and anchor used in the study, as well as the range of validity of the adimensional variables.

Heinlein and Sandall [30] used the heating of pseudoplastic fluids (aqueous solutions of Carbopol) and Binghan fluids (watery sludges) in a tank with an internal diameter of 0.18 m and anchor-type impeller, as shown in Eq. (40). The average shear rate used was based on the concept of Metzner and Otto [15].

$$Nu = C_1 (Re_{MO})^{0.50} (Pr_{MO})^{0.33} (Vi_{MO})^{0.18} \tag{40}$$

The values of the constant C_1 of Eq. (40) for some tank diameter ratios by impeller diameter is provided in **Table 4**.

Mitsuishi and Miyairi [31] considered that the rheological model described by the law of powers is not comprehensive enough to represent the entire amplitude of shear stress variation with deformation rates. Thus, they proposed to conduct an experimental study in the investigation of the heat transfer of non-Newtonian fluids in agitated tanks based on the Ellis rheological model, as presented in Eq. (41).

Impulsor	K	a	b	c	d	Reynolds	Prandtl	φ (%vol)
Radial	0.536	0.66	0.33	0.14	0.065	196–79,800	2.2–702	0–0.0113
Âncora	0.315	0.66	0.33	0.20	0.072	336–94,800	2.2–621	0–0.0116

Table 3.
Parameters of Eq. (39).

D_t/D_a	C_1	Error (%)
1.027	0.691	17.4
1.100	0.531	29.7
1.220	0.546	29.7

Table 4.
Constant values C₁ of Eq. (40).

$$\tau = -\left\{\frac{1}{\eta_0}\left[1 + \left(\frac{\sqrt{\frac{1}{2}(\tau : \tau)}}{\tau_{\frac{1}{2}}}\right)^{\alpha'-1}\right]\right\}\nabla\vec{v} \qquad (41)$$

The rheological model of Ellis can estimate shear stresses for shear rate values from 0.01 to 1000, due to the model having three adjustment constants; η_0, the apparent viscosity at the limit of the shear rate tending to zero, $\tau_{1/2}$; the shear stress corresponding to 50% of the variation of the deformation rate; and α, a constant of adjustment of the experimental data.

The tank under study has a diameter of 0.4 m and two radial impellers with two paddles with two diameters were used, one of 0.20 m and the other of 0.322 m. The tank doesn't contain baffles. The heating of the aqueous solutions of carboxymethyl-cellulose (CMC) and polyethylene (PO) was given by a simple jacket. Due to the complexity of Eq. (41), the authors arrived to a conclusion that the predominant flow with the radial impeller is of the tangential type. By similarity analysis, they deduced that the flow between two coaxial cylinders (as in a Searle-type-viscometer), as shown in **Figure 5**, was the same thing that happened during the agitation.

In **Figure 5**, an inconvenience is observed, referring to the hypothesis adopted by the authors regarding the similarity of agitation with the flow between the coaxial cylinders. As the smallest impeller used has 50% of the tank diameter and the other around 80.5%, a large part of the flow present in the agitation will be without phenomenological explanation, which in terms of design, is difficult to apply. The authors presented an equation for the apparent tangential viscosity, as shown in Eq. (42).

In the case of non-Newtonian fluids with high viscosities, the torque generated by the mechanical impeller when moving the fluid inside the tank should be considered; thus the apparent viscosity is defined in relation to the voltage generated in the tank wall, considering the hypothesis of not slipping the fluid near the wall. Thus, this apparent viscosity is also called apparent viscosity of potency (Eq. (43)):

$$\eta_{tan} = \frac{\eta_0}{2}\left\{\frac{[1 - (D_a/D_t)2]/(D_a/D_t)2}{\frac{1}{2}\left(\frac{1}{(D_a/D_t)2} - 1\right) + \frac{(T_{or})^{\alpha-1}}{2\alpha}\left(\frac{1}{(D_a/D_t)^{2\alpha}} - 1\right)}\right\} \qquad (42)$$
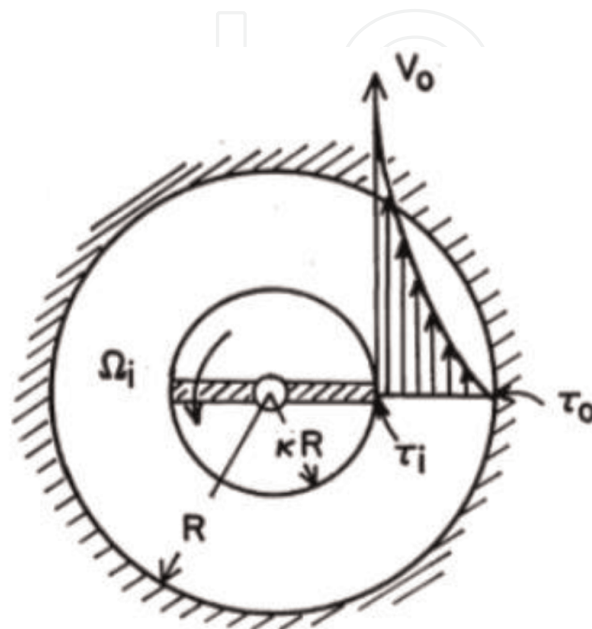

Figure 5.
Flow between two coaxial cylinders [31].

$$\eta_{pot} = \eta_0 \left/ \left\{ 1 + \frac{4}{\alpha+3} \left[T_{orw} \left(\alpha 8N \frac{\eta_0}{\tau_{\frac{1}{2}}} \right) \right]^{\alpha-1} \right\} \right. \tag{43}$$

With the two apparent viscosities defined by Eqs. (42) and (43), the expressions for the number of Nusselt tangential and on the wall, are obtained by Eqs. (44) and (45), respectively.

$$Nu_{tan} = K_{tan} \left(Re_{tan} \right)^{a''} \left(Pr_{tan} \right)^{0.33} \left(Vi_{tan} \right)^{0.14} \tag{44}$$

$$Nu_{pot} = K_{pot} \left(Re_{pot} \right)^{a'''} \left(Pr_{pot} \right)^{0.33} \left(Vi_{pot} \right)^{0.14} \tag{45}$$

The values of Nusselt tangential and Nusselt on the wall (power) are obtained graphically through a dimensionless variable defined as j_{Tan} and j_{Pot} respectively. You can observe in **Figure 6** the graphic ratio for the impeller with 50% of the tank diameter. The study is conducted by Mitsuishi and Miyairi [31] and also analysis of the heat transmission with a helical impeller wrapped in a *draft tube*.

Shamloo and Edwards [32] studied the heat transfer in the agitation of Newtonian and non-Newtonian fluids with high viscosities in a tank with a diameter of 0.15 m and another tank of 0.40 m. The mechanical impeller chosen was the helical type, and the heating jacket was the spiral type. The fluids used were chocolate, aqueous solutions of carboxymethylcellulose, glycerin, lubricating oils, silicon, sucrose solution, and solutions of Carbopol 940.

The authors proposed a model based on Eq. (29) for the Nusselt number adding two terms referring to the helical impeller: (1) Term referring to the number of impeller blades and (2) term referring to the distance between the impeller tip to the tank wall, as shown in the Eq. (46).

$$Nu = 0.568(Re_o)^{0.23}(Pr_o)^{0.23}(n_b)^{0.23}(C'/D_t)^{-0.54} \tag{46}$$
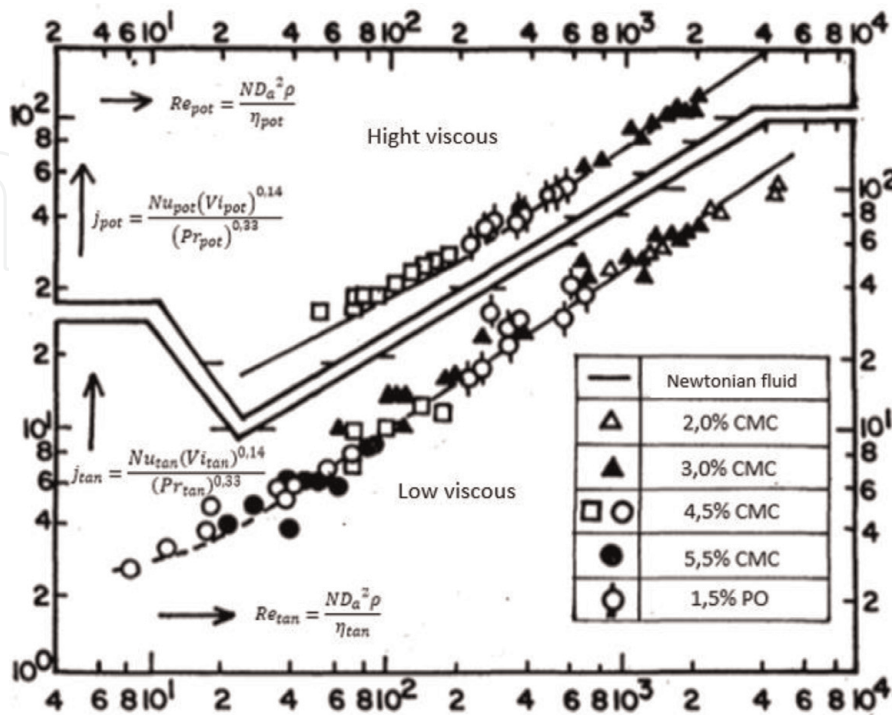

Figure 6.
Graphical relations for Eqs. (44) and (45) [31].

The Reynolds number and the number of Prandtl are given by Eqs. (47) and (48), respectively. It is noteworthy that these equations are proposed modified by Shamloo and Edwards [32], in which the average shear rate is initially based on Metzner and Otto [15] and adapted for helical impeller as presented in Eq. (49).

$$Re_o = ND_a{}^2\rho/\{[k34 - 144(c'/D_t)]N\}^{n-1} \tag{47}$$

$$Pr_o = c_p\{[k34 - 144(c'/D_t)]N\}^{n-1} \tag{48}$$

$$\nabla\vec{v} = [34 - 144(C'/D_t)]N \tag{49}$$

It is observed in Eq. (46) that the term for apparent viscosity variation was not included due to the narrow range of Reynolds number used that was between 0.01 and 10, which characterizes a fully laminar flow so that in this type of flow, the variation between the viscosities at the temperature *bulk* and the wall is negligible.

Suryanarayanan et al. [33] conducted a study of heating and cooling of pseudoplastic fluids (carboxymethylcellulose solutions) in a tank with baffles containing a jacket and a tank containing a helical serpentine. The impeller used in the study was a turbine type with four flat blades. The authors varied the impeller diameter in 78, 118 and 152 mm, the liquid level in the tank between 90 and 214 mm, the diameter of the helical serpentine in the range of 169–278 mm and the serpentine tube in the range of 15.9–22.2 mm. The average shear rate used in this work was based on the concept of Calderbank and Moo-Young [16].

In this study, the authors aimed to determine the effect of the variation of the level height, the diameter of the serpentine and the heat exchange itself, in addition to the traditional analysis of the number of Reynolds and Prandtl in the number of Nusselt. Eq. (50) presents the model obtained for the radial impeller with the use of the helical serpentine, with an experimental error of 7.1%, and in an analogous way and Eq. (51) provides to the model for the jacket with a 7.8% error:

$$Nu = 0.21(Re_{CM})^{0.66}(Pr_{CM})^{0.33}\left(\frac{D_a}{D_t}\right)^{0.17}\left(\frac{H}{D_t}\right)^{0.13}\left(\frac{D_c}{D_t}\right)^{-0.29}\left(\frac{D_i}{D_t}\right)^{-0.45} \tag{50}$$

$$Nu = 0.22(Re_{CM})^{0.63}(Pr_{CM})^{0.33}\left(\frac{D_a}{D_t}\right)^{0.14}\left(\frac{H}{D_t}\right)^{0.09}\left(\frac{D_c}{D_t}\right)^{-0.21}\left(\frac{D_i}{D_t}\right)^{-0.35} \tag{51}$$

It is observed in Eqs. (50) and (51) that the authors neglected the effects of the apparent viscosity ratio at the temperature *bulk* apparent viscosity at the wall temperature, since they added to the model proposed in Eq. (29) four terms referring to the effect of the stirring system geometry. The relationship of these geometric terms generates very small numbers, although the exponents are significant, in such a way that more than 90% of the response of the Nusselt number in these equations is given only by the variation of the number of Reynolds and Prandtl. The validity range of the 62 equations for Reynolds between is 200 and 21,700, Prandtl in the range of 49–1220, and consistency index between 0.47 and 1.0.

Regarding the range of application of Eqs. (50) and (51), it should be satisfied for the Reynolds and Prandtl numbers and the employment of turbine type impeller with four flat blades.

Kai and Shengyao [34] did a study of heating and cooling of non-Newtonian fluids in agitated tanks with a different approach from the authors. Eq. (29) was modified in the following respects: (1) The Reynolds number contemplates the power consumed by the mechanical impeller as well as the Prandtl number, (2) a

term relative to the influence of the impeller diameter ratio by the tank diameter, and (3) a term relative to the Impeller blades, quantity of impellers on the shaft and blade angulation. In Eq. (52) these modifications are in a condensed manner:

$$Nu = K(Re')^a (Pr')^b (Vi')^c \left(\frac{D_a}{D_t}\right)^d \left(\frac{n_b n_n b sin\omega}{H}\right)^e \qquad (52)$$

The Reynolds number is calculated from Eq. (53) based on the concepts provided by Kai and Shengyao [34] presented in Eqs. (54), (55) and (56) and similarly, the Prandtl number in Eq. (57).

$$Re' = \frac{N D_a^2 \rho}{k} 0.4^{(1-n)} N^{[2-f(2-n)](1-n)/n} \qquad (53)$$

$$f' = \exp\left(-m Re_m\right) \qquad (54)$$

$$\nabla \vec{v} = k''^{1/n} N^{[2-f'(2-n)]/n} \qquad (55)$$

$$k'' = K_p D_a^5 \rho / 2\pi k_1 Vk \left(D_a^2 \rho/k\right)^{f'} \qquad (56)$$

$$Pr' = \frac{c_p k}{k'} 0.4^{(1-n)} N^{[2-f(2-n)](1-n)/n} \qquad (57)$$

The parameters m and k depend on the type of flow, the viscosity of the fluid and the type of impeller, however, the authors found the values of 0.00705 and 0.4 for m and k, respectively, based on the best conditions for adjusting the mathematical models.

As an example of Eq. (52), for cooling with vertical tube baffles and radial impeller, with a range of Reynolds between 26 and 6310, according to Eq. (58).

$$Nu = 1.19 (Re')^{0.67} (Pr')^{0.33} (Vi')^{0.17} \left(\frac{n_p n_n b sin\omega}{H}\right)^{0.74} \qquad (58)$$

Hai Devotta and Rao [35] studied the heat transfer with Newtonian and non-Newtonian fluids in a stirring system with the use of helical impeller and heating jacket. Two tanks were used, one with 75 mm and the other with 80 mm diameter, the height of the liquid level and the number of impeller blades ranged. The average shear rate was calculated from Eq. (59).

In Eq. (59), the expression for the calculation of the Nusselt number is displayed, which is valid for Reynolds in the range of 10–1000 and the consistency index between 0.45 and 1.0.

$$Nu = 0.55 (Re)^{0.48} (Pr)^{0.33} (Vi)^{0.14} (H/D_t)^{-0.44} \qquad (59)$$

Triveni et al. [5] conducted a study in the heating and cooling of castor oil, liquid soap, carboxymethylcellulose solutions and calcium carbonate solutions. The heat transmission was promoted in a tank with a diameter of 0.29 m, by a single-type helical serpentine immersed in the tank, and the impellers used were an anchor-type and a radial turbine. The average shear rate was based on the concept of Calderbank and Moo-Young [16].

In this study, the authors incorporated Eq. (29), the term concerning the formation of vortices (number of Froude), the influence of natural convection (number of Grashof) and, finally, the influence of aeration on heat transmission, as presented in Eq. (60).

$$Nu = K(Re_{CM})^a (Pr_{CM})^b (Vi_{CM})^c (Fr_0)^d (Gr_{CM})^e \left(\frac{v_g}{ND_a}\right)^f \qquad (60)$$

The influence of aeration, described by the term $\frac{v_g}{ND_a}$ is irrelevant in increasing heat transmission by both the axial impeller and the radial impeller. However, the effects of natural convection are considerable in the anchor-type impeller due to its low rotation of operation, favoring the formation of natural convection currents in the system. However, in relation to the turbine-type impeller, this effect is negligible due to the great turbulence achieved by this impeller. The Froude number presents significance only in systems without baffles or with low rotations, as in the case of the anchor-type impeller.

Eq. (61) presents the prediction of the number of Nusselt with the turbine-type impeller, despising the effects of natural convection and aeration:

$$Nu = 0.514 Re_{CM}{}^{0.598} Pr_{CM}{}^{0.335} Vi_{CM}{}^{0.112} Fr_0{}^{-0.179} \qquad (61)$$

Several works present countless forms of the Nusselt equation for each type of non-Newtonian fluid in agitation in the tanks, for example, Pimenta and Campos [36], who studied the effects of viscoelasticity on the heat transmission of Non-Newtonian solutions in tanks equipped with helical serpentine and laminar flow. Viscoelasticity can be represented by the Weissenberg number (Eq. (62)), which represents the relaxation time of the fluid after the application of a shear rate.

The authors also incorporated the Nusselt number prediction model with the effects of secondary flows occurring around the surface of the helical serpentines, characterized by the number of Dean (Eq. (63)). The proposed Nusselt equation (Eq. (64)) is valid for the heating of carboxymethylcellulose and xanthan gum, with Prandtl between 17 and 203, with a 30% error in the calculation of the Nusselt number. The average shear rate was based on the concept of Metzner and Otto [15]:

$$Wi = \lambda'' v / D_{ic} \qquad (62)$$

$$De = Re_{MO}(D_{ic}/D_c)^{1/2} \qquad (63)$$

$$Nu = 0.486 \left[\left(\frac{3n+1}{4n}\right)^{0.275n} \left(0.717 + 0.993\frac{D_{ic}}{D_c}\right) (Re_{MO}Pr_{MO})^{0.275} \right] \qquad (64)$$
$$(1 + 0.728\, De^{0.225})(\, Wi + 1)^{0.011(n-1)}$$

5. Example of project

One 1 m^3 internal diameter tank and containing four baffles will be used for the heating of 794 kg of an aqueous solution of carboxymethylcellulose (CMC) concentration of 1.0% (w/w) from 20 to 40°C. The heating will be carried out through a simple jacket, in which you will go through hot water with inlet temperature of 60°C with a flow rate of 2000 kg/h. The mechanical impeller employed is of radial turbine type with six flat blades, which provides a rotation of 100 rpm. Calculate the thermal exchange area (design) that the jacket must possess to make the solution warm up in 40 min. The tank is insulated with rigid polyurethane foam, so that heat loss to the external environment can be considered negligible. The dimensions of the tank and its internals follow the proposals by Rushton et al. [12]—See **Figure 4**. The solution of CMC is pseudoplastic type, following the model of the law of the powers for variation of apparent viscosity. **Table 5** shows the physical and

transport properties of cold and hot liquids in the average heating and cooling temperatures.

The process described occurs in transient and batch regime, so the thermal change area of the jacket is calculated from Eqs. (65) and (66) previously presented in Eqs. (24) and (25) (Item 4.1).

$$ln\left(\frac{K_1 w_h c_{ph}(T_1 - T_b) - W_{vc}}{K_1 w_h c_{ph}(T_1 - T_{b_0}) - W_{vc}}\right) = -\frac{K_1 w_h c_{ph}}{M c_{pc}}\theta \tag{65}$$

$$A = \frac{w_h c_{ph}}{U} ln\left(\frac{1}{1 - K_1}\right) \tag{66}$$

The non-Newtonian solution for being pseudoplastic was treated rheologically by the model of the power law and the relationship between the rotation of the mechanical impeller and the shear rate, described by the model of Metzner and Otto [15].

Initially, the Reynolds number of the solution at the average temperature of the non-Newtonian liquid should be calculated, in this case 30°C, according to Eq. (67).

$$Re_{MO} = \frac{N^{2-n}.D_a^{\;2}\rho}{k.k_s^{\;n-1}} = 340.83 \tag{67}$$

With N of (100/60) rps, n of 0.66, D_a of 0.33m (1/3 de D_t), ρ of 1010 kg/m³, k of 1.468 and k_s of 11.5 (**Table 1**). Usually, the number of power is obtained in graphs according to the type of impeller and the presence or not of baffles in the tank. In this example, the impeller is of radial type with six flat blades, so that the work of Metzner and Otto [15] shows the power number curve with the Reynolds number in the laminar flow.

Therefore, the power number read in the graph is presented in Eq. (68) and the work provided by the impeller the solution in Eq. (69).

$$N_p = 3.8 \tag{68}$$

$$W_{vc} = N_p \rho N^3 D_a^{\;5} = 69.5W \tag{69}$$

With ρ of 1010 kg/m³, N of (100/60) rps e D_a of 0.33 m

Replacing the provided data and the calculated work in Eq. (69), by trial and error, you get the constant K_1 with w_h of 0.556 kg/s, C_{ph} of 4180 J/kg°C (in T, de 60°C), T_1 of 60°C, T_b of 40°C, T_{b0} de 20°C, C_{pc} of 4580 J/kg°C e M of 794 kg.

Property	Water (45°C)	CMC 1% (30°C)
ρ (kg/m³)	985.62	1010
C_p JKG°C)	4189.9	4580
K′ (W/M°C)	0.686	0.624
μPa.S	0.000609	—
K (Pa. S^n)	—	1.468 (30°C); 0.95 in (45°C)
n	—	0.66 (30°C); 0.69 in (45°C)
β (1/s)	0.00013	—

Table 5.
Physical and transport properties [22].

$$K_1 = 0.47 \tag{70}$$

Therefore, Eq. (70) can be rewritten as:

$$A = \frac{1475.51}{U} \tag{71}$$

The coefficient U is obtained from the internal and external convection coefficients, as presented in Eq. (72) (previously presented in Eq. (26)).

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} \tag{72}$$

The internal coefficient convection (h_i) is referring to hot water traversing the jacket. According to Silveira [37], the coefficient h_i can be calculated for a simple jacket with the heating fluid seeping upward according to Eq. (73).

$$h_i = k' 0.15 \left(\frac{c_p \mu}{k'}\right)^{\frac{1}{3}} \left[\frac{\rho^2 g \beta' LMTD}{\mu^2}\right]^{\frac{1}{3}} = 836.9 W/m^2{}^\circ C \tag{73}$$

With properties on the T_m of 45°C, β' of 1.3×10^{-4}°C^{-1} and LMTD obtained by $\frac{60-30}{\ln\left[\frac{60-40}{30-20}\right]} = 43.3$°C.

The external coefficient convection will be determined by the model provided by Hagedorn and Salamone [27] for the turbine-type radial impeller (Eq. (74)). Noting that Eq. (74) already has the exponents referring to the radial impeller, obtained previously in **Table 2**.

$$\frac{h_o D_t}{k'} = 3.57 Re_{MO}{}^{0.75} Pr_{MO}{}^{0.24} (Vi_{MO})^{0.30} n^{0.78} \tag{74}$$

The number of Prandtl and the relationship between viscosities are calculated in Eqs. (75) and (76), respectively.

$$Pr_{MO} = c_p k (k_s N)^{n-1} / k' = 3926.16 \tag{75}$$

$$Vi = \eta/\eta_w = \left[k(k_s N)^{n-1}\right]_{t_b} / \left[k(k_s N)^{n-1}\right]_{T_W} = 1.42 \tag{76}$$

with n of 0.66 for T_b of 30°C and n of 0.69 for T_w 45°C.

In Eq. (76), the "bulk" temperature was the average heating of the non-Newtonian solution and the temperature in the wall (T_w) was considered the average temperature of the heating fluid (hot water at 45°C). The jacket or any other heat exchanger in tanks is made of copper or some other metal that has high thermal conductivity, in order to resist the heat transmission between the jacket wall and the tank being negligible.

Replacing Eqs. (67), (75) and (76) in Eq. (74) with K from 0.624 W/m°C and D_t from 1 m, you have:

$$h_o = 1052.86 \ W/m^2{}^\circ C \tag{77}$$

Soon, the coefficient U by Eq. 72:

$$U = 466.27 \ W/m^2{}^\circ C \tag{78}$$

Finally, the thermal exchange area is obtained by replacing the coefficient U in Eq. (71):

$$A = 3.16 \, m^2 \tag{79}$$

The total height of the tank is calculated by the sum of the useful height (referring to the liquid level) and a safety margin to avoid transshipments. The height of the tank has the same value of the inner diameter, in this case 1 meter. Therefore, in addition to 15%, it is

$$H_{total} = H + 0.15H = 1.15m \tag{80}$$

The tank has a total lateral area of 3.61 m², while the jacket had its design area in 3.16 m², which shows the coherence of the calculation obtained.

6. Conclusion

In order to present a compact and practical way for undergraduate students and professionals in the field, the basic concepts of agitation with non-Newtonian liquids in tanks with mechanical impellers were presented in the text. The text also included a brief review of the design equations for tanks with heat exchange for non-Newtonian liquids. The work was completed with an example of a heating design with a pseudoplastic liquid jacket.

Acknowledgements

Nomenclature

A	thermal exchange area (m²)
Cp	specific heat (J/kg°C)
Cp_c	specific heat of the cold fluid (J/kg°C)
Cp_h	hot fluid specific heat (J/kg°C)
D_a	diameter of mechanical impeller (m)
D_t	internal diameter of tank (m)
E	distance from impeller to bottom of tank (m)
E_{VC}	total energy in control volume (J)
g	gravitational acceleration (m/s²)
h_e	specific enthalpy of the input mass flows in the control volume (J/kg)
h_s	specific enthalpy of output mass flows in control volume (J/kg)
h_i	internal convection coefficient (W/m²/°C)
h_o	external convection coefficient (W/m²/°C)
H	height of liquid level (m)
J	width of baffles (m)
k	consistency factor of the law model of the Powers (Pa.sⁿ)
k'	thermal conductivity (W/m°C)
L	blade length of mechanical impeller (m)

LMTD	logarithmic average of temperature differences (°C)
M	fluid mass in tank (kg)
n_b	number of mechanical impeller blades
n	powers Law consistency index
N	rotation of mechanical impeller (rpm)
p	pressure (Pa)
P	power consumed by mechanical impeller (W)
Q	heat generation rate (W/m²)
Q	heat transfer rate (W)
Q_{VC}	heat transfer rate between control volume (W)
T	hot fluid outlet temperature (°C)
T_1	hot fluid inlet temperature (°C)
T_m	average hot fluid temperature (°C)
t_b	"bulk" temperature (°C)
U	overall heat transfer coefficient (W/m²°C)
v	velocity (m/s)
V	tank volume (m³)
$\dot{V}$	volumetric flow rate (m³/s)
w_h	hot fluid mass flow (kg/s)
W	blade width of mechanical impeller (m)
W_{Vc}	work provided to control volume by impeller rotation (W)

Greek letters

ρ	specific mass (kg/m³)
μ	dynamic Viscosity (Pa.S)
μ_b	Binghan model Viscosity (Pa.S)
μ_d	differential viscosity (Pa.sⁿ)
μ_{Dw}	differential viscosity at wall temperature (Pa.sⁿ)
η	apparent viscosity (Pa.S)
η_w	apparent viscosity at wall temperature (Pa.S)
η_0	apparent viscosity with shear rate at zero (Pa.S)
θ	time (min)
β	coefficient of thermal expansion
Φ	viscous dissipation (J/kg)
∇	operator Nabla
τ	shear stress (Pa)
τ_0	initial shear stress (Pa)
ω	angle of the blades of the mechanical impeller
λ	fluid relaxation time (s^{-1})

Adimensionals Numbers

Fr_0	Froude number, $Fr = N^2 D_a/g$
Gr	number of Grashof, $Gr = \beta' g \Delta T D_a{}^3 \rho^2/\mu^2$
N_P	power number, $N_P = P/\rho N^3 D_a{}^5$
$Naked$	Nusselt number for stirring system, $Nu_o = h_o D_t/k'$
Pr_{Mo}	Metzner and Otto's Prandtl number, $Pr_{MO} = c_p k (k_s N)^{n-1}/k'$
Pr_{Cm}	Prandtl number of Calderbank and Moo-Young,
	$Pr_{CM} = c_p k \left(BN \left(\frac{4n}{3n+1}\right)^{n/(1-n)} \right)^{n-1}/k'$

Pr_o Prandtl number of Shamloo and Edwards
$$Pr_o = c_p\{[k34 - 144(c'/D_t)]N\}^{n-1}$$

Re Reynolds number for agitation, $Re = ND_a{}^2\rho/\mu$

Re_{Mo} Reynolds number of Metzner and Otto,
$$Re_{MO} = N^{2-n}D_a{}^2\rho/k(k_s)^{n-1}$$

Re_{MR} Reynolds number of Metzner and Reed, $Re_{MR} = \frac{\rho v^{2-n}D_i{}^n}{8^{n-1}k}\left(\frac{4n}{3n+1}\right)^n$

Re_{Cm} Reynolds number of Calderbank and Moo-Young,
$$Re_{CM} = ND_a{}^2\rho/k\left(BN\left(\frac{4n}{3n+1}\right)^{n/(1-n)}\right)^{n-1}$$

Re_m modified Reynolds number, $Re_m = N^{2-n}D_a{}^2\rho/k$

Re_{Carr} Reynolds Number of Carreau, $Re_{Carreau} = \dfrac{N^{2-n}D_a{}^2\rho}{(k/8)\left(\frac{6n+2}{n}\right)^n}$

Re_o Reynolds Number of Shamloo and Edwards
$$Re = ND_a{}^2\rho/\{[k34 - 144(c'/D_t)]N\}^{n-1}$$

Vi relationship between the apparent viscosity of the fluid in the "bulk" temperature by the viscosity of the fluid in the wall temperature, $Vi = \eta/\eta_w$

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