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Chapter

Optimum Efficiency on Broadcasting Communications

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Abstract

This chapter is devoted to review a set of new technologies that we have developed and to show how they can improve the process of broadcasting in two principal ways: that is, one of these avoiding the loss of transmission signals due to abrupt changes in sign of the diffraction index and the other, preventing the mutual perturbation between signals generating information leak. In this manner, we propose the join of several of the mentioned technologies to get an optimum efficiency on the process of broadcasting communications showing the theoretical foundations and discussing some experiments that bring us to create the plasma sandwich model and others. Despite our very innovative technology, we underline that a complete recipe must include other currently in use like multiple-input multiple-output (MIMO) simultaneously. We include some mathematical proofs and also give an academic example.

Keywords: wireless communications, optimal broadcasting, information packs, negative refraction index, communication theory, wave propagation through plasma

1. Introduction

Nowadays, one of the most innovative procedures to improve communications is the random scattering of microwave or radio signals that may enhance the amount of information that can be transmitted over a channel. This fact, from a mathematical point of view, is due to the growth of the phase space available for that channel, which provides a more rich mathematical base to define every single signal. In many recent papers, a common subject is the use of a broad range of base functions to span each signal. The hope is that every single collision of the initial signals will be scattered and reaches another phase space region providing additional information, but the increase of phase space involves a more complicated set of describing functions. A multiple scattering of the obstacles enlarges the effective aperture in a time-reversed process for acoustic or electromagnetic signals when they are placed in random manner.

Another current tool is time reversal, or phase conjugation in the frequency domain, where a source at one location transmits sound or electromagnetic waves, which are received at another place, time reversed (or phase conjugated), and retransmitted. The effect is to eliminate noise pollution.

Despite the existence of the mentioned resources and others like multiple-input multiple-output (MIMO), many problems survive, but fortunately, we have proposed some additional ways to improve the broadcasting by diminishing the information loss. Some of our results are based on communication theory and others in the mathematical properties of particular integral equations and their solutions.

Through the present chapter, we introduce for convenience a hypothetical discrete system in order to write finite matrices. But we can certainly extend the validity of our expressions as we will see, even for both discrete and continuum systems provided the involved potentials fulfill very general conditions not discussed in the present work.

In the same manner, because the formalism we have developed for the study of time reversibility refers to acoustic systems, we recall that the scalar wave equation for acoustic signals can be written as:

$$k(\mathbf{r})\frac{\partial^2 f(\mathbf{r},t)}{\partial t^2} = \nabla^2(f(\mathbf{r},t)/\rho(\mathbf{r}))$$

(1)

We now describe the quantities appearing in Eq. (1), $\rho(\mathbf{r})$ represents the mass density and $k(\mathbf{r})$ the compressibility of the propagation medium, while $f(\mathbf{r},t)$ is the acoustic signal.

Because the wave equation is of second order in time, we can talk about time reversibility, and then allows solutions, which travel toward the future or the past. An efficient time reversal requires to ensure that the system be ergodic, making possible that the signal may travel both senses in time. To improve focusing, we must describe the signal propagation towards the future or past by means of equations of the same type [18, 22, 27] that is both directions inhomogeneous or both homogeneous. Linearity permits that a signal traveling toward the past can be written with the aid of the integral equation:

$$f(\mathbf{r};T-t) = f^{(\circ)}(\mathbf{r};T-t) + \int_V \int_{-\infty}^{\infty} U^*(\mathbf{r}')G^{(\circ)*}(\mathbf{r}',\mathbf{r};T-t',t)f(\mathbf{r}';T-t')dt'dV'$$

(2)

In Eq. (2), $G^{(\circ)*}(\mathbf{r}',\mathbf{r};T-t',t)$ is the free Green function, $U^*(\mathbf{r}')$ depicts the complex dispersion coefficients, and $f(\mathbf{r};T-t)$ is the returning signal that has traveled toward the past. The inhomogeneous term $f^{(\circ)}(\mathbf{r};T-t)$ is known as a sink term and makes both the outgoing and returning equations inhomogeneous integral equations. In Eq. (2), the parameter T represents the time during which the outgoing signal (the one traveling toward the future) is being considered and recording. It is observed experimentally [9] that the time-reversed signal has a definition of a 14th of , the wavelength of the used signal for acoustic signals but this is also true for electromagnetic waves. On several experiments [9, 10], Lerosey, de Rosny, Tourin, and Fink have shown that when such a source term is included, the apparent cross section is increased in two ways: first, the multiple scattering also multiplies the available phase space so when the time is reversed, the information is increased, and second, in the electromagnetic case, the sink term stimulates and triggers the braking of the confinement of the evanescent waves that also raise the information and in consequence the definition to level of about $\lambda/14$. In acoustics,

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the sink term consists in the operation of the source in reverse order; in the electromagnetic case, the sink term can be implemented with a crest of fine wires around the antennas.

2. Recovering the matrix equations

As we have said above and considering that from a strictly mathematical point of view, both the acoustic and electromagnetic waves achieve the same wave equation type (with a vector version in the electromagnetic case). Then, we can regain, without further ado, the vector matrix formalism [1–7, 11–14] which generalizes the discrete scalar time reversal acoustic model and includes an original model for discrete broadcasting systems that we have called the plasma sandwich model (PSM) [8, 16–18] and we put some associated parameters appeared on it into the named vector matrix formalism (VMF) [8, 20, 24]. But we must underline that is the resonant behavior the one must be considered for increasing efficiency on communications and to achieve extraordinary resolution. To this end, we remember that a three-dimensional version of Eq. (1) can be written as the Fourier transform of an integral generalized homogeneous Fredholm's equation (GHFE) [21–24] for resonances, and does not matter if for acoustic or electromagnetic ones. To analyze the resonant behavior, we must eliminate the inhomogeneous term so we can write the following algebraic equation satisfied by the Fourier transform of the resonant waves:

$$[1 - \eta_R(\omega)\mathbf{K}^{(\circ)}(\omega)]_n^m \mathbf{w}_R^n(\omega) = 0 \tag{3}$$

where the kernel $\mathbf{K}^{(\circ)}(\omega)$ is the product of the Fourier transform of the free Green function $\mathbf{G}^{(\circ)}(\omega)$ with the interaction U (without loss of generality we can suppose that U does not depend on ω), so this can be written explicitly as:

$$\left[1 - \eta_R(\omega)\mathbf{G}^{(\circ)}(\omega)U\right]_n^m \mathbf{w}_R^n(\omega) = 0 \tag{4}$$

At this point, we must say that we could obtain a transfer matrix description [16–18] instead Eq. (4), but our last equation represents the core of the VMF version. The fact is there are important differences between the two formalisms; for example, VMF makes the time-reversal process easy. Of course, we are moving over a frequency domain and not over a time-dependent one, the former the appropriate in agreement with information theory applications. And certainly, the most important difference is that VMF formalism includes the concept of the resonant solutions.

3. Introducing the PSM parameters

One of the methods we have proposed is based on experiments executed by Xiang-kun Kong, Shao-bin Liu, Hai-feng Zhang, Bo-rui Bian, Hai-ming Li et al. [8] in which they put three layers of plasma joined and alternated with one of them magnetized in the core and the other two unmagnetized in the extremes of the device; when this plasma sandwich is submitted to an external electric potential, it is observed that for a range of values of the external potential, the refraction index is negative [15, 19]. When we analyzed those experiments, we conclude that for this range of the electric potential, the plasma sandwich brakes the confinement of the evanescent waves as occurs in a left-hand material and we proposed a model named

the plasma sandwich model for the behavior of the propagation media. Depending on the particular conditions of the propagation media, that is, depending of the values of the plasma sandwich parameters, and for particular conditions of the external electric potential, the propagation media may behave like the plasma sandwich and acquire a negative refraction index. In this section, we introduce the PSM parameters and find the resonant frequencies for a specific problem, underlying that resonant frequencies can be used only to associate an interval of frequencies of a real signal to a device that could be an antenna and not to a single emitted frequency by them; this is because resonant waves are released evanescent waves that vanish in the resource sites and not precisely information carriers. The frequency bands we can build from the resonant frequencies can be considered as convenient highways for the transit of information. Every kernel depends on the response of the media in circumstances that can vary for different time intervals. In this manner, we present an example very easy to work but in which is not relevant the particular behavior of the signal we used to get it. Next, we can find the resonant frequencies for an academic example. First, we choose an appropriate discrete kernel $\mathbf{K}^{(\circ)}(\omega)$, for convenience; in this particular kernel, we do not take into account the three components of the electromagnetic field (usually represented for the indices n and m). However, we propose a system constituted by two emitting antennas. One possible may be written [1, 3–7]:

$$\mathbf{K}^{(\circ)}(\omega) = \begin{pmatrix} \frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} & -i\frac{\cos(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} \\ i\frac{\cos(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} & \frac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} \end{pmatrix} \tag{5}$$

In kernel (5), we have introduced the plasma sandwich model (PSM) parameter δ, which is defined as:

$$\delta \, .. \, \kappa \bar{d}_M \tag{6}$$

Definition (6) involves κ with the physical meaning of the wave number of an incident beam that interacts with the magnetic and electric fields in a way that the whole kernel is the expressed in Eq. (5); $\bar{d}_M$ is the average thickness of a plasma-magnetized layer that generates this interaction; parameter ω_p is the average value for the plasma frequency in the magnetized plasma layer which can be written in terms of the local electron concentration in the layer as:

$$\omega_p = \frac{1}{2\pi}\left(\frac{Ne^2}{m\varepsilon_0}\right)^{\frac{1}{2}} \tag{7}$$

In this definition, N is the electron concentration, e is the electronic charge, and ε_0 is the permittivity of vacuum.

It is possible to note that any change in the parameter values gives different broadcasting conditions [5]. PSM suggests that there is not a single stationary set of iterated layers but a bunch of sets evolving in time and in consequence with different effects for each frequency. We must remember that the equation to solve is Eq. (3) where,

$$\mathbf{K}_m^{n(\circ)}\left(\mathbf{r}', \mathbf{r}; \omega\right) = \begin{cases} 0 \ \textit{if } \mathbf{r}' = \mathbf{r} \\ U^{nm}(\mathbf{r}')\mathbf{G}_\omega^{nm(\circ)}(\mathbf{r}', \mathbf{r}) \ \textit{if } \mathbf{r}' \neq \mathbf{r} \end{cases} \tag{8}$$

The last two ubiquitous conditions to achieve resonance are the vanishing of Fredholm's determinant for Eq. (4), and that Fredholm's eigenvalue λ equals to 1 [6, 11, 22, 23]. The last two conditions give us the expected resonant frequencies for the system constituted by two antennas dependent on the PSM parameters. Now, we must remember that resonances have a special behavior that can be represented by a complex frequency:

$$\omega = K - i\Lambda \tag{9}$$

The transformation of the evanescent waves for traveling ones is due precisely to the imaginary part Λ. In addition, the relation between ω and the wave number κ is:

$$\kappa = \sqrt{\mu\varepsilon}\omega \tag{10}$$

Substituting expressions (9) and (10) into Eq. (3), we can write the resonance condition as:

$$\Delta \begin{pmatrix} \mathcal{M} & \mathcal{N} \\ \mathcal{N} & -\mathcal{M} \end{pmatrix} = 0 \tag{11}$$

The abbreviated components of the matrix in (11) are explicitly

$$\mathcal{M} = \rho_p[\sin(\rho_p)ch(\gamma_p) - \lambda_p] + \gamma_p sh(\gamma_p)\cos(\rho_p) \tag{12}$$

$$+i[\rho_p sh(\gamma_p)\cos(\rho_p) + \gamma_p\lambda_p] \quad (12)$$

and

$$\mathcal{N} = \gamma_p\cos(\rho_p)ch(\gamma_p) + \rho_p\sin(\rho_p)sh(\gamma_p) \tag{13}$$

$$+i[\rho_p\cos(\rho_p)ch(\gamma_p) - \gamma_p\sin(\rho_p)sh(\gamma_p)]$$

In Eqs. (12) and (13), we have used the following definitions:

$$\sigma_M = \bar{d}_M\sqrt{\mu\varepsilon} \tag{14}$$

$$\rho_p = \sigma_M(K^2 - \Lambda^2 - \omega_p K) \tag{15}$$

$$\gamma_p = \sigma_M\Lambda(\omega_p - 2K) \tag{16}$$

$$\lambda_p = \lambda(\rho_p^2 + \gamma_p^2) \tag{17}$$

To have an image of the solutions of Eq. (11) (see **Figure 1**), we can make $K = x$ and $\Lambda = y$ those are the real and imaginary parts of ω, and fix the value for the plasma frequency ω_p so we have the following image:

We obtain for the particular conditions:

$$K = \Lambda \tag{18}$$

$$\omega_p = 10^6 \, Hz \tag{19}$$

The solutions (resonances):

$$x_1 = 5.009 \times 10^5 \, Hz \tag{20}$$

$$x_2 = -985.99 \, Hz \tag{21}$$

In this case only, x_1 is properly a resonance and x_2 has not physical meaning but maintain their orthogonality properties.

4. Communication theory measurement of information loss

Because we have now a wide vision of the loss of information and we know that this is the reason that the images are not perfect, we can use the results of Shannon, Nyquist, Wiener, Hartley, Hopf [25–29], and other authors that have formulated a measure of the loss of information in communication systems. We support our mathematical results on related works [6, 11, 24, 26, 28], which give us a solid theoretical frame to our present and future papers. Indeed, because the *capacity* of a channel and *entropy* are very close concepts, we can use some of the results we have cited above to answer the problem for TRT and LHM.

Basically, we recall two theorems:

Theorem I.

If the signal and noise are independent and the received signal is the sum of the transmitted signal and the noise, then the rate of transmission is:

$$R = H(y) - H(n) \tag{22}$$

This means that the rate of transmission is the entropy of the received signal less the entropy of the noise. The channel capacity is:

$$C = \underset{P(x)}{Max} H(y) - H(n) \tag{23}$$

Theorem II.

The capacity of a channel of band Θ perturbed by white thermal noise power N when the average transmitter power is limited to P is given by:

$$C = \Theta \log \left(\frac{P + N}{N} \right) \tag{24}$$

In this expression, P is the average power of the transmitted signal and N is the average noise power.

From these two theorems, we make our proposal for a channel where we have lost information in three ways. That is, we have limitations on the maximum frequency Θ (band), the presence of different classes of noise, and on a limited time

T available for a time-reversal process. Then, defining a joint average for the power $Q(n, T)$, the channel capacity is:

$$C_T = \Theta \log \left(\frac{P + Q(n, T)}{Q(n, T)} \right) \qquad (25)$$

This remains equal to zero when $P = 0$. The very significant feature of this proposal is the explicit dependence on T, in both the joint average power and the channel capacity, as opposed to the conventional treatment of the signal time duration that is considered as a limit process which tends to infinity. This is a consequence of the explicit form of the Fourier transform of the time-reversed Green function that changes with a factor $e^{i\omega T}$, so even if we are not forced to do so, we can think of it as a parameter that defines the channel. We can think of an arbitrary channel but, when we use it to reverse any signal in time, we follow a different process depending on the time T we decide to fit. Then, we can label the channel with each T as a different one and of course with a different capacity with those corresponding to other values of T. Because of the arguments expressed previously in this work, we can use this measure to the same extent on LHM, ATR, and TRT. For a related discussion of the equivalence of the time-reversal methods and the employment of left-hand materials, we can see ref. [30], and for the use of time reversal on antennas, we can see also ref. [16].

5. An academic example

In order to give an insight into information measurement applied to TR, let us propose that our system behaves like a filter. So, in this particular example, we have no loss if we select $t < T$. We also propose that we have a signal like [12]:

$$\frac{\sin(2\pi\Theta t)}{2\pi\Theta t} \qquad (26)$$

And, that we have instead of the incoming signal in Eq. (15) another like [10]

$$\frac{1}{2} \frac{\sin^2(\pi\Theta t)}{(\pi\Theta t)^2} \qquad (27)$$

The input function Eq. (26) is a sample of a more general function generated by the sum of a series of shifted functions

$$a \frac{\sin(2\pi\Theta t)}{2\pi\Theta t} \qquad (28)$$

where a, the amplitude of the sample is no greater than $\sqrt{S}$ (S is the peak allowed transmitter power).

The channel capacity would be [23] approximately (provided that S/N is small)

$$C_T = \Theta \log \left(\frac{S + Q(n, T)}{Q(n, T)} \right) \qquad (29)$$

In the time-reversal process, we have shown that for each Fourier component, we should add a complex exponential factor dependent on T. But we know now that the tool is the same and that only the numerical value of channel capacity C_T

changes. We see how in practice the time-reversal parameter T appears explicitly but also that when we cut the time duration of reversed signal, it is possible to consider them as an additive contribution to $Q_-(n,T)$. But the form of Eq. (25) suggests a generalized measure of a blend or mix channel capacity when sharing the same band W and differ only by the recording time $T_1, T_2, \cdots, T_n$

$$C_{T_1, T_2, \cdots, T_n} = \Theta \log \left(\frac{S + Q(n, T_1, T_2, \cdots, T_n)}{Q(n, T_1, T_2, \cdots, T_n)} \right) \qquad (30)$$

The fact that we are using the same band but different cutting limits $T_1, T_2, \cdots, T_n$ also suggests that we can design an appropriate filter that can distinguish between signals according to the recording time that is we can superpose signals with the same frequency range but with different recording times. In a previous work, we have sketched a filter, but now we give a better-defined device, so we propose (see **Figure 2**) as a hint to get the filter, the following steps for both the transmitter and the receiver:

5.1 Transmitter

First, increase the n frequencies on the unique entrance band $B(\omega_0)$ (that is centered in frequency ω_0) incoming from the inverse of $T_1, T_2, \cdots, T_n$, then the n new top frequencies $\omega_1, \omega_2, \omega_Q$ are used to create n transformed signals with the rule suggested by communication theory and these last signals enter a blender. Then, the mixed signal is taken by a band generator and projected in Q new bands centered at the frequencies $\square_1, \square_2, \square_Q$ (each corresponds to a resonant frequency). Finally, each band enters this signal transmitter.

5.2 Receiver

The Q traveling signals enter the mirror band amplifier, so called because it knows that there are Q resonant frequencies and then can create (or separate the signal in Q resonant bands) Q sub-bands and amplify the signal in each band (at this moment, each band carries a piece of the original n different signals); after this,
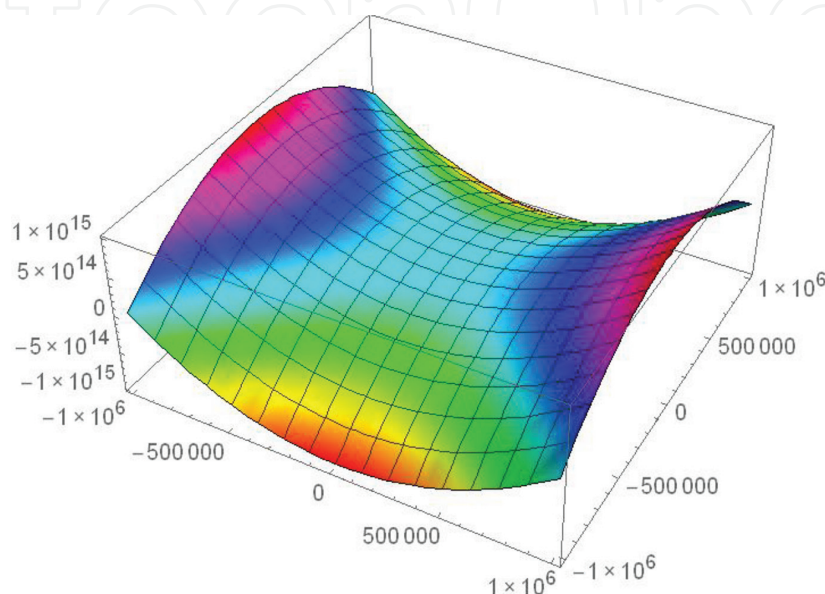
Figure 1.
Image of the solutions of Eq. (11) when the related equation is 987.93 (x²-y²−10⁶) = y (10⁶-2x).
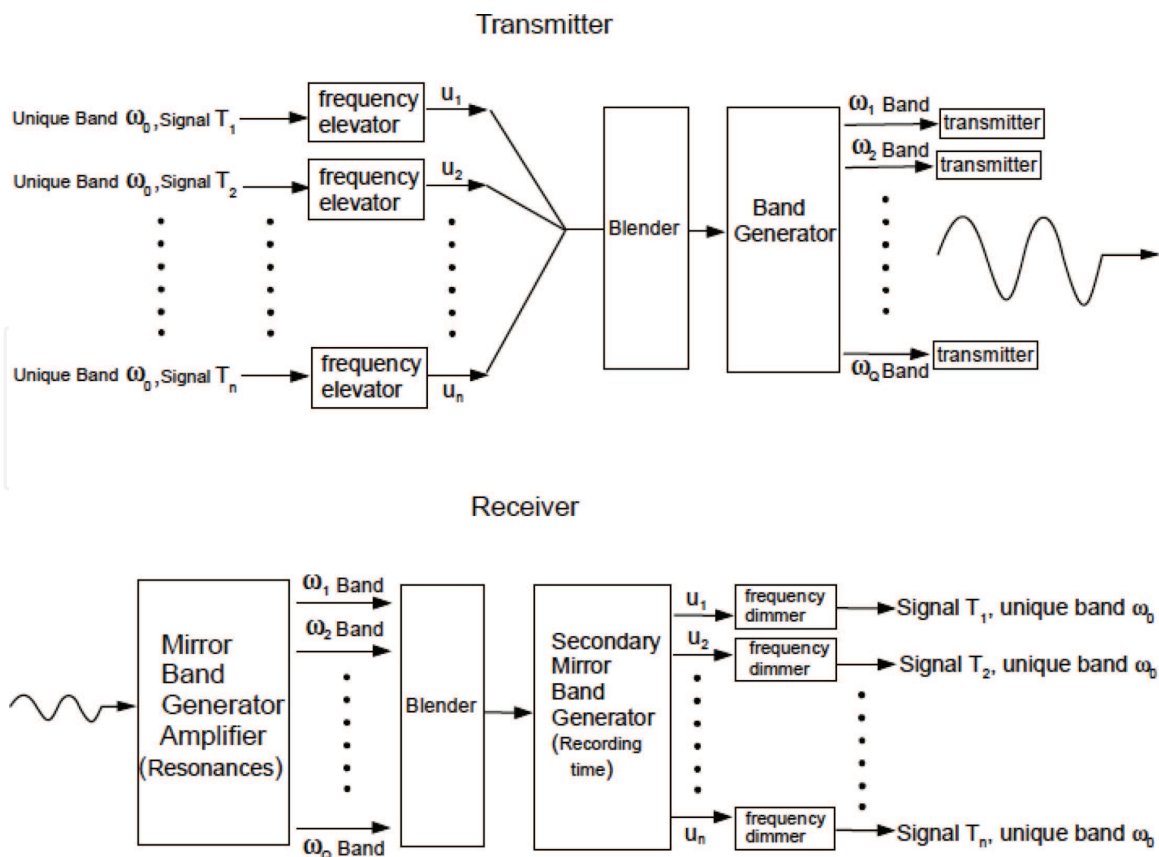
Figure 2.
Flow chart for a proposal device. This can emit and read the blended messages with recording times $T_1, T_2, \cdots, T_n$ beneath to the unique band ω_0

the Q signals are blended and then sending to a secondary mirror band generator which knows that there are n recording times $T_1, T_2, \cdots, T_n$ and because of that it can create n bands with the higher central frequencies $u_1, u_2, \cdots, u_n$ (these last signals could be amplitude-modulated signals) and distribute the blended signal among them. Then, every signal on each band enters a frequency dimmer (the inverse operation performed by the frequency elevators in the transmitter), so we retrieve the n original signals corresponding to the unique band $B(\omega_0)$. For example, in Section 3, we have that the total number of resonances is $Q = 2$, and the two resonant frequencies are $\omega_1 = 5.009 \times 10^5\, Hz$ and $\omega_2 = -985.99\, Hz$.

At this point, it is important to say that the key point on the use of the proposed device is the build of information packs described in another place in order to diminish mutual interference between different signals.

6. Error in time reversing and a related theorem

Based on the equivalence of the TRT and the properties of the Green function, we can trust that any discussion about the interaction of metamaterials with electromagnetic field can be done through this function and simultaneously observe the effect of a time reversal. For this reason, we can now describe the error in terms of the Green function by the hypothesis that LHM can be put to test by forward and backward in time signals and read the results with two points of view: first, the direct effect of the loss of information because of the limited record time T or second, how the negative refraction index helps to preserve information. Now, we can review our previous results and generalize using the kernels, so we can characterize the capacity of a channel in many different circumstances. So, we have made

use of the analogies [30] between the TRT and the employment of LHM to propose that we can express the capacity of any of these negative refraction index materials in the same terms or procedures as those of TRT. Also, we can propose an identical description for the channel capacity that is Eq. (24) and its generalization Eqs. (25) and (30). Then, the matrix formalism for discrete systems can be used to characterize the channel capacity of transmission of information in a process of time reversibility using the Fourier transforms of the Green functions (properly we use the kernels with the interaction matrix $\mathbf{V} = \mathbf{1}$) forward and backward. That is, by the first step, the signal transforms like (in the following equations I and F stand for initial and final places):

$$\mathbf{Y}_F = [\mathbf{1} + \mathbf{R}(\omega)]\mathbf{X}_I \tag{31}$$

then in the second step, it returns to the initial place by means of the operation.

$$\mathbf{Z}_I(\omega) = [\mathbf{1} - \mathbf{K}^{(\circ)}(\omega)]\mathbf{Y}_F(\omega) \tag{32}$$

Then, the complete signal trip would be:

$$\mathbf{Z}_I(\omega) = [\mathbf{1} - \mathbf{K}^{(\circ)}(\omega)][\mathbf{1} + \mathbf{R}(\omega)]\mathbf{X}_I(\omega) \tag{33}$$

So that by defining the error in the time-reversing process by:

$$\delta\mathbf{X}_I = \mathbf{X}_I - \mathbf{Z}_I \tag{34}$$

We can write this like:

$$\delta\mathbf{X}_I(\omega) = \mathbf{X}_I(\omega) - [\mathbf{1} - \mathbf{K}^{(\circ)}(\omega)][\mathbf{1} + \mathbf{R}(\omega)]\mathbf{X}_I \tag{35}$$

or

$$\delta\mathbf{X}_I(\omega) = -[\mathbf{R}(\omega) - \mathbf{K}^{(\circ)}(\omega) - \mathbf{K}^{(\circ)}(\omega)\mathbf{R}(\omega)]\mathbf{X}_I(\omega) \tag{36}$$

Eq. (36) is a corollary that shows explicitly the role of both the forward and backward Fourier transforms of the Green function (we have done $\mathbf{V} = \mathbf{1}$ on Eq. (8) for convenience and also for the complete kernels $\mathbf{K}(\omega)$ and $\mathbf{R}(\omega)$). Eq. (36) is very clear about the origin of the errors because we can see, for example, that in the case that the forward and backward Green functions are mathematically one the transpose conjugated of the other for a perfect time reversal (when acting the first on a column vector and on a row vector the other), we get that the error is zero and that the error increases as the differences of both functions also increase. In a very special case, we can then propose that $\mathbf{K}(\omega)$ and $\mathbf{R}(\omega)$ only differ by the factor $e^{i\omega T}$ or $e^{2\pi i\frac{\omega}{\omega_T}}$ when the only source of error is the recording time T, so that we obtain from Eq. (36) that:

$$\delta\mathbf{X}_I(\omega) = -\left[e^{-2\pi i\frac{\omega}{\omega_T}}\mathbf{K}(\omega) - \mathbf{K}^{(\circ)}(\omega) - \mathbf{K}^{(\circ)}(\omega)e^{-2\pi i\frac{\omega}{\omega_T}}\mathbf{K}(\omega) \right]\mathbf{X}_I(\omega) \tag{37}$$

In Eq. (37), the function $e^{-2\pi i \frac{\omega}{\omega_T}} \mathbf{K}(\omega)$ has the form of the Fourier transform of the Green function but with the argument translated by an amount equal to the recording time T that appears explicitly in Eq. (19) that is the Fourier transform of:

$$\mathbf{K}(t - T) \qquad (38)$$

But with the time running backward, so, as we will show in a moment, if T is very short, the error will be very huge. On the contrary, if the time goes to infinity, the error will go to zero. Resuming, the new Eqs. (33)–(38), make possible a characterization of the lost information in left-hand materials not only for micro-wave range, but also for visible frequencies because we have extended recently the time-reversal techniques (see ref. [3, 12]).

Now, we can define:

$$\mathcal{K} = e^{-2\pi i \frac{\omega}{\omega_T}} \mathbf{K} \qquad (39)$$

So, we can write Eq. (37) like:

$$\delta \mathbf{X}_I(\omega) = -[\mathcal{K}(\omega) - \mathbf{K}^{(\circ)}(\omega) - \mathbf{K}^{(\circ)}(\omega)\mathcal{K}(\omega)]\mathbf{X}_I(\omega) \qquad (40)$$

and because the kernel of the Fourier transform of the generalized inhomoge-neous Fredholm's equation (GIFE) satisfies the following integral equations:

$$\mathcal{K} = \mathbf{K}^{(\circ)} + \mathbf{K}^{(\circ)}\mathcal{K} \qquad (41)$$

$$\mathcal{K} = \mathcal{K}^{(\circ)} + \mathcal{K}^{(\circ)}\mathcal{K} \qquad (42)$$

While Eq. (41) exactly represents the problem with a finite recording time T, Eq. (42) represents a hypothetical problem in which the recording time is infinite. Substituting Eq. (41) into Eq. (40), we have:

$$\delta \mathbf{X}_I(\omega) = -[\mathcal{K}(\omega) - \mathcal{K}(\omega)]\mathbf{X}_I(\omega) \qquad (43)$$

Then, we can suppose that the two kernels in Eq. (40) represent the real and the hypothetical problem described above. Of course, we see that if real conditions approximate the ideal ones, the error is clearly zero. But we can factorize the interaction matrix in Eq. (43):

$$\delta \mathbf{X}_I(\omega) = -\mathbf{V}[\mathcal{G}(\omega) - \mathcal{G}(\omega)]\mathbf{X}_I(\omega) \qquad (44)$$

But Eq. (44) says clearly that the error does not depend on the form of the interaction, only depends on the recording time T. Even we have supposed that the only source of error was the recording time, we do not suppose any particular behavior for the interaction. So, we have enunciated and proved a theorem:

Theorem III.

In the time-reversal problem and for left-hand material conditions, the normal-ized error:

$$\frac{\delta \mathbf{X}_I(\omega)}{\|\mathbf{V}\|} \tag{45}$$

is independent of the explicit form of the interaction provided the last is isotropic.

$$(\mathbf{V}^{-1} = \mathbf{V}^{t*})$$

Returning to the time representation, for the time-dependent retarded isotropic (remember that in the following expression, the indices m and n indicates components of the field and can be omitted), free Green function related to $\mathbf{K}^{(\circ)}(\omega)$, we can write explicitly.

$$G^{mn(\circ)+}(\mathbf{r},t;\mathbf{r}',t') \equiv G^{(+)}(\mathbf{r},t;\mathbf{r}',t') = \frac{\delta[t'-(t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})]}{|\mathbf{r}-\mathbf{r}'|} \tag{46}$$

and for the advanced time-dependent free Green function related to $\mathbf{R}^{(\circ)}(\omega)$:

$$G^{mn(\circ)-}(\mathbf{r},t;\mathbf{r}',t') \equiv G^{(-)}(\mathbf{r},t;\mathbf{r}',t') = \frac{\delta[t'-(t+\frac{|\mathbf{r}-\mathbf{r}'|}{c}-T)]}{|\mathbf{r}-\mathbf{r}'|} \tag{47}$$

That is the recording time appears explicitly in the advanced Green function and we can show that its value makes possible to blend many signals on the same channel without interference. It is important to note that for resonances, the relevant Green functions are precisely the free ones and not the complete ones as we can see in Eqs. (5) and (6).

7. Information packs

In this section, we present the support and the definition of the information packs that are required for the adequate performance of the device shown in Section 6 and that by him constitute a method to improve the broadcasting efficiency. To this end, we must remember that on communication theory [9, 10] are defined the so-called ensembles of functions dependent on time. One of their properties is really a group one from the mathematical point of view and lies in that any ensemble transforms into another member of the same ensemble when we change the function at any certain amount of time. To illustrate this property, we shift by an amount t_1 the argument of all the members of the ensemble defined as follows:

$$F_\theta(t) = \sin(t+\theta) \tag{48}$$

where θ is distributed uniformly from 0 to 2π.
Then, we have:

$$F_\theta(t+t_1) = \sin(t+t_1+\theta) = \sin(t+\varphi) \tag{49}$$

where φ is distributed uniformly from 0 to 2π.

Then, each function has changed individually, but the ensemble as a whole is invariant under the transformation. Also, if we apply the operator T which gives for each member

$$S_\alpha(t) = TF_\alpha(t) \tag{50}$$

It implies that

$$S_\alpha(t + t_1) = TF_\alpha(t + t_1) \tag{51}$$

It is possible to prove that if T is an invariant operator and the input ensemble $F_\alpha(t)$ is stationary, the output ensemble $S_\alpha(t)$ is also stationary. Now, for communication purposes, the operator T, which could be a modulation process, is not invariant because of the phase carrier that gives certain time structure, but if the translations are multiples of the periods of the carrier, then the modulation will be invariant. At this stage, it is important to remember that Wiener [6] has pointed out that if a device is linear as well as invariant (in the sense of the last definition), then the Fourier analysis is the appropriate mathematical tool for dealing with the problem. Now, suppose in addition that we are interested on functions that are limited to the band from 0 to Θ cycles per second, then we have the following theorem [10]:

Let $F(t)$ contain no frequencies over Θ. Then:

where,
$$F(t) = \sum_{-\infty}^{\infty} X_n \frac{\sin \pi(2\Theta t - n)}{\pi(2\Theta t - n)} \tag{52}$$

$$X_n = F\left(\frac{n}{2W}\right) \tag{53}$$

In this expansion, $F(t)$ is represented as a sum of orthogonal (basis) functions. The coefficients X_n of the various terms can be considered as coordinates in an infinite dimensional "functions space." We will take the last theorem (Eqs. (52) and (53)) as a very suggestive rule to consider the recently obtained resonant frequencies. If we use physical arguments about the reasons of the presence of a resonance, we can be sure that channels available for broadcasting are also limited in number. Indeed, in a recent paper, we have generalized the procedure for electromagnetic scalar and vector potentials [30] and we have established that we can use either the electromagnetic field or the potentials for obtaining the resonances and also for the use of the recording time as a resource to optimize communications. And now, we can build information packs (IP) that are functions, which represent a part of the signal we want to send with the minimum loss of information. The resultant expression is:

$$F_e(t) = \sum_{-\infty}^{\infty} X_{n,e} \frac{\sin[\pi(2\omega_e t - n)]}{\pi(2\omega_e t - n)} \tag{54}$$

where,

$$X_{n,e} = F_e\left(\frac{n}{2\omega_e}\right) \tag{55}$$

Every ω_e allows us to build a decomposition like (54) but we expect that only a few terms are necessary for a well representation of $F_e(t)$. Next, we send separately each $F_e(t)$ by its own device and it is all we need for broadcasting. To receive the signal, we need a separate device for each ω_e.

A very important feature is that because of the properties of the modulation process stated in Eqs. (50) and (51), we can recover, for any arbitrary signal, the behavior under spectral representation and under separated pack representation. So we can either talk about $F_e(t)$ in Eq. (54) as the representation of some element of the basis function for the spectral representation or directly as the e component of an arbitrary signal $S(t) = TF(t)$. Now, we recall the two resonances founded in another work [3]:

$$\omega_1 = \frac{\pi}{4d} + \omega_0 \tag{56}$$

and

$$\omega_2 = \frac{3\pi}{4d} + \omega_0 \tag{57}$$

Suppose that $S(t)$ is the signal

$$S(t) = \frac{\sin[\pi(2\Theta t)]}{\pi(2\Theta t)} \tag{58}$$

Then, we have the first pack:

with

$$S_1(t) = \sum_{-\infty}^{\infty} X_{n,1} \frac{\sin[\pi(2\omega_1 t - n)]}{\pi(2\omega_1 t - n)} \tag{59}$$

$$X_{n,1} = S\left(\frac{n}{2\omega_1}\right) \tag{60}$$

And, we have the second pack

$$S_2(t) = \sum_{-\infty}^{\infty} X_{n,2} \frac{\sin[\pi(2\omega_2 t - n)]}{\pi(2\omega_2 t - n)} \tag{61}$$

with

$$X_{n,2} = S\left(\frac{n}{2\omega_2}\right) \tag{62}$$

We can see that if $\Theta = \omega_1$, the only coordinate distinct to zero is $X_{0,1} = 1$ and if $\Theta = \omega_2$, only survives the term $X_{0,2} = 1$. So, we remark self-consistency of the method.

Even VMF has a broad application on the microwave range, maybe it would be more useful to apply for larger frequencies. But even the great technological boom, there is not any device that could manipulate visible light at length as happens with microwaves. Whatever we can recall some of the basic early ideas on radio broadcasting when the option was sending information by means of modulating the wave's amplitude as appears in **Figure 3**. However, we can take our definition of information packs and put it in a modulated visible-light signal taking the enveloping of the signal we name the wrapping signal (WS) as the information that can be injected inside Eq. (54). Technically, we rewrite Eqs. (50) and (51) in the form:

$$H_\beta(t) = \Omega S_\beta(t) \tag{63}$$
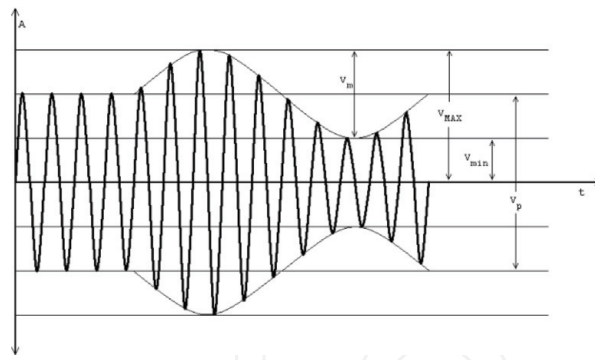
Figure 3.
The former radio broadcasting procedure: modulated amplitude. Image given by Pérez-Martínez [31].

It implies that

$$H_\beta(t + t_1) = \Omega S_\beta(t + t_1) \tag{64}$$

Now, the operator Ω is a generic operator like T but acting over the ensemble $S_\beta(t)$. Some care must be taken when reading the WS information, because the translations stated in Eqs. (63) and (64) were multiples of the periods of the carrier, and then as we said above, the modulation will be invariant. The resonant frequencies will be obtained by the same procedure.

In order to complete the methodology, we recall the concept of group velocity $c_g(t)$ and construct this inherent quotient between them and the enveloping frequency ω_g which results in the wave number κ_g, so we associate them with the resonance frequencies in a similar form as we styled with microwaves, but now these last signals come from the measured properties of the Green's function associated with the modulated signal. In this way, in Eq. (54), we put directly the WS first for a non modulated beam:

$$S_{e'}(t) = \sum_{-\infty}^{\infty} X_{n,\,e'} \frac{\sin\left[\pi(2\omega_{e'}t - n)\right]}{\pi(2\omega_{e'}t - n)} \tag{65}$$

in which the coefficients are given by:

$$X_{n,\,e'} = S_{e'}\left(\frac{n}{2\omega_{e'}}\right) \tag{66}$$

The signal $S_{e'}(t)$ in (65) can be viewed as the representation of some element of the new basis functions or as the e' component of an arbitrary amplitude-modulated signal $H_{e'}(t)$. Now, we can give an example where we use the same values for the resonances on Eqs. (56) and (57) and where we propose an arbitrary amplitude modulated or WS (for a modulated visible light beam) signal given as follows:

$$H(t) = a\cos\left(\Theta_A t + \delta\right) \tag{67}$$

In Eq. (67), $\Theta_A = \Theta_p \pm \Theta_m$ is an arbitrary frequency, and in a same manner, a and δ are preconceived constants but otherwise arbitrary.

With these preliminaries, we can build the first IP:

$$H_1(t) = \sum_{-\infty}^{\infty} X_{n,\,1} \frac{\sin\left[\pi(2\omega_1 t - n)\right]}{\pi(2\omega_1 t - n)} \tag{68}$$

where explicitly the coefficients are:

$$X_{n,1} = H\left(\frac{n}{2\omega_1}\right) \tag{69}$$

And taking expression (67)

$$X_{n,1} = a \cos\left[\Theta_A\left(\frac{n}{2\omega_1}\right) + \delta\right] \tag{70}$$

In a similar manner, the second IP will be:

$$H_2(t) = \sum_{-\infty}^{\infty} X_{n,2} \frac{\sin\left[\pi(2\omega_2 t - n)\right]}{\pi(2\omega_2 t - n)} \tag{71}$$

in which

$$X_{n,2} = H\left(\frac{n}{2\omega_2}\right) \tag{72}$$

Also, by taking Eq. (67):

$$X_{n,2} = a \cos\left[\Theta_A\left(\frac{n}{2\omega_2}\right)\delta\right] \tag{73}$$

As we said above, the resonances must come also for the WS. By this procedure, we have enlarged the scope of the formalism we named vector-matrix or VMF [1–3].

In order to complete our example, we put explicit values of the resonances for the two visible light IP:

$$H_1(t) = \sum_{-\infty}^{\infty} X_{n,1} \frac{\sin\left[\pi\left(2\left[\frac{\pi}{4d} + \omega_0\right]t - n\right)\right]}{\pi\left(2\left[\frac{\pi}{4d} + \omega_0\right]t - n\right)} \tag{74}$$

And explicitly

$$X_{n,1} = a \cos\left[\Theta_A\left(\frac{n}{2\left[\frac{\pi}{4d} + \omega_0\right]}\right) + \delta\right] \tag{75}$$

For the second IP

$$H_2(t) = \sum_{-\infty}^{\infty} X_{n,2} \frac{\sin\left[\pi\left(2\left[\frac{3\pi}{4d} + \omega_0\right]t - n\right)\right]}{\pi\left(2\left[\frac{3\pi}{4d} + \omega_0\right]t - n\right)} \tag{76}$$

in which

$$X_{n,2} = a \cos\left[\Theta_A\left(\frac{n}{2\left[\frac{3\pi}{4d} + \omega_0\right]}\right) + \delta\right] \tag{77}$$

8. Conclusions

In Eqs. (25), (29), (30), (34)–(40), we have shown that it is possible to use an operator language and the properties of the Green function to define the capacity of a channel, the loss of information, and finally, the error in the time-reversal process. Therefore, we can use our results to describe the behavior of LHM interacting with electromagnetic field whether forward or backward in time. Thanks to our interpretation of a resonance in the broadcasting problem with the left-hand material conditions, and the application of the model PSM, we make up a broadcasting system that has the power for distinguishes between signals according to their recording time, and allows to superpose signals in the same frequency range having different recording times with the minor loss because of resonance technology; to this end, we have presented a detailed support and definition of the information packs (IP) and the possibility of application for visible light. In addition, we have enunciated and proved a theorem (theorem III) that establishes: for the TRT and LHM, the normalized error is independent of the particular behavior of the interaction. Summarizing, we give a complete recipe for optimizing communications efficiency.

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