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Chapter

Feedback Control of Rayleigh Convection in Viscoelastic Maxwell Fluids

Ildebrando Pérez-Reyes, Cornelio Alvarez-Herrera and Jonathan Rodríguez-Campos

Abstract

Control of Rayleigh convection in a viscoelastic Maxwell fluid is addressed here by considering a feedback from shadowgraphic visualizations. Here, a theoretical approach is made to the problem of the onset of convective motion through a source term in the lower thermal boundary condition. A numerical Galerkin technique is then used to study the linear hydrodynamic stability. Small relaxation times are considered for Prandtl numbers 1 and 10. Interesting results for the Rayleigh, the wavenumber, and the frequency of oscillations are presented along with discussion on the physical mechanism. In short, the linear hydrodynamic stability analysis states that suppression of convection may be favored.

Keywords: viscoelastic fluid, feedback control, Rayleigh convection

1. Introduction

The idea of control in physics and engineering, and other areas of applications, is not new. Several examples and interesting advances can be found in chemical processes where multivariable problems may appear in the way to obtain desired product quality or to guarantee safety of operators [1, 2]. At the same time, technology has been developed to meet the needs of the industry such as proportional integral derivative controllers, sensors, etc. On the other hand, new technology and lab developments needing control of variables may use different hardware. Onset of thermal convection and thermoconvective pattern formation are examples of phenomena aimed to be controlled.

Control of Rayleigh convection is a subject that has called the attention of researchers because of its connection to applications in electronics [3] and genetics [4], for example. In these cases understanding and modulation of convective motions are important so that the problem of Rayleigh convection may be coupled to a feedback control scheme. It is straight that heat transfer along the fluid layer should be monitored and controlled in order to modulate the convective motions.

The present manuscript is devoted to give an approach to the control of the Rayleigh convection phenomena in a viscoelastic fluid layer by a feedback scheme based on shadowgraphic visualization. This is a theoretical approach to the problem which actually introduces a modification in the lower thermal boundary conditions to modulate the heat transfer so that convection may be changed [5]. Since

convective motions are undoubtedly connected to the hydrodynamic stability of the fluid layer, this theoretical framework is combined with the idea of a control feedback provided by shadowgraph technique measurements. The results presented in this work contribute to further developments in applications involving viscoelastic fluids.

Several investigations have been conducted in previous decades by authors interested in the selection of convective patterns or in avoiding convective motions, for example. However, to the best knowledge of the authors, perhaps the works of Singer et al. [6] and Singer and Bau [7] triggered interesting studies on the control of thermal convection. Singer et al. [6, 7] worked on the laminarization of chaotic convective motions by using a feedback control signal based on temperature measurements both theoretically and experimentally. Petrov et al. [8] reported results on the nonlinear control of convective motion in a liquid bridge which is based on temperature measurements that feed a nonlinear control algorithm controlling a thermoelectric element. Other works were devoted to the suppression of Rayleigh convection as motivated by the results (Singer et al.) [6, 7]. Later, Howle reported interesting results for the control of Rayleigh convection in Newtonian fluids both experimentally [9] and theoretically [5, 10]. In these works, Howle [5, 9, 10] has studied the coupling of linear hydrodynamic stability in a Newtonian fluid with the problem of controlling the supplied heat transfer to the fluid by using the familiar shadowgraph visualization technique. Howle [9] used an experimental arrangement for the visualization of convective motions to get information useful in the feedback of a set of heaters installed at the bottom of the lower boundary.

The mentioned investigations along with the possible improvements to applications [3, 4] have motivated the present study related to viscoelastic fluids. The main idea is to extend the results of Howle [5, 9, 10] about the control of thermal convection for viscoelastic Maxwell fluids. The thermal hydrodynamic stability of a viscoelastic fluid layer is investigated, while heat flux at the lower boundary is modulated by a feedback signal obtained from shadowgraphic visualization. Then, results of a numerical analysis are presented for the linear hydrodynamic stability.

The manuscript has been organized as follows. A brief introduction to the Rayleigh convection in viscoelastic fluids is given in Section 1. In Section 3 the governing equations of the problem in hand are presented along with an explanation of the physical nature of the system. In Section 3.1 the corresponding boundary conditions are introduced. Next, the linear hydrodynamic stability is shown in Section 4. General comments based on experimental tests on the shadowgraph visualization of the Rayleigh convection are presented in Section 5. Section 6 is devoted to expose results and discussion of the findings. Finally, the main conclusions are given in Section 7.

2. On the Rayleigh convection in viscoelastic fluids

The problem of Rayleigh convection in viscoelastic fluids has been a subject of interest for researches [11, 12] since several decades ago. The early works on this matter came up from the concern on the usage of viscoelastic fluid models by taking advantage of already known results and theory about Rayleigh convection in Newtonian fluids. In this way, the problem of Rayleigh convection in viscoelastic fluids can be represented as shown in **Figure 1**.

Although there are several models for the representation of viscoelastic fluids a few have been widely considered for thermal convection. These are the Maxwell

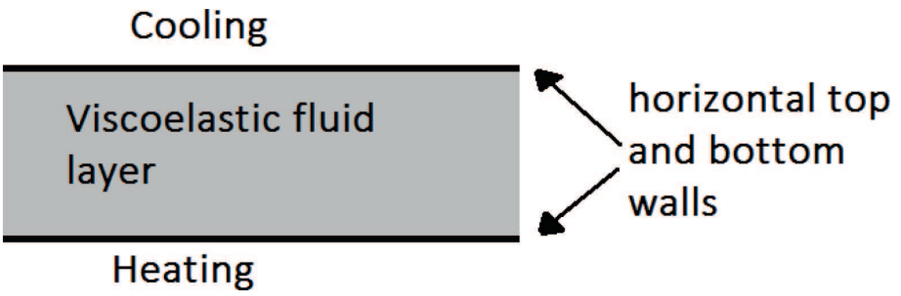

Figure 1.
The problem of Rayleigh convection in a viscoelastic fluid layer. As the critical conditions for the Rayleigh convection are achieved, convective fluid motions set in.

and the Jeffreys viscoelastic models which have linear and nonlinear constitutive equation representations. The following linear constitutive equation,

$$\left(1+F\frac{\partial}{\partial t}\right)\tau = 2\eta_0\left(1+EF\frac{\partial}{\partial t}\right)\mathbf{e}$$

where τ is the stress tensor and $\mathbf{e}$ is the share rate tensor [13], corresponds to the Jeffreys viscoelastic fluid model and reduces to the Maxwell viscoelastic fluid model when $E = 0$. These two viscoelastic fluid models predict time-dependent thermal convection also known as oscillatory convection [12, 15], which is of interest due to changes in the instability of the fluid. In other words, convective motions in the fluid may set in at lower critical conditions [12, 15] than for the Newtonian case. Finally, the mentioned findings represent a start point for developments on convective motion control.

3. Mathematical formulation

Here, the onset of thermal convection in a horizontal infinite viscoelastic Maxwell fluid layer heated from below is considered. The schematic of the system is shown in **Figure 2** where the optical setup along with proper data processing could give insight on the modulation of the heat supplied at the lower wall. This is in fact a feedback control scheme.

From the mathematical point of view, the linear hydrodynamic stability problem can be represented by the equation for the balance of momentum, the continuity

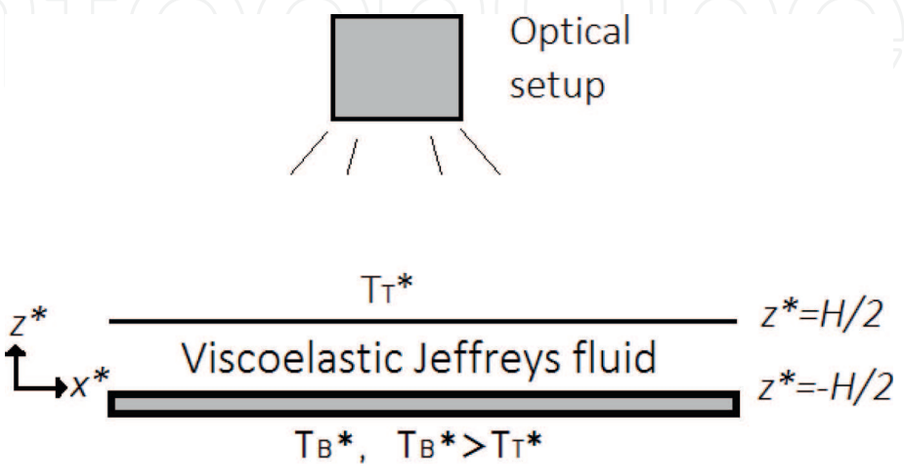

Figure 2.
*General schematics of the problem. The box for optical setup represents the setup for fluid visualization by the shadowgraph technique. Both walls are rigid but made of different materials since the one in the top should be a transparent media. * indicates dimensional variables.*

equation, the heat diffusion equation, and the constitutive equation for the viscoelastic Maxwell fluid. For short some familiar steps are avoided (see the publications of [14–16] for details). Thus, the governing equations are

$$(1+F\sigma)\left[\frac{\sigma}{Pr}\left(\frac{d^2}{dz^2}-k^2\right)W-Rk^2\Theta\right]=\left(\frac{d^2}{dz^2}-k^2\right)^2 W \tag{1}$$

$$\left[\sigma-\left(\frac{d^2}{dz^2}-k^2\right)W\right]\Theta=W \tag{2}$$

where Eqs. (1) and (2) were obtained after a process of perturbation and nondimensionalization of the governing equations. The nondimensionalization of the variables was made with H for lengths, κ/H for velocity, $\kappa\mu/H^2$ for pressure, ΔT^* for temperature, and H^2/κ for time. Also, in Eqs. (1) and (2) $Pr=\nu/\kappa$, $F=\lambda\kappa/H^2$, and $R=\beta g H^3\Delta T^*/\nu\kappa$. Notice that pressure and velocity fields were separated by operating twice the rotational operator on the momentum balance equation. Also, the constitutive equation for the viscoelastic Maxwell fluid has been combined with the momentum balance equation. Next, normal modes $\exp\left[i\left(k_x x+k_y y\right)+\sigma t\right]$ were introduced so that the problem in hand could be reduced to a system of ordinary differential equations as shown in Eqs. (1) and (2). Then, Eq. (1) corresponds to the balance of momentum, and Eq. (2) corresponds to heat.

3.1 Boundary conditions

Eqs. (1) and (2) should be subjected to a proper set of boundary conditions. For the system in hand, the boundaries of the fluid layer are both rigid and solid walls so that the following mechanical conditions shall be used:

$$W=\frac{dW}{dz}=0 \quad \text{at} \quad z=-\frac{1}{2},\frac{1}{2}$$

Thermal conditions are adapted from those used in the study in the Rayleigh convection of a fluid layer bounded by a good thermal conducting wall at the top and by an insulating wall at the bottom [17]. This configuration of the thermal boundary conditions is preferred since control of convection can be made, in an experimental setup, by changing the heat flux at the bottom. Thus, the ideas of [10] are embraced here. For short, adjustments made in the supplied heat flux can be introduced in the model as an inhomogeneity in the bottom thermal boundary condition through the controller gain or magnitude of the made adjustments.

The boundary conditions for θ are

$$\Theta=0 \quad \text{at} \quad z=\frac{1}{2} \tag{4}$$

$$\frac{d\Theta}{dz}=\gamma k^2\int_{-1/2}^{1/2}\Theta dz \quad \text{at} \quad z=-\frac{1}{2} \tag{5}$$

4. Linear stability analysis

Eqs. (1)–(5) represent an eigenvalue problem for the Rayleigh number. The solution to this problem is then made by the Galerkin method which allows the

calculation of the eigenvalue without completely solving for W and Θ. Then, this technique is numerically implemented to study the hydrodynamic stability of the viscoelastic fluid layer as follows (see the book of [18] or the monography of [19], for example).

First, a trial function for W, satisfying boundary condition (Eq. (3)), is proposed. This is

$$W_n = \sum_{n=1}^{N} (2z-1)^n (2z+1)^n \tag{6}$$

Next, W_n is substituted into the equation for Θ. The result of that substitution allows to solve for Θ from Eq. (2) subject to the corresponding boundary conditions (Eqs. (4–5)). The previous step gives the advantage of carrying more information on the solution of the problem and improving accuracy. Therefore, back in with Eq. (1) for W, the residual is calculated multiplying that equation by W_n as given in Eq. (6) and then integrating across the fluid layer. This orthogonalization process outputs a solvability condition from which R can be obtained. This is

$$\left| (1+F\sigma)\frac{\sigma}{\mathrm{Pr}} \left\langle W_n \left(\frac{d^2}{dz^2} - k^2 \right) W_m \right\rangle - \left\langle W_n \left(\frac{d^2}{dz^2} - k^2 \right)^2 W_m \right\rangle - (1+F\sigma) R k^2 \langle W_n \Theta_m \rangle \right| = 0 \tag{7}$$

where the angle brackets indicate the integral from $z = -1/2$ to $z = 1/2$. Eq. (7) is a matrix of size (N, M) with $N = M = 1$, the first approximation. As the size of the matrix is increased, the accuracy is improved and so is the complexity of the calculations. The condition given in Eq. (7) demands that the determinant of that matrix should be zero. From the resulting mathematical expression, the Rayleigh number R can be determined.

4.1 Numerical analysis

The hydrodynamic stability of the viscoelastic layer as modified by a varying heat flux supply at the bottom is studied numerically. First, a comparison between the results obtained from condition (Eq. (7)) and those reported by previous authors was carried out showing very good agreement. The critical Rayleigh, wavenumber, and frequency of oscillation were obtained by first fixing γ and ω and then minimizing the Rayleigh number with respect to the wavenumber. Critical R_c and k_c give critical ω_c. The process was repeated for a range of values of γ, and since Eq. (7) is valid for different viscoelastic fluids, the properties E, F, and Pr were mapped for some representative cases.

The validation of the present results was made by comparison of the results obtained from Eq. (7) with those reported by previous authors [10, 17]. Besides, these comparisons help to establish the order of approximation to be used in Eq. (7). For the case of Rayleigh convection in a Newtonian fluid layer the agreement with the results of reference [17], when the lower boundary is kept at constant heat flux and the top boundary is kept at constant temperature, is very good. In the present work, the critical Rayleigh and wavenumber are 1303.44 and 2.56, respectively, with a maximum error of 3.7%, for this case.

For the same conditions used by [10], from the condition (Eq. (7)), the critical Rayleigh and wavenumber are 3976.59 and 3.97, respectively, for $\gamma = 100$. The maximum error for this case is 2%. Here, the results of [10] were slightly extended. Curves of criticality for the Rayleigh and wavenumber are shown in F**igures 4–5** for a wide range of γ. These curves show that the Rayleigh number of very strong

changes can be obtained at magnitudes of γ between 0 and 10. These are important results and have triggered further investigation in viscoelastic Maxwell fluids.

Comparison of the data for the hydrodynamic stability of viscoelastic Maxwell fluids was made with the results reported by [20]. In their work, Sekar and Jayalatha [20] made calculations for the hydrodynamic stability of a viscoelastic fluid heated from below subjected to different mechanical and thermal boundary conditions. Very good agreement for the corresponding case was obtained. However, an unexpected result is that the critical Rayleigh number is larger for the case of bottom rigid insulation wall and top rigid isothermal wall than for the case of two rigid isothermal walls [20]. Despite the large amount of results, published by [20], the physical mechanism of the mentioned finding is not explained. This change in the hydrodynamics of the fluid can be attributed solely to the viscoelastic nature of the fluid since the Newtonian fluid does not show this behavior.

For the numerical computations of this work, third-order, $n = 3$, approximations were used since a very good convergence was found at this order. Investigated higher-order approximations only give improvements smaller than 10^{-2}. Perhaps, the very good convergence was due to the contributions given by the solution of the heat equation instead of using trial functions for Θ.

The effect of heat flux modulation at the bottom wall on the hydrodynamic stability for the viscoelastic fluid was investigated as follows. Viscoelastic Maxwell fluids with $F = 0.1$ were investigated, while the Prandtl number was fixed at 1 and 10. Then, the critical Rayleigh number, the wavenumber, and the frequency of oscillation were determined.

5. An experimental possibility

In this section, some general comments are given based on preliminary results on an experimental work made on this problem. In view of the previous theoretical analysis to the problem of controlling the thermal convection in a viscoelastic Maxwell fluid layer, an experimental setup may be proposed. Since a feedback control strategy has been considered for this purpose, the scheme shown in **Figure 2** can be extended to include other features. **Figure 3** shows a very general experimental setup where some optomechanical and electromechanical parts are missing, but it still shows the experimental counterpart of the theoretical approximation.

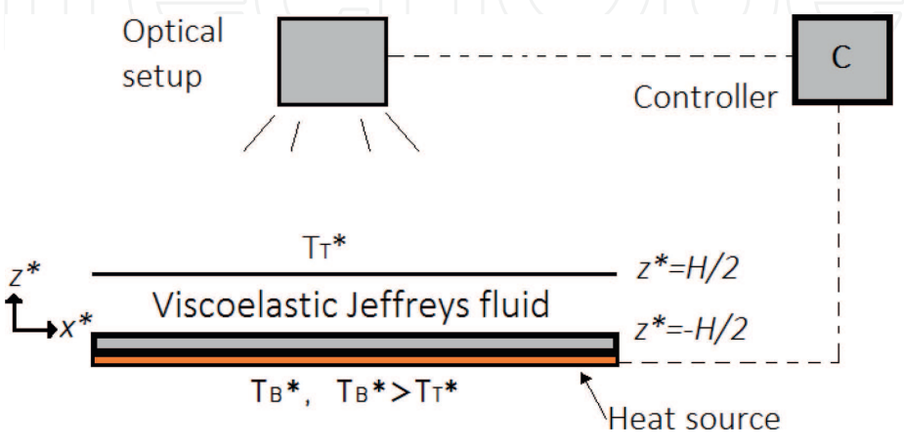

Figure 3.
Extension to the Rayleigh convection visualization schematics shown in ***Figure 2****. Dashed lines indicate communication between the given hardware.*

Two possible goals may be set for the experiment: detection of the beginning of the Rayleigh convection and tuning for favoring the formation of a given convective pattern. For the present case, some experiments have been performed in an open loop for the setup shown in **Figure 3** where the detection of the beginning of fluid motion is visualized in the screen of laptop computer attached to a shadowgraph optical setup and further adjustments are manually introduced in a PID temperature controller.

Although convective motion was detected on the range of experimental conditions reported by previous authors, more work is needed to have a closed loop. In other words, the PID temperature controller should be changed by a PID shadowgraph controller able to feedback the heat supplied by the source (see **Figure 3**). This idea is also based on that early mentioned by Howle [5] who used a proportional control algorithm based on shadowgraph images of the experiment.

On the light of the experimental test, some comments can be made on the connection between theoretical predictions and experimental data. First, the controller output is related to a precise amount of Watts used by the heat source so that dynamical behavior is expected and has been found with the open loop tests, and as a consequence, the value of the parameter γ would change in time as well. Second, a thermal fluid may be used to supply the needed heat to the layer instead of an arrangement of heaters [5] which could give a delayed change in the Rayleigh number but more uniform behavior in the fluid layer horizontal extent.

6. Results and discussion

Very interesting results were found. **Figures 5a–5c** and **6a–6c** present the main findings on the linear hydrodynamic stability of the Maxwell viscoelastic fluid layer. Weak viscoelastic fluids, with F = 0.1, are investigated for understanding of the role played by the controller gain γ.

For the case of Pr = 1, the critical Rayleigh, the wave number, and the frequency of oscillation show unexpected behavior since at small values of γ the magnitude of

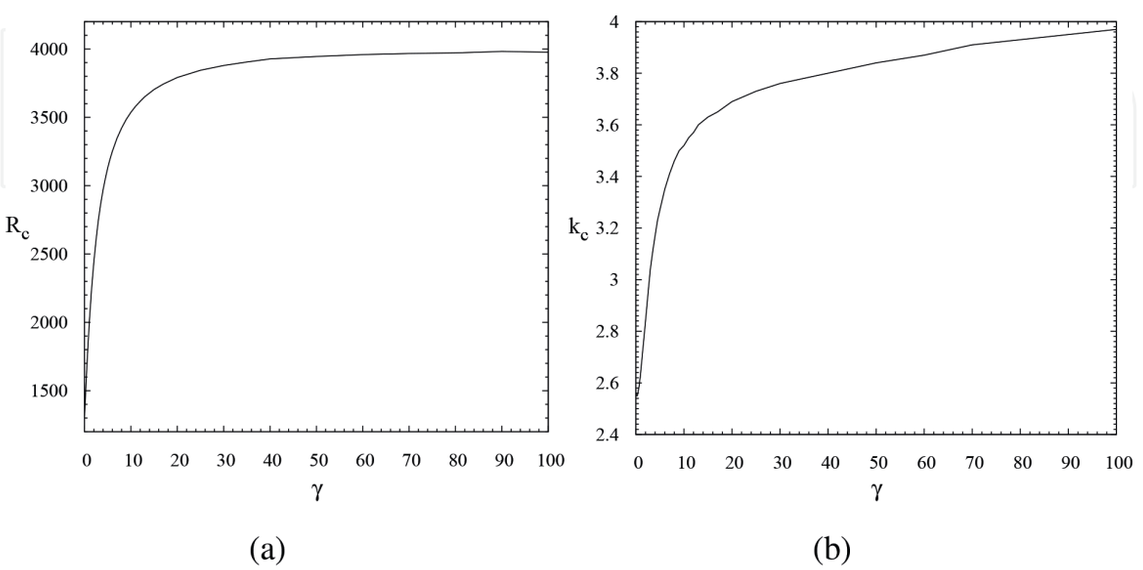

Figure 4.
Curve of criticality for the Rayleigh convection in a Newtonian fluid corresponding to R_c against γ (a) and to k_c against γ. This curve of criticality extends the results of [10]. At γ = 100, R_c = 3976.59, and k_c = 3.97.

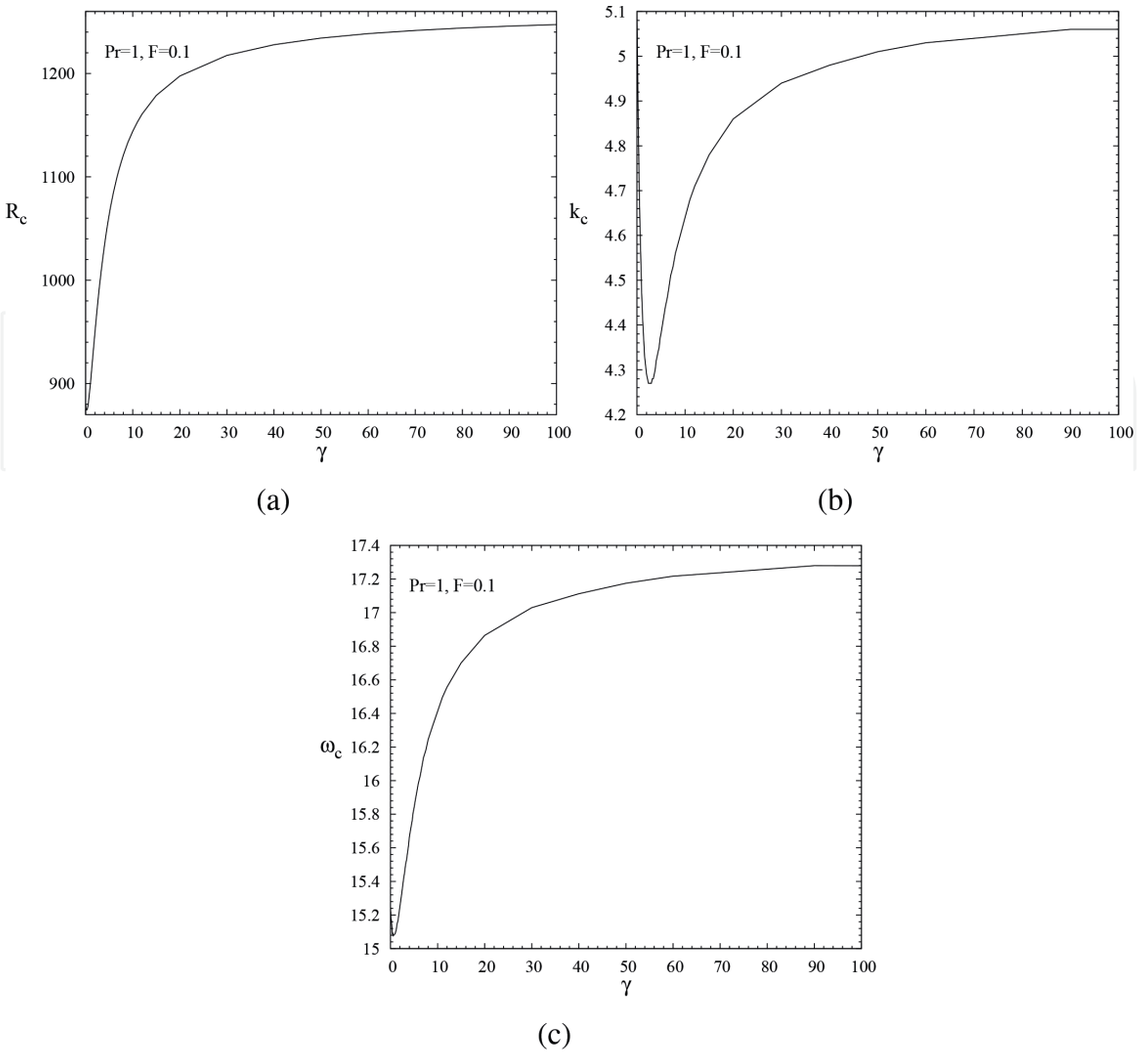

Figure 5.
Curve of criticality for the Rayleigh convection in a Maxwell viscoelastic fluid with Pr = 1 and F = 0.1. These curves correspond to R_c, k_c, and ω_c against γ.

R_c, k_c, and ω_c decreases. However, at certain values of γ, the same parameters start growing monotonically. On the other hand, for the case of Pr = 10, only the critical Rayleigh number behaves as for Pr = 1. The critical wavenumber and frequency of oscillation decrease monotonically with γ for Pr = 10.

The results on the hydrodynamics are unexpected and can be attributed to a coupling of the viscoelastic property *F* and to the nonzero heat flux bottom boundary condition. For the two values of the Prandtl number investigated, the fluid layer always stabilizes after certain critical value of γ. From the comparison with the curves for the Newtonian case, it can be said that fluid viscoelasticity triggers stronger nonlinear behavior of R_c, k_c, and ω_c (**Figures 4–6**).

The physical interpretation of the present results is as follows. Increasing γ means that temperature at the bottom is increased too. As the heat flux is increased, viscoelasticity helps to destabilize the system. At the same time, there must be a limit for the effect of small γ since the Rayleigh number *R* depends on the temperature difference which cannot be indefinitely increased. If temperature is increased with no limit, along with larger values of γ, the thermal energy should be released or converted into fluid motions, for example. Then, the oscillations in the fluid would help to diffuse the heat very quickly, while the layer becomes more stable with γ. This behavior is found in both systems (**Figures 5** and **6**).

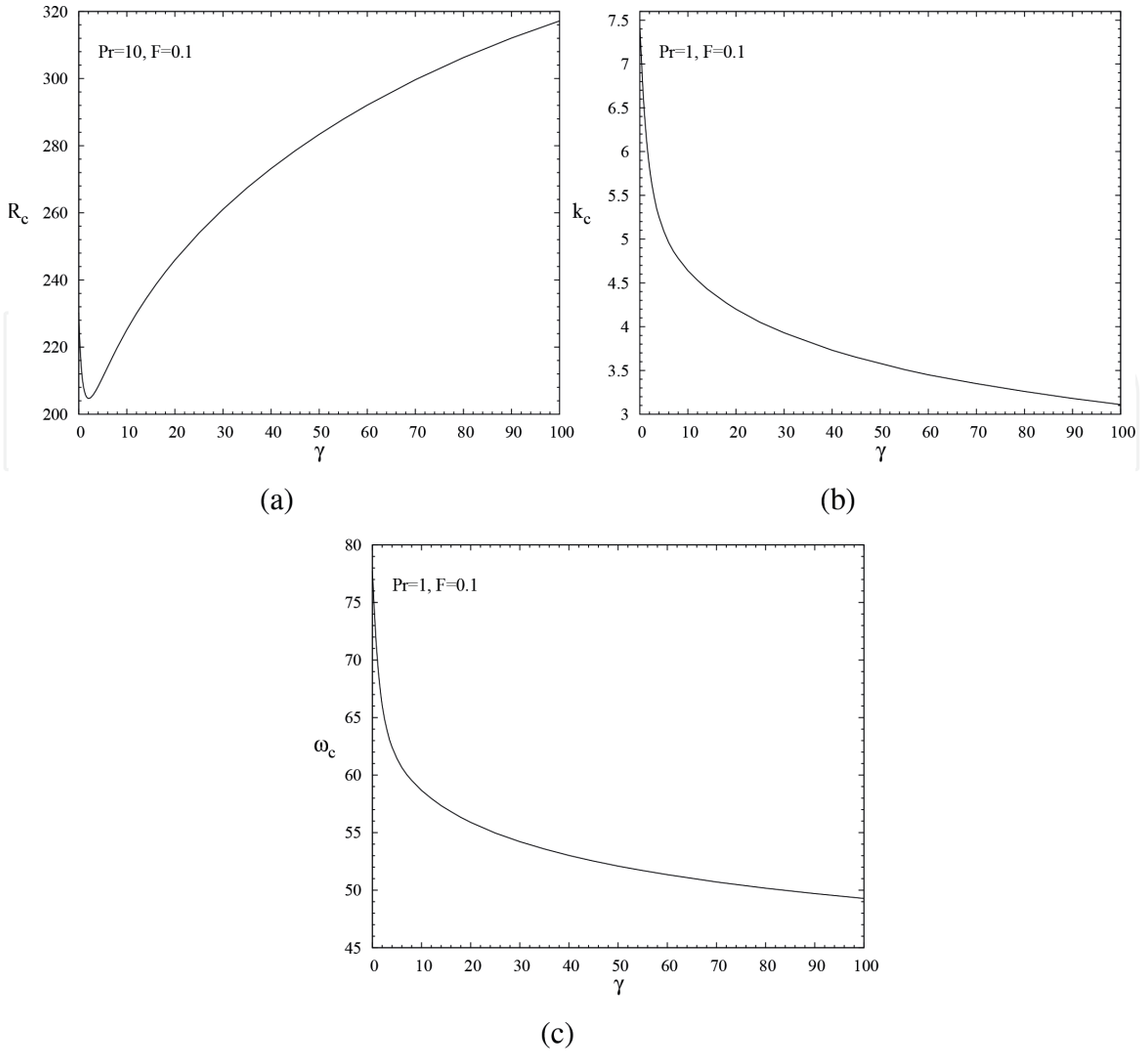

Figure 6.
Curve of criticality for the Rayleigh convection in a Maxwell viscoelastic fluid with Pr=10 and F=0.1. These curves correspond to R_c, k_c, and ω_c against γ.

7. Conclusions

In the present work, the effect of controller gain in the linear hydrodynamic stability of a viscoelastic Maxwell fluid was studied.

The main conclusion of this work is that convection in the fluid layer can be controlled, or at least it can be suppressed. This is a direct conclusion since the curves of criticality state that the hydrodynamic stability of the fluid layer is increased with γ. The coupling of the stability parameters gives unexpected behaviors at small γ, but to the best knowledge of the authors, it could happen experimentally.

Acknowledgements

I. Pérez-Reyes and J. Rodríguez-Campos would like to thank the financial support from CONACyT through the project Ciencia Básica-255839. Numerical computations were performed using Maple™.

Nomenclature

Greek letters	
F	dimensionless relaxation time
g	acceleration due to gravity
H	fluid layer depth
k	perturbation wavenumber
Pr	Prandtl number
R	Rayleigh number
T_B^*	bottom wall temperature
T_T^*	top wall temperature
W	vertical velocity perturbation
β	thermal expansion coefficient
γ	controller gain
Θ	temperature perturbation
κ	thermal diffusivity
λ	stress relaxation time
ν	fluid kinematic viscosity
μ	fluid dynamic viscosity
σ	complex parameter
σ_R	perturbation growth rate
ω	frequency of oscillation

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