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# Vortex-Induced Vibration of a Marine Riser: Numerical Simulation and Mechanism Understanding

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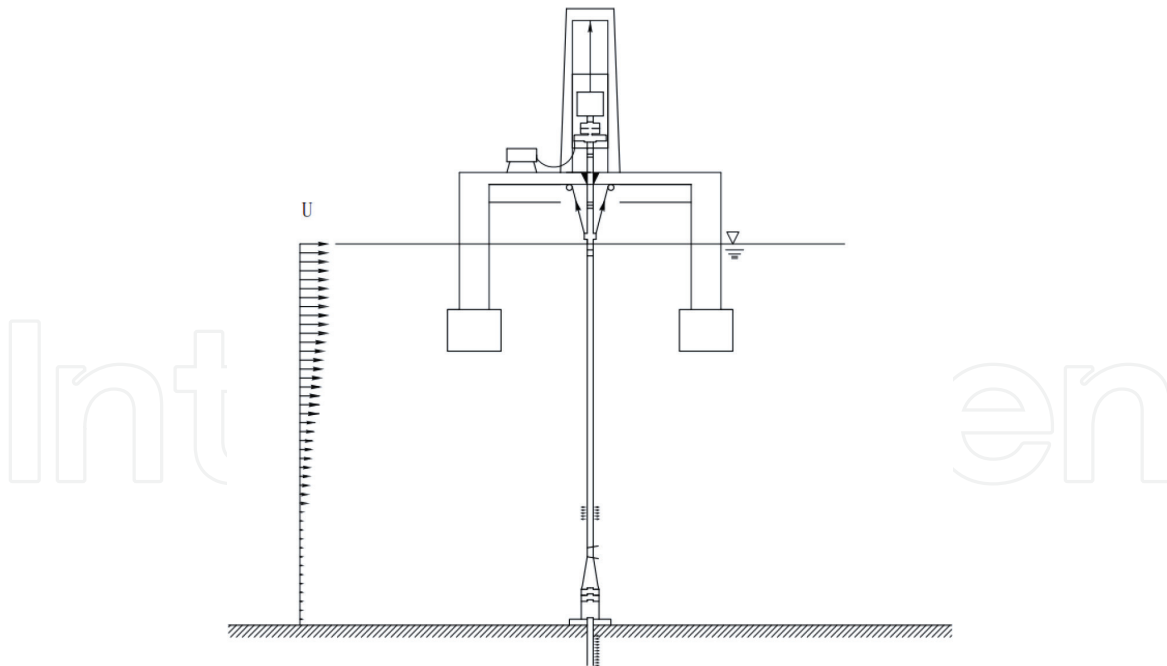
## Abstract

Marine riser is a key equipment connecting a floating platform and a seabed wellhead. Vortex-induced vibration (VIV) is the main cause of the fatigue damage of the riser. The prediction of marine riser VIV is very difficult because of its strong non-linearity, instability and uncertainty. In recent years, many numerical models of VIV of marine riser have been developed to explore the mechanism of marine riser VIV, providing scientific theoretical basis and practical engineering methods for vibration control and engineering design of marine riser. Combined with the authors' own recent research, this chapter discusses the research progress on marine riser VIV in the ocean engineering, including phenomenon mechanism analysis and different numerical research methods.

**Keywords:** fluid-structure interaction, marine riser, numerical simulation, structural dynamics, vortex-induced vibration

## 1. Introduction

As a key part of the deep-sea oil and gas exploitation system, the marine riser known as the “life line of offshore oil” is an important transport component connecting the undersea oil field with the ocean platform, as shown in **Figure 1**. In the process of operation, the marine riser bears the impact, erosion and other damage of seawater, and it needs to be repaired or renewed frequently. Its service life is only 2–3 years. Marine riser related technology has become one of the main obstacles to human being's deep-sea development. As the key structure connecting floating platform and underwater production system, marine riser generally bears the axial tension. The lower end connects with the seabed wellhead through the universal joint, and the upper end is coordinated with the slip joint on the platform or the bottom of the ship to carry out drilling, liquid guiding, mud guiding and other work. It is a necessary equipment for the deep-sea oil and gas development and transportation. In deep-sea environments, currents passing through the riser may induce the rear edge of the riser releasing the vortex alternately, known as “Karman vortex street”. The presence of “Karmen vortex street” causes the riser structure vibrating simultaneously in the cross-flow (CF) and the in-line (IL) directions, and the vibration of the



**Figure 1.**  
*Schematic diagram of marine riser operation.*

riser will in turn change the vortex characteristics of the flow field. This phenomenon of fluid structure interaction is called “VIV”. When the frequency of vortex release is close to the natural frequency of the riser structure, the phenomenon of “Lock-in” occurs. With the significant increase of the vibration amplitude of the riser, fatigue damage and even failure of the riser structure are often caused. In 2011, the “Gannet Alpha” drilling platform of shell oil company on the North Sea area of the United Kingdom suffered a sudden accident, resulting in the fatigue fracture of the oil pipeline connected with the platform. As a result, about 21.6 billion tons of crude oil leaked into the sea, causing serious pollution to the surrounding environment and causing major economic losses. However, the strong non-linearity, non-stationarity and uncertainty of the VIV of marine riser make it extremely difficult to predict the VIV of marine riser. How to accurately predict the VIV of marine riser and provide a more comprehensive theoretical basis for the design and daily maintenance of marine riser has been an ideal goal pursued by engineers and scientists.

For the study of VIV of marine riser, early scholars started from the simplest riser section, and now they have carried out a very rich experimental and numerical simulation study on two-dimensional (2D) rigid cylinder, including the study of flow around fixed cylinder, forced vibration and self-excited vibration of cylinder. The study of a rigid cylinder VIV (2D VIV) establishes many basic concepts of VIV, explores the law of rigid cylinder VIV, and lays a foundation for the construction of three-dimensional (3D) riser VIV algorithm model. However, the vibration of marine riser shows the characteristics of high order vibration and the vortex released by the rear edge of the riser has obvious 3D characteristics. Therefore, the research results based on the 2D VIV fail to provide complete technical support for the 3D marine riser. This prompted the researchers to study the VIV of 3D marine riser based on the research results of a rigid cylinder VIV. Due to the different emphases of the study on the VIV of a rigid cylinder and slender flexible riser, there are also many differences in the research methods and research hotspots. So far, there have been abundant researches on VIV, and many reviews have summarized the results of studies on the VIV of a rigid cylinder or a riser. For example, Williamson [1], Williamson and Govardhan [2, 3], Sarpkaya [4], Gabbai and Benaroya [5], Bearman [6], Wu et al. [7] have summarized the research results of a cylindrical structure or a

riser VIV in recent decades. In this chapter, the numerical simulation study of marine riser VIV in recent decades are summarized, and the development process, advantages and disadvantages of different numerical research methods are analyzed and summarized. Finally, the research progress of current basic research and hot issues in engineering practice is discussed. The numerical analysis methods of marine riser are mainly divided into two categories: one is the empirical model method based on experimental data, the other is the numerical simulation method based on computational fluid dynamics (CFD) and computational structure dynamics (CSD).

## 2. Empirical model

The numerical prediction method based on empirical model is suitable for solving practical engineering problems, because the physical model is clear, and the calculation is simple. Empirical models mainly include wake oscillator model, modal superposition model, statistical model, etc. Among them, wake oscillator model and its improvement model are most widely used. Bishop and Hassan [8] and Hartlen and Currie [9] proposed the wake oscillator model and described the hydrodynamic force with the nonlinear Van de Pol equation. In the model, the hydrodynamic equation is coupled with the vibration equation which describes the transverse VIV of the structure. This model can simulate many phenomena in the experiment qualitatively. Iwan and Blevins [10] and Iwan [77] deduced the coupling equation of wake oscillator and structural motion based on momentum conservation principle. The model has clear physical meaning and well reflects the hydrodynamic characteristics of VIV. The empirical parameters in the equation are determined by the test results of forced vibration. Therefore, the wake oscillator model is highly dependent on the empirical coefficient, and the empirical parameters selected by different wake oscillator models are greatly different. Therefore, even for the same research object, the prediction results of different wake oscillators are also significantly different. As demonstrated by Chaplin et al. [11], the amplitude response predicted by fourteen empirical models for the prediction of VIV in the same physical experiment is smaller than the experimental value, and the prediction results of each empirical model are also significantly different. The wake oscillator model has been widely accepted as a mainstream semi-empirical and semi-theoretical algorithm for a long time. However, the inherent deficiency of the algorithm lies in that the hydrodynamic frequency of the algorithm is obtained mainly through the calculation of the St number (St) obtained by the experiments of fixed circular cylinder. However, the frequency of hydrodynamic force of circular VIV of cylinder differs greatly from the frequency corresponding to the St number in the flow around the fixed cylinder. Therefore, in most cases, this algorithm can only obtain qualitative results, and its accuracy is greatly different from the experimental results. Facchinetti et al. [12] improved the dynamic characteristics of the wake oscillator model and, respectively, considered the coupling effect of displacement, velocity and acceleration on the wake oscillator. By comparing the prediction results of different coupling modes with the experimental results, it is found that the coupling of acceleration and lift induced by vortices can reflect the VIV characteristics of rigid cylinders quantitatively. Facchinetti et al. [13] further extended the model and applied it to VIV response of slender flexible cables. Mathelin and Langre [14] extend the work of Facchinetti et al. [13] to predict VIV response in shear flow. Furnes and Sørensen [15], Ge et al. [16] and Li et al. [17] proposed the simulation of dual-coupling oscillator model to simulate flow direction and transverse pulse dynamics, but the wake oscillator processed separately in both directions could not accurately predict the phase difference between flow direction and transverse response. Srinil [18]

and Srinil and Zanganeh [19] used a double duffing-van der Pol oscillator to predict VIV responses to flow and lateral coupling. The model can predict the amplitude response of flow direction and transverse VIV more successfully. In general, the empirical model approach does not consider the specific flow field structure, but directly considers the flow field and structure as a whole system. A set of equations is used to describe the characteristics of the whole system, which has the advantages of simple model, wide application range, and low requirement for computing power and storage capacity of the computer. However, this set of equations has many parameters, and the selection of these parameters is determined by experience or experiment. The choice of these parameters is crucial to the result, and the choice of parameters of different empirical models varies greatly. How to apply forced and self-excited vibration test data under specific conditions has not been completely solved, which constitutes a major shortcoming of the empirical model method. In addition, the empirical model is mainly applied to the prediction of VIV response of riser, which is not suitable for the study of VIV mechanism, wake vortex law and fluid-solid coupling characteristics of riser.

### **3. Numerical model based on CFD method**

#### **3.1 Numerical simulation of 2D VIV**

Numerical simulation of 2D VIV can be divided into two aspects. On the one hand, how to simulate the numerical results and phenomena which match the physical experiments; on the other hand, the intrinsic mechanism of VIV is studied by numerical simulation. In the study of a cylinder VIV, there are two topics that attract most attention: one is the super upper branch, and another is phase jump.

##### *3.1.1 Super upper branch*

Jauvtis and Williamson [20, 21] studied the VIV characteristics of a cylindrical structure at low mass and damping with 2 DOF and found that when the mass ratio  $m^*$  dropped to 2.6, the 2 DOF system got a super upper branch of transverse response amplitude, the largest response amplitude reaching 1.5 D, which all previous experiments had failed to get, and the “2T” vortex mode corresponding to the maximum transverse amplitude is observed.

Blevins and Coughran [22] adopted the physical experiment method to study comprehensively the VIV of cylindrical structure for 1 DOF and 2 DOF with variable mass ratio, damping, and found that the measured maximum amplitude was 1.75 D for a smooth 2 DOF cylinder at Reynolds number of 139,000. Based on experimental study, many scholars have carried out numerical study of VIV. Most scholars have used 2D numerical simulation to study the VIV of a cylindrical structure with low mass-damping [23–26]. For  $Re = 100–200$ , numerical simulations [23–26], including our previous research [27–29], gave similar amplitude results ( $A_y/D = 0.6$ ) which were lower than the expected values. Subsequently, some scholars simulated the VIV of a cylindrical structure with  $Re = 1000$ , with the amplitude of transverse vibration reaching a value of  $A_y/D = 0.7$ . All these numerical simulations were carried out at low Reynolds numbers, well below the Reynolds numbers used in the classic experiment [21]. For this reason, numerical simulation seemed to capture only the initial and lower branches, whereas the upper and super upper branches were absent. Recently, some scholars began to use numerical simulation methods to study the VIV of a cylindrical structure at Reynolds numbers matched with the classic experimental values. Guilmineau and

Queutey [30] used the incompressible two-dimensional Reynolds-Averaged Navier-Stokes (RANS) scheme to simulate the VIV of an elastically mounted rigid cylinder with low mass-damping, constrained to oscillate transversely to a free stream and compared their results with compared with the 1996 experimental results of Khalak and Williamson [31]. According the initial condition used, the simulations predict correctly the maximum amplitude. On the other hand, the numerical results do not match the upper branch found experimentally. However, these results are encouraging, because no simulations have yet predicted such a high amplitude of vibration. Wanderley et al. [32] used the Roe-Sweby scheme to solve the compressible RANS equations to simulate the VIV of an elastically mounted rigid cylinder for 1 DOF with the mass ratio  $m^* = 1.8$ . The numerical results obtained in the present work agree remarkably well with experimental data obtained from the literature (Khalak and Williamson [31]) which the mass ratio was equal to 2.4 and captured the corresponding response branch and vortex patterns. Pan et al. [33] also adopted the incompressible two-dimensional RANS scheme to simulate the VIV of an elastically mounted rigid cylinder for 1 DOF with the mass ratio  $m^* = 2.4$ , and the result was compared with the experimental data reported by Khalak and Williamson [31]. The absence of the upper branch in RANS simulations is explained in depth because of discrepancies, which exist between experiments and RANS simulations. Srinil et al. [34] presents an experimental and numerical investigation of an elastically mounted rigid cylinder for 2 DOF with variable nature frequency ratios  $f_x/f_y$  and reported that when  $f_x/f_y = 1.0$  and mass ratio  $m^* = 3.5$ , the transverse amplitude of the numerical simulation was much smaller than that of the experimental value. Gsell et al. [35] investigated the VIVs of an elastically mounted circular cylinder using a direct numerical simulation method. In the upper branch, the maximum amplitude predicted by the simulation at  $U^* = 6.5$  (about  $1.2D$ ) is lower than the amplitude measured experimentally [21] at the same reduced velocity. The author thinks that some deviations are expected due to the difference in the value of  $Re$ . Zhao and Cheng [36, 37] used the incompressible two-dimensional Reynolds-Averaged Navier-Stokes scheme to obtain the super upper branch and 2T vortex shedding pattern with initial conditions of velocity increasing constantly, which was in a good agreement with the experimental results [21]. Li et al. [38] adopted two typical turbulent models to simulate the VIV of a cylindrical structure for

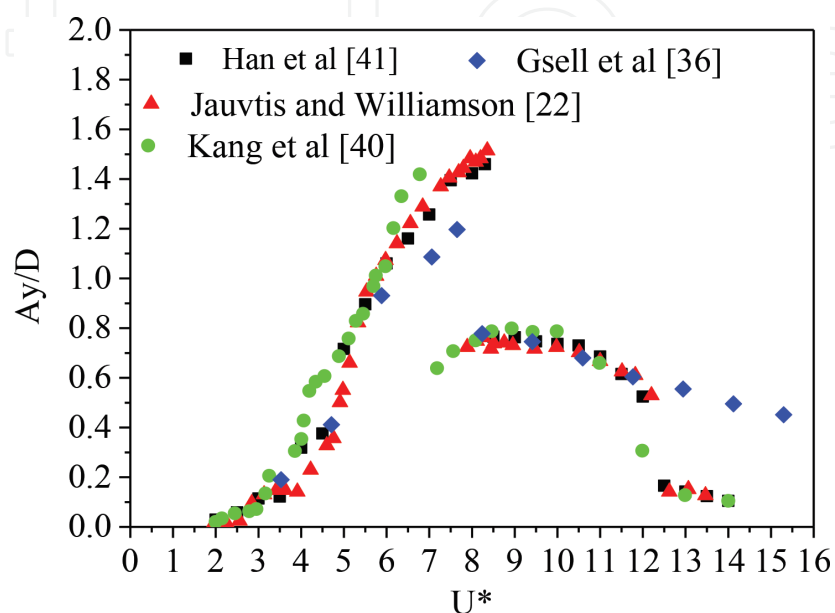


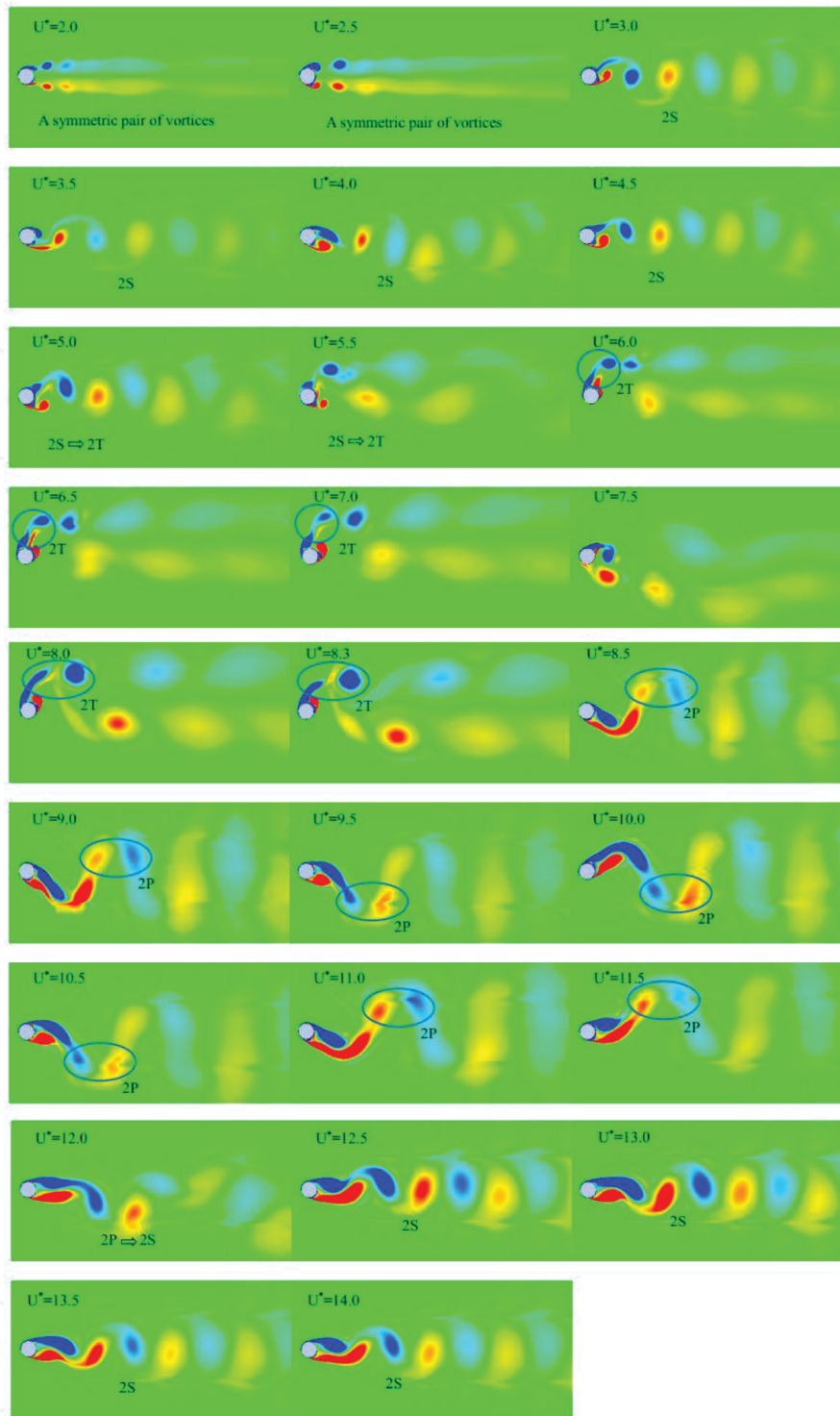
Figure 2.  
Response amplitudes of the transverse directions under different reduced velocities [40].

1 DOF, and the predicted maximum amplitude was lower than that obtained by Khalak and Williamson [31]. Kang et al. [39] applied a modified SST model based on OpenFOAM to simulate experimental results [21] and it captured the maximum amplitude reaching values of 1.4 D when the entrance velocity was made to increase constantly in the process of numerical simulation. In previous studies [36, 37, 39], the initialization of numerical simulation started from the entrance, the inflow velocity is increased gradually from 0 or one low velocity value to the target value at a constant acceleration value and then is kept constant. The results show that the maximum transverse amplitude can be captured more accurately with appropriate inflow acceleration value. The value of this acceleration is usually very small, it takes a long time to accelerate to the target value, and how to determine the appropriate value of acceleration needs to be tested. Using this method to simulate the upper branch, the workload and computation time of is very large. Han et al. [40] successfully capture the initial branch, the lower branch, and the super upper branch with more accurate results, as shown in **Figure 2**. The units of each coordinate axis in the figure are dimensionless. The corresponding reduced velocity range of each branch is consistent with the classical experimental results [21]. The maximum value of the super upper branch is 1.46 D.

The vortex pattern at different reduced velocities is simulated successfully, as shown in **Figure 3**. The VIV of the streamwise direction is in a resonance state at a low reduced velocity and the vortex pattern in the wake is a symmetric pair in vortex modes. With the increase in reduced velocity, the transverse amplitude increases continuously, the cylindrical structure response enters the super upper branch, and the vortex pattern switches from 2S to 2T. When the reduced velocity increases further, the cylindrical structure's response enters the lower branch and the vortex pattern becomes 2P. With further increase in the reduced velocity, the transverse amplitude decreases continuously, and the vortex pattern becomes 2S.

### *3.1.2 Phase jump*

Previous work has also shown phase differences between lift and CF displacements. In 1964, Bishop and Hassan [8] experimented on the lateral forced oscillation of a cylinder in a uniform flow, identifying the important phenomenon that when a cylinder oscillation frequency is near a natural vortex shedding frequency, the phase difference of the cylinder lift and the CF response undergoes a "sudden" jump from an "out-of-phase" mode to an "in-phase" mode. Then, Sarpkayab [41, 42], Bearman and Currie [43], Gopalkrishnan [44], and Carberry et al. [45] conducted similar experiments and reported the same phenomenon. The experimental study of Carberry et al. showed that in the process of a cylinder oscillating from a low frequency area (lower than the natural vortex shedding frequency) to a high frequency area (higher than the natural vortex shedding frequency), there is a transition zone. In the transition zone the mode of the vortex wake changes and at the same time the phase difference of the lift and CF responses undergoes a "sudden" jump. Sarpkaya [41] concluded that the phase difference between the lift and CF responses was closely related to an energy transfer between the cylinder and fluid. Zdravkovich [46] used visual means to analyze the previously studied flow field and found that a phase difference between the lift and CF responses was related to the vortex shedding time. Ongoren and Rockwell [47] used visualization analysis and reached a similar conclusion. Gu et al. [48] pointed out that with an increase in the frequency ratio, at a certain critical frequency, compared with the initial time, the cylindrical vortex shedding jumped from one side to another. Previous research has also shown that a phase difference between the cylinder lift and displacement, which determines

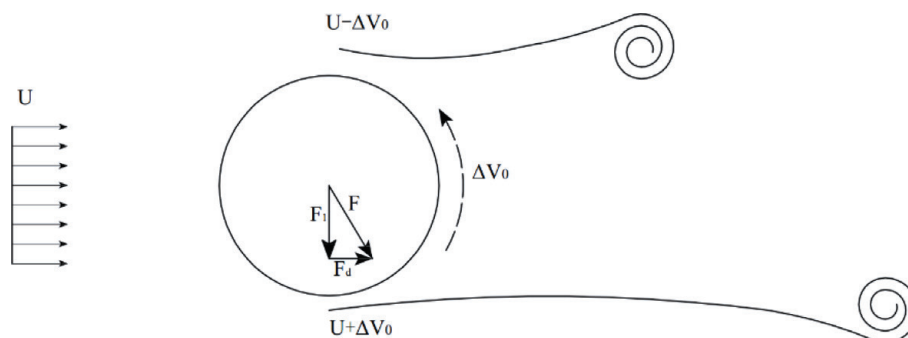


**Figure 3.**  
 Vorticity contours under different reduced velocity [40].

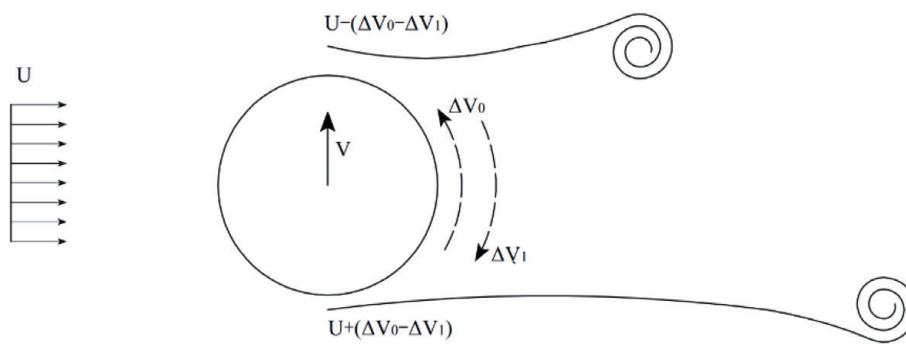
the amplitude of a cylinder vibration, is related to the energy transfer between the cylinder and fluid. Govardhan and Williamson [49] used the particle image velocimetry method to study transverse VIVs and highlighted that the phase remained slightly above throughout the initial and upper branch regimes and jumps of almost occurred throughout the upper and lower branch. Guilmineau and Queutey [30] used the RANS method to simulate a cylindrical CF forced oscillation and also found that the phase difference jumped about  $180^\circ$ . Li et al. [50] studied the nonlinear characteristics of VIV at and successfully captured phase-switch, lock-in, and beat phenomena. Wang et al. [51] has demonstrated the importance higher harmonic flow forces and phase mechanisms related to relative velocity and has study the



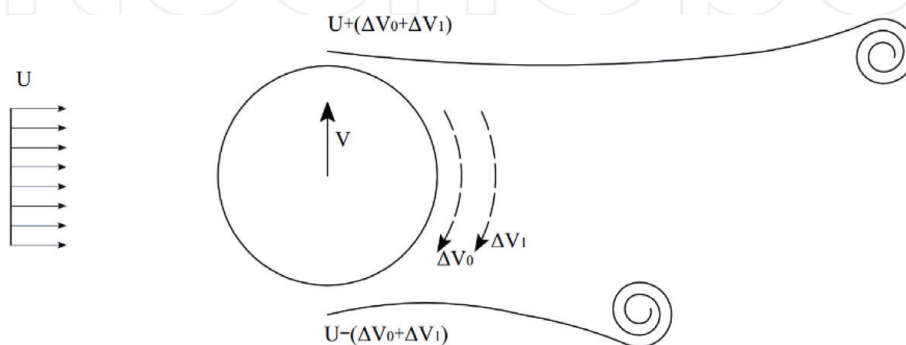
effects of the relative velocity of the cylinder's oscillation with respect to the flow. Vortex formation modes been studied by 2D and 3D simulations before [6, 52–56]. The abovementioned research [6, 46–50, 52–56] highlighted typical behaviors in the relation between CF responses and lift and vortex shedding modes of the both elastically mounted cylinders and forced VIVs but did not clarify the science behind the law of the relationship between vortex shedding and phase difference, or how the vortex affects the phase difference. Despite numerous studies of typical behaviors of the VIV of an elastic cylinder, the underlying mechanisms governing typical behaviors such as phase difference remain to be elucidated. Han et al. [28] investigates the VIV of an elastically mounted cylinder at various frequency ratios. According to differences in the vortex shedding location, the vortex wake can be characterized by two kinds of mode, that is, the “first mode” and the “second mode”. The mechanisms behind the phases of the first mode and the second mode vortex wakes are investigated and it is found that the flow speed induced by a cylindrical transverse vibration and the position of a vortex release are the root causes of the phase difference between the lift coefficient and transverse displacement. The speeds caused by a cylinder vibration and a cylinder shed vortex are the reasons that the lift amplitude of an oscillatory cylinder is different from that of a fixed cylinder. For a CF VIV, when a cylinder sheds the vortex, in addition to producing a flow similar to that of a fixed cylinder with a velocity of  $\Delta v_0$ , it also produces flow caused by the cylinder vibrating with a velocity of  $\Delta v_1$ . **Figure 4** shows the schematic diagram of the flow field around a fixed cylinder. As shown in the figure, when the vortex is released from the upper surface of the cylinder, flow velocity  $v$  is generated around the cylinder, making the upper surface velocity change to  $U - \Delta v_0$  and the lower surface velocity change to  $U + \Delta v_0$ . At this point, the pressure on the upper surface is greater than the pressure on the lower surface due to the small upper surface velocity and the large lower surface velocity, resulting in a downward lift. **Figure 5** shows the schematic diagram of the “first mode” flow field around a cylinder moving to the maximum displacement for the CF VIV. The “first mode” means when a cylinder moves to the maximum displacement, the cylindrical upper surface begins to shed vortex. At this time,  $\Delta v_1$  and  $\Delta v_0$  were in the opposite direction. The direction of the lift depended on the positive or negative condition of  $\Delta v_0 - \Delta v_1$ . When  $\Delta v_0 - \Delta v_1 > 0$ , the cylinder formed a downward lift and when  $\Delta v_0 - \Delta v_1 < 0$ , the cylinder formed an upward lift. **Figure 6** shows the schematic diagram of the “second mode” flow field around a cylinder moving to the maximum displacement for CF VIV. The “second mode” means when a cylinder moves to the maximal displacement and the cylindrical lower surface begins to shed vortex. At this time,  $\Delta v_1$  and  $\Delta v_0$  are in the same direction and the upper surface velocity is higher than the lower surface velocity, causing lowering of the cylinder's upper surface pressure compared to that of the lower surface pressure, thereby producing an upward lift [28].



**Figure 4.** Schematic diagram of the flow field around a fixed cylinder [28].



**Figure 5.**  
 Schematic diagram of the “first mode” flow field around a cylinder for CF VIV [28].



**Figure 6.**  
 Schematic diagram of the “second mode” flow field around a cylinder for CF VIV [28].

### 3.2 Numerical simulation of 2.5D VIV

The basic idea of the multi-strip method is to take a certain number of 2D slices of the riser in the axial direction to simulate the flow field and extract the load acting on the riser. Then, the load is applied to the riser to obtain the motion response of the riser, and the load is applied to the riser to obtain the movement response of the riser, which is carried out repeatedly, and finally to predict the VIV of the riser. This multi-strip method can effectively reduce the calculation time, and many scholars [57–61] compare the prediction data obtained by the multi-strip method with the experimental data, and the results are relatively consistent. Willden and Graham [59] used the multi-strip method to construct the quasi-three-dimensional riser model to simulate the VIV of riser in the transverse direction under shear flow and found that the fluid controlled the vibration frequency of the structure by influencing additional mass. Yamamoto et al. [57] established a quasi-three-dimensional CFD model, solved the hydrodynamic force of each section with the discrete vortex method, and calculated the vibration response of marine riser based on the Euler Bernoulli beam theory. The vibration response of the riser at different reduction velocities was calculated and compared with the test results. It was found that there was a vortex in the form of “2P” in the larger amplitude and a vortex in the form of “2S” in the smaller amplitude. Based on the theory of slicing method and the radial basis function dynamic mesh technology. Professor Wan [62–66] led the team to use OpenFOAM to solve the fluid field at each section and used the structural finite element method to simulate the motion response of the riser. The effects of various parameters (such as mass ratio, tip pretension, flow velocity and flow profile) on VIV of a riser were investigated. However, the VIV of a marine riser is a 3D problem in nature, and it is impossible to consider the influence of the flow along the axial direction by using the multi-strip method, and the description of the vibration mechanism of multi-modal coexistence is relatively vague. In addition, for

the slicing method, it needs some interpolation algorithms to deal with the relationship between the slices, but this interpolation algorithm does not have a uniform standard [67, 68]. Hovor et al. [69] studied the three-dimensional effect of the wake of oscillating cylinder and the correlation of the flow force along the axial direction and proposed that the three-dimensional flow field characteristics of the vertical tube VIV are closely related to its structure amplitude, and the three-dimensional effect of the wake limited the further increase of the structure response amplitude.

### **3.3 Numerical simulation of 3D VIV**

In recent years, due to the improvement of computer hardware, several fully 3D numerical simulations of a marine riser VIV are generated. Constantinides et al. [67] used a finite element Navier-Stokes (NS) solver to study a high L/D riser model. This method overcame the shortcomings of the Q3D method and correctly estimated the 3D effect. The response amplitude of the numerical simulation was compared with experiment to verify the rationality of the algorithm and vortex shedding modes were briefly analyzed. Holmes et al. [68] used fully 3D CFD methods to simulate a straked riser VIV. The resulting solutions were compared with available experiment data on a 38 m long riser model based on RMS displacements. Xie et al. [70] used a finite-volume method to study the multi-mode VIV of a flexible circular cylinder. Huang et al. [71, 72] used a RANS method and an overlapping mesh technique to simulate the VIV of a riser with the length to diameter ratio of 482 in uniform flow and shear flow environments and compared numerical calculation results with the experiment to verify the accuracy of the calculation procedure. Bourguet et al. [73–76] used direct numerical simulation of the 3D incompressible NS equations and a beam model to simulate the VIV of a cylindrical tensioned beam with the aspect ratio of 200 in linear and exponential shear flows at a Reynolds number equal to 330. Phasing mechanisms between the IL and CF, mono- and multi-frequency, and lock-in of VIV were explained.

In recent studies, many researches have investigated the VIV of marine risers but most of their research has been limited to 2D or rigid cylinder VIV. For the 3D flexible riser VIV, researchers have mostly adopted the 2.5D method. Only a few studies have adopted a bidirectional fluid-structure interaction method to simulate the VIV of the riser. In such studies, the displacement of the riser was solved using a mode superposition method or the Euler-Bernoulli bending beam equations. Most of these studies have concentrated on quantification of the vibration response. Nevertheless, the frequency characteristics of the vibration response and the universal rule of vortex shedding modes and trajectories in the spanwise direction have been less addressed, so further research is needed. Meanwhile, some studies have focused only on the VIV of a riser in linear and exponential shear flows at low Reynolds numbers, and further study is needed for the VIV of a riser in uniform flows at higher Reynolds numbers.

## **4. Research focus and prospect**

### **4.1 Study on the nonlinear mechanism of VIV**

Studies on VIV in recent decades are often accompanied by the discovery of new phenomena and locked-in, jump, lower branch, upper branch, super upper branch, lower branch, and various wake vortex forms (such as 2P, 2T, P + S) are found in cylindrical VIV; in the study of 3D marine riser VIV, the phenomenon

of traveling-standing wave interaction propagation, multi-modal vibration and high-order modal vibration was found. Scholars have been continuously exploring the conditions and internal mechanism of these new phenomena, but it is difficult to give a conclusion at present. The goal is to further understand the phenomenon of VIV and extract the internal logic relations of major factors affecting VIV, to establish a more reasonable prediction model to serve ocean engineering practice.

#### **4.2 Improve the prediction model of marine riser VIV**

The prediction model of marine riser VIV still needs to be improved. Different from the numerical simulation based on CFD and FEM, these prediction models applied to practical engineering should be of high computational efficiency and can be easily applied to the structural design of marine riser. Perfect forecast model mainly from the following two aspects, that is, one is using simple parameters reflect the basic characteristics of VIV problem and special phenomenon of VIV (jumping, the response of each branch, etc.), and the selection of its value cannot rely too much on user's personal experience, avoid parameters selection process cumbersome and lose operability. Another is building empirical models that can be validated on a broader scale. At present, although there are various models, and each of them can be verified well under certain conditions, there will be great differences in the calculation of back-to-back verification. No model has absolute advantage, and it is difficult for one model to match the results of multiple experiments.

#### **4.3 Fluid-structure interaction model based on CFD and CSD**

With the continuous improvement of computer operation speed, numerical methods have been developed rapidly, and now have become one of the main research methods in parallel with theoretical analysis and tank test. The fluid-solid coupling numerical methods based on CFD and CSD are playing more and more important roles in the field of ocean engineering, especially in the design of offshore platforms and structural safety reliability analysis. The development of efficient large-scale parallel numerical analysis system will be an important direction to enhance the competitiveness in the field of ocean engineering in the future.

The 3D numerical simulation of the VIV of marine riser requires a hydrodynamic computational program and a structural mechanical computational program. In the coupling interface, fluid dynamic data and structural displacement data are exchanged, which involves grid data mapping, interpolation and fluid mesh reconstruction of fluid and solid media, resulting in the reduction of precision. In addition, the computing technology also involves some key technical problems, such as efficient interactive use between different solvers, economic and practical parallel algorithm and parallel programming, etc., and exploring more effective algorithms to further improve precision and efficiency is still one of the focuses of recent research. With the increase of the length-diameter ratio of marine riser and the increase of Reynolds number (the actual Reynolds number of marine engineering is mostly within the range of  $10^5$ – $10^6$ ), the above problems become more difficult to solve. Research on circular VIV within the range of high Reynolds number is still rare, so it is necessary to conduct research on VIV under the condition of high Reynolds number, further enrich the research results, explore new fluid-solid coupling phenomena and theories, and lay a theoretical foundation for the conceptual design of marine engineering riser.

## 5. Conclusions

This chapter summarizes the research status of various numerical calculation methods, including empirical model and CFD numerical calculation. In the aspect of experience model, this chapter briefs the development process of experience model and the deficiency of current experience model is proposed. In the aspect of 2D numerical simulation, the progress of 2D numerical simulation of VIV is summarized combining the author's own research work on the two hot phenomena of super upper branch and phase jump. The development process of the multi-strip method for 3D marine riser VIV simulation is introduced in the aspect of 2.5D numerical simulation. In the aspect of 3D numerical simulation, numerical simulation of 3D marine riser VIV is analyzed with the limitations of current 3D marine riser research. At the end of the chapter, the future research direction is proposed, including the nonlinear mechanism of VIV, prediction models, and CFD and CSD fluid-solid interaction models.

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## Conflict of interest

No conflict of interest declared.

## Appendices and nomenclature

$A_y$	the amplitude of the transverse vibration
$D$	the cylinder diameter
$U$	the inlet velocity
$m^* = m/m_d$	ratio of oscillating mass over displaced mass
$m_a = \rho\pi D^2/4$	added mass in still water
$St = f_0 D/U$	Strouhal number in still water
$Re = UD/\nu$	Reynolds number
$\nu$	kinematic viscosity coefficient
$U^* = U/(f_n D)$	reduced velocity in water
$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_a}}$	natural frequency of elastic cylinder in still water
$f_{nx}$	natural frequency of elastic cylinder of x direction in still water
$f_{ny}$	natural frequency of elastic cylinder of y direction in still water

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