Chapter

Remote Computing Cluster for the Optimization of Preventive Maintenance Strategies: Models and Algorithms

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Abstract

The chapter describes a mathematical model of the early prognosis of the state of high-complexity mechanisms. Based on the model, systems of recognizing automata are constructed, which are a set of interacting modified Turing machines. The purposes of the recognizing automata system are to calculate the predictors of the sensor signals (such as vibration sensors) and predict the evolution of hidden predictors of dysfunction in the work of the mechanism, leading in the future to the development of faults of mechanism. Hidden predictors are determined from the analysis of the internal states of the recognizing automata obtained from wavelet decompositions of time series of sensor signals. The results obtained are the basis for optimizing the maintenance strategies. Such strategies are chosen from the classes of solutions to management problems. Models and algorithms for self-maintenance and self-recovery systems are discussed.

Keywords: turing machine, maintenance optimization, preventive maintenance, remaining useful life, remote calculating cluster

1. Introduction

This chapter describes a mathematical model that allows to unify the multiplicity of approaches to the creation of intelligent maintenance systems on the one hand and also allows more to accurately formalize and then algorithmize the optimization tasks of maintenance strategies.

Consideration of problems in the management of maintenance is useful to begin with the formalization of the basic tasks of PHM. The main trend of today is the development of prognostics and health management (PHM) in the sequence condition-based maintenance (CBM)-predictive maintenance (PdM) [1]. This concept may be called “CBM +” or “proactive management of materials degradation,” and then on the horizon, there are new concepts, bearing a semantic load, in particular self-maintenance and self-recovery systems. However, all these concepts need further formalization (mathematical). It should also be noted that the effectiveness of any maintenance strategy depends on how reliably the PHM system is able to predict the state of technical objects of high complexity. Construction of the effective maintenance strategies is possible on the basis of a reliable prognosis.

Let us dwell in more detail on this question. Any prognostic system is based on the statistical processing of the signals received by sensors mounted on the technical objects. This can be a variety of vibration sensors, sensors of pressure, and measurement of currents and voltages. In this chapter, we will appeal to the examples of prognosis of the technical state of rotating machinery and the reciprocating action mechanisms, demonstrating the commonality of models.

Also, the chapter will pay attention to medical applications, in particular, to remote cardiac monitoring systems. Here, the task of the prognosis consists of three sub-tasks: a prognosis of the state of the heart on the basis of wearable or implanted in the body-miniaturized ECG recorders; construction of the management model of the heart state with the help of variable parameters of implantable devices in the human body, such as ICD and CRT devices; and prognosis and estimation of the remaining useful life (RUL) of the implanted devices [2, 3]. It should be noted that not only statistical methods are the basis of the prognosis, but also, more importantly, in this basis, physical models of the monitored object and its subsystems should be contained. Ultimately, we are talking about digital counterparts, that is, accounting for all components and processes occurring in a working device is necessary.

Thus, there is a task of prognosis of the technical state of the object and a time estimation under the general name remaining useful life (RUL). The prognosis of the technical condition and RUL estimation are the bases for constructing cost-effective maintenance strategies.

It is on the basis of the prognosis and RUL estimates is possible formalization of the task of determining the cost-effective maintenance strategies, taking into account the conditions of goal setting. Goal setting involves taking into account the requirements for the technology and determining the ultimate goal of its operation. For example, in monitoring vehicles, extending the time of operation with a minimum change in operating parameters, for example, the engine and its subsystems, is a natural requirement that determines the strategy for calculating optimal operating conditions. Obviously, the calculated strategy is unsatisfactory for military applications, since it will explicitly prohibit operation with violation of optimal speed regimes and all kinds of extreme exploitation. When calculating maintenance strategies for military systems, the goal setting conditions change, where the determining strategy is the delivery of one's own weapons to a given point of space at any cost, taking into account the impact of enemy-striking factors.

The noted condition directly points to the fact that a maintenance strategy with necessity must be determined in a number of cases in real-time conditions, while the goal setting itself will change during operation. The transition to earlier prediction methods that can be called the diagnosis or prognosis of the root causes, hidden predictors of prognosis, etc. creates the conditions to search for more effective maintenance strategies. And, finally, the creation of self-maintenance and self-recovery systems requires the presence of a physical model of processes, within which functional dependencies between the parameters of management of process and its state are determined.

2. Basis

For precise algorithmized formulations of optimization tasks for maintenance strategies, mathematically rigorous formalizations of the basic concepts of prediction tasks and tasks of management of the state of technical object are necessary.

We will assume that the system is equipped with all the necessary sensors, registering the vibration of the engine housing, the sensors of the angle of rotation of the shaft or crankshaft, pressure sensors in high-pressure fuel lines and other necessary sensors, most of which are included in the system of traditional onboard diagnostics or control systems. It also means the possibility of transmitting sensor signals (time series) to a remote computing cluster.

Further input to the computing cluster signals or time series is presented in the form of their wavelet coefficients. The fixation of all indices of the wavelet coefficients except for the quasi-period index, that is, the current number of the cycle of turbine engine, etc., determines the so-called cascades. The entire set of cascades is considered. Their number is equal to the number of wavelet coefficients of the decomposition of the time series multiplied by the number of sensors from which signals are received. A set of finite segments of fixed cascade defines a state vector in its sequence. The evolution of the state vector at successive change of segments determines the vector of trajectory. The multi-trajectory is determined by the vector of trajectories of all cascades, that is, a set of state vectors determines the multistate or state of the entire system.

The first prognosis problem is reduced to the definition of the evolution equations describing the evolution of the state vector.

Depending on the properties of the process, these equations are known in the sections of nonequilibrium thermodynamics called the "basic kinetic equation." The basic kinetic equation is reduced depending on the properties of the cascade (stationarity, ergodicity, nonstationarity, Markovity, non-Markovity, etc.) and reduces to equations such as the Fokker-Planck equation, the Schrodinger equation, the balance equation, to single-step processes, etc. [4, 5].

The prognosis task is formulated as a definition of the probability of a transition from the initial state vector to the final one and preassigned [6, 7], for example, preassigned on the boundary of the failure region, on the boundary of the region of the nucleation of failure predictor, or on the boundary of failure predictors. The development or evolution of predictors or hidden predictors is also described by evolutionary equations of the type listed above. Thus, there is a set of trajectories or multi-trajectories of sequences of states of the system. Further formalization requires the classification of trajectories in order to determine the trajectories leading to the boundaries of failure. The boundaries between classes of different trajectories may not be physically observable. However, these boundaries affect the trajectories, changing their characteristics. For example, in the case of interpreting a trajectory as a random walk in a multidimensional lattice or its continual counterpart, the evolution equations themselves and, consequently, the RUL estimates change.

Thus, there arises the problem of classifying a set of physically feasible trajectories or the task of representing trajectories in the form of a set of classes and the task of describing the boundaries between classes. Separation of the set of trajectories into classes is a rather ambiguous task, and often there are problems with changing the classification when changing the types of processes. However, it follows from the constructed model that the separation of trajectories into classes is related to the transformation of the topological characteristics of the state space and trajectory spaces.

In the case under consideration, each class is characterized by its own group of symmetries of the probability density of transitions between vectors and/or the group of symmetries of the generating functional. Factorization of the symmetry group by the isotropy subgroup, leaving the vector state in place, generates a homogeneous space [8]. It is in this space that the vector process wanders. In the process of operation of the mechanism and degradation of the material, the topology of the homogeneous space is changed; this change generates the boundary between classes.

Let us return to the tasks of management. In the concepts defined above, the task of management is formalized as follows in which variations of the controlled parameters preserve the trajectory of the states of the system in the given class for as long as possible. The following formulation concerns the estimates of RUL as an estimate of the time to reach the class boundary when the controlled parameters are varied. When crossing class boundaries, the task of evaluating the RUL and maximizing the time of stay in the class is solved again. In this case, the evolution equations change.

The model described above makes it possible to formalize the problem of finding optimal maintenance strategies as a task of determining control parameters or more precisely determining the range of admissible control parameters under which the trajectory is kept as long as possible in a given class.

3. Model

The search of a way to formalize maintenance management tasks and building models and algorithms for searching for optimizing strategies is useful to start with the formalization of the management process in an extremely general setting. The approach presented here is rather complicated, but it is useful for the development of further formalizations and construction of algorithms.

To do this, we will present the task of managing, using the following definitions. Let the considered technical object have in its arsenal several parameters, the variation of which affects the state of the mechanism, changing all the permissible modes of its operation. More precisely, the variation of the control parameters allows the mechanism to be switched from one operating mode to another physically acceptable mode. Next, consider some abstract mathematical space; often, these are certain subsets of a multidimensional space $\boldsymbol{R}^{\boldsymbol{N}^*}$. Further constructions show that these subsets of space $\boldsymbol{R}^{\boldsymbol{N}^*}$ are topological manifolds with a complex topology.

Each point of such space determines the state of the mechanism at a fixed time; the sequence of states defines a trajectory in the state space. We also accept, as an empirically understandable assumption, that when the control parameters are varied, the continuous trajectories change in the same way without discontinuities. That is, a small perturbation of the parameters also causes a slight perturbation of the trajectory; in other words, for small perturbations the new trajectory is in some sense close to the original trajectory.

As a result, a continuous mapping from the parameter space to the state space is determined. **Figure 1** demonstrates the mapping of the management loop Ω, consisting of two management parameters to the state space. It is assumed that all values of the parameters inside the circuit are physically realizable. When mapping the management interval $I = \{\lambda_i\} \stackrel{\text{def}}{=} [0, 1]$, the path is formed from the initial to the final state in the state space. As a result, at the change of parameters and with changes in the state of the mechanism during operation, many paths are generated. The set of paths in the state space **X** defines a new space [9], designated as $\Omega\boldsymbol{X}$—the loop space of space $\boldsymbol{X}$. In this case, the next parameter determines already the mapping of the management interval:

$$I \to \boldsymbol{\Omega X} \tag{1}$$

thus defining a twofold loop space $\Omega^2\boldsymbol{X}$ in the space $\boldsymbol{X}$.

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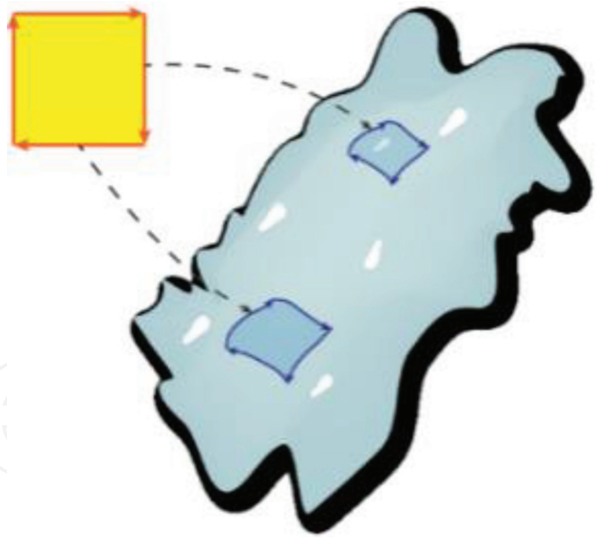

Figure 1.
Mapping the management loop X $\mathbf{R}^n \times \mathbf{R}^1$ in state space; $\lambda_1 \ldots \lambda_n$;—Variable parameters, t*-time.*

$$\Omega^2 \boldsymbol{X} \stackrel{\text{def}}{=} \boldsymbol{\Omega}(\boldsymbol{\Omega X}) \tag{2}$$

$$\Omega^k \boldsymbol{X} \stackrel{\text{def}}{=} \Omega^{\boldsymbol{k-1}}(\Omega \boldsymbol{X}) \tag{3}$$

Further presentation will require some information from algebraic topology, more precisely the homotopy theory. In view of the complexity of mathematical constructions, one must sacrifice a mathematically rigorous exposition in favor of simplification and clarity. Thus, the above arguments necessarily lead to an analysis of the set of paths $\Omega \boldsymbol{X}$ in the state space $\boldsymbol{X}$ defined by the mapping Eq. (1) for one management parameter. With an increasing number of parameters, the path space is generated in the path space of the previous parameter Eq. (3), and so on with the growth of the number of control parameters. As a result, taking into account all control parameters leads to consideration of the k-fold space of paths, more precisely to the k-fold loop space [9].

Returning to the basic concepts of homotopy theory, it should be noted that the methods mentioned here were used in the 1970s in the physics of a condensed state for the analysis of singularities in condensed media, including superconductors (Abrikosov vortices), superfluid liquids, and liquid crystals. The methods of homotopy topology are effective not only for general analysis and classification of singularities of condensed media [10] but also transferred to the analysis of processes expressed in the form of multiple spaces of loops. This fact can be explained as follows. The management contour at the mapping to the state space defines a contour in the state space itself or on the corresponding loop space, the multiplicity of which is determined by the number of management parameters. The following problem arises, solved by the homotopy theory methods. Can the image ∂I^k under the mapping and defined by the contour in the state space or constructed Eq. (4, 5) on it the loop space be continued from the boundary of the set I^k, ∂I^k, to its interior I^k in a continuous manner? Or such continuation is impossible, that means the presence of topological obstacles, expressed by the nontriviality of the topological (homotopy) type of the state space, the loop space. In the case of obstacles, any continuation will undergo a discontinuity in the corresponding topology of the loop space. In the case when the mapping F to the loop space is topologically nontrivial, that is, corresponds to a nontrivial element of the homotopy group of the state space or loop spaces, then a discontinuity will occur when the management parameters are varied. This means that it is not possible to continue the regularity from the boundary of the management loop to its interior without discontinuities. Physically, with small variations in the management parameters, a transition from the initial process to the final process will take place abruptly. This, depending on the specific physical content of the management model, leads to dramatic changes in the state of the mechanism that is accompanied by a sharp change in the operating conditions and extreme loads, leading to accelerated degradation of the material: the nucleation and growth of microcracks, the development of abnormal wear in the corresponding mechanical junction and other troubles, the precursor of creation of avalanche changes in the material, and so on.

$$F : \partial I^k \to X \tag{4}$$

$$I^k = \{\lambda_i : i = 1, \ldots k\} \stackrel{\text{def}}{=} \prod_k [0, 1] \tag{5}$$

The above results need more precise definitions of the state space X, the space of trajectories, and the identification of physical causes for the appearance of homotopically nontrivial state space.It is appropriate here again to use the analogy with topological defects of condensed media. It has already been noted above that when using analogies of this kind, it is only necessary to redefine the notion of a degeneracy space. The redefined degeneracy space in this case and thanks to the work [9] is nothing more than a k-fold loop space.

Topological singularities in condensed media are provided by the homotopy nontriviality of the so-called degeneracy space of the free energy functional of a condensed medium. The presence of the degeneracy group of the free energy and its further factorization with respect to the isotropy subgroup gives the required degeneration space, in mathematics called the homogeneous space [10]. In the task under consideration, the analog of the construction of the degeneracy space is in the most general case the characteristic functional of the stochastic process. The symmetry groups of such a functional are considered in [11]. To understand the methods of constructing degeneration spaces, one can consider the density of function of the distribution of the process. If we return to the cascades of the wavelet coefficients of the observed signal and then to the vector processes, then we consider the vector process or segments of length N or the set of such segments or vectors under certain assumptions about the properties of the observed process, for example, if the process reduces to a random walk in a multidimensional lattice or on a continuum. In the example under consideration, the group of probability density function (PDF) of the process has a Gaussian distribution, and hence the symmetry group of such a process is the group $SO(N)$.

For example, in the problem of walk of $\boldsymbol{R}^N$ [6, 12], the Gaussian function for the density of probability of falling into a point $R \in \boldsymbol{R}^N$ after traversing the path of length L is the following:

$$G(R; L) = \left(\frac{N}{2\pi lL}\right)^{\frac{N}{2}} \exp\left(-\frac{N\|R\|^2}{2lL}\right). \tag{6}$$

The subgroup of isotropy is in this case the subgroup of rotations of the vector R about its axis, that is, $SO(N-1)$. The result of the factorization $SO(N)$ of the group with respect to the subgroup $SO(N-1)$ is the N-1-dimensional sphere:

$$SO(N)/_{SO(N-1)} \cong S^{N-1} \tag{7}$$

Taking into account other symmetries existing in the observed process changes the degeneracy space. For example, the vector processes under consideration can in some cases have symmetry with respect to time reversal, and then the vector field becomes a field of directors as in a nematic liquid crystal. In this case, the degeneracy space is transformed from a sphere into a projective space of dimension:

$$ {}^{SO(N)}\!/_{SO(N-1)\times Z_2} \cong PR^{N-1} \tag{8} $$

The presence of trends or dynamic predictors removes such degeneracy, and the degeneracy space again becomes a sphere. The permutation group acting on the components of the vectors, that is, changes their places, turns the sphere into an even more complex homogeneous space, where the gluing takes place in the discrete orbits of the group of permutations during factorization, generating a space homotopically equivalent to a bouquet of spheres of different dimensions (**Figure 2**):

$$ \mathrm{DS} = \mathrm{S}^{\mathrm{N}-1} \bigvee\nolimits_{\{\mathrm{i}\}} \mathrm{S}^1 \tag{9} $$

The transition to space trajectories (spaces of k-fold loops) determines in the final analysis ultimately a classification of trajectories, representing each class from the set of admissible trajectories as a set of homotopy equivalent trajectories. The set of homotopy classes of such spaces is denoted as in [9]. This set has a structure of group as follows from the given examples.

Useful relations for computing homotopy groups of homogeneous spaces and loop spaces are given below, along with examples of homotopy groups of spheres and other homogeneous spaces:

$$ [W, \Omega X] \leftrightarrow [\Sigma W, X] \tag{10} $$

$$ \pi_i(\Omega X) \cong \pi_{i+1}(X) \tag{11} $$

$$ \Sigma W \overset{\text{def}}{=} ((W \times I))/((W \times 0) \cup (w\_0 \times I) \cup (W \times 1) \tag{12} $$

ΣW—cited superstructure over W;

$$ \pi_{n+15}(S^n) = Z_{480} \oplus Z_2 \tag{13} $$

$$ \pi_1(SO^n) = Z_2 \tag{14} $$

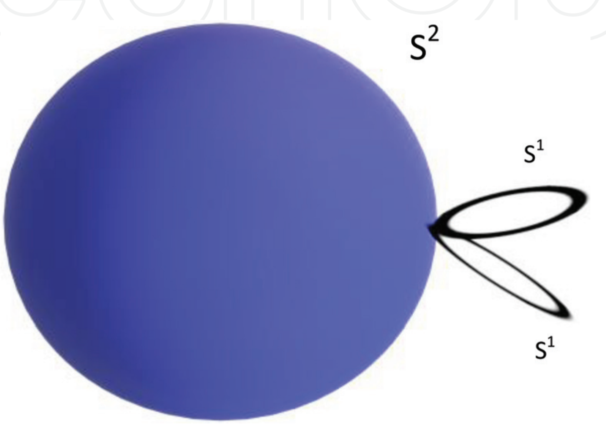

Figure 2.
A bouquet of two-dimensional and two one-dimensional spheres.

Part of the Hopf fibration is a mapping $S^3 \to S^2$ [13–15]. This mapping is the generator of the homotopy group $\pi_3(S^2) = Z$.

This is under the assumption that the degeneracy space does not change. When the homotopy type of the degeneration space changes, the classification of trajectories also changes. The change in the topology of the degeneracy space is due to a change in the symmetry groups of the process. There is a violation of symmetry due to a change in the characteristic features of the process, as already noted, for example, with the appearance of trends. Predictor of the trend is, in fact, a change in the structure in the set of transition probabilities, as will be discussed below.

Symmetry breaking or removal of degeneracy by isotropy subgroups can occur for various reasons. One such mechanism is associated with noise-induced transitions. In [16] examples of this kind are given. The reason for removing the degeneracy and, consequently, changing the topological type of the degeneracy space is the presence of multiplicative noise. As a result of the growth of the amplitude of such noise, a change occurs in the characteristics of the process, in particular, the density of the distribution function changes.

Returning to the tasks of management, the following should be noted. Thus, a space of degeneracy for the system and constructed on it k-fold loop space that is homotopy equivalent to the space of paths on the degeneracy space are defined sufficient roughly. The classification of paths is determined by the set of classes of homotopy equivalent paths. The transition from one class of paths to another class of paths is accompanied by symmetry breaking. Very conditionally the process of development of failure and dysfunctions of the mechanism can be shown in **Figure 3**. The colored concentric rings represent different types of homogeneous spaces on which it is necessary to keep the trajectory as long as possible. At the same time, the time to reach the boundary of the RUL class is estimated.

That is, the process of the development of faults as a result of operation passes from one class to another, reaching at the end of the failure field. In this case, the intersection of the conditional boundary is determined by a violation of the symmetry of the process. Further, the degeneration space itself and the character of the transition from one class to another change.

The mathematical model described above allows us to make the first step in the formulation and formalization of optimization of the maintenance strategy. The optimization task is reduced to determining the number of management parameters and determining the image of the management loop in the state space or the k-fold space of paths that hold the trajectory of the process in a given homotopy class or in a given degeneracy space. If necessary, homotopic obstacles are overcome by

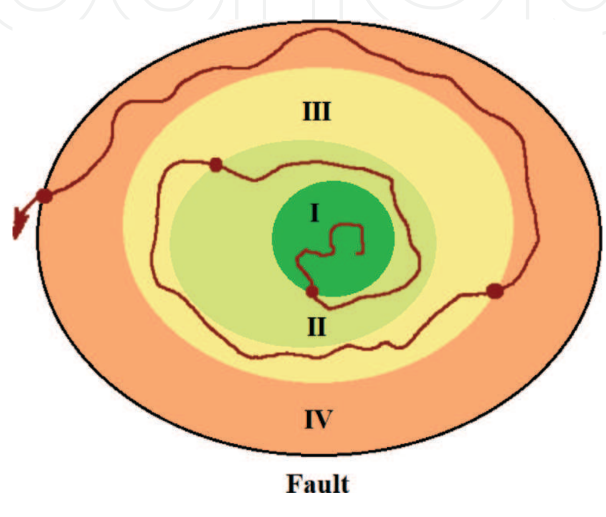

Figure 3.
Interpretation of optimization problem and maintenance strategies.

increasing the dimension of the management loop, that is, by increasing the number of management parameters. The operation of the restructuring of the degeneracy space is inevitable, therefore with each new restructuring task literally reformulated.

Restructuring degeneracy space is associated with symmetry breaking. To explain all of the above, we can use a very simplified example. Consider the degeneration group *SO(3)* and the isotropy group *SO(2)* $* Z_2$. Moreover, the degeneracy space is a projective space RP_2. Removing degeneracy by Z_2 is associated with the violation of time-reversal symmetry. Such loss of symmetry is possible with the appearance of the trends of the initial cascade of wavelet coefficients of the observed time series. In this case, the degeneracy space is transformed into a sphere. Homotopy groups of projective space and spheres differ and are represented by Eqs. (15) and (16):

$$\pi_1\left(S^2\right) = 0 \tag{15}$$

$$\pi_1\left(RP^2\right) = Z_2 \tag{16}$$

The homotopy groups of k-fold loop spaces are represented by Eqs. (10) and (11). The nontriviality of homotopy groups generates a classification of paths, that is, their division into classes of homotopically nonequivalent paths in a management task with two or three management parameters.

When removing degeneracy and the transformation of the degeneration space itself into the space of another homotopy type, respectively, the homotopy classes of paths also change. In this case, new hidden predictors of failure will appear. An example is the predictors of turbine surging, described in [17]. It is noted that the early predictor of surging is destroyed by the mixing of wavelet coefficients. In this approach this means that in the observed process (the observed signal) from the pressure sensor the violation symmetry with respect to the permutation group occurred.

The above general model is based only on two statements that need concrete implementation for the further use of such model in prognosis and management tasks. Two said assumptions are as follows: there exists a space of states realized as a vector multidimensional space then taking into account the symmetry groups the set of states of the system, and the set of trajectories was represented in the form of degeneracy space and multiple path spaces that take into account the management parameters.

4. Review of the solutions of the prognosis and management task from abstract models to implementation

The constructed model on the basis of the introduced assumptions using the concepts of homotopy topology gives a general classification of admissible trajectories, their evolution. The model demonstrates the complexity of the prognosis in view of the need to take into account symmetry breaking, in other words, the removal of degeneracy by one or more subgroups of process symmetries or cascade of wavelet coefficients. That is, the description of the topological transformation of the degeneracy spaces and the association with its spaces of k-fold paths allow one to look at the tasks of prognosis and management in a different interpretation. The model allows us to describe all admissible types of topological transformations, defines classes of admissible trajectories, determines all possible transformations of classes of trajectories and subsets, and characterizes subsets in the state space and trajectory spaces related to regions of failure. Further, the same methods describe the evolution of such regions, their interaction, and pair interaction on the basis of the group structure of homotopy classes. Involving physical models from the physics of failure makes it possible to determine the physical meaning of topological nontriviality and to connect topological obstacles and topological prohibitions with physical mechanisms that ensure topological transformations.

The next step to the construction of computational algorithms for prognosis and management is the transition from topological dynamics described above to the construction of the evolution equations of trajectories and states and finally to the construction of algorithms for prognosis and management. Moreover, the conclusions and results of the homotopy model must be taken into account with necessity.

To do this, it is necessary to determine the specific content of the above concepts such as the state space and path space. A detailed exposition of this construction is contained in the works [18–23]. The observed signal is represented as the coefficients of its wavelet transformation:

$$\left\{{}^{k}_{Hist}W^{N}_{i,j}\right\}, N = 1, 2, 3, \ldots, N^{*} \tag{17}$$

N—number of cycle; i, j—indices of wavelet decomposition.

Hist—duration of cycles (unevenness of stroke) of histogram column index.

k—numbering of vectors from wavelet coefficients of dimension N^{*}.

This takes into account the fact that the mechanism under consideration is a reciprocating or rotational mechanism for all fixed indices except that N is determined by stochastic process with discrete time, N cascade. Further, fixing the limiting value N as N^{*} is determined by a set of vectors of dimension N^{*}, chosen from the consistent values of the process under consideration with discrete time. As a result, the space $\boldsymbol{R}^{N^{*}}$ is determined, consisting of all possible finite segments of dimension размерности N^{*}.

The state space is defined as follows:

$$\{\boldsymbol{R_k}\} \stackrel{\text{def}}{=} \left\{{}^{k}_{Hist}W^{N}_{i,j}\colon k\,N^{*} \leq N \leq (k+1)N^{*}, k = 0, 1, 2, 3\ldots\right\}, \boldsymbol{R_k} \in \boldsymbol{R}^{N^{*}} \tag{18}$$

Thus, a vector space of dimension H is defined. The numerical value of H is not yet specified. It is determined in the process of preprocessing. The task of prognosis here reduces to determining the probability of transition from an initial vector to a finite vector in j steps. In this case, such task is solvable either by an explicit solution of the evolution equations or by calculating the moments, mainly of the dispersion, that is, second moment. Similar calculations are given in [6] and are reduced in most cases to the calculation of the Feynman integral along trajectories [6, 18–24] or to the solution of evolution equations such as the Fokker-Planck equation. In this case, the trajectory of states is represented as a walk along a multidimensional lattice or its continual analog, that is, R^{N} space [6], in those cases when the observed process possesses certain properties, for example, the Chapman-Kolmogorov condition, the Markov property, stationarity and ergodicity are hold. The above properties of the process determine the evolution equations for the transition probability in the form of the Fokker-Planck equations already mentioned or Hamilton-Jacobi type equations, Schrödinger equations, and so on.

Topological prohibitions, implying the existence of such prohibition by physical mechanisms, determine other scenarios for the evolution of trajectories, that is, the probability of transition from one vector to another for a fixed number of steps. Moreover, in the interpretations of the process as a random walk on a lattice or continuum, processes are realized with allowance for the prohibitions imposed by the nontriviality of the homotopy types of the degeneracy space. Evolution equations at the same time are complicated. And to obtain evolution equations, it is required to introduce three-point, four-point, etc. density of the distribution function for transition probabilities. In the present, brief review of approaches should be mentioned often; there are cases where the probability density function of the process or the transition probabilities are not Gaussian but have a so-called heavy tail in its distribution, expressed as

$$P(R, L) \sim \frac{L}{\|R\|^{\alpha+1}}. \tag{19}$$

Thus, the approaches described above lead to the evolution equations with fractional derivative [25].

5. Construction of automata

This section is devoted to the description of a family of automata, analogs of Turing machines, allowing to formalize and, ultimately, take into account the above difficulties in the development of algorithms of reliable prognosis and numerical estimates of RUL, without which it is sometimes impossible to build and optimize the maintenance strategy and also implement the management task when choosing a class of trajectories that optimize the operation modes of technical objects.

The transition from abstract models to the construction of recognizing and predictive automata occurs in two stages. By recognizing automata in this case, we mean a set of single-tape Turing machines. At the first stage, the state space described in the previous section and in more detail in [7, 22] is constructed. The second stage involves the construction of the symbolic space described in the work [18–21]. For this, the transition from the initial state to the final state on the state space is represented as the product of matrices of special form acting in the affine space:

$$\Omega_{i,k}\begin{pmatrix} n_1 \\ n_2 \\ \cdot \\ n_i \\ n_k \\ 1 \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ \cdot \\ n_i \\ n_k \\ 1 \end{pmatrix} = \begin{pmatrix} n_1 \\ n_2 \\ \cdot \\ n_i + 1 \\ n_k - 1 \\ 1 \end{pmatrix} \tag{20}$$

The product of such matrices eventually transforms the initial state vector into the final state:

$$\mathbf{\Omega}_{N^*} \stackrel{\text{def}}{=} \left(\prod_{\mathbf{1}}^{\mathbf{N*}} \mathbf{\Omega}_{\mathbf{i},\mathbf{k}}\right) \text{ for } \forall\, (i, k) \tag{21}$$

The symbolic space in this case is a space P whose dimension is equal to the number of columns of the frequency histogram of the vectors of the initial or final. For definiteness, the dimension is increased by adding columns with zero values, thereby encompassing the physically permissible range of values of the observed signal. The successive multiplication of matrices of elementary steps (coordinatewise) with subsequent renormalization determines transition probabilities in the form of a matrix $\widehat{w}_{k,m}$ defined by the operator Ω_N .

The equation describing the evolution of certain states of the Turing machine in this way is a known balance equation [5]:

$$\frac{\partial p_m}{\partial t} = \sum_k \left[\widehat{w}_{k,m} p_k - \widehat{w}_{m,k} p_m\right] \tag{22}$$

or the basic kinetic equation [4].

The renormalization, which determines the transition from the frequency representation to the probabilistic one, also allows us to interpret the change of states as a walk in an N^*—dimensional simplex Σ^{N^*}. Since each one-tap automaton processes only one cascade of wavelet coefficients, then for the entire set of cascades, the number of automata is equal to the number of wavelet coefficients of the signal in one period. Since in the general case there is a scatter of periods over lengths, the set of recognizing automata is determined for each interval in the histogram of the lengths of the periods, depending on the values of the lengths of the elementary intervals and the probable transition to packet wavelet decomposition. At a signal sampling frequency of 2 kHz, the number of automata is estimated from below by a number equal to 40×10^3. If vibration signals are analyzed in a wide frequency range, then the number of automata is estimated by orders 10^6.

6. Accounting for topological dynamics and how the automata work

Thus, the operation of an automaton reduces to change in its internal state when it shifts by one step in the input cascade of wavelet coefficients of the observed signal. It is assumed here that the equations describing the changes in the state of the automaton are independent of time, that is, the internal states of the automaton are stationary. In practice, small deviations from stationarity in the Levy metric are allowed [26]. The value of the permissible deviations is determined on the basis of the chronological database. Thus, the quasi-stationary nature is verified by checking the approximate fulfillment of the stationarity conditions of the basic kinetic equation Eq. (22):

$$\widehat{w}_{k,m} = \sum_m \widehat{w}_{m,k} \tag{23}$$

If the quasi-stationary conditions are violated, for example, a trend appeared in one of the columns of the histogram or in several columns, the prognosis of the evolution of the internal states of the automaton is determined already by solving the nonstationary basic kinetic equation. Meanwhile, under the quasi-stationary conditions, the change in the internal states of the automaton is possible. As an example, we can mention the noise-induced transitions [16]. In this case, the change in the type of internal states is connected with a slow evolution of the coefficients of polynomial approximation of stationary solutions of the basic equation [3, 20]. Thus, in the transition from the initial state vector in the state space or in the symbol space introduced above, there are many transition paths in the formalism of birth-death process. However, all paths under quasi-stationary conditions reduce to permutations in the commutative subgroup of matrices of elementary transitions $\Omega_{i,k}$. That is, the transition from one vector to another takes place under the condition that the form of the symbolic histogram is stationary or that small deviations in the Levy metric are assumed. Moreover, the set of transition paths are in the same homotopy class of k-fold loops of the degeneracy space. The scenario described is valid for Gaussian processes with zero correlation length over the time variable. As noted in this case and taking into account the symmetry with respect to time reversal, the degeneracy space is an m-dimensional projective space. When time correlations of nonzero length appear, the degeneracy of the group Z_2 occurred. When a trend appears in the signal, degeneration by subgroups of the permutation group is removed. The admissible class of paths, on which the transition from one state vector to another takes place, is narrowed. Further dynamics of the internal states of the automaton is associated with the creation of new filling cells and the destruction of some old ones. At the same time, new ways of transition from one state to another are determined. And with the appearance of new ways, the estimate of the time to reach the class boundary or the field of the failure is changed. A complete reconstruction of the whole class of paths occurs when the homotopy type of the degeneracy space is reconstructed, for example, under noise-induced transitions reminding the second-order phase transitions in condensed media.

7. Reproduction and birth of automata with increasing complexity

However, the constructed set of automata is still not enough for an effective prognosis. This deficiency is closely related to topological dynamics, in particular, to the already mentioned effects of excluded volume. In other words, taking into account the homotopy classes of paths, that is, in those important cases, when the processes under consideration are not Markovian, the Chapman-Kolmogorov identity is not satisfied. In the analytic approach, three-particle and then many-particle distribution functions are considered in such cases. Here, the system of equations for these functions can have infinite dimension. Another analog of such equations is the transition from evolution equations in PDF to equations in moments, where an infinite-dimensional system of equations also appears. Most often, such equations are solved by truncating in dimension, assuming that the finite-dimensional part of the system of differential equations approximates an infinite chain of equations in some sense. In the case when multiparticle distribution functions are introduced to obtain solutions, it is assumed that, beginning with a certain number, many-particle functions are assumed to be approximately Markovian, that is, are represented as a product of multipoint functions of lower orders.

The examples given represent some analogies for completing the construction of a set of automata. The family of constructed automata with the necessity for complete account of topological dynamics must be supplemented by some additional properties. The analogies described above demonstrate what properties a family of automata should possess.

Additional properties of the family of automata:

1. Automata must be pairwise interacting.

2. In a number of cases, automata must analyze the described situations, that is, construct automata by merging one-tape automata, thereby passing to automata with increasing complexity.

A pair interaction between automata can be introduced in different ways, depending on the language describing these automata in terms of evolution equations or in constructing the Feynman path integral. In this case, the simple way of constructing interacting automata is to introduce interaction through a statistical interaction, taking into account this interaction from the first principles, when the measure of interaction is $d(P, P')$ [11, 26]:

$$d(P, P') = \left\{ \int_{\Omega} |\psi(\omega) - \psi'(\omega)|^2 dQ(\omega) \right\}^{1/2} \tag{24}$$

where P, P' are two probability measures:

$$Q = \frac{1}{2}(P + P') \tag{25}$$

$$\psi = \left(\frac{dP}{dQ}\right)^{1/2} \psi = \left(\frac{dP'}{dQ}\right)^{1/2} \tag{26}$$

is the Radon-Nicodym density.

There are more complex forms of interaction by introducing a potential or some vector field, as described in the work [6]. With the pair interaction between automata taken into account, it becomes possible to construct a graph whose automata are placed in 0-dimensional vertices. Since we are talking about automata on the wavelet cascades, the resulting graph allows us to analyze the situation with a root cause prognostics if the interaction is defined for automata with different time indices. Or, if automata interact with different scaling indices, then an analysis of multiscale processes or processes that occur at different scale levels and are interconnected becomes available.

8. Conclusions

The result of the completed constructions is the family of predictive automata. The set of automata is large, and depending on the sampling frequencies of incoming signals from sensors installed on the mechanisms, it is estimated from 10^6 to 10^9. The automata themselves represent some analog of the Turing machine. In this case, the set of automata interacts in pairs. The interaction leads to the construction of more complicated automata and is an analogy of transitions to many-particle distribution functions or their densities. Thus, the family of interacting automata generates the next generation of automata with increased complexity. Many features of the operation of automata and methods for constructing state spaces and degeneration spaces remain outside the scope of this chapter. It is only necessary to note that as the state space it is considered a sequence nested with respect to the dimension of spaces, as is the sequence of path spaces whose multiplicity can tend to infinity. In such limiting cases, infinite-dimensional symmetry groups appear. And when implementing limit transitions, there are mathematical models that allow us to algorithmize the problem in some sense.

The complexity of family of automata is determined by the complexity of topological dynamics. If we are talking about the observed signals, then the account of symmetry groups, the appearance, and the removal of degeneracy in different subgroups are determined by the complexity of the signals themselves, reflecting in turn the complexity of physicochemical processes occurring in complex mechanical and electronic systems at various scale levels. Symmetry groups appear in all existing time series. Most often the groups of symmetries of incoming signals are caused by the concrete physical processes of the failure physics occurring in complex mechanisms in the presence of friction, gas hydrodynamics, physical and chemical processes, etc. In the overwhelming number of cases, the physical models of the failure physics confirm this fact. Turning to concrete implementations of the family of prognostic automata, it should be noted that, in spite of the complexity of topological dynamics, specific algorithms prescribe fully realizable requirements for the costs of supporting their work in parallel architectures. In this case, as practice has shown in the operation of automata, their structures are optimized.

In addition, in the optimization process, there are opportunities that allow some automata to be transferred to the computing power of onboard computers when it comes to, for example, monitoring of mobile objects, in particular, monitoring of all kind of transport. At the same time, onboard automata perform not only signaling functions but also are able to manage remote computing cores, thereby optimizing the computational processes on the remote computing cluster. And to the contrary, the automata of computing cluster can change the structure and functionality of peripheral automata, located on onboard computers or other computational capacities inherent in microcontrollers of onboard electronics. Returning to the complexity of topological dynamics, it should be noted that the automata for prognosis that support this complexity allow us to formalize and algorithmize the models of the root cause prognosis and, in the end, algorithmize the tasks for the intellectual self-maintenance and self-recovery systems.

From the model constructed above, a certain hierarchy of prognostic problems also follows, since the set of physically acceptable trajectories is divided into the classes of homotopic equivalence. In turn, the classes themselves are changed during transformations of the space of degeneracy; other admissible trajectories and their classes appear that differ from the previous ones and have their own predictors and time estimates of the RUL. This means that with each transformation of the space of degeneration there is a change in the prognosis and changes in the RUL estimates. Thus, with RUL estimates, it is necessary to take into account not only the time to reach the class boundary but also the time to reach the moment when the transformation of degeneration space begins, for example, the time to reach the bifurcation set in the bifurcation tasks of the stationary solution of the Fokker-Planck equation. Another example is the time to reach a certain critical value of the amplitude of multiplicative noise. And each time the task of determining the RUL is updated. A good example of such an update is the calculation of the probability density of the transition from the original value to the preassigned one when determining the RUL in the representation of the probability density of the transition for a fixed time in the form of the Feynman path integral. At the same time, a change in the class of admissible transition trajectories, when the degeneracy by one of the isotropy groups or by its subgroups is removed, changes the evolution equation itself. In some cases, with the emergence of new topological obstacles, the Smoluchowski-Chapman-Kolmogorov identity does not hold the system and becomes non-Markov etc.; finally, an integro-differential equation appears as the Fokker-Planck evolution equation. In this case, all the previous predictors are changed, as well as all the time estimates. And so it happens with each new transformation of the space of degeneration.

The family of interacting automata presented here changes traditional approaches to the learning of automata in recognizing early predictors of failure, in other words, in identifying the characteristics of the trajectories, the movement along which leads to the boundaries of the failure regions. In the examples described above and from the general model, it should be that learning is reduced to a set of segments of wavelet coefficient cascades as long as the automaton output to the quasi-stationary regime is not going to happen. For simple automata, the segment length is estimated at about 1000 full cycles of the engine operation or the number of revolutions of the turbine shaft. Further, the algorithm during monitoring verifies compliance with the conditions of quasi-stationarity. At the birth of more complex automata with an increase in their dimension, each added dimension increases the length of the segments.

And in conclusion, it is necessary to say a few words about the set of early predictors of failure. In accordance with the hierarchical construction of the prognostic model, when removing the next degeneration by one of the isotropy subgroups in each new class of trajectories, their predictors are determined. That is, the inheritance of predictors in the transition of the trajectory from one class to another is not necessary. And this non-obligation is connected with the mapping of the gopotopic groups of the previous space of degeneration into the space of degeneration after its transformation. Part of the predictors may persist, another part may disappear, and new predictors may appear. The listed mutations of the set of predictors are determined by the specific structure of the degeneration space, that is, by the set of symmetry groups and isotropy groups. In this case, the trajectory itself or its characteristics, for example, configurational entropy, can act as a predictor, along with other types of the Kullback type of entropy.

Another type of predictor exists, and here again the analogy between the topological singularities of condensed media and the singularities of multiple loop spaces is appropriate. We are talking about the structure of the singularity core. Conducting the noted analogy, if the trajectory passes through the core of the singularity, then the effect of changing the permissible number of trajectories gives rise to changes, for example, of pointwise holder regularity of the trajectory.

In terms of the evolution of the internal states of a set of interacting automata, the above conclusions are expressed as additional conditions imposed on the densities of the distribution functions and transition probabilities for automata in any dimension.

This chapter is mainly devoted to the presentation of theoretical prognostic models and the basic ideas of constructing predictive automata. Demonstration of examples of the work of predictive automata and more detailed description of the predictors of the early prognosis will be continued in the next edition of IntechOpen book *Prognostics* edited by Prof. Fausto Pedro García Márquez.

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