Chapter

Derivation of the BG Model

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Abstract

The BG model (a model for predicting 3D beach changes based on the Bagnold's concept) was introduced, and the fundamental aspects of the model were explained. The BG model is based on the concepts such as (1) the contour line becomes orthogonal to the wave direction at any point at the final stage, (2) similarly, the local beach slope coincides with the equilibrium slope at any point, and (3) a restoring force is generated in response to the deviation from the statically stable condition, and sand transport occurs owing to this restoring force. The same concept has been employed in the contour-line-change model and N-line model. In these studies, the movement of certain contour lines was traced, but in the BG model, 3D beach changes were directly calculated.

Keywords: BG model, derivation, physical meaning

1. Introduction

The BG model is based on the concepts such as (1) the contour line is orthogonal to the wave direction at any point at the final stage, (2) similarly, the local beach slope coincides with the equilibrium slope at any point, and (3) a restoring force is generated in response to the deviation from the statically stable condition, and sand transport occurs owing to this restoring force. The same concept has been employed not only in the contour-line-change model [1] but also in the N-line model [2–7]. In these studies, the movement of certain contour lines was traced, but in the BG model, the depth change on the 2D horizontal grids was directly calculated. Falqués et al. [8, 9] developed a medium- to long-term model for beach morphodynamics named Q2D-morfo. In their model, similar expressions regarding crossshore and longshore sand transport equations as the BG model were employed. In particular, they used the beach slope measured on a real coast as the equilibrium slope, similar to the BG model, and the prediction of beach changes bounded by two groynes was carried out, but the prediction period was as short as 35 days [8]. van den Berg et al. [10] predicted the development of a sand wave associated with large-scale beach nourishment due to shoreline instability under the oblique wave incidence at a large angle using the model proposed by Falqués et al. [8]. Larson et al. [11] proposed the crossshore sand transport equation in the swash zone using the concept of the equilibrium slope and predicted the foreshore evolution. Similarly, Larson and Wamsley [12] proposed crossshore and longshore sand transport formulae in the swash zone, and they used the similar equations as the BG model. Their equations were also employed in the 3D beach change model in the swash zone by Nam et al. [13].

2. Derivation of the BG model

In the derivation of the sand transport equation of the BG model, we referred Bagnold [14] and the previous studies after Bagnold (Inman and Bagnold [15], Bowen [16], Bailard and Inman [17], and Bailard [18]). Bagnold [14] derived the sand transport equation for a unidirectional steady flow with an explicit expression of the seabed slope by applying the energetics approach. Inman and Bagnold [15] assumed that sand transport in a wave field is the sum of the components caused by shoreward flow during the motion of incoming waves and those caused by seaward flow during the motion of outgoing waves and defined the slope satisfying zero net onshore or offshore sediment transport as the equilibrium slope. Their equilibrium slope is the slope when upslope effect due to the asymmetry in action of incoming and outgoing waves and downslope effect due to the gravity balance each other.

Regarding the sand transport equation under waves, Bowen [16], Bailard and Inman [17], and Bailard [18] formulated the instantaneous sand transport flux on the basis of the sand transport equation for a unidirectional steady flow by Bagnold [14], assuming that the wave dissipation rate is proportional to the third power of the instantaneous velocity. Then, the net sand transport formula was derived by integrating the instantaneous sand transport flux over one wave period. Furthermore, they derived the equilibrium slope equation using the wave velocity parameters. Out of these studies, the sand transport flux formula by Bailard and Inman [17] considers both bed load and suspended load, and this formula has been extensively used in the models for predicting beach changes. Kabiling and Sato [19] calculated the wave and nearshore current field using the Boussinesq equation and predicted 3D beach changes using the Bailard formula. Long and Kirby [20] also carried out the numerical simulation of beach changes using the Bailard formula and Boussinesq equation. However, the application of their model to the long-term prediction of the topographic changes in an extensive calculation domain is limited because the recurrent calculations in solving the time-dependent equation of the wave field are time consuming. On a real coast, a longitudinal profile maintains its stable form as a whole, as a result of the wave action for a long period of time, apart from the short-period seasonal variation of the beach, suggesting the existence of an equilibrium slope on a real coast. In contrast, in their studies, the beach slope does not necessarily agree with the equilibrium slope after long-term prediction, even though an equilibrium slope exists, and it is difficult to explain the phenomena really observed on a coast.

In this study, we return to the starting point of Bagnold's basic study, and simple sand transport equations are derived. Then, a model for predicting 3D beach changes by applying the concept of the equilibrium slope introduced by Inman and Bagnold [15] and the energetics approach of Bagnold [14] is developed [21].

Figure 1 shows the definitions of the variables. Consider Cartesian coordinates (x, y) and the seabed elevation $Z\ (x, y, t)$ with reference to the still water level as a variable to be solved, where t is the time. Assume that waves are obliquely incident on a coast with a slope of $\tan\beta$. n and s are the local coordinates taken along the directions normal (shoreward) and parallel to the contour lines, respectively. The n-axis makes an angle of θ_n measured counterclockwise from the x-axis, $\vec{e_n}$ is the unit vector normal to the contour lines (shoreward), $\vec{e_s}$ is the unit vector parallel to the contour lines, $\vec{e_w}$ is the unit vector in the wave direction, and θ_w is the wave direction measured counterclockwise from the x-axis. α is the angle between the wave direction and the direction normal to the contour lines. These unit vectors are expressed as

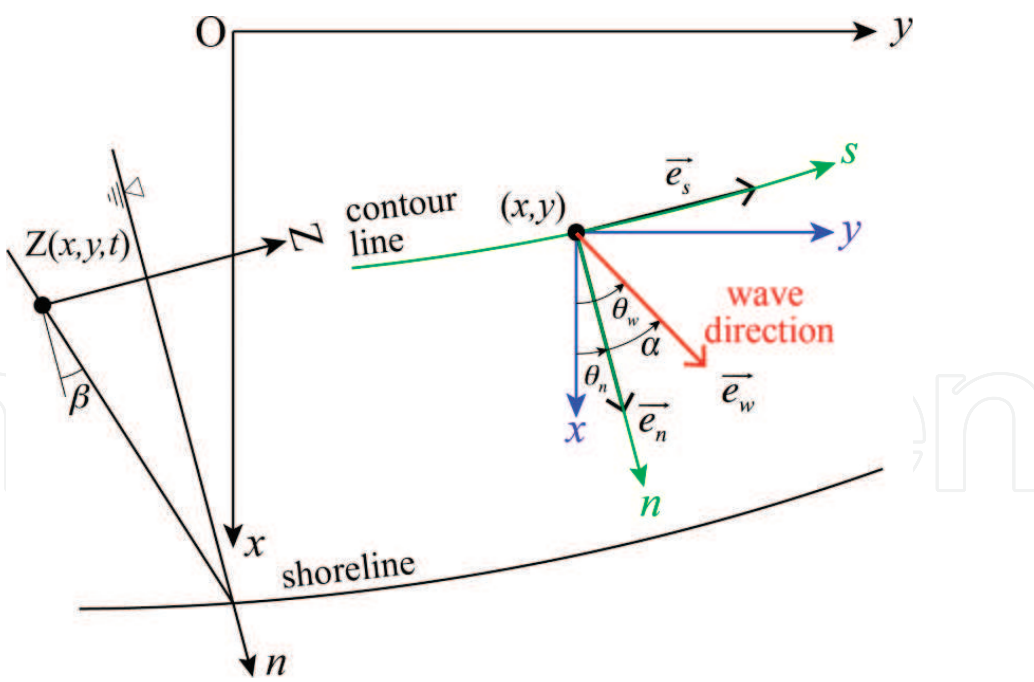

Figure 1.
Setup of the coordinate system and definition of variables.

$$\vec{e_n} = (\cos\theta_n, \sin\theta_n) \tag{1}$$

$$\vec{e_s} = (-\sin\theta_n, \cos\theta_n) \tag{2}$$

$$\vec{e_w} = (\cos\theta_w, \sin\theta_w) \tag{3}$$

The components of the sand transport vector $\vec{q}$ are expressed as Eq. (4), the direction and magnitude of which give the direction of sand transport and a volumetric expression for the sand transport rate per unit width normal to the direction of sand transport and per unit time, respectively. In addition, $\vec{q}$ can be expressed as the vector sum of the crossshore and longshore components in each direction of n and s as in Eq. (5), and by taking the inner products of $\vec{e_n}$ and $\vec{q}$ and of $\vec{e_s}$ and $\vec{q}$, the crossshore and longshore components of sand transport, q_n and q_s, are given by Eqs. (6) and (7), respectively.

$$\vec{q} = \left(q_x, q_y\right) \tag{4}$$

$$\vec{q} = q_n \vec{e_n} + q_s \vec{e_s} \tag{5}$$

$$q_n = \vec{e_n} \cdot \vec{q} \tag{6}$$

$$q_s = \vec{e_s} \cdot \vec{q} \tag{7}$$

When the gradient vector of Z is defined as Eq. (8), $\vec{\nabla Z}$ becomes a vector, the direction and the absolute value of which are along n-axis and $\tan\beta$, respectively, with the component form of the expression in Eq. (9) in (x, y) coordinates. Furthermore, Eqs. (10)–(15) are satisfied.

$$\vec{\nabla Z} = \tan\beta\ \vec{e_n} \tag{8}$$

$$\vec{\nabla Z} = (\tan\beta\cos\theta_n, \tan\beta\sin\theta_n) = (\partial Z/\partial x, \partial Z/\partial y) \tag{9}$$

$$\left|\nabla\vec{Z}\right| = \sqrt{(\partial Z/\partial x)^2 + (\partial Z/\partial y)^2} = \tan\beta \tag{10}$$

$$\tan\beta\ \vec{e_s} = (-\partial Z/\partial y, \partial Z/\partial x) \tag{11}$$

$$\theta_n = \tan^{-1}\left(\frac{\partial Z}{\partial y} \Big/ \frac{\partial Z}{\partial x}\right) \tag{12}$$

$$\alpha = \theta_w - \theta_n \tag{13}$$

$$\begin{aligned}\cos\alpha &= \vec{e_w} \cdot \vec{e_n} \\ &= \left(\vec{e_w} \cdot \nabla\vec{Z}\right)/\left|\nabla\vec{Z}\right| \\ &= [\cos\theta_w(\partial Z/\partial x) + \sin\theta_w(\partial Z/\partial y)]/\tan\beta\end{aligned} \tag{14}$$

$$\begin{aligned}\sin\alpha &= \vec{e_w} \cdot \vec{e_s} \\ &= \left(\vec{e_w} \cdot \tan\beta\, \vec{e_s}\right)/\tan\beta \\ &= [-\cos\theta_w(\partial Z/\partial y) + \sin\theta_w(\partial Z/\partial x)]/\tan\beta\end{aligned} \tag{15}$$

The fluid motion due to waves near the sea bottom becomes oscillatory, and a sand particle moves back and forth in the crossshore direction. Sand transport in a wave field is assumed to be the sum of the components caused by shoreward flow during the motion of incoming waves and those caused by seaward flow during the motion of outgoing waves, as suggested in [15], and the sand transport equation for a unidirectional steady flow introduced by Bagnold [14] can be applied to each component.

Assuming that $\tan\beta$ is infinitesimal, the flow makes a sand particle move in the direction of the flow, and gravity causes downslope action; the sand transport flux of a unidirectional flow is expressed by Eq. (16) as a linear approximation in terms of $\tan\beta$ [15–18].

$$\vec{q_u} = a_0\,\vec{e_u} - a_1\ \nabla\vec{Z} \qquad (a_0>0, \quad a_1>0) \tag{16}$$

Here, the subscript u denotes the unidirectional flow, $\vec{q_u} = \left(q_{ux}, q_{uy}\right)$ is the sand transport vector, the direction and magnitude of which give the direction of sand transport and a volumetric expression for the sand transport rate per unit width normal to the direction of sand transport and per unit time, respectively, $\vec{e_u}$ is the unit vector in the direction of the flow, $\nabla\vec{Z} = \tan\beta\ \vec{e_n} = (\partial Z/\partial x, \partial Z/\partial y)$ is the gradient vector of Z, and $\tan\beta$ is the seabed slope. The sign of the coefficients a_0 and a_1 is always positive. The first and second terms in Eq. (16) represent the action produced by the flow and the downslope action due to gravity, respectively. In the equation in [17] based on the bedload equation of Bagnold [14], the coefficients a_0 and a_1 are described in terms of the angle of the internal friction of sand and the flow velocity. On the other hand, the sand transport equations in [16, 18] based on the suspended load equation of Bagnold [14] can be expressed in the same form as Eq. (16), although the coefficients a_0 and a_1 are described in terms of the falling velocity of a sand particle and the flow velocity. Thus, Eq. (16) is satisfied for not only the bedload but also suspended load, that is, total load.

The net sand transport flux due to waves, $\vec{q} = \left(q_x, q_y\right)$, is the sum of the components due to incoming and outgoing waves, as shown in Eq. (17), when the time-averaged sand transport rate in a period involving the action of incoming and outgoing waves is expressed by Eqs. (18) and (19).

$$\vec{q} = \vec{q^+} + \vec{q^-} \tag{17}$$

$$\vec{q^+} = a_0^+ \vec{e_w^+} - a_1^+ \vec{\nabla Z} \quad \left(a_0^+ > 0, a_1^+ > 0\right) \tag{18}$$

$$\vec{q^-} = a_0^- \vec{e_w^-} - a_1^- \vec{\nabla Z} \quad \left(a_0^- > 0, a_1^- > 0\right) \tag{19}$$

Here, the subscripts + and – denote the values corresponding to incoming and outgoing waves, respectively, and $\vec{e_w^+}$ and $\vec{e_w^-}$ are the unit vectors in the directions of the shoreward and seaward flows of waves, respectively. Modifying Eq. (17) under the assumption that the directions of waves propagating shoreward and seaward are opposite, as given by Eq. (20), defining the slope satisfying zero net onshore or offshore sediment transport when waves are incident from the direction normal to the slope as the equilibrium slope, $\tan \beta_c$ (Eq. (21)), and defining the coefficient A by Eq. (22), the sand transport flux is given by Eq. (23).

$$\vec{e_w^-} = -\vec{e_w^+} \tag{20}$$

$$\tan \beta_c = \left(\frac{a_0^+ - a_0^-}{a_1^+ + a_1^-}\right) \tag{21}$$

$$A = \left(a_1^+ + a_1^-\right) \tag{22}$$

$$\vec{q} = A\left[\tan \beta_c \, \vec{e_w} - \vec{\nabla Z}\right] \tag{23}$$

Here, $\vec{e_w} = \vec{e_w^+}$ is the unit vector in the wave direction θ_w (Eq. (3)). Bowen [16], Bailard and Inman [17], and Bailard [18], based on the Bagnold's concept, formulated the equilibrium slope of Eq. (21) using the oscillatory flow velocity due to waves, the angle of the internal friction of sand, and the falling velocity of a sand particle. Hardisty [22, 23] also formulated the equilibrium slope using the wave parameters on the basis of the same concept. Furthermore, Dean [24, 25] formulated the equilibrium profile in terms of energy dissipation rate due to wave breaking, and Larson et al. [26] gave a theoretical formulation of the equilibrium profile, the derivative of which is equal to the equilibrium slope, with the combination of wave parameters.

In this study, we used the seabed slope measured on real coasts as the equilibrium slope instead of using the formulated results of the equilibrium slope. The measured slope is assumed to be given a priori because the real seabed topography includes every effect of past events, and it has a stable form, except for seasonal short-period variations, in the long term.

Applying the energetics approach [14] and assuming that the coefficient A in Eq. (23) is proportional to the wave energy dissipation rate Φ, the total longshore sand transport rate is obtained by integrating Eq. (23) over the depth. This has the same form as the CERC-type formula given by Komar and Inman [27], and the coefficient A is determined as Eq. (24) from the equivalence of both results as mentioned later. Finally, the fundamental equation of sand transport flux due to waves is given by Eq. (26).

$$A = C_0 \frac{K_1 \Phi}{\tan \beta_c} \tag{24}$$

$$C_0 = \frac{1}{(\rho_s - \rho) g (1 - p)} \tag{25}$$

$$\vec{q} = C_0 \frac{K_1 \Phi}{\tan\beta_c}\left[\tan\beta_c\, \vec{e_w} - \vec{\nabla Z}\right] \tag{26}$$

Here, K_1 is the coefficient of longshore sand transport, Φ is the wave energy dissipation rate per unit time and unit seabed area, C_0 is the coefficient through which the sand transport rate expressed in terms of the immersed weight is related to the volumetric sand transport rate, ρ_s and ρ are the sand and water densities, respectively, g is the acceleration of gravity, and p is the porosity of the sediment.

3. Physical meaning of the sand transport equation of the BG model

3.1 Statically stable condition

When we set $\vec{q} = \vec{0}$ in Eq. (26), Eq. (27) is derived as a statically stable condition.

$$\vec{\nabla Z} = \tan\beta_c\, \vec{e_w} \tag{27}$$

This equation demonstrates that the directions of the vectors on both sides of Eq. (27) and their absolute values are equivalent. When $\tan\beta$ and $\vec{e_n}$ are set to the seabed slope and the unit vector normal to the contour lines (shoreward), respectively, the relation $\vec{\nabla Z} = \tan\beta\ \vec{e_n}$ holds. Thus, Eq. (27) is equivalent to the following relationships being satisfied.

$$\vec{e_n} = \vec{e_w},\ \tan\beta = \tan\beta_c \tag{28}$$

Finally, the conditions required for the formation of a statically stable beach are (1) the contour line is orthogonal to the wave direction at any point and (2) the local beach slope coincides with the equilibrium slope at any point. This concept was also employed in the model for predicting a statically stable beach [28]. According to Eq. (26), a restoring force is generated in response to the deviation from the statically stable condition, and sand transport occurs owing to this restoring force.

3.2 Topographic changes

Topographic changes can be determined from the mass conservation equation.

$$\frac{\partial Z}{\partial t} = -\nabla\bullet\,\vec{q} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} \tag{29}$$

When the sand transport fluxes in Eq. (26) are expressed by the components in (x, y) coordinates and are substituted into Eq. (29), the following two-dimensional diffusion equation is obtained, assuming that the coefficient A, equilibrium slope, and wave direction are constant:

$$\frac{\partial Z}{\partial t} = A\left(\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2}\right) \tag{30}$$

The left term and the terms in the parenthesis on the right represent the rate of topographic changes and the spatial curvature of the topography, respectively. In other words, beach changes cause the smoothing of an uneven topography, and in a closed system of sand transport, a statically stable beach is obtained such that the direction of the contours at any point becomes orthogonal to the wave direction, and the local slope is equivalent to the equilibrium slope. These characteristics are the same as those of the contour-line change model [1].

3.3 Dynamically stable beach

In addition to the formation of a statically stable beach, a stable beach can also be dynamically stable, which occurs when the divergence of the sand transport flux in Eq. (30) becomes 0. The dynamically stable beach topography satisfies the Laplace equation, and the relationship between the dynamically stable topography and the sand transport flux has an analogy with the two-dimensional potential flow in fluid dynamics [29].

3.4 Crossshore sand transport

Using Eqs. (6) and (26), the crossshore component of sand transport q_n is obtained as

$$q_n = \vec{e_n} \cdot \vec{q} = C_0 \frac{K_1 \Phi}{\tan \beta_c} \left(\tan \beta_c \cos \alpha - \tan \beta \right) \tag{31}$$

Here, n is the coordinate in the crossshore direction and $\vec{e_n}$ is the unit vector in the cross-shore direction, as defined by Eq. (1). α is the angle between the wave direction and the direction normal to the contour lines, as in Eq. (13). When waves are incident from the direction normal to the shoreline, Eq. (31) becomes

$$q_n = C_0 \frac{K_1 \Phi}{\tan \beta_c} \left(\tan \beta_c - \tan \beta \right) \tag{32}$$

Eq. (32) has the characteristics that crossshore sand transport diminishes when waves are incident from the direction normal to the shoreline, and the local slope is equal to the equilibrium slope ($\tan \beta = \tan \beta_c$). Shoreward transport is generated when the local slope is smaller than the equilibrium slope and vice versa (**Figure 2**). This represents the balance between the upslope flow asymmetry and the downslope component of gravity [15–18].

Figure 2(a) shows the stabilization mechanism of a beach profile based on sand movement during one wave period, a sand particle moves from point 1 to point 2 during incoming waves and from point 2 to point 3 during outgoing waves, and it returns to the same position after one wave period. The net movement of the sand particle is zero, resulting in the formation of a stable beach profile. The seabed slope $\tan \beta$ under this condition is equivalent to the equilibrium slope $\tan \beta_c$. **Figure 2(b)** shows the movement of a sand particle in the case that the local slope is larger than the equilibrium slope $\tan \beta_c$. Because of the increase in the effect of gravity, the sand particle moves seaward as from point 1 to point 2 and then to point 3, resulting in net seaward sand transport and the beach attains a stable slope. To the contrary, when the local slope is gentler than the equilibrium slope, the sand particle is transported landward because of the decrease in the effect of gravity, as schematically shown in **Figure 2(c)**. In **Figure 2**, an extremely simple condition that a sand particle sets on the slope is assumed as an imaginary case to enhance the understanding of the physical meaning of sand movement. In fact, a sand body to be transported due to waves was conceptually regarded as "a sand particle," instead of the method of tracking one sand particle.

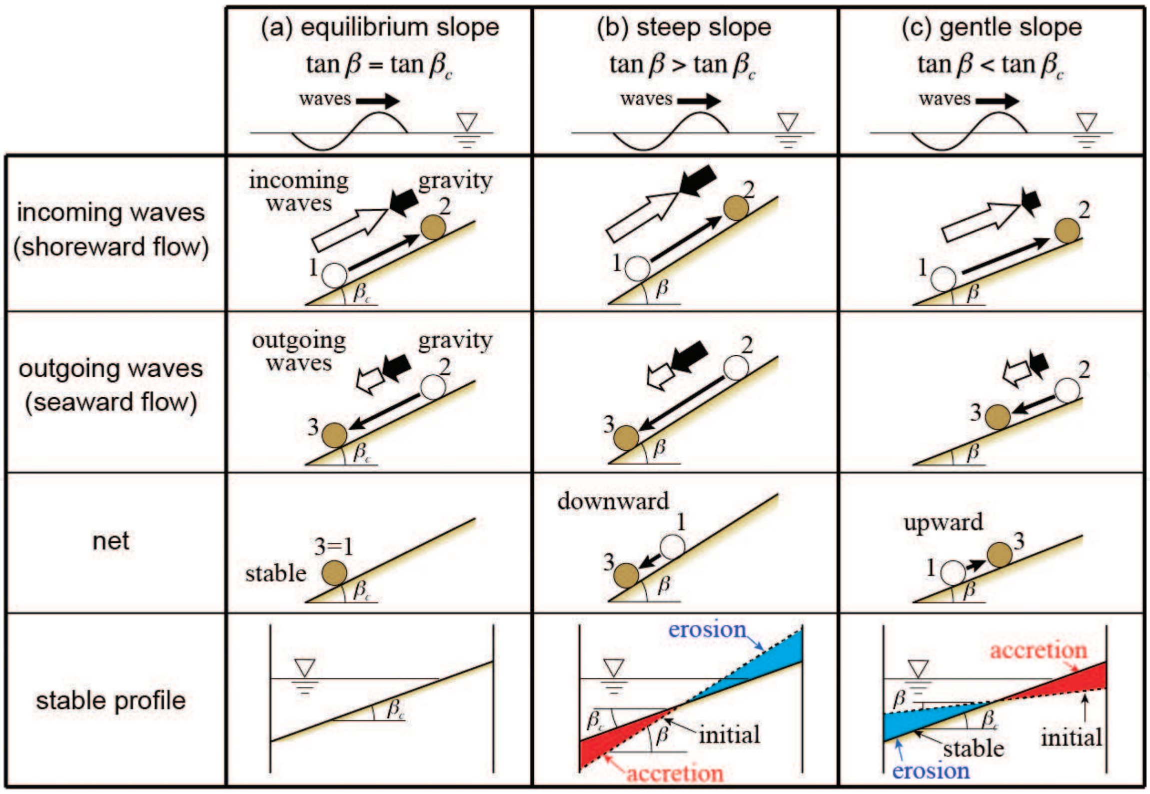

Figure 2.
Stabilization mechanism of the beach profile based on the sand movement during the one wave period.

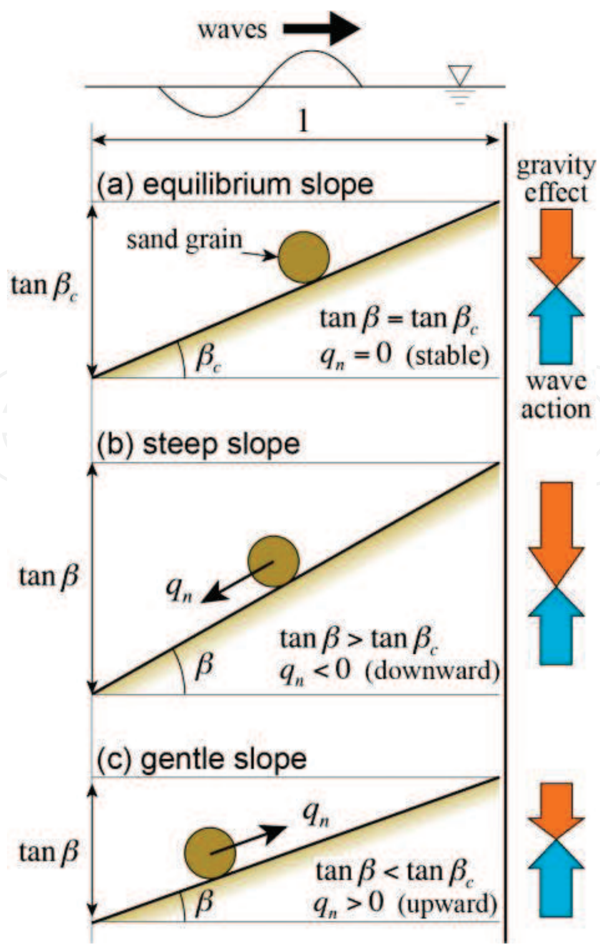

Figure 3.
Equilibrium between the gravity effect and the wave action.

Figure 3 shows a summary of the movement of a sand particle. Crossshore sand transport is zero when the local seabed slope is equivalent to the equilibrium slope, similar to the stabilization mechanism of the longitudinal profile described by Serizawa et al. [1]. Offshore (shoreward) sand transport occurs when the local slope is larger (smaller) than the equilibrium slope.

When waves are obliquely incident to the shoreline, the equilibrium slope $\tan\beta_c'$ can be obtained as Eq. (33) after setting $q_n = 0$ in Eq. (31). Here, the breaker angle α_b is substituted into α as an approximation.

$$\tan\beta_c' = \tan\beta|_{q_n=0} = \tan\beta_c\cos\alpha \approx \tan\beta_c\cos\alpha_b \tag{33}$$

Although $\tan\beta_c'$ is smaller than $\tan\beta_c$ by a factor of $\cos\alpha_b$, the approximation of $\cos\alpha_b \approx 1$ holds, because α_b normally takes a value within 20°, and $\tan\beta_c'$ can be regarded as $\tan\beta_c$.

3.5 Longshore sand transport

Using Eqs. (7) and (26), the longshore component of sand transport q can be expressed as

$$q_s = \vec{e_s}\cdot\vec{q} = C_0 K_1 \Phi \sin\alpha \tag{34}$$

Here, s and $\vec{e_s}$ are the coordinate and the unit vector in the longshore direction, as defined in Eq. (2), respectively. This equation shows that the longshore sand transport q_s becomes 0 when the wave direction coincides with the direction normal to

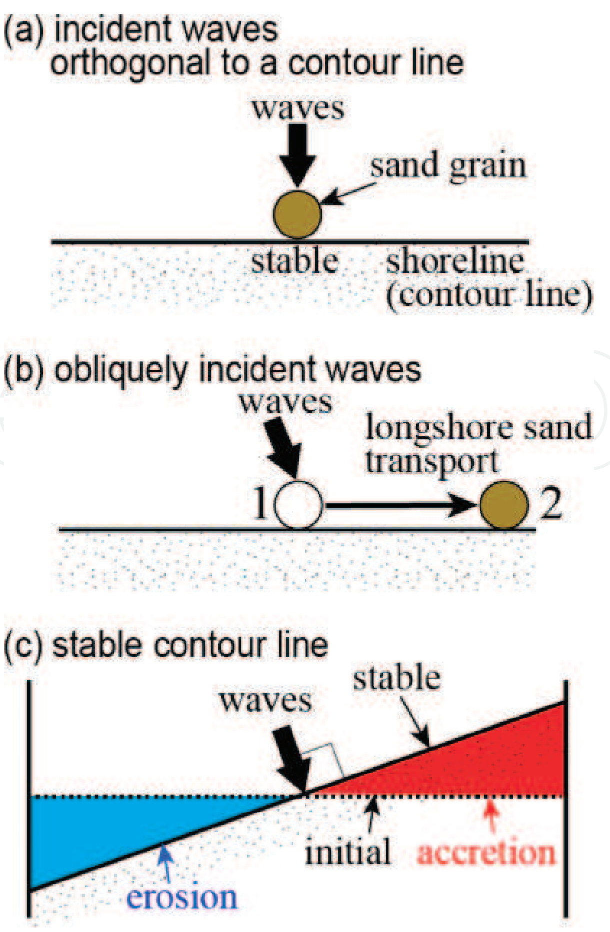

Figure 4.
Formation of stable contour lines.

the contour lines. Under other conditions, longshore sand transport is induced, as schematically shown in **Figure 4**.

When the total sand transport Q_s is calculated by integrating Eq. (34) in the crossshore direction, it coincides with the CERC-type formula [1], as in Eq. (35), under the assumptions that the integral of Φ in the crossshore direction is equal to the energy flux per unit length of the coastline at the breaking point and that α is approximately given by the breaker angle α_b.

$$\begin{aligned} Q_s &= \int q_s dn = C_0 K_1 \int \Phi \sin\alpha \, dn \\ &= C_0 K_1 (EC_g)_b \cos\alpha_b \sin\alpha_b \\ &\left(\alpha \approx \alpha_b, \int \Phi dn = (EC_g)_b \cos\alpha_b\right) \end{aligned} \tag{35}$$

Here, $(EC_g)_b$ and α_b are the wave energy flux at the breaking point and the breaker angle, respectively. This CERC-type formula [30] has been employed in the one-line model in practical engineering [31, 32]. Thus, the integral of the longshore component of sand transport in the BG model in the crossshore direction is equivalent to the total longshore sand transport, so both bedload and suspended sediment transport are automatically included in the total sand transport. The fundamental equations of the BG model are compatible with the CERC total sand transport formula, which has often been employed in the prediction of beach changes in the practical applications.

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References

[1] Serizawa M, Uda T, San-nami T, Furuike F, Kumada K. Improvement of contour-line change model in terms of stabilization mechanism of longitudinal profile. In: Coastal Sediments '03. World Sceintific Publication Co.; 2003. pp. 1-15

[2] Hanson H, Aarninkhof S, Capobianco M, Jimenez JA, Larson M, Nicholls R, et al. Modelling coastal evolution on early to decadal time scales. Journal of Coastal Research. 2003;**19**:790-811

[3] Perlin M, Dean RG. A numerical model to simulate sediment transport in the vicinity of coastal structures, Misc. Report, No. 83-10. Coastal Engineering Research Center, U.S. Army Corps of Engineers; 1983. p. 119

[4] Steetzel HJ, de Vroeg JH. Application of a multilayer approach for morphological modelling. In: Coastal Sediments' 99. Vicksburg: ASCE; 1999. pp. 2206-2218

[5] Steetzel HJ, de Vroeg H, van Rijn LC, Stam JM. Long-term modelling of the Holland coast using a multi-layer model. In: Proceedings of the 27th International Conference on Coastal Engineering. Sydney, Australia: ASCE; 2000. pp. 2942-2955

[6] Hanson H, Larson M. Simulating coastal evolution using a new type N-line model. In: Proceedings of the 27th International Conference on Coastal Engineering. Vicksburg: ASCE; 2000. pp. 2808-2821

[7] Dabees MA, Kamphuis JW. Efficient modeling of 3D beach change. In: Proceedings of the 27th International Conference on Coastal Engineering. Sydney, Australia: ASCE; 2000. pp. 2700-2713

[8] Falqués A, Garnier R, Ojeda E, Ribas F, Guillen J. Q2D-morfo: A medium to long term model for beach morphodynamics. River, Coastal and Estuarine Morphodynamics. 2007;**1-2**: 71-78

[9] Falqués A, van den Berg N, Calvete D. The role of cross-shore profile dynamics on shoreline instability due to high-angle waves. In: Proceedings of 31st ICCE; 2008. pp. 1826-1838

[10] van den Berg N, Falqués A, Ribasz F. Long-term evolution of nourished beaches under high angle wave conditions. Journal of Marine Systems. 2011;**88**(1):102-112

[11] Larson M, Kubota S, Erikson L. Swash zone sediment transport and foreshore evolution: Field experiment and mathematical modeling. Marine Geology. 2004;**212**:61-79

[12] Larson M, Wamsley TV. A formula for longshore sediment transport in the swash. In: Proceedings of Coastal Sediment'07. ASCE; 2007. pp. 1924-1937

[13] Nam PT, Larson M, Hanson H, Hoan LX. A numerical model of beach morphological evolution due to waves and currents in the vicinity of coastal structures. Coastal Engineering. 2011; **58**(9):863-876

[14] Bagnold RA. Mechanics of marine sedimentation. In: Hill MN, editor. The Sea. Vol. 3. New York: Wiley; 1963. pp. 507-528

[15] Inman DL, Bagnold RA. Littoral processes. In: Hill MN, editor. The Sea. Vol. 3. New York: Wiley; 1963. pp. 529-533

[16] Bowen AJ. Simple models for nearshore sedimentation: Beach profiles and longshore bars. In: The Coastline of Canada, The Geological Survey of

Canada, McCann SB, Paper 80(10); 1980. pp. 1-11

[17] Bailard JA, Inman DL. An energetics bedload model for a plane sloping beach: Local transport. Journal of Geophysical Research. 1981;**86**(C3):2035-2043

[18] Bailard JA. An energetics total load sediment transport model for a plane sloping beach. Journal of Geophysical Research. 1981;**86**(C11):10938-10954

[19] Kabiling M, Sato S. Two-dimensional nonlinear dispersive wave-current model and three-dimensional beach deformation model. Coastal Engineering in Japan. 1993;**36**(2): 195-212

[20] Long W, Kirby JT. Cross-shore sediment transport model based on the Boussinesq equations and an improved Bagnold formula. In: Proceedings of Coastal Sediments ’03. Clearwater Beach: ASCE; 2003

[21] Serizawa M, Uda T, San-nami T, Furuike K. Three-dimensional model for predicting beach changes based on Bagnold’s concept. In: Proceedings of 30th ICCE; 2006. pp. 3155-3167

[22] Hardisty J. A note on negative beach slopes and flow asymmetry. Marine Geology. 1985;**69**:203-206

[23] Hardisty J. A morphodynamic model for beach gradients. Earth Surface Processes and Landforms. 1986; **11**:327-333

[24] Dean RG. Equilibrium beach profiles: Characteristics and applications. Journal of Coastal Research. 1991;7(1):53-84

[25] Dean RG, Dalrymple RA. Coastal Processes with Engineering Applications. New York, NY: Cambridge University Press; 2002. p. 474

[26] Larson M, Kraus NC, Wise RA. A equilibrium beach profiles under breaking and non-breaking waves. Coastal Engineering. 1999;**36**:59-85

[27] Komar PD, Inman DL. Longshore sand transport on beaches. Journal of Geophysical Research. 1970;**75**: 5914-5927

[28] Sakai K, Uda T, Serizawa M, Kumada T, Kanda Y. Model for predicting three-dimensional sea bottom topography of statically stable beach. In: Proceedinsg of 30th ICCE; 2006. pp. 3184-3196

[29] Uda T, Serizawa M, San-nami T, Furuike K. Model for predicting dynamically stable beach based on analogy with two-dimensional potential flow. In: Proceedings of 30th ICCE; 2006. pp. 3911-3923

[30] CERC. Shore Protection Manual. Vicksburg, Mississippi: U.S. Army Coastal Engineering Research Center, Corps of Engineers

[31] Komar PD. Beach Processes and Sedimentation. London: Prentice Hall International; 1998. p. 544

[32] Horikawa K, editor. Nearshore Dynamics and Coastal Processes. Tokyo: University of Tokyo Press; 1988. p. 522