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Parrondian Games in Discrete Dynamic Systems

Steve A. Mendoza and Enrique Peacock-López

Abstract

An interesting problem in nonlinear dynamics is the stabilization of chaotic trajectories, assuming that such chaotic behavior is undesirable. The method described in this chapter is based on the Parrondo's paradox, where two losing games can be alternated, yielding a winning game. The idea of alternating parameter values has been used in chemical systems, but for these systems, the undesirable behavior is not chaotic. In contrast, ecological relevant map in one and two dimensions, most of the time, can sustain chaotic trajectories, which we consider as undesirable behaviors. Therefore, we analyze several of such ecological relevant maps by constructing bifurcation diagrams and finding intervals in parameter space that satisfy the conditions to yield a desirable behavior by alternating two undesirable behaviors. The relevance of the work relies on the apparent generality of method that establishes a dynamic pattern of behavior that allows us to state a simple conjecture for two-dimensional maps. Our results are applicable to models of seasonality for 2-D ecological maps, and it can also be used as a stabilization method to control chaotic dynamics.

Keywords: chaos control, Parrondo's paradox, switched dynamic systems, ecological maps, seasonality

1. Introduction

In population dynamics, discrete dynamic systems have been used to model the dynamics of ecological systems. One of the first maps used in ecology that suggested to study the new, (X_{n+1}), and the old, (X_n), non-overlapping populations is the logistic map. [1] Although a simple one-dimensional (1-D) map, the logistic map shows complex dynamics including chaos. Furthermore, the analyses of the logistic map gave us a better understanding of the properties of chaotic dynamics [2–7].

In the case of 1-D discrete dynamics, for the last 18 years, alternate dynamics strategies have been the center of attention due to the so-called Parrondo paradox [8–10], where two losing games can be combined to yield a winning game. Furthermore, the idea that "lose + lose = win" has been extended to "chaos +chaos = periodic" in one-dimensional maps [11]. Just recently and for the first time, we were able to find the Parrondo dynamics in two 2-D maps [12]. In the contest of seasonality, we consider the alternation of undesirable dynamical behaviors yield a desirable behavior [13, 14]. So in the context of population dynamics we have considered cases where "undesirable + undesirable = desirable" dynamical behaviors occur as a result of a simple alternation of parameters [15–21].

In our present discussion, we extend our seasonality modeling strategy to several two-dimensional ecologically relevant maps and find that the "undesirable + undesirable = desirable", the "chaos + chaos = periodic", as well as, the "periodic + periodic = chaos" behaviors are not unique to 1-D maps. In Section 2, we consider a delayed logistic map, and in Section 3, we analyze a Lotka-Volterra map. In Section 4, we study a modified 2-D Ricker map, and in Section 5, we analyze the Beddington map. In Section 6, we discuss a modified Lotka-Volterra map, which includes a logistic prey growth. We conclude in Section 7 with a discussion and a summary of our results.

2. Delayed logistic equation

In our analysis of two-dimensional maps, we begin with the extended logistic map that incorporates a delay in population growth, defined by the following relation:

$$X_{n+1} = Y_n \tag{1}$$

$$Y_{n+1} = C\,Y_n(1 - X_n) \tag{2}$$

where C is our bifurcation parameter. For the Lagged Logistic Equation, we consider C values from 0 to 2.27 in the original map, although with alternation, we can obtain a bifurcation diagram showing larger C values. **Figure 1**, shows the regular bifurcation map of the lagged logistic model. For all of the maps we study, both the X and Y graphs for a given show the same dynamics; for instance, parameters associated with chaotic dynamics in the X map are also associated with chaotic dynamics in the Y map; since we focus on a map's dynamics, we only show the X function map.

From **Figure 1**, we define our parameter value regions associated with complex or non-complex dynamics. The map on the left of the figure shows the whole bifurcation map; while the right magnifies the complex region. On the right hand figure, which is the magnified map, we can clearly see some periodic windows, but we pick parameter values associated with complex dynamics.

Next, we switch, or alternate, the parameter values between even and odd iterations through the following relation:

$$X_{n+1} = \begin{cases} f_n(X_n, Y_n) = Y_n & \text{if } n \text{ even} \\ f_n(X_n, Y_n) = Y_n & \text{if } n \text{ odd} \end{cases} \tag{3}$$

$$Y_{n+1} = \begin{cases} g_n(X_n, Y_n) = C_e\,Y_n(1 - X_n) & \text{if } n \text{ even} \\ g_n(X_n, Y_n) = C_o\,Y_n(1 - X_n) & \text{if } n \text{ odd} \end{cases} \tag{4}$$

The equation above describes our switching strategy in which we pick one parameter for every odd iteration, which we name C_o, and use the even parameter, C_e, as our bifurcation parameter, for every even iteration. For the first type of behavior, we pick one parameter associated with complex dynamics as our C_o value and switch it with our even parameter, C_e, in areas associated with chaotic dynamics. In our case, we see chaotic dynamics for C values greater than 2.0 when we construct the bifurcation diagram for Eqs. (1) and (2). In the resulting alternated, or switched, bifurcation diagram, represented by Eq. (3) for the current section, we

look for regions that have periodic oscillations that are normally associated with chaos, thus resulting in the case "chaos + chaos = order." Hence, for every switching map that we study, in the figures we also show the unswitched map for the same C_e parameter space. We make a note that the alternation of parameter values as defined by our switching strategy may result in an extension of C parameters yielding oscillations; that is we may see oscillations for C values greater than 2.27 as in the case for lagged logistic map. To compare our maps with Eqs. (1) and (2), we only study bifurcation maps using Eqs. (3) and (4) from $C = 0$ to 2.27.

For our analysis, we pick a C_o as in the "chaos + chaos = order" case; however, we may choose a parameter within the periodic windows in the chaotic region, and when using the switched map, we focus on the C_e values that are less than the onset of chaos, which for the case of the lagged logistic pap is $C = 2$. We use the analysis discussed above for all of the cases in this paper.

For our first example of "chaos + chaos = periodic", we consider the parameter value, $C_o = 2.10$ and Eqs. (3) and (4). In our bifurcation diagram for Eqs. (3) and (4), in **Figure 2**, we consider C_e greater than 2 and look for C_e values that give us periodic oscillations. **Figure 2** shows two maps at once, the left hand showing Eqs. (1) and (2), and the right hand graph shows Eqs. (1) and (2) with $C_o = 2.1$. In this case, from **Figure 2**, we can see one region of periodicity from $C_e = 2.26$ to 2.27.

Another combination of parameters yielding "chaos+chaos = periodic" uses $C_o = 2.15$ and Eqs. (3) and (4), where **Figure 3** shows a range of C_e values for which "chaos + chaos = periodic" holds, from $C_e = 2.36$ to 2.38. The same figure also shows other values for which the "chaos + chaos = periodic" relation holds, but these bands are not as prominent as the one we focus on. Through out the paper, we make a point that different C_e values give widely different behaviors and these differences in dynamic behaviors reveals the differences in the C_e values that give us desirable behaviors. For the rest of the paper, the approximate ranges of C_e will be given for
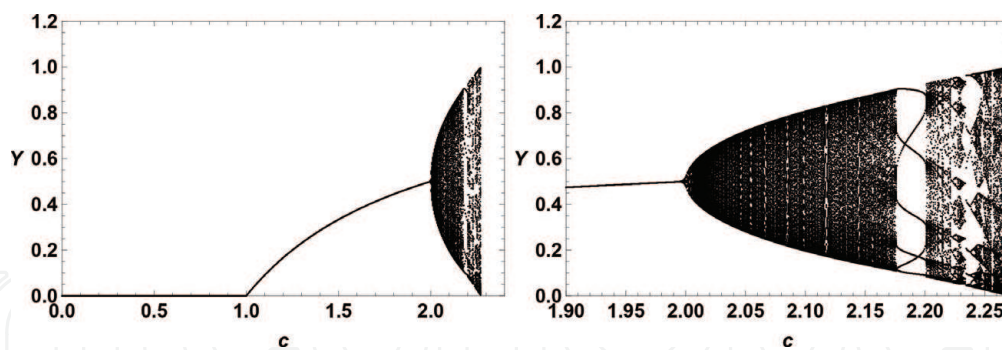

Figure 1.
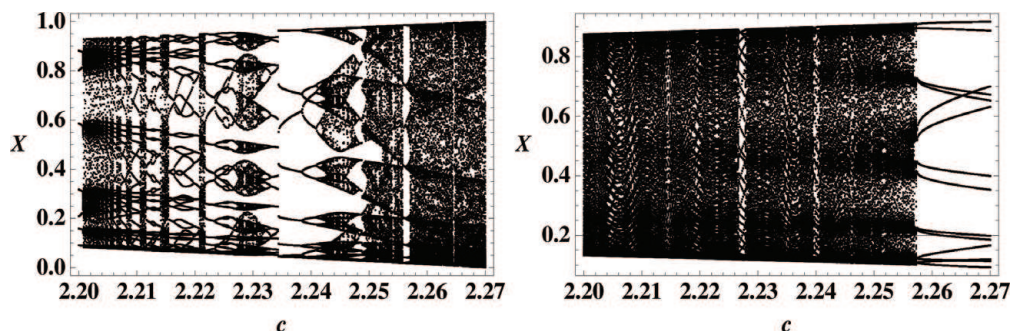*Lagged logistic map model, Eqs. (1,2), with C = 0 to C = 2.27 and the region from C = 1.90 to C = 2.27.*

Figure 2.
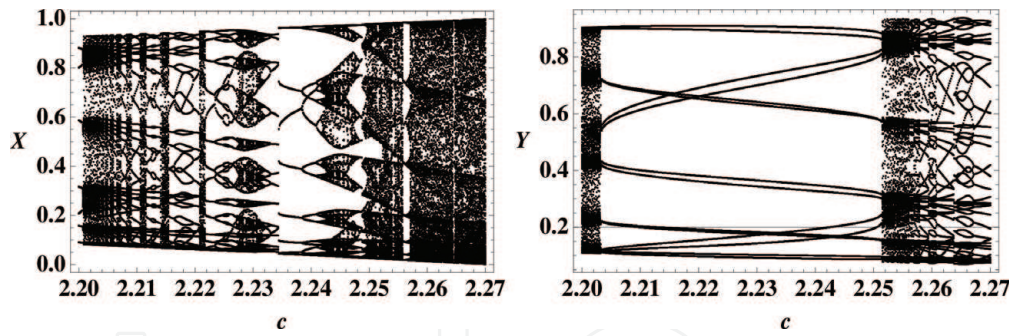*Bifurcation diagram for Eqs. (1) and (2) and Eqs. (3) and (4), using C$_o$ 2.1.*

Figure 3.
Bifurcation diagram for Eqs. (1) and (2), and Eqs. (3) and (4), using $C_o = 2.15$.
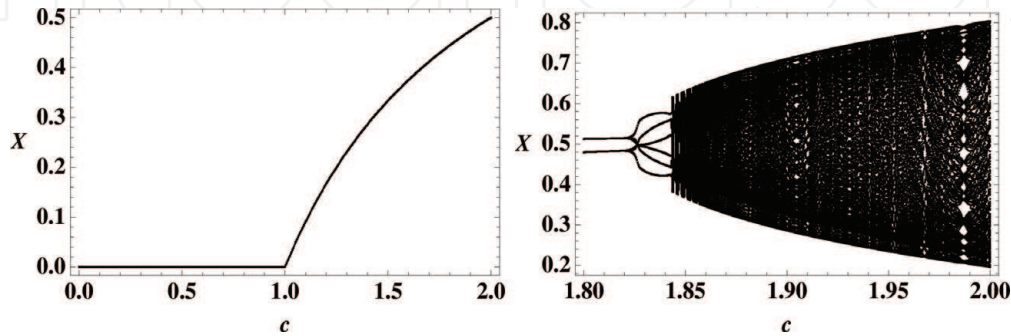

Figure 4.
Bifurcation diagram for Eqs. (1) and (2), and Eqs. (3) and (4), using C_o 2.19.

one window that satisfies the "chaos + chaos = order" or "periodicity + periodicity = chaos", since there are sometimes a variety of parameters meeting the relevant criteria for switching.

We complete our analysis of delayed logistic map with one case in which "periodic + periodic = chaos". As mentioned beforehand, we pick our value associated with periodic trajectories from the area associated with chaotic trajectories, and focus on C_e values less than the chaotic region for comparison. In particular, we choose $C_o = 2.19$ as our periodic parameter for Eqs. (3) and (4); **Figure 4** shows the corresponding bifurcation map, and, from the figure, we see one prominent example of "periodic+periodic = chaos" for $C_e = 1.85$ to 2.00.

3. Lotka-Volterra model

We begin our next section by discussing a discretized form of the Lotka-Volterra model. The Lotka-Volterra map describes predator prey interactions, assuming that the prey has a relatively high initial population, and that the predator's growth rate is directly proportional to the prey's growth rate.

The model follows a relation defined by the map below

$$X_{n+1} = (1+r)X_n - rX_n^2 - C X_n Y_n \tag{5}$$

$$Y_{n+1} = C X_n Y_n \tag{6}$$

In **Figure 5**, showing Eqs. (5) and (6), we look at the unswitched map, defined by showing the ranges of periodic and aperiodic behavior. As in the previous section, we use the unswitched bifurcation map as a comparison to the switched map when using certain parameters. For this section, we focus on the interval $C = 0$ to 2.8, and set $r = 2$ for this map and the rest of the maps that have an r parameter.

$$X_{n+1} = \begin{cases} f_n(X_n, Y_n) = (r+1)(X_n) - r(X_n)^2 - C_e X_n Y_n & \text{if n even} \\ f_n(X_n, Y_n) = (r+1)(X_n) - r(X_n)^2 - C_o X_n Y_n & \text{if n odd} \end{cases} \quad (7)$$

$$Y_{n+1} = \begin{cases} g_n(X_n, Y_n) = C_o X_n Y_n & \text{if n odd} \\ g_n(X_n, Y_n) = C_e X_n Y_n & \text{if n even} \end{cases} \quad (8)$$

As before, we pick a C_o value associated with a chaotic trajectory and alternate with C_e, using Eq. (7), which we use as the bifurcation parameter, illustrated in **Figure 6**. For this figure, we use $C_o = 2.1$, and we can easily find conditions in which "chaos + chaos = order." In particular, we see this phenomena for parameter values of $C_e = 2.33$–2.40. **Figure 7**, shows another example of "chaos + chaos = order" using Eqs. (7) and (8) with a C_o value of 2.22, and in the corresponding bifurcation diagram for roughly $C_e = 2.58$–2.65.

We conclude the present section with an example of "periodicity + periodicity = chaos", using Eqs. (7) and (8) and $C_e = 2.44$. In this case, in **Figure 8**, we see
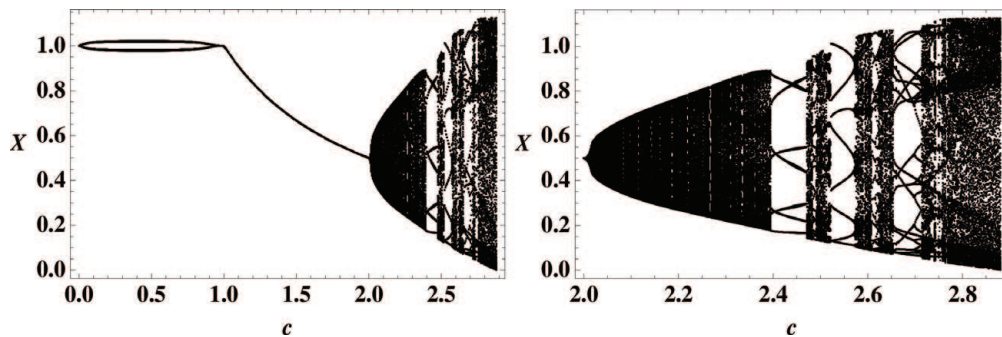
Figure 5.
Bifurcation diagram for Eqs. (5) and (6), showing the interval studied, as well as a close up of the chaotic region.
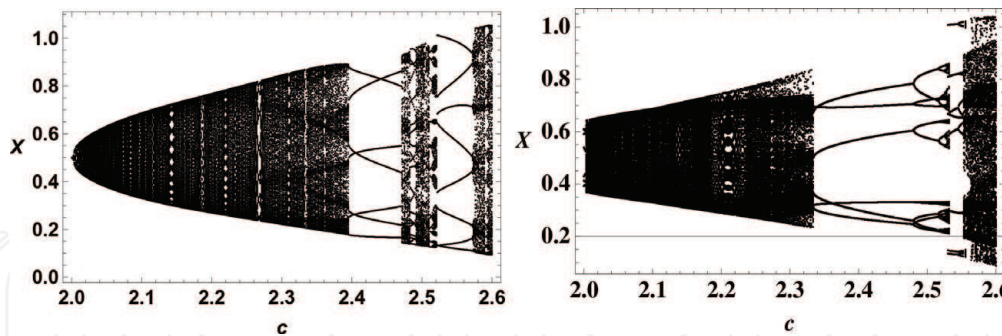

Figure 6.
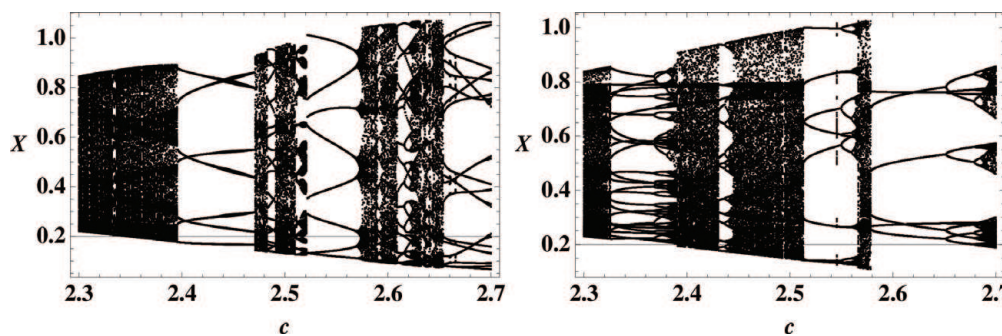*Bifurcation diagram for Eqs. (5) and (6) and Eqs. (7) and (8) with the C_o value 2.1.*

Figure 7.
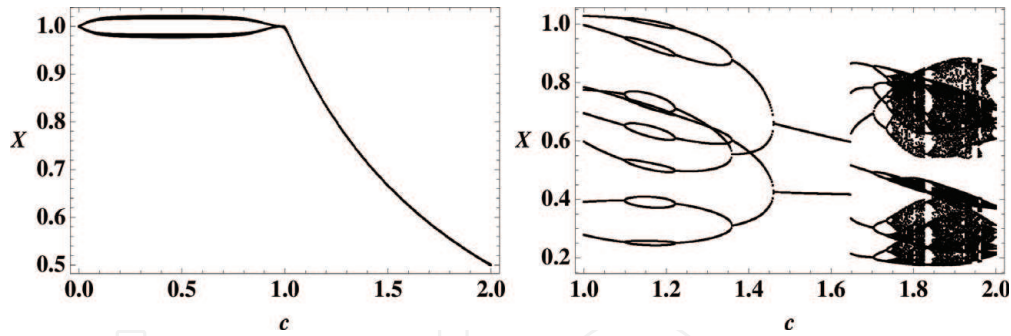*Bifurcation diagram for Eqs. (5) and (6) and Eqs. (7) and (8) with the C_o value 2.22.*

Figure 8.
Bifurcation diagram for Eqs. (5) and (6) and Eqs. (7) and (8) with the C_o value 2.44.

some chaotic behavior for values of C_e in the interval $[1.75, 2.00]$, a region that is periodic when using Eqs. (5) and (6).

4. Modified 2-D Ricker map

Another interesting map includes an exponential term, describing the prey growth, with a simple predator–prey interaction term. The map is determined by the following equations:

$$X_{n+1} = X_n \ Exp\left[r(1 - X_n - Y_n)\right] \tag{9}$$

$$Y_{n+1} = C \, X_n(Y_n) \tag{10}$$

which is in essence modified and extended to 2-D Ricker-like map [22]. The corresponding switched map is defined below:

$$X_{n+1} = \begin{cases} f_n(X_n, Y_n) = X_n \ Exp\left[r(1 - X_n - Y_n)\right] & \text{if n even} \\ f_n(X_n, Y_n) = X_n \ Exp\left[r(1 - X_n - Y_n)\right] & \text{if n odd} \end{cases} \tag{11}$$

$$Y_{n+1} = \begin{cases} g_n(Y_n) = C_e X_n Y_n & \text{if n even} \\ g_n(Y_n) = C_o X_n Y_n & \text{if n odd} \end{cases} \tag{12}$$

Figure 9, showing Eqs. (9) and (10), considers the range of C values we focus on, from C = 0 to 2.8. We want to remark however, that this map also shows some interesting behavior beyond the interval of study, but we choose this interval to get a close up of the intervals of periodicity, since this interval is where we find our relevant behavior. For the X function we study, at higher values, the function stays at unity for values of C = 28 and higher, while the Y function stays at extinction, or Y = 0.

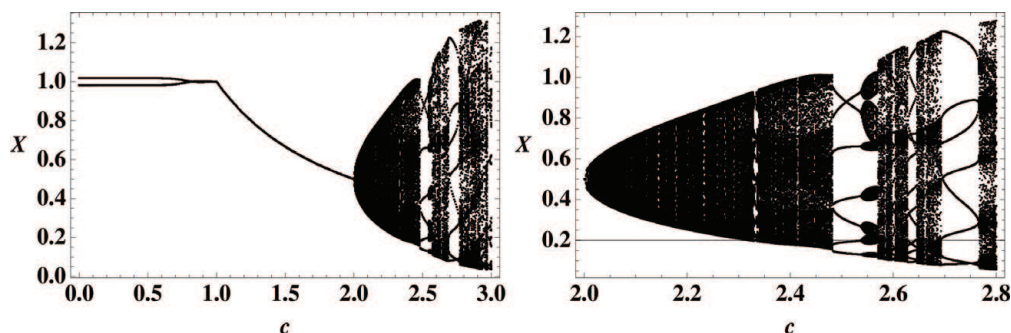**Figure 9.**
Bifurcation map for Eqs. (9) and (10) showing the interval studied, as well as a close up of the chaotic region.

As in previous cases, we start with finding parameter values satisfying the "chaos + chaos = order" relation. To begin, we use Eqs. (11) and (12) with $C_o = 2.10$, associated with aperiodic dynamics. **Figure 10** zooms into the region for which "chaos + chaos = order" holds. From this diagram, we see a narrow region of periodicity from C = 2.775 to 2.790.

We then use Eqs. (11) and (12) with $C_o = 2.26$, for which **Figure 11** hones in on the relevant C_e parameter values. The interval of C_e values is significantly wider in this case than the previous one, since we find "chaos + chaos = order" for 2.74–2.80.

We finish this section by introducing one case in which "periodic + periodic = chaos". We pick the periodic parameter $C_o = 2.333$. In some maps, it is harder to find periodic windows, although they could usually be found sometimes but an extra significant figure is necessary such as in this case. We find chaos in this map from $C_e = 1.71$ to 2.00, as shown in **Figure 12**, periodic values when using Eqs. (11,12).
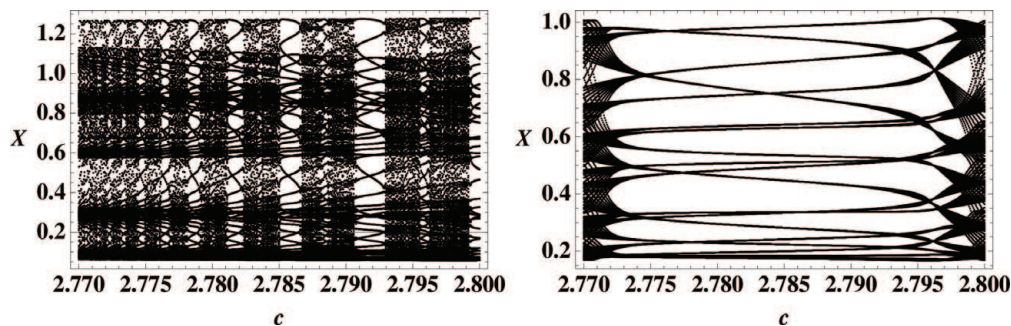

Figure 10.
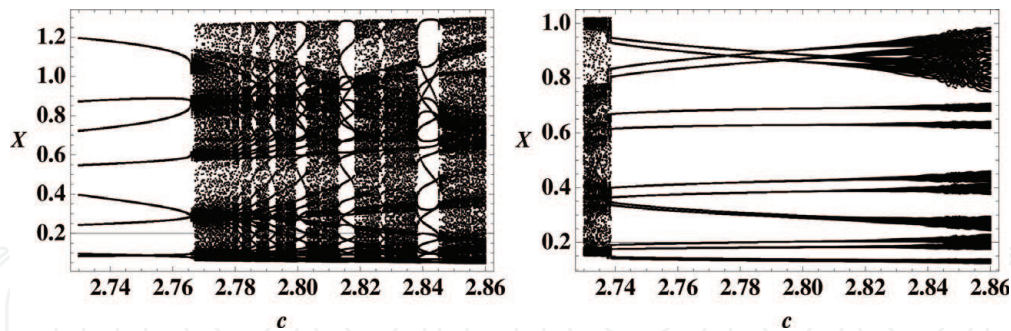*Bifurcation diagram for Eqs. (9) and (10) and Eqs. (11) and (12) with the C_o value 2.10.*

Figure 11.
Bifurcation diagram for Eqs. (9) and (10) and Eqs. (11) and (12) with the C_o value 2.26.
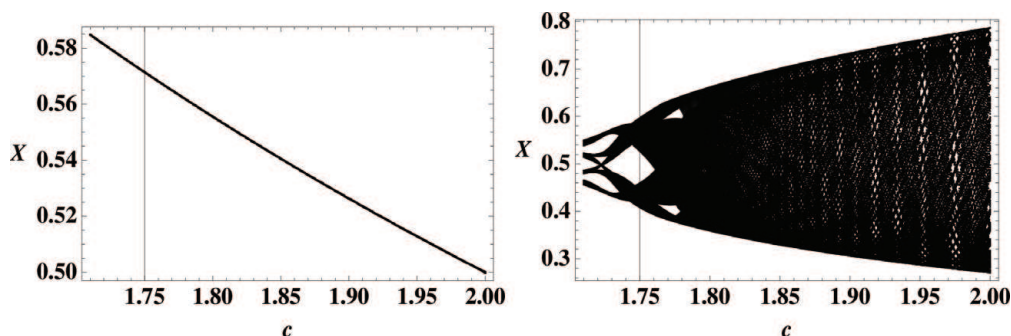

Figure 12.
Bifurcation diagram for Eqs. (9) and (10) and Eqs. (11) and (12) with the C_o value 2.333.

5. Beddington model

Our next map is the Beddington 2-D map defined by the following equations:

$$X_{n+1} = X_n \, Exp \, (r(1 - X_n) - Y_n) \tag{13}$$

$$Y_{n+1} = C \, X_n (1 - Exp \, (-Y_n)) \tag{14}$$

along with the corresponding alternation equation.

$$X_{n+1} = \begin{cases} f_n(X_n, Y_n) = X_n \, Exp \, (r(1 - X_n) - Y_n) & \text{if n even} \\ f_n(X_n, Y_n) = X_n \, Exp \, (r(1 - X_n) - Y_n) & \text{if n odd} \end{cases} \tag{15}$$

$$Y_{n+1} = \begin{cases} g_n(X_n, Y_n) = C_e \, X_n(1 - Exp \, (-Y_n)) & \text{if n even} \\ g_n(X_n, Y_n) = C_o \, X_n(1 - Exp \, (-Y_n)) & \text{if n odd} \end{cases} \tag{16}$$

Figure 13, showing Eqs. (13) and (14), shows the parameter range we use to analyze the map. We pick points between 0 and 14, and show the corresponding bifurcation diagrams within that range. We pick 14 as our maximum value because above that parameter, there are only steady state solutions.

We start with describing our first chaotic value, C_o = 10, for Eqs. (15) and (16). **Figure 14** shows the corresponding bifurcation diagram, and we see a relatively wide range of C_e values for which we have "chaos + chaos = order". We find this behavior for most points of C_e between 4.54 and 4.7.

We then use Eqs. (15) and (16), with C_o = 4.0, and here we also see a relatively wide range of parameters in which we find that "chaos + chaos = periodicity". Specifically, we see that alternating with C_e = 10.7–10.88 gives us the desired behavior, shown in **Figure 15**. Our last figure pertaining to this map, **Figure 16**, shows the
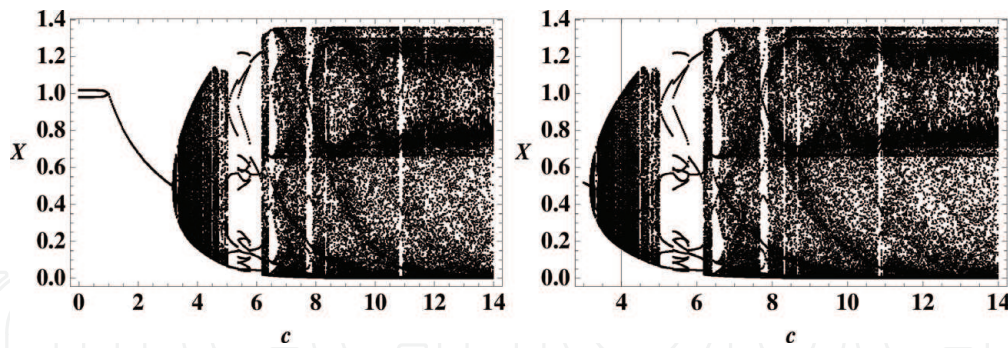

Figure 13.
Bifurcation diagram for Eqs. (13) and (14) showing the interval studied, as well as a close up of the chaotic region.
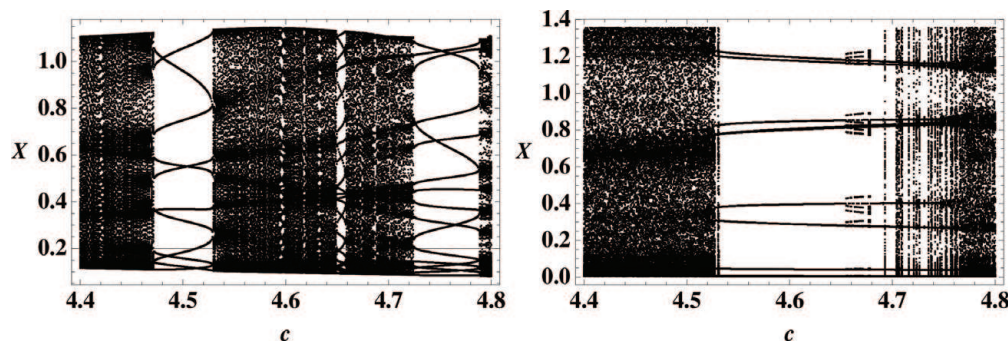

Figure 14.
Bifurcation diagram for Eqs. (13) and (14) and Eqs. (15) and (16) with the C_o value 10.
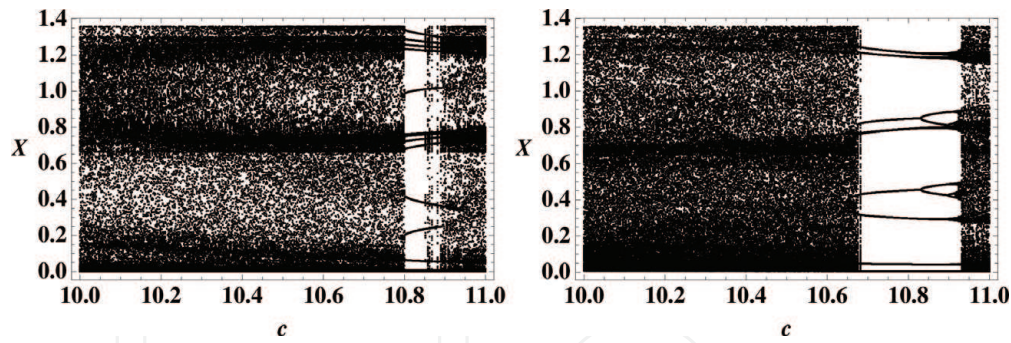
Figure 15.
Bifurcation diagram for Eqs. (13) and (14) and Eqs. (15) and (16) with the C_o value 4.0.
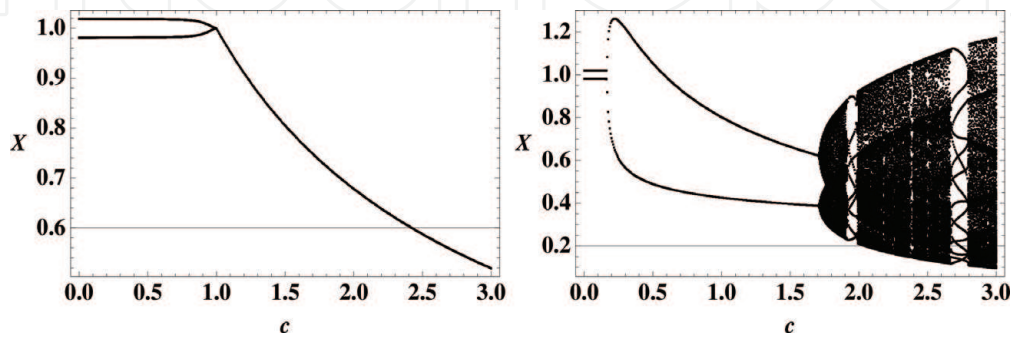

Figure 16.
Bifurcation diagram for Eqs. (13) and (14) and Eqs. (15) and (16) with the C_o value 6.0.

"periodic + periodic = chaos" behavior, for C_o = 6.0. **Figure 16** shows the area of the map that is normally periodic, and shows characteristic chaotic behavior from C_e = 1.7–3.0, although this particular map shows some periodic windows than the other "periodic + periodic = chaos" maps.

6. Modified Lotka-Volterra map

Our last 2-D map considers a logistic growth, and an interaction term, and only a predation term for the predator. The dynamics of this map is considerably different than the previous two maps,

$$X_{n+1} = (1+r)X_n - r X_n^2 - \frac{CX_nY_n}{X_n + h} \tag{17}$$

$$Y_{n+1} = \frac{CX_nY_n}{X_n + h} \tag{18}$$

As before, the switched map is shown below.

$$X_{n+1} = \begin{cases} f_n(X_n) = (1+r)X_n - rX_n^2 - \dfrac{C_eX_nY_n}{X_n + h} & \text{if n even} \\[3mm] f_n(X_n) = X_n(r+1) - r(X_n)^2 - \dfrac{C_oX_nY_n}{X_n + h} & \text{if n odd} \end{cases} \tag{19}$$

$$Y_{n+1} = \begin{cases} g_n(Y_n) = \dfrac{C_oX_nY_n}{X_n + h} & \text{if n odd} \\[3mm] g_n(Y_n) = \dfrac{C_eX_nY_n}{X_n + h} & \text{if n even} \end{cases} \tag{20}$$

Aside from the r parameter, this map also has the h parameter, which we set equal to unity. Unlike the previous two maps we study that have relevant behaviors past C = 10, the max value of the unswitched map is C = 3.85, but chaos is only present above C = 3.0, as shown in **Figure 17**.

Our first chaotic point is C_o = 3.3, and the corresponding bifurcation diagram is shown in **Figure 18**. There is a somewhat small region of periodicity from C_e = 3.704 to 3.724.

The second to last figure, **Figure 19** shows our final odd switching parameter, C_o = 3.1 and the corresponding bifurcation diagram, which shows a similar range of periodic parameter values, specifically, C_e = 3.70–3.72.

Our last figure, **Figure 20**, shows an example of "periodic + periodic = chaos", where we switch with C_o = 3.57 and see chaos for most values between C_e = 2.5 and 3.0.
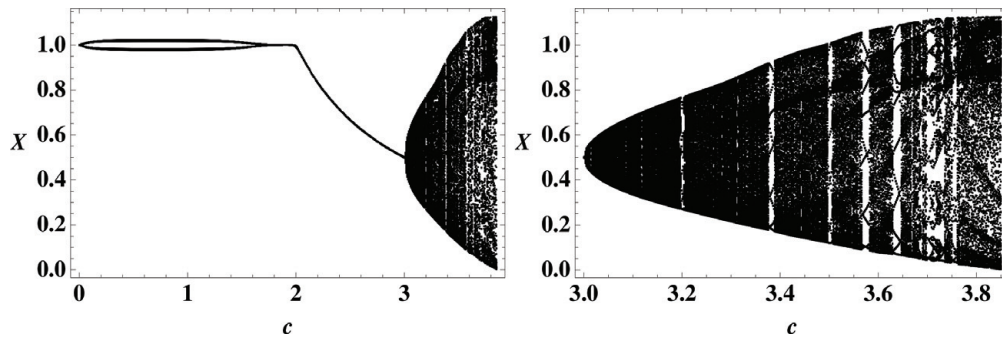

Figure 17.
Bifurcation diagram for Eqs. (17) and (18) showing the interval studied, as well as a close up of the chaotic region..
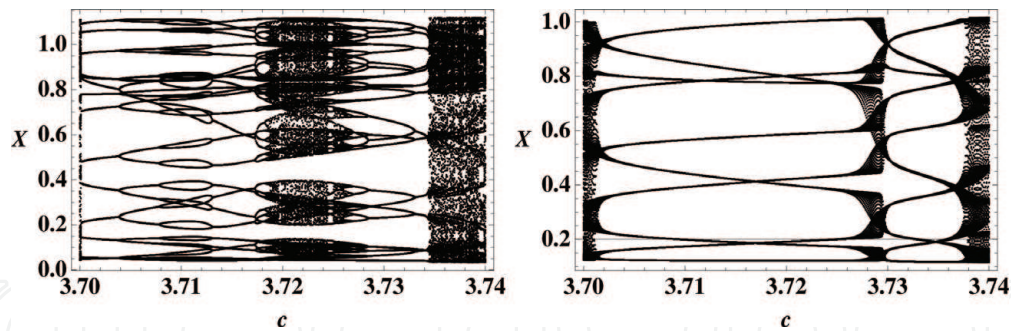

Figure 18.
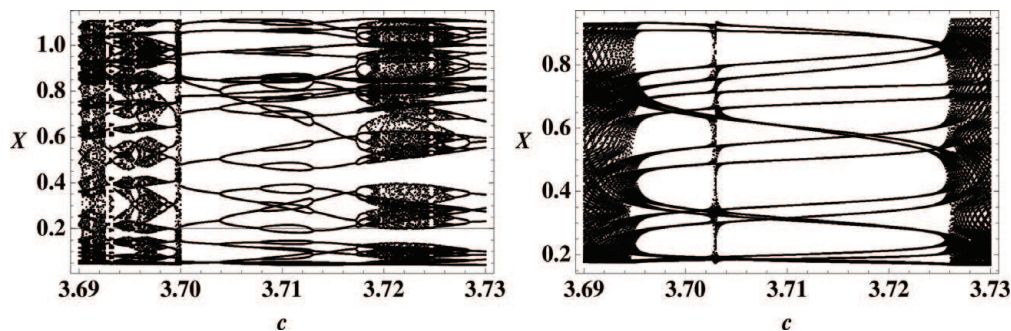*Bifurcation diagram for Eqs. (17) and (18) and Eqs. (19) and (20) with the C_o value 3.3.*

Figure 19.
Bifurcation diagram for Eqs. (17) and (18) and Eqs. (19) and (20) with the C_o value 3.1.
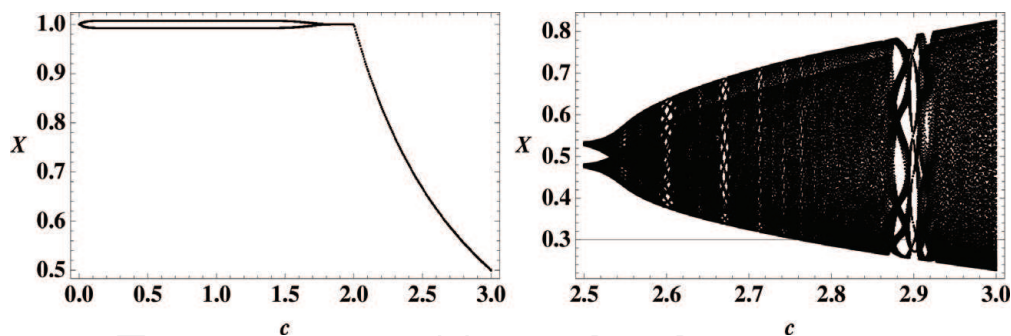
Figure 20.
Bifurcation diagram for Eqs. (17) and (18) and Eqs. (19) and (20) with the C_o value 3.57.

7. Discussion

In previous sections, we have analyzed five relevant ecological 2-D maps, setting a pattern of dynamic behavior similar to the well studied "chaos + chaos = periodic" in switched 1-D maps. Therefore, with the results discussed in this chapter, we can extend the 1-D maps conjecture to 2-D maps. The conjecture asserts that given a map with chaotic dynamics, we can find two parameters associated to chaotic trajectories that, when alternated yield a periodic trajectory. In general, we can consider these kinds of maps as nonautonomous maps because one of the parameters is a function of the iterations. In most case, we pick a parameter value for the even iterations and a different parameter for the odd iterations. But the connection with the Parrondo's paradox is associated with the kind of alternating parameters, which in the conjecture are parameter associated with chaotic, or, in general, complex trajectories.

The case of "chaos + chaos = periodic" was presented for the first time by Almeida et al. [16] for simple 1-D maps, and just recently for 2-D maps by Mendoza et al. [12]. The implication of the so-called Parrondo's dynamics has been used to model seasonality, but with the observation that, under the Parrondo dynamics, the case of "periodic + periodic = chaos" is also possible [15]. As generalization we have consider cases of "undesirable + undesirable = desirable" dynamics behaviors to analyze simple models of seasonality [23–25], which include migration or immigration [13, 14].

In the present analysis, we emphasize the use of bifurcation diagrams to find intervals of values in parameter space that could satisfy the "undesirable + undesirable = desirable" or "periodic + periodic = chaos" dynamics. Although we are interested in modeling ecological systems and in particular the effect of seasonality, one could use our results to look at the switched maps as a way to control chaotic dynamics. In particular an extension to continuous dynamic systems may be relevant or applicable to chemical and mechanical systems [26].

In summary, our approach of building bifurcation diagrams readily yield intervals of parameter values that can show the so-called Parrondian dynamics for 1-D and 2-D maps. We have concentrated on ecological relevant maps, but the approach applies to any kind of maps. In particular, we can easily find parameters that show desirable dynamics in switched maps, controlling complex or undesirable dynamics, with the by product that we can also avoid the alternation of desirable dynamics that could yield undesirable dynamics in switched maps. Finally, we believed that we have stablished a pattern of dynamic behavior that supports the conjecture described in previous paragraphs.

8. Conclusions

In previous sections, we have stablished a pattern of dynamic behavior for 2-D maps, which have been used to model ecological systems. The dynamic pattern allows to state that for any 2-D maps that shows chaotic dynamics for a set of parameters, we can always find two of such parameters that, when alternate, yield a periodic trajectory. This conjecture is an extension of the so-called Parrondo's paradox, in the sense that two undesirable dynamics can be alternate to yield a desirable dynamics. In other words, we can always find a region in parameter space, where we can select a pair of such parameters. Therefore, we the developed methodology can be use, in general, as a chaos control approach, and, in particular, we can use it to model, in the case of ecological maps, seasonality. Although we interested in ecological relevant 2-D maps, we believed that our conjecture can be extended to other type of 1-D and 2-D maps. Finally, we consider that the major application of the methodology is in controlling chaotic dynamics.

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