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Chapter 1

Probabilistic Modelling in Solving Analytical Problems of System Engineering

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Abstract

This chapter provides some aspects to probabilistic modelling in solving analytical problems of system engineering. The historically developed system of the formation of scientific bases of engineering calculations of characteristics of strength, stability, durability, reliability, survivability and safety is considered. The features of deterministic and probabilistic problems of evaluation of the characteristics of strength, stiffness, steadiness, durability and survivability are considered. Probabilistic problems of reliability, security, safety and risk assessment of engineering systems are formulated. Theoretical bases and methods of probabilistic modelling of engineering systems are stated. The main directions of solving the problems of ensuring security and safety according to the accident risk criteria are determined. The possibilities of probabilistic modelling methods in solving the problems of strength, reliability and safety of engineering systems are shown in practical examples.

Keywords: engineering system, multi-level concept, probability, modelling, safety, survivability, security, safety, risk

1. Introduction

Sustainable development of social systems and the natural environment is determined by the state and prospects of the development of engineering and technology. Modern engineering and technology are created on the basis of the achievements of fundamental scientific research. Particular importance is the development of fundamental foundations of mechanics, which is the basis for the design and produce of engineering systems. New machines and structures are creating, based on achievements of construction mechanics, theories of elasticity, plasticity and strength of materials. Multivariance of design and engineering solutions to engineering problems and increase of uncertainties associated with the manifestation of complex combinations of dangerous natural, technical and social factors in the creation and operation of technical objects require the application of new approaches. These approaches will increasingly be based on a combination of traditional deterministic and developing statistical and prospective probabilistic methods of modelling, calculation and testing. Of particular importance is the development of methods of statistical mechanics, probabilistic fracture mechanics, reliability theory and safety theory of engineering systems [1, 2].

At the present time, a multi-level concept has been developed to ensure the safe operation of engineering systems (**Figure 1**). This concept includes specific stages, requirements, criteria, calculated parameters and directions of development. Each higher level is created and developed on the achievements of the lower levels. At the first stages, the methodology of modelling engineering systems and the calculation and experimental validation of operability were based on deterministic methods, with elements of statistical analysis (stages I–III). Understanding the role of random factors in the disruption of operability led to the use of probabilistic methods of modelling and analysis (stages IV, V). At the end of the twentieth century, operability analysis of complex engineering systems began to use parameters of safety S and risk R of disasters. These parameters take into account natural, technical and social hazards (stages VI, VII). On this basis, by the end of the twentieth century, a complex of interconnected multi-level deterministic and probabilistic requirements to engineering systems and their parameters was formed: "strength R_σ → stiffness R_δ → steadiness R_λ → durability $R_{N,}$ τ → reliability $P_{P,\ R}$ → survivability $L_{l,\ d}$ → safety S". Each stage in the development of fundamental research and requirements in this structure corresponds to a certain practical result in the design, creation and operation of engineering systems: "indestructibility - preservation of size and shape - durability - fault tolerance - survivability - risk of disasters". Risk is considered as a quantitative probabilistic measure of safety.

The basic equation for determining these characteristics of engineering systems can be written in the following form [1, 2]:

Years	Stages	Characterictics	Criterias	Parameters
2030-2016	VIII	*Security*	*Acceptable risk*	$Z(\tau)$
2000	VII	*Risk*	*Acceptable loses*	$R(\tau)$
1990	VI	*Safety*	*Acceptable hazards*	$S(\tau)$
1980	V	*Survivability*	*Stability of damages*	$L_{d,l}$
1970	IV	*Reliability*	*Fault tolerance*	$P_{P,R}$
1960	III	*Durability*	*Operation life time*	$R_{N,\tau}$
1940	II	*Stiffness Steadiness*	*Saving of form*	R_δ R_λ
1920	I	*Strength*	*Indestructibility*	R_σ

Figure 1. Structure of system for ensuring operability of engineering systems.

$$\{R, S, L_{l,d}, P_{P,R}, R_{N,\tau}, R_\delta, R_\lambda, R_\sigma\} = \Psi\left\{\varphi_Q(Q, N, t, \tau); \varphi_\sigma(\sigma_y, \sigma_b, E, \nu, m, \psi, K_{1c}); \varphi_A(\alpha_\sigma, l, A)\right\} \quad (1)$$

where Ψ is a generalized function of technical state; $\varphi_Q(.)$ is loading functional that takes into account load parameters Q, number of cycles N, temperature t, time τ of loading; $\varphi_\sigma(.)$ is functional of physical and mechanical properties of structural materials, taking into account the yield strength σ_y, ultimate strength σ_b, fatigue limit σ_r, modulus of elasticity E, Poisson's ratio ν, hardening ratio m, ultimate deformation ψ, critical stress intensity factor K_{1c}; $\varphi_A(.)$ is a functional of constructive forms, taking into account cross sections area A, lengths l of the defects, and stress concentrators α_σ.

Expression (1) can be considered for limiting states, under which the engineering system ceases to meet the requirements of operation, and for admissible states, determined by the system of safety factors n.

The modern stage of research of engineering systems takes into account the largest man-made accidents and disasters of nuclear, hydraulic and thermal power engineering objects, transport systems and in chemistry objects from twentieth to twenty-first centuries. Taking this into account, stages VII–VIII consider the protection of technical objects based on according risk criteria. The defining equation of this new direction of the engineering methodology of probabilistic modelling, calculation and experimental justification for security Z becomes the functional of the following form:

$$Z(\tau) = F_Z\{R, S, L_{l,d}, P_{P,R}, R_{N,\tau}, R_\delta, R_\lambda, R_\sigma\} \quad (2)$$

The probabilistic characteristics play a decisive role in the structure of the functional (1) and (2). Therefore, for their analysis, further development of probabilistic modelling methods of engineering systems is necessary.

2. Theoretical foundation of probabilistic modelling for engineering systems

2.1. Statement for probabilistic modelling problems of engineering systems

The peculiarity of the above multi-level concept (**Figure 1**) ensuring operability of engineering systems in the form (1) is that each of the stages I–VIII considers its own, specific, calculating situations (**Figure 2**). At each stage, special fundamental problems of the mechanics of solids are solved:

- boundary problems for stress determining in the most loaded elements, in cross sections A and local volumes $V(x, y, z)$

$$\left\{\sigma_{ij}, e_{ij}\right\} = F_\sigma\{Q, A, V(x, y, z)\}$$

- experimental problems of obtaining metal deformation diagrams (equations of state)

$$\{\sigma_{max}, e_{max}\} = F_{\sigma,e}\{\sigma_{ij}, e_{ij}\}$$

- experimental problems of estimating the criterial values of stresses and deformations corresponding to the achievement of the conditions for breaking strength (fracture)

$$\{\sigma_c, e_c\} = F_c\{\sigma_b, e_f\}.$$

The nominal stresses σ_n and deformations e_n, local stresses σ_l and strains e_l, fracture stresses σ_f and deformations e_f, as well as actual and critical dimensions of technological and operational defects l and l_f are used as the determining parameters. The values of fracture stresses and deformations characterize the limiting states and are determined taking into account the loading regime in terms of the number of cycles N and time τ, the fatigue diagrams $(\sigma_f - N_f)$ and $(e_f - N_f)$; long-term strength $(\sigma_f - \tau_f)$ and $(e_f - \tau_f)$; fracture toughness $(\sigma_f - l_f)$ and $(e_f - l_f)$.

The basic characteristics of strength R_σ, stiffness R_δ, steadiness R_λ, and durability $R_{N,\tau}$ are considered for design situations when the values of all parameters are in the deterministic limits established by the project:

$$\{R_\sigma, R_\delta, R_\lambda\} = F_R\{(\sigma_n, e_n); (\sigma_l, e_l); (\sigma_f, e_f)\} \tag{3}$$

$$R_{N,\tau} = F_{N,\tau}\{(N,\tau); (R_\sigma, R_\delta, R_\lambda)\} \tag{4}$$

If statistical properties are taken into account for combinations (σ_n, e_n); (σ_l, e_l); (σ_f, e_f), the characteristics of strength, stiffness and resource can be determined with use quantile of probability

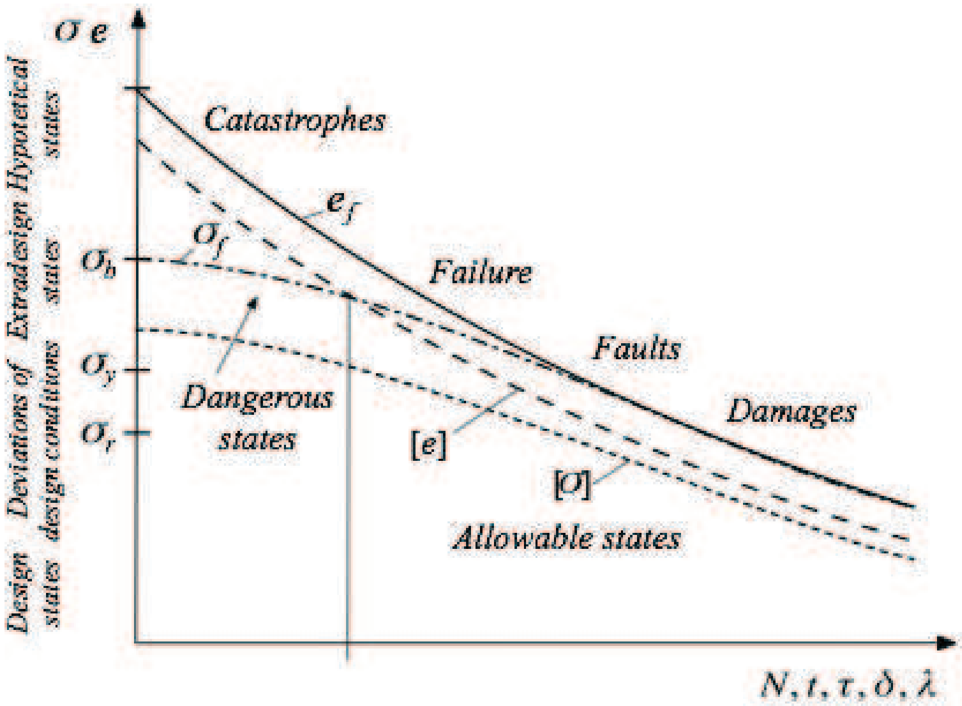

Figure 2. State diagram of engineering systems.

U, given by margin factors for nominal and local stresses n_σ and $n_{\sigma l}$, nominal and local strains n_e and n_{el}, destroying stresses $n_{\sigma f}$, and deformations of n_{ef}:

$$\{R_\sigma, R_\delta, R_\lambda\}_U = F_R\{(\sigma_n/n_\sigma, e_n/n_e); (\sigma_l/n_{\sigma l}, e_l/n_{el}); (\sigma_f/n_{\sigma f}, e_f/n_{ef})\} \quad (5)$$

$$\{R_{N,\tau}\}_U = F_{N,\tau}\{(N/n_N, \tau/n_\tau); (R_\sigma, R_\delta, R_\lambda)\} \quad (6)$$

As probabilistic modelling methods evolved, problems (3) and (4) were considered in a probabilistic formulation, when the basic parameters are given by probability distribution functions. In this case, by the methods theory of probability and reliability theory, one can obtain diagrams of limiting states in the coordinates $(\sigma_f, e_f) - (N, \tau, \delta, \lambda, l)$ for different probabilities P of their realization (1%, 50%, 99%) (**Figure 3**).

The reliability of engineering systems is determined in the presence of probability distribution functions of the basic parameters of operability. In a general case, reliability is estimated by the given probabilistic properties P of the characteristics of strength, stiffness, steadiness, durability:

$$P_{P,N,\tau} = F_P\{P|\ (Q, N, \tau); (\ R_\sigma, R_\delta, R_\lambda, R_{N,\tau})\} \quad (7)$$

The survivability of engineering systems is considered for design situations and beyond design situations with considering damage accumulation processes D. In engineering, practice damage is characterized by the sizes of the technological and operational defects L or scalar measure of accumulated damage d:

$$D = \{d, l\}; d = F_d\left\{\frac{N}{N_f}, \frac{\tau}{\tau_f}, \frac{l}{l_f}, \frac{\delta}{\delta_f}, \frac{\lambda}{\lambda_f}, \frac{\sigma}{\sigma_f}, \frac{e}{e_f}\right\}; l = F_l\{l_i(N, \tau), i = 1, m\} \quad (8)$$

Survivability can be estimated in deterministic and probabilistic formulation by using expression:

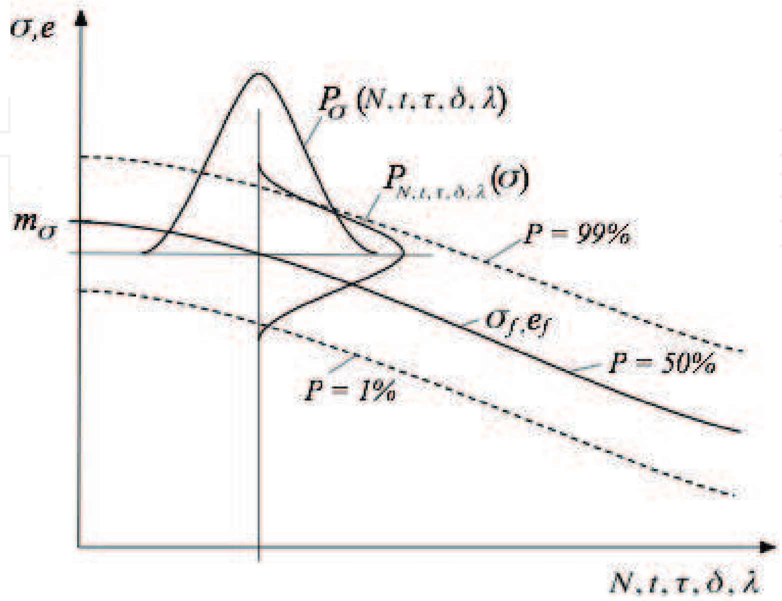

Figure 3. Probabilistic diagrams of limiting states.

$$L(\tau) = \{l_i(N, \tau), i = 1, m\} \tag{9}$$

In the probabilistic formulation, solution of problem (8) consists of obtaining probability distribution function of the survivability $F(L_{d,l})$ for given probabilistic properties of accumulated damage. Here it should be noted that at stresses above the yield point, the calculation of damages in the stress values $d = \sigma/\sigma_f$ is significantly more complicated. Therefore, the damage is calculated in terms of relative deformations, $d = e/e_f$.

From these positions, in the analysis of survivability, new calculation cases are considered such as deviations from design situations, beyond design situations, and hypothetical situations that characterize the transition from failures to accidents and disasters (**Figure 2**).

Currently, national and international programs to ensure the safety of engineering systems, engineering infrastructures, and natural environment (Rio-1992, Johannesburg-2002, Kobe-2005, Hyogo-2015) focus attention on security characteristics S. Quantitative assessments of safety characteristics are based on complex analysis of reliability and survivability of engineering systems [2, 3]:

$$S(\tau) = F_S\{P_{P,N,\tau}, L_{d,l}\} \tag{10}$$

The safety is the ability of the engineering systems to remain operative in damaged states and fracture states. In engineering practice, quantitative safety characteristics have become associated with the risks of accidents and disasters. Risk in quantitative form is defined as a function of the probabilities P_f of accidents and catastrophes and the associated losses U_f:

$$R(\tau) = F_R\{P_{P,N,\tau}, L_{d,l}; U_f\} = F_c\{P_f, U_f\}, P_f = F_f\{P_{P,N,\tau}, L_{d,l}\} \tag{11}$$

It is important to note that the probabilities P_f are estimated for beyond design and hypothetical situations, with the extreme values of Q^{extr} operation parameters and extreme strength and resource characteristics, not envisaged by the project:

$$P_f = F_P\{P|\ (Q^{extr}, N, \tau); (\ R_\sigma, R_\delta, R_\lambda, R_{N,\tau})^{extr}\} \tag{12}$$

The presented analysis shows that probabilistic models and probabilistic methods acquire an increasingly important role in ensuring the operability of engineering systems.

2.2. The development of traditional probabilistic methods

The development of theoretical foundations' probabilistic approaches to the analysis of the operability of engineering systems covers a significant historical period (stages I–VI, **Figure 1**). The first studies in this direction were carried out by M. Mayer (1926), N.F. Hotsialov (1929), and W. Weibull (1939). In these studies, the significant variation of strength characteristics for structural materials was shown, and the idea of introducing safety factors was proposed. Essential development of these studies was the work of N.C. Streletsky (1935). In his studies, the strength characteristics of materials (σ_f, e_f) and load parameters (σ_n, e_n) were considered as random variables. The further development of this approach was made by A.R. Rzhanicyn (1947). In his works, the relationship between safety factor n_σ and reliability P was established.

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Theoretical basis for calculating the reliability of structures in form (7) was formulated for case of two random variables: the load q and the strength r (**Figure 4a**).

In the 1960s and 1970s, Polovko et al. developed methods for probabilistic calculation of fatigue life $R_N\tau$ of and reliability of machine parts $P_{P,N,\tau}$ according to expressions (5) and (6). These methods have been used to calculate the probability diagrams of fatigue characteristics of aircraft, transport, and other equipment.

According to F. Freudenthal (1956) and M. Shinozuki (1983), the reliability problem was formulated for a finite number of random variables $X = \{x_i, i = 1, n\}$ (geometrical parameters, material properties, loads, environmental factors, etc.). In this case, it was assumed that probabilistic properties of random variables are determined by joint probability distribution function $f(X)$. Reliability is determined by computing a multi-fold integral on given security region Ω_S:

$$P(X) = P\{X|G(X) > 0\} = \iiint_{\Omega_S} f(X)dX, \Omega_S = \{X|G(X) > 0\} \quad (13)$$

Difficulty in computing this probability has led to the development of various approximation methods: the first-order reliability method (FORM) (Hasofer, Lind, 1974) and the second reliability method (SORM) (Tvedt L., 1988). These methods are widely used in engineering practice [4, 5]. The main drawback of these methods is that they relate to cases when changing random parameters in time does not exceed the limits of their statistical variability characterized by function f(X).

The further development of probabilistic methods was obtained in the 1970–1980s of the twentieth century on the basis of three conceptual ideas [6]. The first idea was that the external conditions of operation structure and its reaction to these conditions are random processes. Therefore, probabilistic methods for calculating structures should be based on methods of the theory of random functions. The second idea was that the failure of the structures in most cases is a consequence of the accumulation of damage (d, l). These damages, reaching a certain value, begin to interfere with the normal operation of structures. The third idea was that the main indicator of reliability should be probability staying of system parameters in a certain permissible region. Violation of normal operation (accident)

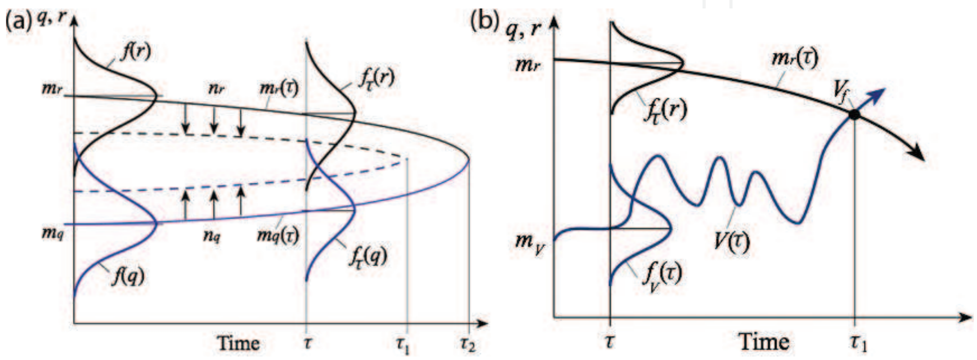

Figure 4. Failure models for calculating reliability.

of the system was interpreted as an output of parameters from this area (**Figure 4b**). Taking these ideas into account, reliability was determined in the form:

$$P\{X(\tau)\} = P\{V(X,\tau) \in \Omega_S, \tau \in [0, \mathrm{T}]\} \tag{14}$$

Thus, the interpretation of reliability as the probability of the fulfilment of some inequality connecting random variables gave way to a more adequate and in-depth interpretation in form emissions of random functions from an admissible region [6–8].

In subsequent years, based on the achievements of reliability theory, extensive studies were carried out to substantiate and improve the normative design calculations using probabilistic methods and reliability theory. Research is carried out in nuclear engineering construction [9], aerospace technology [10], and other industries. An important role in this was played by the achievements of fracture mechanics, taking into account the presence of technological and operational defects and structural damage [11, 12]. The development of probabilistic models of fracture mechanics made it possible to create a concept and methods for probabilistic risk analysis of engineering systems [13, 14].

At the present time, probabilistic modelling and probabilistic methods of calculation have become an integral part of a wide class of problems of statics and dynamics of engineering systems, in which randomness plays an essential role and is introduced by the variations of their geometric and physical properties. To this class belong the problems of strength of micro-inhomogeneous materials, composite materials, and structures, including nanomaterials and microstructures. Significant progress in this direction is associated with the development of numerical methods of analysis and computational technologies [15, 16].

2.3. New directions for solving engineering problems of security and safety by risk criteria

Engineering systems, with rare exceptions, are complex structures of elements of different nature. The problems of probability modelling of such structures turn out to be multivariate and lead to ambiguous solutions. In conditions of complexity and statistical diversity of states of the engineering systems, the diversity of elements, the multiplicity of the mechanisms of catastrophes, it seems unlikely that an integrated comprehensive risk model will be constructed in the near future. A more promising direction can be development individual models of risk, based on the representation of the engineering systems in the form of a structure Σ, consisting of subsystems σ and elements e [13].

$$\Sigma = \underset{i}{\cup}\sigma_i\left(\underset{j}{\cup}e_{ij}\right), \quad i = 1, n, \quad j = 1, m. \tag{15}$$

These models must realize the decomposition of R-characteristics (risk-decomposition) of structure (13) in the next form [14].

$$R_\Sigma \to \{R_i\} \to \{R_{ij}\} \to \{R_{ijk}\} \tag{16}$$

where R_Σ is integral (system) risk, R_i is complex (subsystem) risk, R_{ij} is elemental risk, and R_{ijk} is criterial risk.

The final level in this expansion is criterial risk, which allows the connection of systemic risk with mechanisms of catastrophes.

When constructing a system of models, implementing decomposition (16), it is necessary to take into account the following problem features of the engineering systems as the objects of risk analysis. First, in most cases, we have to analyse situations that have not been seen before, since the coincidence of all circumstances of disasters is an almost impossible event. Second, the analysis is carried out under conditions of high uncertainty associated with both the random nature of external influences and processes in the elements of systems, and with the ambiguity of objectives and safety criteria, as well as alternatives to decisions and their consequences. Third, the analysis is performed with time limit. At the stage of analysis of design decisions, these restrictions are determined by the design time, at the stage of operation—by the time of response to an emergency or emergency situation.

These features make specific requirements for model representations, the computer, and the information base for risk analysis. The development of model representations and a computational technology is connected with the solution of a number of specific problems. The first task is to describe the engineering systems from the standpoint of integrity and hierarchy. The creation of a substantial and compact model with a large number of significant parameters belongs to the number of difficult tasks, even with the use of modern mathematical and computational technologies.

The second task is to formulate information support for risk analysis. This task has two aspects. The first aspect is related to the task of processing information. Information in the hierarchical system comes in the language of the level that is being analysed. For conclusions at a higher hierarchical level, generalization is required, and at a lower level, detailing this information is needed. In both cases, this translation is ambiguous. The second aspect is related to the need to construct hypotheses about the states of elements on base the available information. The reliability of such hypotheses depends on the level of completeness of information and its reliability.

The third task is connected with the choice of the risk criterion. It can be solved on the basis of an analysis or development of special indicators that have the necessary properties of indicators of limit states of engineering systems. This choice can also be ambiguous or multicriteria.

Finally, the fourth task is to create theory and methods for risk analysis at given parameters. This apparatus can be considered as a set of mathematical models that reflect the mechanisms of catastrophes in a given sequence of the process of risk analysis. Here it is necessary to take into account the accidental nature of the catastrophe event of the system and the possibility of a formalized description and measurement of the random parameters of the systems.

A separate and difficult task is modelling the processes of accumulation of damage. In the general case, it is necessary to consider multicriterial damage (MCD) for each element and multi-structural damage for system (MSD).

To take into account multifocal character of damages and their structural hierarchy, we use the principle of selective scale and select the hierarchy of scales $M = \{\cup M_i,\ i = 1, n\}$ on which damages develop. Each scale M_i is considered as internal for a given level and is analysed by appropriate methods. For example, for the scale level of structural elements can be used by the methods of fracture mechanics, and for the level of construction, by the methods of structural mechanics. It should be noted that if fracture of individual elements, caused by MCD, can be considered as independent events, then at structure level there is an agreed redistribution of loads, and formation of the focus of MSD should be considered as a cooperated process.

2.4. Statistical information for risk analysis and safety

The safety and risk analysis is carried out using statistical information on dangerous events and damages. The systematization of data on major natural disasters and man-made disasters is carried out at the international and national levels [2, 3, 17]. Statistical studies show that modern global, national, sectoral and object security problems are the result of centuries-old quantitative and qualitative transformations both in social development and in the system "nature-machine-human." The uneven growth of damage from major disasters creates a real threat to the economy not only of individual regions but also for the planet as a whole. The scale and consequences of natural disasters and man-made disasters today are very tangible not only for developing countries but also for technologically advanced countries. The total losses currently for developed countries are 5–10% of GDP or more. In value terms, the total losses exceed 350 billion dollars (**Figure 5**).

Extreme losses are also attributed to individual catastrophic events. The losses from the hurricane Katrina (USA, 2005) amounted to 140 billion dollars. The accident at the Sayano-Shushenskaya hydroelectric power station resulted in the death of 75 people and damaged over 7.5 billion Roubles. Tsunami and the accident at the nuclear power plant "Fukushima" (Japan, 2011) led to the death of 20,000 people and damage of over $ 300 billion.

Based on the analysis of statistical data and estimates, modern man-caused hazards are characterized by the following values [2]:

- in the frequency of occurrence of failures, accidents, and disasters (1/year): objects of technical regulation 10^1–10^0; hazardous industrial facilities 10^0–10^{-1}; critical objects 10^{-1}–10^{-2}; strategically important objects 10^{-2}–10^{-3};
- in economic losses (dollars): objects of technical regulation 10^3–10^5; hazardous industrial facilities 10^4–10^7; critical objects 10^5–10^9; strategically important facilities 106–1011;
- in the risks of failures, accidents, and disasters: objects of technical regulation 10^3–10^5; hazardous industrial facilities 10^4–10^6; critical objects 10^4–10^7; strategically important objects.

The given statistical data can serve as a basis for categorizing man-made hazards according to the levels of risks of accidents and catastrophes. Risk diagrams can be represented by a power law of the following type:

$$R(\tau) = C_u\{U(\tau)\}^m \tag{17}$$

where C_U is a coefficient that depends on the dimension of the coordinates; m is an indicator that depends on the type of object.

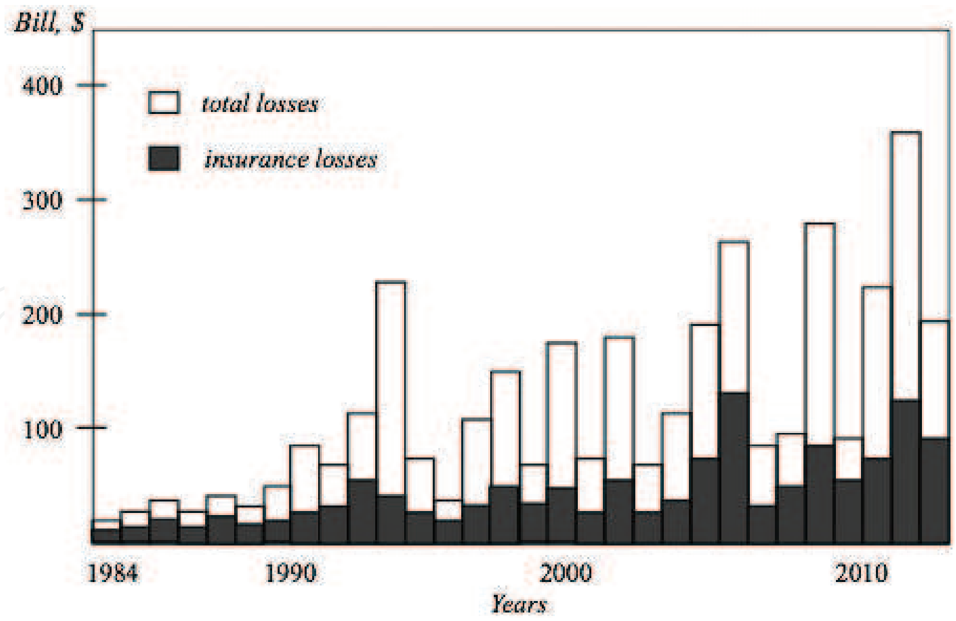

Figure 5. Losses from catastrophes of recent decades [3].

For natural disasters, natural-technogenic, and technogenic accidents and disasters, the value of m is in the range 0.3–1.0. For technogenic accidents and disasters, m = 0.55–0.60. The principal feature of distribution of losses according to the probabilities is that for critical and strategically important objects, large losses occur, leading to “heavy tails” of distributions.

The development and implementation of large infrastructure projects based on the achievements of science and technology not only dramatically increased opportunities in all areas of the world community but also created high risks of man-caused and natural-technogenic catastrophes at a global level. Modern engineering systems have destructive energy potential comparable to those of natural disasters. At the same time, the possibilities of parrying and localizing technogenic catastrophes are limited, despite the achievements of scientific and technological progress.

3. Solving engineering problems using probabilistic modelling

3.1. Probabilistic modelling of safe crack growth and estimation of the durability of structures

Crack growth up to a critical size under cyclic and long-term static loading is a rather complex process, which can be described by various crack growth equations. Methods for estimation of the lifetime of structures containing defects can be developed on the basis these equations. However, there are insufficient studies of the probabilistic aspects of crack growth, which greatly limit the opportunity for practical applications of these methods. To overcome this restriction, probabilistic models of the crack growth have been developed. This part presents the results, in a generalized manner, of these studies involving the probabilistic modelling of safe crack growth and the estimation of the durability of a structure [18, 19].

Probabilistic factors of crack growth are present both at the micro- and macro-levels of deforming materials. At a micro-level, these factors are the structural heterogeneity of materials and the heterogeneity of the stress-deformed conditions of local zones at the level of grain size. The important factors at the macro-level include the heterogeneity of intensely deformed zones of structural elements, the uncertainty of form, size, and orientation of cracks, and the dispersion in the evaluation of the cyclic crack growth resistance of materials. It is an extremely complex problem to develop probabilistic models of crack growth that reflect all levels of the process. Therefore, our main attention is directed to probabilistic models that handle macro-level factors.

Three models can represent crack growth: a discrete model with casual moments of time; a continuous model with casual increments at fixed time intervals; and discrete continuous model with casual increments of both types. In all cases, the conditions of irreversibility $\delta l_\tau \geq 0$ and kinetic conditions apply

$$\frac{dl}{d\tau} = \varphi(\Delta\sigma, l_\tau) \tag{18}$$

The problem of probabilistic modelling of the crack growth consists of the assignation of probabilistic features of trajectories $l(\tau)$, which adequately describe real processes. The problem of the probabilistic estimation of functions $f(l|\tau)$ and $f(\tau|l)$ on given probabilistic features of the trajectories. Modelling trajectories can be carried out on the basis of models of the theory of casual processes, empirical models, and probabilistic models of fracture mechanics.

The theory of casual processes offers a wide spectrum of models. Among the analytical models, it is possible to consider diffusive models as being the most respective. The use of diffusive models allows one to write the kinetic function in the manner of:

$$l(\tau) = a(l,\tau)d\tau + b(l,\tau)dw(\tau) \tag{19}$$

Modelling of processes with jumps requires that the kinetic function given by Eq. (19) must contain an additional component, that is:

$$l(\tau) = l(0) + \int_\tau a\{l(\tau),\tau\}d\tau + \int_\tau b\{l(\tau),\tau\}dw(\tau) + \int_\tau \theta(l,\tau)d\tau \tag{20}$$

The models represented by Eqs. (18)–(20) allow one to directly obtain the densities of the distribution of defects $f(l|\tau)$ or durability $f(\tau|l)$. In particular, Eq. (19) creates a diffusive durability distribution of kind:

$$f(\tau|l_f) = \Phi\left\{\frac{a\tau - l_f}{b\sqrt{a\tau/l_f}}\right\} + \exp\left\{\frac{2a^2}{b^2}\right\}\Phi\left\{-\frac{a\tau + l_f}{b\sqrt{a\tau/l_c}}\right\} \tag{21}$$

It is necessary to point out that such diffusive models present difficulties with respect to a physical interpretation of the parameters. So, if the physical sense of functions a, b, θ are

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understood, then the sense of component w remains unclear. Nevertheless, as will be shown later, the practical use of this approach gives rather efficient results.

Using a Monte Carlo method can be considered as another effective approach. The advantage of this method is the possibility to use determined forms of the equations of the crack growth with casual parameters. Let us consider an example of the kinetic equation. As is known, it has the form:

$$\frac{dl}{dN} = C(\Delta K)^m = C\left\{\Delta\sigma\sqrt{\pi l}\varphi(l)\right\}^m \tag{22}$$

If one accepts that parameters C, m, $\Delta\sigma$, l are casual variables with given probability distribution functions, use of the Monte Carlo method allows one to obtain casual realizations of the process trajectory $l(N)$ for a given number N of loading cycles:

$$l(N) = \left[\int_{l_0}^{l_f} l^{-m/2}(\varphi(l))^{m-1}\right]^{-1}\left\{C\sum_{j=1}^{N}\sigma_j^m\right\} \tag{23}$$

Distribution densities $f(l\,|\,\tau)$ or $f(\tau\,|\,l)$ in the case are a result of frequent realizations of the model expressed by Eq. (23). Obviously, the determination of specified probability distributions is a complex and labour-consuming task, and representative statistics on the parameters of cyclic crack resistance are not available at present.

The most productive approach is a creation of special probabilistic models, the parameters of which can be determined by means of fracture mechanics. Assuming the fundamental basis of the mechanics of the crack growth, a probabilistic kinetic model can be formulated in the manner of:

$$l(\tau) = \varepsilon_N P_N + \varepsilon_\tau p_\tau \tag{24}$$

Increments can be calculated by means of fracture mechanics. As a first approximation, it is possible to suppose that these values are the parts of the plastic zone at the crack front, where deformations reach the fracture state e_f. Using an energy notation of crack growth, the probabilities can be written as follows:

$$p_N = 1 - \exp\left\{-\left(\frac{W_N}{W_{fN}}\right)^\alpha\right\}, p_\varepsilon = 1 - \exp\left\{-\left(W_\varepsilon/W_{f\varepsilon}\right)^\beta\right\} \tag{25}$$

Obviously, the values of W_N and W_ε are widely variable, while W_{fN} and $W_{f\varepsilon}$ only slowly change. It is possible to estimate values of W_N and W_ε by means of computational fracture mechanics.

The presented models contain initial defect sizes $l(0)$. Therefore, probabilistic models of the distributions of the defect sizes must be included into the main models. Statistical studies of defects in welded joints show that it is possible to use a two-parameter Weibull distribution as a basic probabilistic model for distribution of defect sizes:

$$f(l) = \frac{\gamma}{\theta}\left(\frac{l}{\theta}\right)^{\gamma-1}\exp\left\{-\left(\frac{l}{\theta}\right)^{\gamma}\right\} \tag{26}$$

An estimation of the parameters of Eq. (26) for structures of different types shows that parameter of form γ changes from 0.4 to 4.0. The second parameters θ changes over rather wide limits (from 1.5 mm to 25 mm and more).

These probabilistic models were used to study crack kinetics in the welded joints for high-pressure vessels and estimation of durability and reliability functions. The results of calculation reflect the character of the model trajectories over a range of dispersion in the function $l(\tau)$, which corresponds to those observed in laboratory experiments.

The empirical Paris-Erdogan model, in combination with the statistical simulation method, was used for modelling kinetic of crack growth in the weld joint of a pipeline in the nuclear reactor VVER-1000. Processing of the results of the probabilistic modelling of the crack kinetics allowed one to estimate reliability functions for a weld joint fracture as is shown in **Figure 6**.

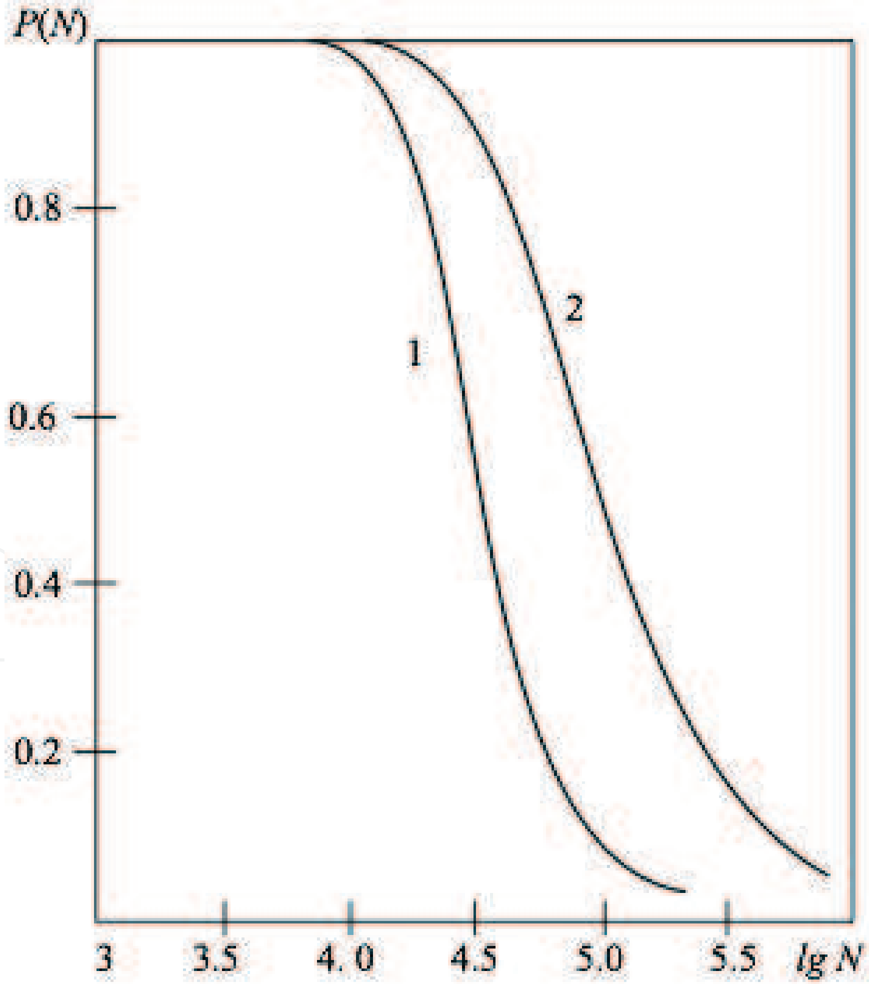

Figure 6. Reliability functions for hermetic breach criterion (1) and fracture criterion (2).

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Producing complete probabilistic diagrams of integrity is a major prospect. These diagrams are based on information gained from the same model. The structure of complete probabilistic diagrams of integrity is shown in **Figure 7**. A complete probabilistic diagram of integrity presents itself as a number of sections of a surface connecting three parameters: probability *P*, safety factor *n*, and durability *N*. They allow one to estimate the durability and probability of its attainment. Additionally, there is a possibility for a decision of an inverse task—the definition of probability that for a chosen safety factor, a certain durability will be achieved. Thus, a complete probabilistic diagram of integrity presents itself as a number of sections of a surface connecting three parameters: probability *P*, safety factor, and durability.

3.2. Reliability and risk assessment of metal-liner composite overwrapped pressure vessels

Metal-composite pressure vessels (MCOPVs) have found a wide application in aerospace and aeronautical industries. Such vessels should combine the impermeability and high weight efficiency with enhanced long-term safety and durability. To meet these requirements, theoretical

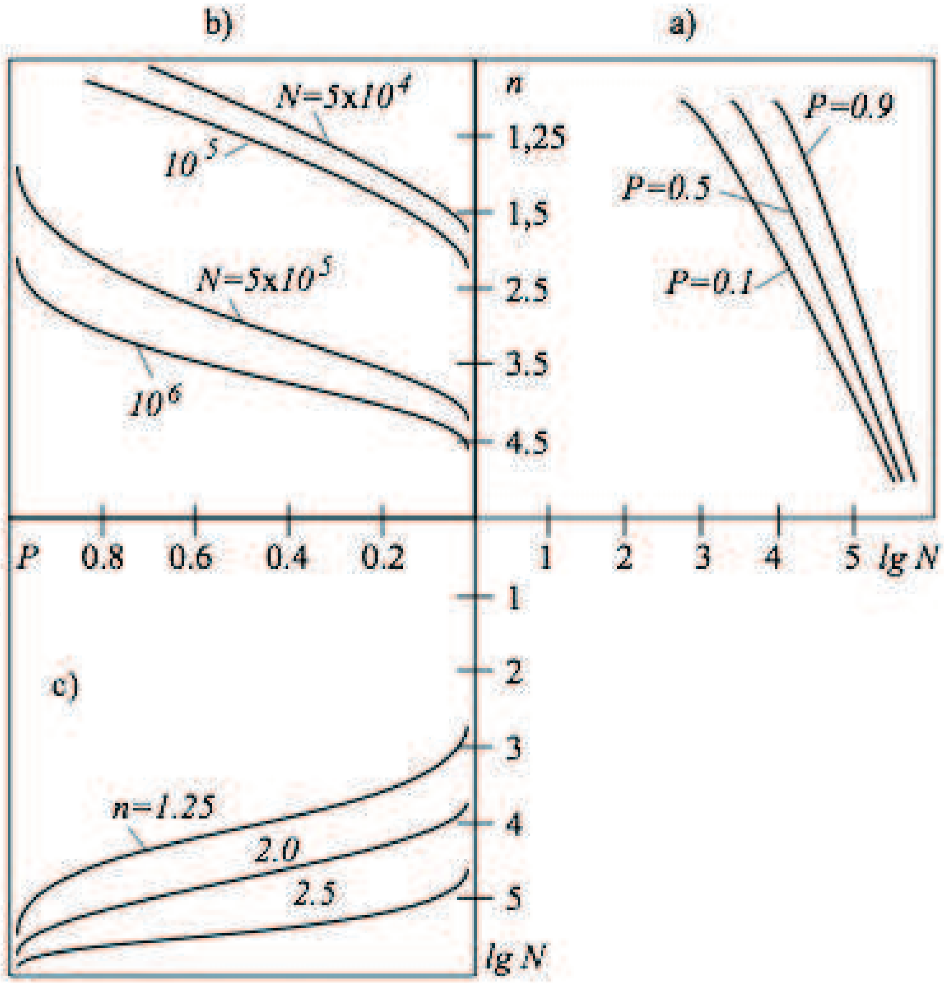

Figure 7. A complete probabilistic integrity diagrams: (a) probability of fatigue, (b) durability distributions, and (c) functions of equal safety factors.

and experimental studies on the mechanics of deformation and failure of MCOPVs are required [20].

Investigate reliability and risk fractures were based on results of numerical stress analysis and experimental tests' full-scale samples of MCOPV. The construction of MCOPV was having an axisymmetric ellipsoid-like shell of revolution with the minor to major diameter ration of about 0.6 (**Figure 8a**). The thin-welded liner was made of VT1–0 titanium alloy. The composite shell was formed by helical winding of IMS-60 carbon fibres impregnated with a polymer matrix. The stress analysis of MCOPV under internal pressure was performed using the finite element method. The calculations were carried out with finite element models developed to reflect all significant geometric and deformation characteristics of the composites vessel (**Figure 8b**).

Actual MCOPVs structures will exhibit a non-uniform distribution of stresses and deformation owing to a number of factors. These include the nuances of liner geometry and its interaction with the overwrap winding pattern, the relative stiffness of the liner to the overwrap, the liner-overwrap interface slips characteristics, and the presence of incompatible curvature changes. Load equilibrium in the bimaterial vessels requires that the total applied pressure be equal to the sum of the pressure carried by the individual components.

Taking this into account, the calculation of reliability function $R(P, \tau)$ included the evaluation of two components: the reliability $R(DM)$ at the beginning of service and the reliability $R(\tau, \sigma)$ during operation—$R(P, \tau) = R(DM) \times R(t, \sigma)$. The component $R(DM)$ was estimated by means of a conventional "load-strength" model, assuming the Gaussian law for load and strength values of MCOPV [20]:

$$R(DM) = Prob\{DM > 1\} = \Phi\left\{\frac{\mu_f - \mu_p}{\sqrt{s_f^2 + s_p^2}}\right\} = \Phi\left\{\frac{DM - 1}{\sqrt{V_f^2 DM^2 + V_p^2}}\right\} \tag{27}$$

where Φ is the standard normal distribution function; μ_f, μ_p are median values for P and P_f; V_f, V_p are coefficients of variations for P and P_f, $DM = P_f/P$ is design safety margins.

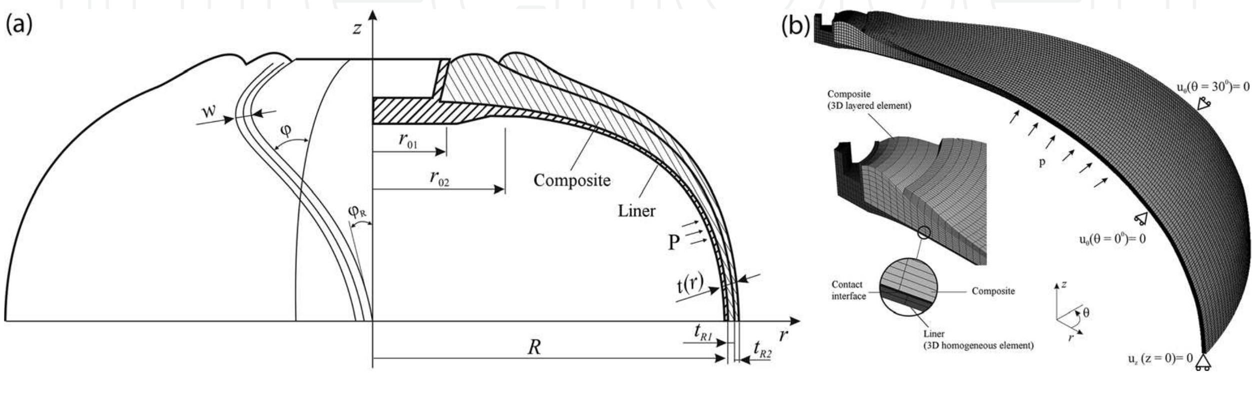

Figure 8. A metal-lined composite pressure vessel: calculation scheme (a), finite element model (b).

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The Phoenix approach based on the Weibull reliability model was used to determine the term $R(\tau, \sigma)$. To account the influence of structural-mechanical heterogeneity of MCOPV, a reference measure M_0 was introduced. It is assumed that within M_0, the deformation of material is uniform. The probability of failure-free operation $R(\tau, \sigma)$ can be expressed as follows:

$$R(\tau,\sigma) = \exp\left\{-\frac{M}{M_0}\left[\frac{\tau}{\tau_c}\left(\frac{\sigma_P}{\sigma_f}\right)^{\alpha}\right]^{\beta}\right\} \tag{28}$$

where M is the total "scale" of MCOPV; τ is the time; τ_c is the characteristic (reference) time, which can be considered as time of failure during static test; σ_p is stress corresponding to operating pressure P; σ_f is stress at burst pressure; and α, β are statistic parameters.

Risk assessment was performed on the basis of an analysis of possible mechanisms of MCOPV destruction. The event of fracture of the MCOPV decays into two events: the destruction event of the liner and the event of destruction of the power composite shell. In the first case, the main parameter controlling the state of the liner is deformation, and in the second case, the stresses in the composite shell are the determining parameter:

$$\{\text{Risk}\} \rightarrow \begin{cases} \{\text{Leakage}\} \rightarrow P_f(\tau, \varepsilon) = P\{\tau|\varepsilon_p \geq \varepsilon_f\} \\ \{\text{Fracture}\} \rightarrow P_f(\tau, \sigma) = P\{\tau|\sigma_p \geq \sigma_f\} \end{cases} \tag{29}$$

Reliability functions $R(t,\sigma)$ for MCOPV in the orbit are shown in **Figure 9**. As can be seen from the figure, while ensuring the homogeneity of the properties of the composite sheath and

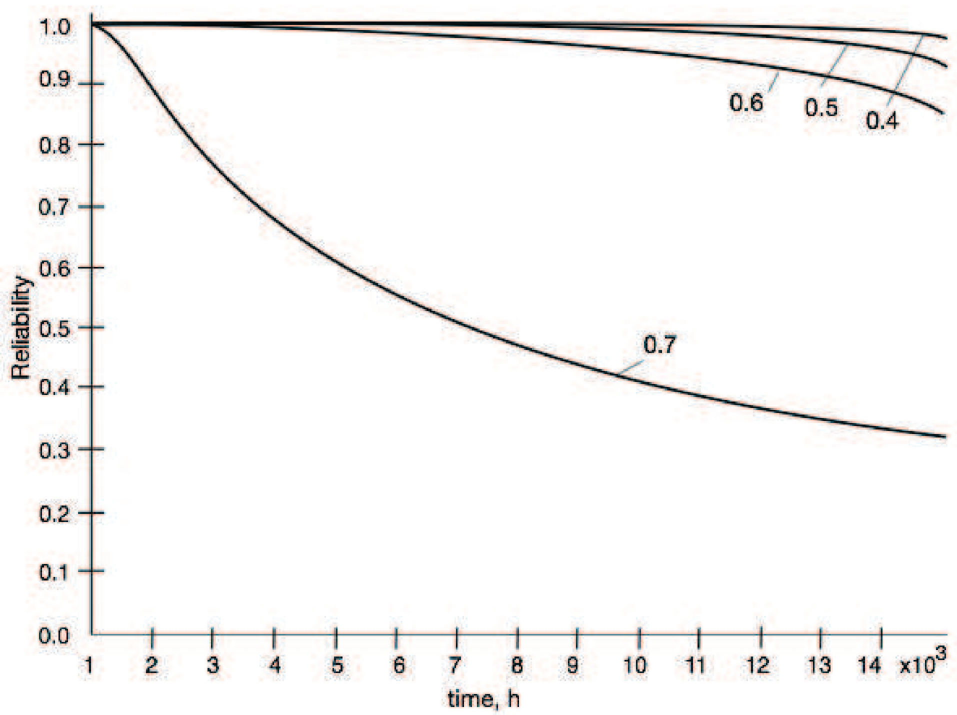

Figure 9. Functions of reliability depending on the time and the stress level σ_p/σ_c.

Type of destruction	Risk fracture of MCOPV	
	7-layer shell	**9-layer shell**
Leakage breach of liner in the test	1.3×10^{-6}	1.1×10^{-6}
Leakage breach of liner in the space (orbit)		
at the beginning work	1.1×10^{-8}	2.5×10^{-9}
at the end work	2.8×10^{-7}	6.3×10^{-8}
Fracture of MCOPV in the space		
at the beginning work	2.4×10^{-8}	4.7×10^{-9}
at the end work	1.8×10^{-6}	3.7×10^{-7}

Table 1. Calculated estimates of risk fracture of MCOPV.

operating stresses not exceeding 0.5 of the strength level of the composite material, the reliability of MCOPV is ensured at a level of at least 0.999 at the end of operation time in orbit. When the relative stresses level is increased to 0.6, high reliability is ensured only in the first 500 h of operation. The load level of more than 0.6 is unacceptable for the MCOPVs.

Quantitative risk assessments were performed for the most dangerous scenarios. The risk calculation was performed for the time moments 1000 h (the beginning work on the orbit) and 15,000 h (at the end work on the orbit). The calculation of the risk fracture of MCOPV was carried out according to the Phoenix approach, replacing the standard fracture stress of the composite σ_f by the numerical or experimental estimate of the actual value of the composite strength σ_c.

The results of the calculation are presented in the **Table 1**. The obtained risk assessments can be regarded as tentative, since they do not take into account possible processes of creep of liner and composite under load during the MCOPV operation. It should be noted that obtained values of the probabilities of destruction of the MCOPV belong to the class of extremely unlikely events. Therefore, the risk of destruction of the MCOPV in the orbit can be considered acceptable.

4. Conclusion

This chapter provided some aspects to probabilistic modelling in solving analytical problems of system engineering. The main tasks of engineering design are analysis of system operation from the moment of the conception and substantiation of the initial idea to the moment of writing off and wrapping the product so that it does not fail during the service period. When solving these problems, variability in the employed materials, loads, manufacturing process, testing techniques, and application inevitably arise. Probabilistic models and probabilistic methods proceed from the fact that various uncertainties are inevitable end essential features of the nature on an engineering system or design and provide ways of dealing with quantities whose values cannot be predicted with absolute certainty. Unlike deterministic methods,

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probabilistic approaches address more general and more complicated situations, in which behaviour of the engineering system cannot be determined with certainty in each particular experiment or a situation. Probabilistic models enable one to establish the scope and limits of the application of deterministic theories and provide a solid basis for substantiated and goal-oriented accumulation and the effective use of empirical data. Realizing the fact that probability of failure of engineering system is never zero, probabilistic methods enable one to quantitative assess the degree of uncertainty in various factors, which determine the safety of system and design on this basis a system with a low probability of failure.

Analysis of the largest man-caused and natural-technogenic catastrophes of recent years indicates the need to improve methods and means of ensuring the safety of the engineering systems. One of the main ways in this direction is to improve the historically established system of forming the scientific basis for engineering calculations of the characteristics of strength, stability, durability, reliability, survivability, and safety. Of decisive importance is the need to switch to a new methodological framework and principles for ensuring the safety of engineering systems by the criteria for risks of accidents and disasters. A special role in this direction is that security defines all the main groups of requirements for engineering systems: strength, rigidity, stability, reliability, survivability, and risk.

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