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A Creative Approach to Isometries Integrating Geogebra and Italc with 'Paper And Pencil' Environments

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Abstract

Creativity is recognized nowadays as a basic skill. However, the educational system fails in promoting their development. On the other hand, a growing acknowledgement of the importance of geometry emerges. Conceptual renewal, namely on isometries, requires new approaches based on mathematically significant tasks. The digital revolution has brought powerful tools but demands changes in the educational process. The use of Dynamic Geometry Environments (DGE), complementing 'paper and pencil', can contribute to provide rich learning environments, enhanced by Classroom Management Systems (CMS) such as iTALC. Indeed, the qualitative case study we carried out suggests that: the creation of an "atmosphere" of cooperation, collaboration and sharing seems to increase creativity dimensions; the use of DGE can facilitate the emergence of more creative productions; development of knowledge and geometrical capabilities seems to benefit from a complementary approach that combines DGE and 'paper and pencil' environments. Different approaches, with a more technological and exploratory nature seem to promote more favourable attitudes towards mathematics in general, and geometry, in particular.

Key words: Mathematics; Isometries; CMS; Creativity; GeoGebra

Introduction

Creativity is seen as a key to a profitable future. Thus, creative thinking is one of the basic skills, transversal to all areas of knowledge, required for this century (Cropley, 2003). It is therefore essential that Education promotes its development in their students (Adams & Hamm, 2010). However, this is not happening (Robinson & Aronica, 2009).

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Several studies (Hiebert 2003; Lu, 2008; Ponte & Serrazina, 2004; Ruthven, 2008) point out that Mathematics remains a subject taught in routinely way. Moreover, some content should be recast. In Portugal, different initiatives tried to reform the curricula. The Basic Education Mathematics Program[2] [PMEB] (Ponte et al., 2007) advocates changes in what and how we learn and teach this subject. In Geometry, which achieves greater importance in the curricula, geometric transformations deserve a central role that also calls for a different approach involving new understandings about isometries and symmetry.

Methodological guidelines of PMEB (id.) suggest the use of computer technologies. In fact, learning Geometry using DGE, such as GeoGebra, is quite different from learning only through traditional instruments in a "paper and pencil" environments. DGE free their users from mechanical and routine tasks such as procedures of measurement, calculation and construction, leaving room for a more active and fruitful work in Geometry.

The design of "technological learning environments" capable of keeping students engaged on tasks, characterized by being truly collaborative and cooperative, is greatly facilitated by the use of CMS. However, despite all institutional efforts to widespread use of computers, particularly in Mathematics, the use of these tools in our schools remains inadequate with limited impact in the classroom.

Creativity and the use of technological tools, including DGE managed by CMS, were the main areas leading to this study, oriented by these questions: *"In which way a* complementary approach that combines DGE and 'paper and pencil' environments *contributes to develop geometric skills and better understanding of geometric concepts? How technology can be used to foster creativity?*

Theoretical Framework

Creativity is a prerequisite for a high level of development in the new (current) global information society (Adams & Hamm, 2010) and studies on creativity reveal that all individuals are creative (Alencar, 2007). The same author states that a significant factor interfering with their creative potential is education. However, its development implies deep and extended educational changes (Cropley, 2003). In fact, education has numerous flaws and distortions where "place for exploration, for discovery, for creative thought is reduced and sometimes nonexistent" (Alencar, 2007, p. 9). Therefore, education should implement favourable practices to improve it in their students (Fleith & Alencar, 2005). This is true for any subject, including mathematics.

Zamir and Leikin (2011) argue that teaching creatively and for creativity can enhance the learning process. Ponte (2005, p. 1) suggests the creation of tasks able to involve students in "mathematically rich and productive activities". According to Berger (2012), mathematical tasks that require "complex and not algorithmic thinking", where students have to determine their own path through the problem, demand students to engage in their exploration using various mathematical concepts, relationships and processes. Computers can free students so they can focus on conceptual aspects connected to a

[2] Although this program has already been repealed in 2013, this research was based on it as it was the official document at the date of completion of the empirical work.

task. Vale, Pimentel, Cabrita and Barbosa (2012) state that confronting students with various resolutions, especially from their own, develops some dimensions of creativity.

The term “creativity” has a wide variety of definitions based on a common idea: the potential to generate original ideas, and therefore unique, as well as useful (Sternberg & Lubart, 1999). In this study, we adopt the definition presented by Torrance (1974), which includes the following concepts:

- Fluency - the ability to generate a great number of ideas, it refers to the continuity of those ideas, use of basic knowledge and flow of associations;
- Flexibility - the ability to produce different categories or perceptions, whereby there is a variety of different ideas about the same problem or thing. It becomes clear when students show the capacity of changing ideas among solutions;
- Originality - the ability to create unique, unusual, brand new or divergent ideas or products. Concerning Mathematics, originality may be manifested when a student analyzes a variety of solutions, methods or answers to a problem and then creates a different one (Silver, 1997; Leikin, 2009; Vale et al., 2012);
- Elaboration - related to the presentation of a large amount of details in one idea (Adams & Hamm, 2010).

In this perspective, creativity is likely to be assessed. Fluency can be measured by the number of correct responses, solutions, proposed by the student during the same task (Silver, 1997; Conway, 1999). Flexibility can be measured by the number of different categories of solutions that students can produce. Originality can be measured analyzing the number of responses in the categories that were identified as original, by comparison with the number of students in the same group that could produce the same solutions.

Thus, it is possible, desirable and urgent to develop a new educational environment able to foster creativity, namely in Mathematics. Furthermore, it can be measured and evaluated.

Mathematics occupies a central role in most advanced societies and in school curricula. Within Mathematics, Geometry has gained further importance (Matos, 2001; NCTM, 2000). Then, it emerges in the PMEB (Ponte et al., 2007), as the main purpose for basic education:

> "Developing students' spatial sense, with emphasis on visualization and understanding of properties of geometric shapes in two and three dimensions, on deeper understanding of geometric quantities and their measurement processes, and on use of such knowledge and skills to solve problems in different contexts" (p.36).

One of the most significant changes is related to the early introduction of isometric transformations, with a special focus on the concept of symmetry. The PMEB also suggests that Geometry approach should be based in tasks that provide opportunities to observe, analyze, relate and construct geometric figures and work with them. Open and complex tasks involving isometries - especially those related to reflections and rotations - require special attention 5^{th} - 6^{th} grades.

Nevertheless, difficulties may arise related with some particular transformation. Several studies have concluded that these difficulties decrease in tasks involving rotation with respect to reflection (Jacobson & Lehrer, 2000). Others found that students’ performance was superior in tasks involving translation regarding rotation (Clements, Battista, Sarama & Swaminhatan, 1996). However, some studies on children’s perceptions (Moyer, 1978; Shah, 1969) showed that they considered translation, particularly horizontal, simpler than reflection, and reflection easier than rotation. Kucheman (1981) found that students considered harder to make rotations when its centre was outside the figure and that oblique reflection constitutes a difficulty as well. Schultz and Austin (1983) stated that students seem to have difficulties when the reflection is underpinned by an oblique axis. Complexity of objects also appears to influence negatively the results. According to Clements (2003), children have early notions on symmetry, thus, an approach to this concept should start from their previous experiences. Schattschneider (2009) states that students begin by learning to recognize symmetry by observing various figures, exploring them with mirrors, folding them, turning them and overlapping them. However, there are some variables that interfere with the ability to perceive symmetry of figures (Hershkowitz, 1990): the orientation of the axis of symmetry, the respective position of different parts of the geometric shape and axis (prototypes phenomenon), and the students' age. Gerkins (1975, cited by Clements, 2003), considers that the vertical bilateral symmetry is more easily understood than the one with a horizontal axis and argues that conceptualisation of symmetry even does not occur in a solid way before the age of twelve.

Tasks must be solved using measuring and drawing instruments, DGE programs and applets, which promote the understanding of geometric concepts and relationships (Ponte et al., 2007). Regarding this software, NCTM (2000) states that:

> "Dynamic geometry software can allow experimentation with families of geometric objects, with an explicit focus on geometric transformations. Similarly, graphing utilities facilitate the exploration of characteristics of classes of functions. Because of technology, many topics in discrete mathematics take on new importa nce in the contemporary mathematics classroom; the boundaries of the mathemati cal landscape are being transformed." (p.27).

According to Breda, Serrazina, Menezes, Sousa, and Oliveira, (2011), "technology extents and enriches quality of research activities, when they provide means to visualize geometric notions on different perspectives" (p.21). Use of technology, particularly computers and DGE, is one of the principles for teaching Mathematics that promotes active and meaningful learning (NCTM, 2008, Veloso 2002). This means to deal with geometry in a completely different way, in a "learning environment that favours the development of other kinds of reasoning because there is an opportunity to work in Geometry in a dynamic way allowing new approaches to new problems (Ponte et al., 2007). Lu (2008) also adds that awareness that interaction between humans and technology can effectively facilitate teaching and learning has been increasing. This does not necessarily imply the marginalization of paper and pencil because the exams are usually held in this environment (Laborde, 2001). There is a need for establishing a compromise using both types of environment, taking advantages from both of them and minimizing their disadvantages. The PMEB (Ponte et al., 2007) itself , suggests, in its methodological hints, a complementary approach.

A computer use that promotes powerful learning environments where students can build their knowledge interacting with objects, with each other and with the world will be much richer and more valuable, passing from an instructional perspective to a constructivist one (Valente, 2001). Regarding DGE, NCTM (2000) states that they "can allow experimentation with families of geometric objects, with an explicit focus on geometric transformations. (…) the boundaries of the mathematical landscape are being transformed." (p. 27). The most commonly used DGE have been Cabri-Géomètre and Geometer's Sktetchpad. More recently GeoGebra has come out, providing an added value when compared with the other applications, as it combines graphical manipulations with their correspondent algebraic representations and calculus (Mehanovic, 2009; Misfeldt, 2009).

Despite obvious benefits, using computerized technology and respective applications can disrupt teaching and learning process (Berliner & Calfee, 1996; Brophy & Good, 1986; Galluch & Thatcher, 2011). Nowadays, many classes take place in Information and Communications Technology [ICT] rooms. Failure to use a system to manage all activities in the room, for example, block access to websites, restrict use of certain applications or closely follow students´ work can negatively affect their progress. A CMS application could be the answer. The CMS should not only be used to monitor students' work but as a catalytic tool: (i) increasing student engagement in tasks; (ii) increasing collaboration, cooperation and sharing; (iii) keeping students focused on the task (especially important for students in the early grades); and (iv) making supervision easier (Joyce & Schmidl, 2008). There are several commercial solutions on the market. We adopt an open source alternative, the iTALC.

The iTALC (intelligent Teaching and Learning with Computers) software allows teachers to monitor and remotely control any workstation in class; show a demo - the teacher's screen is shown on all students' computers in real time; lock workstations; send text messages; and home schooling. Efficient management of a class involves clear communication, from a behavioural and academic point of view, and the establishment of a collaborative and sharing environment. The focus of this study is precisely on these two aspects and their influence in both creativity dimensions and the ability to understand complex abstract concepts.

Methodology

Within a constructivist paradigm, we selected a qualitative case study (Bogdan & Biklen, 1994), focused on two groups of two students (the privileged way of working in classroom) and one group of one; all students were in the 5th grade. They were selected because they had different school performances and expectations regarding Mathematics and they attended every moment of the instructional sequence. At the end of the study, they were eleven years old and had no retentions. The group G1 consisted of one student, Catarina; G2, Tiago and Luísa and G3, Gabriela and Francisca[3].

For this study an instructional exploratory task sequence was designed (Ponte, 2005; Stein & Smith, 2009), on "reflection, rotation and translation" topic, in the "Geometry" theme. The teacher/researcher took an active part in this study, as he planned and led all the events resulting from this research.

To collect data we used: i) participant observation carried out by the teacher/researcher, supported by field notes and Logbook; ii) survey, through questionnaires and interviews with the case students and iii) a documentary analysis of a variety of documents - students' task resolutions, Initial and Final tests and some official documents produced by the school.

[3] These are fictional names to preserve students' true identity.

First, we applied an Initial Questionnaire (IQ) to obtain information mainly about student habits and some basic knowledge of computer use, including DGE. Then, we implemented a small test on technological skills, the results of which served afterwards to adjust the sequence structure. An Initial Test (IT), solved in paper and pencil and with GeoGebra, checked previous knowledge held by students on the topic. Result analysis advised possible changes in planning tasks. Subsequently, it allowed to assess the evolution of student performance when compared with the same Final Test (FT) at the end of the module.

We implemented the instructional sequence in eight sessions consisting in seven sequential tasks (Coelho, 2013), with increasing complexity, both mathematical and technical, previously validated (Cabrita et al., 2011). This approach, since the beginning of the implementation of the module, relate to a socio-constructivist paradigm.

The first task was set to remind some concepts related with isometric transformations taught in earlier grades and to provide students with an informal exploration on reflections, rotations and translations. Traditional materials and instrumentation in paper and pencil and DGE environments were used.

The second task focused on reflection. It was designed to evolve from a more informal approach, with traditional materials and instrumentation in paper and pencil environment, to another one, still informal, with GeoGebra, ending again with paper and pencil in a more formal way.

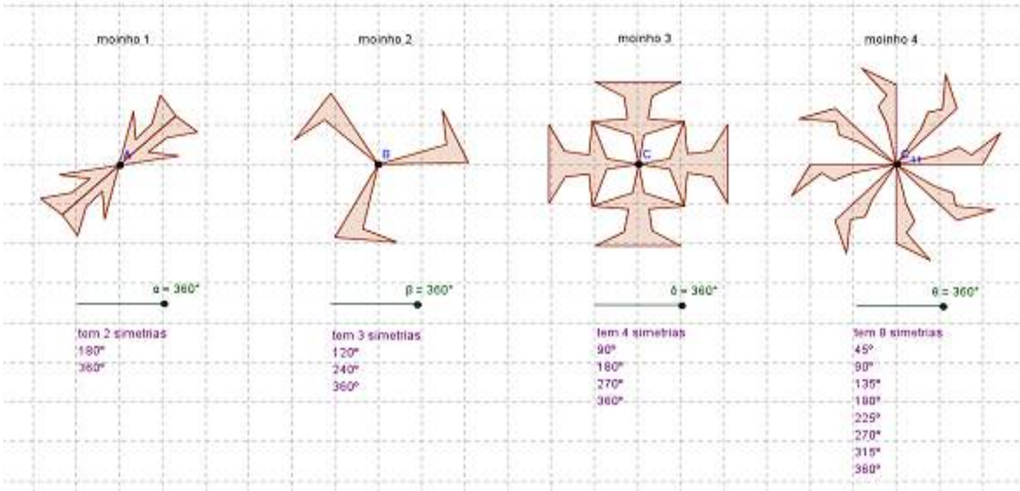
The third task pursued a different approach. Centred in rotation, it aimed at students' learning of the concept through GeoGebra and evolved to a "paper and pencil" environment with higher formal requirements.

The fourth task focused on translation, required the use of traditional instruments in a "paper and pencil" environment in a more formal way. A final open-ended task, using GeoGebra, invited them to create without restrictions and express their creativity.

Glide reflection, approached in the fifth task, was an unknown concept to students. It consisted in an open-ended task, allowing multiple solutions and introducing a slightly more formal notion on composition of isometries.

The last two tasks (adapted from Cabrita et al. 2011) were related to the concept of symmetry. In these tasks, students evolve from a "pencil and paper" environment, exploring several images, to a computer assisted one, to support their earlier conclusions (see Figure 1).

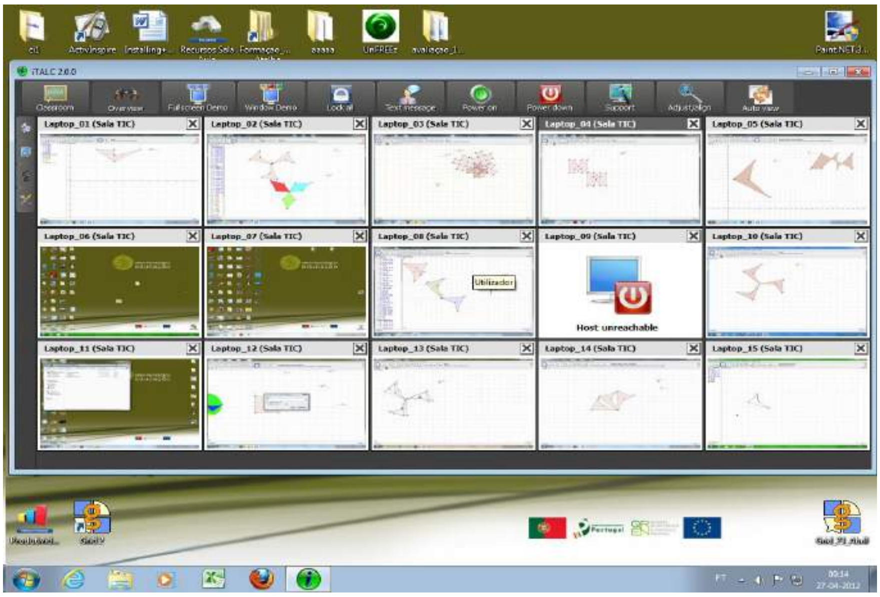
Figure 1 – Exploring rotational symmetries in task VI using GeoGebra



Task: For each windmill, try to find every rotation angle that keeps them "unchanged".

The classroom environment was mediated by a CMS, which was installed by the teacher/researcher in the ICT room. The teacher's workstation screen, which contained the "Master", had constantly been shown in the interactive whiteboard visible to everyone, in real time, together with a projection of any single student computer (see Figure 2).

Figure 2 - iTALC "Master" application is teacher's desktop



In the selection/creation process we aimed to find exploratory activities, with several open-ended tasks, each implemented in four different stages (Stein, Engle, Smith & Hughes, 2008). In the first stage, task was presented orally and some aspects, considered relevant or solicited by students, were clarified by the teacher. In the next stage, all groups solved their tasks autonomously but under teacher's supervision through the CMS when tasks were performed in the computer. In the third stage, the working groups presented their results, both in terms of processes and solutions. Likewise, in computer assisted tasks, they used CMS features to show their own desktop in the interactive whiteboard.

It was very important to create an appropriate atmosphere where students could have room for mistakes, time to think at their own pace, the opportunity to discuss with their classmates and teacher, and also to share ideas. The classroom was a place of confrontation and discussion where technology was used as a social collaborative learning tool.

Finally, in the fourth stage, students drew conclusions writing short reports on daily notebook. They had to present their solutions and discuss the underlying strategies, allowing everybody to reflect on the work carried out by each pair. They took note on main ideas. At the end of each session, we collected students' work and analysed the field notes to improve the Logbook. All these documents were assessed before the next session, so that the plan could be changed, if necessary.

After the module implementation, we used the FT and a Final Questionnaire (FQ). This questionnaire was intended, essentially, to gather data on student opinions about the approach used for this topic. The IT and FT had a double aim: the initial one gave us an image on the knowledge and skills pupils had before module implementation and the final one allowed us to assess their learning concerning isometric transformations and symmetry.

All collected data were subjected to content analysis using categories related to: i) Geometry - isometries and symmetry and ii) dimensions of Creativity – fluency, flexibility and originality.

Results

Direct observation and the analysis of students' answers to the FQ showed the importance they give to the high technological approach to the topic, as well as the nature of tasks and how they were addressed and discussed and their contribution to the development of their creativity. As suggested by Stein and Smith (2009), teaching for creativity must be, at the same time, creative, where the challenging nature of the tasks, based on the formulation and problem solving, exploration and research, can promote creative thinking (Vale et al., 2012).

The same techniques allowed us to conclude that one variable assumed great preponderance in this study playing a decisive role: the construction of a classroom "atmosphere" that allowed truly exploratory activities with open-ended tasks and where students felt "safe" from destructive criticism (Fleith & Alencar, 2005).

"This group felt the necessity to do things in a different way when they saw other approaches when solving the task. They developed two new strategies to complete it,

embedding new elements and ideas. This clearly came from the moment of confrontation mentioned above." (Logbook entry, 16/04/2012).

Supporting this idea, several students answered, in the FQ, that they "lost their fear of making mistakes" realizing that trial and error strategies were part of the process.

Using iTALC in an ICT room seemed to help build this environment. The main goal was to promote sharing and collaboration among the different actors, keeping proper control of a room full of computers connected to the Internet.

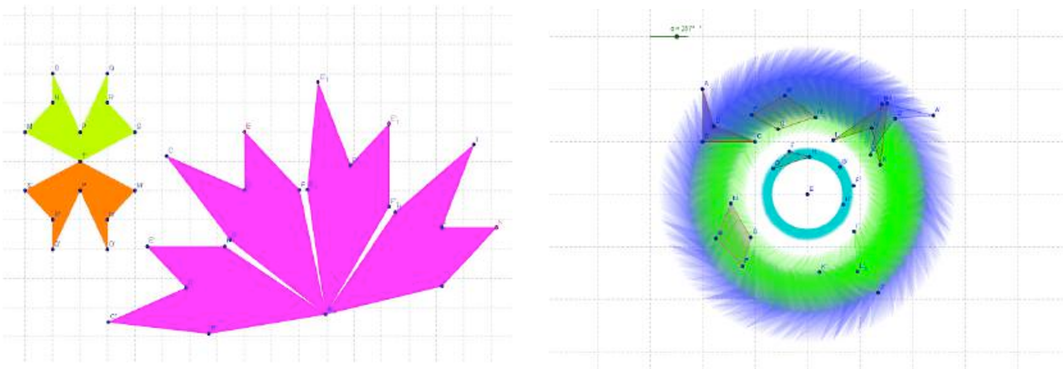
All students also declared in the FQ that iTALC encouraged teamwork. They strongly disagreed with the idea that this software's purpose was for controlling students.

With regard to the influence in the development of creativity of this approach on the topic using GeoGebra in a CMS monitored environment, it is important to note that the classroom "atmosphere" had the same features described above. Students actively shared their knowledge and findings. Discussion moments seemed to trigger new motivation and, consequently, new strategies and outputs.

"The group resolution was shown to the whole class, triggering wonder. This inspired other students. They felt motivated and committed to improve their work." (Logbook entry, 16/04/2012)

The five students also declared in the FQ that observing other students' work (the most creative ones) motivated them to be more creative themselves, although Catarina said that, despite having felt this necessity, she could not be more creative. Although, her work was quite original both in solutions and in adopted strategies (see Figure 3).

Figure 3 – Catarina's creative productions[4] in task I



Task: With GeoGebra, use different isometries to create free "compositions".

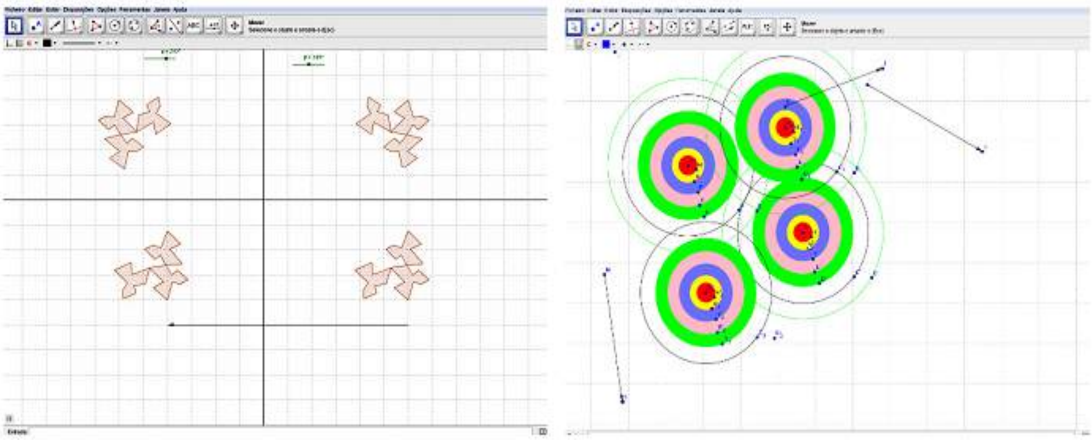
Also Tiago and Luísa and Francisca and Gabriela reacted in a similar way, predisposing them to reassess their approaches. In this cases, and in contrast with Catarina, feedback from classmates was seriously taken into account, which led them to often incorporate new elements that were absent in their original ideas (see Figure 4*).*

[4] Concerning colour, in strictly mathematical terms, the construction should be monochromatic. Attending that it is a student's construction of an early educational level its use was accepted in all cases.

"These groups of students were very receptive to suggestions from their classmates and teacher and they frequently changed their processes and strategies, showing flexibility (Logbook entry, 20/04/2012).

Hence, more original "constructions" progressively arose. This seems to confirm the idea formulated by Levenson (2011) that creativity can be built collectively, while being individually developed. The ability to share, at any time, any approach, process or solution on a computer was provided by iTALC.

Figure 4 – Tiago and Luísa reassess of task IV. Initial (left) and final construction (Wright)



Task: With GeoGebra, use translations to create a free "composition".

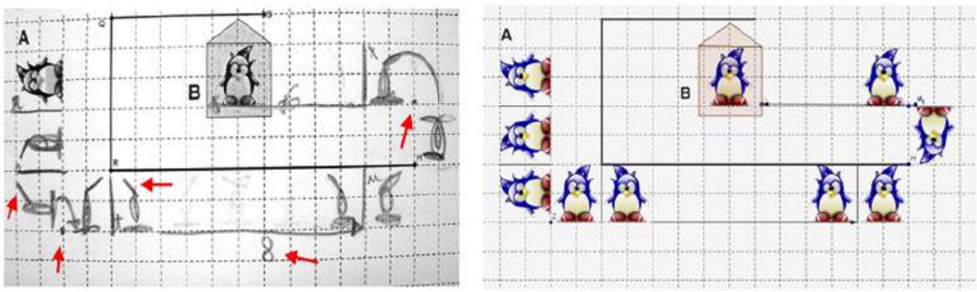
The work of the three selected groups shows unique and distinctive features, therefore great originality (see Figure 5).

Figure 5 - Creative students' productions in different free open-tasks in GeoGebra involving mainly rotations

It was also seen that G2 and G3 tended to have a higher number of more elaborate and original answers when they resorted to GeoGebra to solve the tasks. In these cases, use of paper and pencil (including traditional instrumentation and manipulatives) seemed to "soak" students in a whirlwind of technical procedures that somehow seemed to prevent them from exploring alternative strategies, thus, limiting their ability to adapt processes.

This aspect, very pronounced in G2 and also noticeable in G3, seemed irrelevant for Catarina (G1). This student often used paper and pencil in early attempts to rehearse procedures for resolving tasks (see Figure 6).

Figure 6 – Not requested initial attempt on “pencil and paper” environment



Task: (With GeoGebra) use any known isometries to take *Tux* from position A to position B, without crossing the black lines.

Concerning fluency, it could be observed in several tasks, that all three groups developed several approaches to the same problem, which, in a likely manifestation of flexibility, they adapted to achieve the desired "effects". There seemed to be improvements in three considered dimensions of creativity.

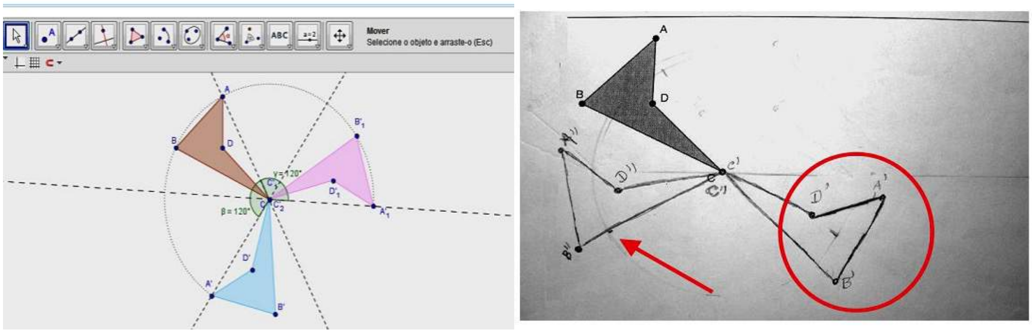
Concerning the impact of this approach developing a stronger ownership of geometric concepts and their application, it could also be seen, earlier in this empirical study, that the three cases showed a very superficial knowledge (or even null), and often conceptual errors on isometries and symmetry. Their final results in the FT were very encouraging revealing a strong evolution for all five students.

It seems clear that, considering the results students had achieved at the end of the study, the use of this software is valuable, establishing itself as a powerful tool in graphical problem solving that allows multiple approaches and solutions (Bardini, Pierce & Stacey, 2004)

Analysis of further responses to the FQ revealed a high degree of agreement on the benefits of using GeoGebra. Any negative aspect wasn’t pointed out. Concerning how the topic had been implemented, some students reported that the program helped them to understand isometries, making Geometry less complex and more fun.

Use of DGE seemed to play an important role, especially for students who had greater difficulties. It was observed that success in solving a task in GeoGebra did not always ensure a similar success when it was performed in a "paper and pencil" environment (see Figure 7).

Figure 7 – Group 2 difficulties when evolving from DGE to "paper and pencil" environment (task III)



Task: Draw images by rotating the quadrilateral [ADCB]: a) centre C, +120º; b) centre C, -120º. Do it first with GeoGebra and then with compass and ruler.

Evolving from "paper" to DGE posed no problem. The reverse was not true. In particular, G2 felt some difficulties in making this transition.

All students expressed their agreement or strong agreement when asked if they considered working with paper and pencil and having used instruments for measuring and drawing to have been important. These findings suggest the importance of a complementary approach (Laborde, 2001; Ponte, 2005; Ponte et al, 2007).

Final remarks

The research undertaken suggests that the appropriate use of CMS in highly technological approaches seems to contribute positively to improve teaching and learning of Mathematics. These applications allow students to remain more focused on their tasks and contribute decisively to building a learning environment where cooperation, collaboration and sharing between all actors in the classroom are indeed possible.

Further deeper and extended studies should be performed to better understand the benefits of its use. Variations in age, topics or themes should be introduced. Knowing CMS' potential for E-learning (when the student cannot attend school) also constitutes an unexplored field. To understand in which context it could be detrimental is another.

This study also aligns with the perception that the creation of a "social atmosphere" seems to elicit increases in dimensions of creativity. However, the limitations of this study, relating primarily to its short period of implementation and the extraordinary complexity of the phenomenon, do not allow more ambitious conclusions.

Also regarding creativity, the use of DGE, seems to promote the emergence of more creative work in Geometry. More studies are needed in both range and depth.

If using DGE appears to have a major influence in some dimensions of creativity when working in Geometry, the development of geometric knowledge and skills seems to benefit more from a complementary approach, which combines DGE with "paper and pencil" environments.

Some of these aspects should be, as suggested, the target of much more extensive and detailed studies. Their relevant role in teaching and learning Mathematics should have implications in teacher training.

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