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Non-Linear Behaviours in the Dynamics of Some Biostructures

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Abstract

Various differentiable models are frequently used to describe the dynamics of complex systems (see the kinetic models, fluid models, etc.). Given the complexity of all the physical phenomena involved in the dynamics of such systems, it is required to introduce the dynamic variable dependencies both on the space-time coordinates and on the scale resolutions. Therefore, in this case an adequate theoretical approach may be the use of non-linear physical models either in the form of the Scale Relativity Theory or of the Extended Scale Relativity Theory, i.e., the Scale Relativity Theory with an arbitrary constant fractal dimension. In the framework of the Extended Scale Relativity Theory, fractal velocity field is described both by topological solitons of kink type and by non-topological soliton varieties of breather type. Applications for the blood flow are proposed. The results revealed the directional flow toward the walls, which can explain the thickening effect which is one of the source of arteriosclerosis.

Keywords: complex system, fractal, non-linear, bio-structure, non-differentiable

1. Introduction

Bio-structure is a complex non-Newtonian fluid made of: plasma and formed cells, cholesterol vesicles and other suspended elements [1]; thus, the laws of fractal physics are completely applicable to sanguine circulation. For conformity, the perfect Newtonian fluid is a fluid in which viscosity is independent of the shear stress, thus having no relation to the sanguine fluid.

However, not only the complex structure of bio-structure justifies the using of fractality, but also the complex structure of the arterial system, with its multiple ramifications, which generate turbulence areas and interruptions of the linear flowing that make classical physics not applicable in this context. We can thus discuss about multi-fractality: a morphological one due to complex structure of the arterial tree as well as a functional one due to bio-structure flow “regimes” [2]. Also, the stress of a visco-elastic fluid, unlike the Newtonian fluid, depends not only on the actually stress applied but on the one applied during previous deformation of the fluid [3, 4].

Standard theoretical models usually used in complex fluid dynamics and particularly that of flow through bio-structure vessels are ambiguous [5]. The assessment that the entities contained by bio-structure move along continuous and differentiable courses proves to be false, as it cannot comprise the entire variety of dynamics that are induced by the flowing of bio-structure through the vessel system (from the separation of bio-structure components through turbulence regimes to bio-structure-bio-structure vessel interactions).

In this context, we propose a new hypothesis according to which bio-structure structural unit move on continuous but non-differentiable curves (and particularly on fractal curves). We cannot predict the entirety of the bio-structure-vessel system, bio-structure-organic tissues, etc., or at elementary level, bio-structure entity-bio-structure structural unit (i.e., lymphocyte – granulocyte, lymphocyte – platelet and others) interactions. This is why accepting the above stated hypothesis comes up as a simple, elegant and efficient solution, the impossibility of predicting all these interactions that take place being substituted by the use of fractality [6].

We are led to the dynamics of a special type of fluid, free of interactions, in which the stream lines are continuous and non-differentiable curves.

Multiple physical models have been developed in the attempt to explain the dynamics of bio-structure flow and its physiological and pathological changes on the course of the entire arterial trunk, starting from the big elastic arteries and continuing with the small arteries and arterioles – resistance vessels -, following with the bio-structure capillaries (with arterial and venous components) and, backwards, with the post capillary veins, then with the middle and large veins – capacitance vessels – ensuring the anti-gravitational mobilization of bio-structure.

Thus, the hypothesis of the geometric risk factor in the development of the circulatory system’s suffering has been proposed. This initially promising theory [7] proved its drawbacks that derive from a non-differentiable, Euclidean approach to the dynamics of bio-structure flow and its effects on the vessels’ wall. The proposed counterbalance is represented by fractal physics laws [4, 8] that offer a great amount of freedom due to accepting the relativity of the complex fluids’ behavior (bio-structure belongs to this category).

Correspondingly, the theoretical models that describe the complex fluids’ dynamics are sophisticated [4, 8]. However, these models can be simplified since the complexity of the interaction process imposes various temporal resolution scales. Also, one should take into account the fact that the pattern evolution imposes different freedom degrees [3, 28].

One could develop new theoretical models assuming the fact that the complex fluids displaying chaotic behavior are recognized to acquire self-similarity in association with strong fluctuations at all possible space-time scales [4, 9]. One can replace the deterministic trajectories by collections of potential trajectories for temporal scales which are large with respect to the inverse of the highest Lyapunov exponent (see, e.g., [10, 11]). Also, the concept of definite positions can be replaced by that of probability density. An interesting example is represented by that of collisions processes in complex system, where the dynamics of the particles can be described by fractal (non-differentiable) curves.

Since fractality (non-differentiability) is a universal property of complex fluids, one should build a fractal (non-differentiable) physics. In this context, replacing the complexity of the interactions processes by non-differentiability, it is not necessary anymore to use the classical “arsenal” of quantities from the differentiable physics. This was developed in the Scale Relativity Theory (SRT) [12, 13] and in the non-standard Scale Relativity Theory (NSRT), i.e., Scale Relativity Theory with arbitrary constant fractal dimension [14]. In the framework of SRT or NSRT let us suppose that the complex system structural unit motions take place on continuous but non-differentiable curves (called fractal curves). In this way, all physical phenomena that are involved in the dynamics depend on the space-time coordinates and also on the space-time scales resolution. In this context, the physical quantities describing the complex systems dynamics could be considered to be fractal functions [13, 14]. Additionally, the complex system entities could be reduced to and identified with their own trajectories. In this way, the complex system behaves as a special interaction-less “fluid” by means of its geodesics in a non-differentiable (fractal) space.

In the present chapter, various bio-structure flow dynamics are analyzed aiming to propose new mechanisms for the genesis and evolution of different bio-structure-related pathologies (arterial occlusion, cholesterol deposition, etc.).

2. Material and method

Assuming that the motions of bio-structure’s structural units take place on continuous but non-differentiable curves (fractal curves), the following consequences emerge [13, 14]:

- (i) Any continuous but non-differentiable curve of bio-structure’s structural units (bio-structure fractal curve) is scale resolution δt dependent. This means that when δt tends to zero, its length tends to infinity.

Let us recall that a curve is non-differentiable if it satisfies the Lebesgue theorem [9]. This means that when the scale resolution goes to zero, its length becomes infinite. In consequence, in this limit, a curve has a zigzag form and consequently it has the property of self-similarity in every one of its points. Since every part reflects the whole, this can be translated into a holography property [9];

- (ii) The physics of bio-structure phenomena are related to the behavior of a set of functions during the zoom operation of the resolution scale δt . Through the substitution principle, δt

can be identified with dt , that is, $\delta t \equiv dt$. Consequently, it will play the role of an independent variable. We shall use the notation dt for the usual time as in the Hamiltonian bio-structure dynamics;

- (iii) The dynamics of bio-structure's structural units are described by means of fractal variables. Since the differential time reflection invariance of any dynamical variable is broken, these functions depend on the space-time coordinates and also on the resolution scale. In consequence, one can define two derivatives of the variable field $Q(t, dt)$ at any point of the bio-structure fractal curve:

$$\begin{aligned}\frac{d_+Q(t, dt)}{dt} &= \lim_{\Delta t \rightarrow 0_+} \frac{Q(t + \Delta t, \Delta t) - Q(t, \Delta t)}{\Delta t} \\ \frac{d_-Q(t, dt)}{dt} &= \lim_{\Delta t \rightarrow 0_-} \frac{Q(t, \Delta t) - Q(t - \Delta t, \Delta t)}{\Delta t}\end{aligned}\quad (1)$$

The “+” sign corresponds to forward processes of bio-structure's structural units, while the “-” sign correspond to the backwards ones;

- (iv) The differential of the spatial coordinate field $dX^i(t, dt)$ by means of which we can describe the bio-structure dynamics, is expressed as the sum of the two differentials, one of them being scale resolution independent (differential part $d_{\pm}x^i(t)$) and the other one being scale resolution dependent (fractal part $d_{\pm}\zeta^i(t)$) i.e.,

$$d_{\pm}X^i(t) = d_{\pm}x^i(t) + d_{\pm}\zeta^i(t) \quad (2)$$

- (v) The non-differentiable part of the spatial coordinate field, by means of which we can describe the bio-structure dynamics, satisfies the fractal equation [14]:

$$d_{\pm}\zeta^i(t, dt) = \lambda_{\pm}^i(dt)^{1/D_F} \quad (3)$$

where λ_{\pm}^i are constant coefficients that help to specify the fractalization type which describes the bio-structure dynamics. Also, D_F defines the fractal dimension of the bio-structure non-differentiable curve.

In this way, the bio-structure processes imply dynamics on geodesics having different fractal dimensions. This variety of fractal dimensions of the bio-structure geodesics is a result of the bio-structure's structure. For $D_F = 2$, quantum type processes are generated in bio-structure dynamics [15]. For $D_F < 2$, correlative type processes are induced and for $D_F > 2$ non-correlative type processes are generated [6, 12, 13].

- (vi) The differential time reflection invariance of any dynamical variable of the bio-structure is recovered by combining the derivatives d_+/dt and d_-/dt in the non-differentiable operator

$$\widehat{d}dt = \frac{1}{2} \left(\frac{d_+ + d_-}{dt} \right) - \frac{i}{2} \left(\frac{d_+ - d_-}{dt} \right) \quad (4)$$

This is a natural result of the complex prolongation procedure applied to bio-structure dynamics [14, 16]. Applying now the non-differentiable operator to the spatial coordinate field, by

means of which we can describe the bio-structure dynamics, yields the complex velocity field of the bio-structure.

$$\widehat{V}^i = \frac{\widehat{d}X^i}{dt} = V^i - iU^i, i = \sqrt{-1} \quad (5)$$

with

$$V^i = \frac{1}{2} \frac{d_+ X^i + d_- X^i}{dt}, U^i = \frac{1}{2} \frac{d_+ X^i - d_- X^i}{dt} \quad (6)$$

The real part V^i of the bio-structure complex velocity field is differentiable and scale resolution independent (differentiable velocity field). The imaginary part U^i is non-differentiable and scale resolution dependent (fractal velocity field).

(vii) If we have no external constraint, one can find an infinite number of fractal curves (geodesics) relating any pair of points. This happens on all scales of bio-structure dynamics. Then, in the fractal space of the bio-structure, all the structural units are substituted with the geodesics themselves so that any external constraint can be interpreted as a selection of bio-structure geodesics. The infinity of geodesics in the bundle, their non-differentiability and the two values of the derivative imply a generalized statistical fluid-like description of the bio-structure dynamics. Then, the average values of the bio-structure variables must be considered in the previously mentioned sense, so the average of $d_{\pm} X^i$ is

$$\langle d_{\pm} X^i \rangle \equiv d_{\pm} x^i \quad (7)$$

with

$$\langle d_{\pm} \zeta^i \rangle = 0 \quad (8)$$

The previous relation (8) implies that the average of the fractal fluctuations is null.

(viii) One can describe the bio-structure dynamics by means of a scale covariant derivative. Its explicit form can be obtained as follows. We assume that the bio-structure fractal curves are immersed in a 3-dimensional space. We also suppose that X^i is the spatial coordinate field of a point on this fractal curve. Let us also consider a variable field $Q(X^i, t)$ and the following Taylor expansion up to the second order

$$d_{\pm} Q(X^i, t) = \partial_t Q dt + \partial_i Q d_{\pm} X^i + \frac{1}{2} \partial_i \partial_k Q d_{\pm} X^i d_{\pm} X^k \quad (9)$$

These relations are valid at any point and more for the points X^i on the bio-structure fractal curve which we have selected in Eq. (9). From here, forward and backward average values for bio-structure variables from Eq. (9) become

$$\langle d_{\pm} Q \rangle = \langle \partial_t Q dt \rangle + \langle \partial_i Q d_{\pm} X^i \rangle + \frac{1}{2} \langle \partial_i \partial_k Q d_{\pm} X^i d_{\pm} X^k \rangle \quad (10)$$

We suppose that the average values of the all variable field Q and its derivatives coincide with themselves and the differentials $d_{\pm}X^i$ and dt are independent. Therefore, the average of their products coincides with the product of averages. Consequently, Eq. (10) becomes

$$d_{\pm}Q = \partial_t Q dt + \partial_i Q \langle d_{\pm}X^i \rangle + \frac{1}{2} \partial_l \partial_k Q \langle d_{\pm}X^l d_{\pm}X^k \rangle \quad (11)$$

Even the average value of $d_{\pm}\zeta^i$ is null, for the higher order of $d_{\pm}\zeta^i$ the situation can still be different. Let us focus on the averages $\langle d_{\pm}\zeta^l d_{\pm}\zeta^k \rangle$. Using Eq. (3) we can write

$$\langle d_{\pm}\zeta^l d_{\pm}\zeta^k \rangle = \pm \lambda_{\pm}^l \lambda_{\pm}^k (dt) \left(\frac{2}{D_F}\right)^{-1} dt \quad (12)$$

where the sign + corresponds to $dt > 0$ and the sign - corresponds to $dt < 0$.

Then, Eq. (11) takes the form:

$$d_{\pm}Q = \partial_t Q dt + \partial_i Q \langle d_{\pm}X^i \rangle + \frac{1}{2} \partial_l \partial_k Q d_{\pm}x^l d_{\pm}x^k \pm \frac{1}{2} \partial_l \partial_k Q \left[\lambda_{\pm}^l \lambda_{\pm}^k (dt) \left(\frac{2}{D_F}\right)^{-1} dt \right] \quad (13)$$

If we divide by dt and neglect the terms that contain differential factors (for details, see the method from [13, 14]) we obtain:

$$\frac{d_{\pm}Q}{dt} = \partial_t Q + v_{\pm}^i \partial_i Q \pm \frac{1}{2} \lambda_{\pm}^l \lambda_{\pm}^k (dt) \left(\frac{2}{D_F}\right)^{-1} \partial_l \partial_k Q \quad (14)$$

where $v_{+}^i = \frac{d_{+}x^i}{dt}$, $v_{-}^i = \frac{d_{-}x^i}{dt}$.

These relations also allow us to define the operators

$$\frac{d_{\pm}}{dt} = \partial_t + v_{\pm}^i \partial_i \pm \frac{1}{2} \lambda_{\pm}^l \lambda_{\pm}^k (dt) \left(\frac{2}{D_F}\right)^{-1} \partial_l \partial_k \quad (15)$$

Using Eqs. (4), (5), and (15), let us calculate the differentiable operator

$$\widehat{\frac{dQ}{dt}} = \partial_t Q + \widehat{V}^i \partial_i Q + \frac{1}{4} (dt) \left(\frac{2}{D_F}\right)^{-1} D^{lk} \partial_l \partial_k Q \quad (16)$$

where

$$D^{lk} = d^{lk} - \overline{d}^{lk} \quad (17)$$

$$d^{lk} = \lambda_{+}^l \lambda_{+}^k - \lambda_{-}^l \lambda_{-}^k, \quad \overline{d}^{lk} = \lambda_{+}^l \lambda_{+}^k + \lambda_{-}^l \lambda_{-}^k$$

Eq. (16) also allows us to define the covariant derivative in the bio-structure dynamics

$$\frac{\widehat{d}}{dt} = \partial_t + \widehat{V}^i \partial_i + \frac{1}{4} (dt) \left(\frac{2}{D_F}\right)^{-1} D^{lk} \partial_l \partial_k \quad (18)$$

Let us now consider the principle of scale covariance (the physics laws – bio-structure dynamics specific – are invariant with respect to scale transformations) and postulate that the passage from the classical (differentiable) physics to the fractal (non-differentiable) physics can be implemented by replacing the standard time derivative d/dt by the non-differentiable operator \widehat{d}/dt . In this way, this operator has the role of a scale covariant derivative. More precisely, it is used to write the bio-structure dynamics fundamental equations in the same form as in the classic (differentiable) case. In these conditions, applying the operator (18) to the complex velocity field (5), with no external constraint, the bio-structure geodesics take the form:

$$\frac{\widehat{d}\widehat{V}^i}{dt} = \partial_t \widehat{V}^i + \widehat{V}^l \partial_l \widehat{V}^i + \frac{1}{4} (dt) \left(\frac{2}{D_F}\right)^{-1} D^{lk} \partial_l \partial_k \widehat{V}^i = 0 \quad (19)$$

This means that the local acceleration $\partial_t \widehat{V}^i$, the convection $\widehat{V}^l \partial_l \widehat{V}^i$ and the dissipation $D^{lk} \partial_l \partial_k \widehat{V}^i$, make their balance at any point of the bio-structure fractal curve. Moreover, the presence of the complex coefficient of viscosity-type $4^{-1} (dt) \left(\frac{2}{D_F}\right)^{-1} D^{lk}$ in the bio-structures dynamics specifies that it is a rheological medium. So, it has memory, as a datum, by its own structure.

If the fractalization is achieved by Markov type stochastic processes, which involve Lévy type movements [9, 13, 17] of the bio-structure structural units, then:

$$\lambda_+^i \lambda_+^l = \lambda_-^i \lambda_-^l = 2\lambda \delta^{il} \quad (20)$$

where δ^{il} is the Kronecker's pseudo-tensor.

Under these conditions, the equation of bio-structure geodesics takes the simple form

$$\frac{\widehat{d}\widehat{V}^i}{dt} = \partial_t \widehat{V}^i + \widehat{V}^l \partial_l \widehat{V}^i - i\lambda (dt) \left(\frac{2}{D_F}\right)^{-1} D^{lk} \partial_l \partial_k \widehat{V}^i = 0 \quad (21)$$

or more, by separating the motions on differential and fractal scale resolutions,

$$\begin{aligned} \frac{\widehat{d}V_D^i}{dt} &= \partial_t V_D^i + V_D^l \partial_l V_D^i - \left[V_F^l + \lambda (dt) \left(\frac{2}{D_F}\right)^{-1} \partial^l \right] \partial_l V_F^i = 0 \\ \frac{\widehat{d}V_F^i}{dt} &= \partial_t V_F^i + V_D^l \partial_l V_F^i + \left[V_F^l + \lambda (dt) \left(\frac{2}{D_F}\right)^{-1} \partial^l \right] \partial_l V_D^i = 0 \end{aligned} \quad (22)$$

Using the standard procedure from [18], the bio-structure dynamics at fractal scale resolution can be described by means of the following equations:

$$\partial_t V_F^i + V_F^l \partial_l V_F^i = \lambda(dt) \left(\frac{2}{D_F}\right)^{-1} \partial^l \partial_l V_F^i \quad (23)$$

$$\partial_i V_F^i = 0 \quad (24)$$

Eq. (23) corresponds to the specific impulse conservation law at fractal scale resolution, while Eq. (24) corresponds to the states density conservation law at fractal scale resolution (we consider that the density of the bio-structure at fractal scale resolution is constant – incompressible bio-structure).

Since this equation system is non-linear, one could find relatively difficult finding the solutions for these equations [19, 20]. However, in the particular case of a stationary flow in a plane symmetry (x, y) , there is an analytical solution of this system. Then, for $V_F = (V_x, V_y, 0)$, Eqs. (23) and (24) take the form:

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial x} = \lambda(dt) \left(\frac{2}{D_F}\right)^{-1} \frac{\partial^2 V_x}{\partial y^2} \quad (25)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \quad (26)$$

The boundary conditions of the flow are:

$$\lim_{y \rightarrow 0} V_y(x, y) = 0, \lim_{y \rightarrow 0} \frac{\partial V_x}{\partial y} = 0, \lim_{y \rightarrow \infty} V_x(x, y) = 0 \quad (27)$$

and the flux momentum per length unit is constant

$$\Theta = \rho \int_{-\infty}^{+\infty} V_x^2 dy = const. \quad (28)$$

Using the method from [18–20] for solving Eqs. (25) and (26), with the conditions (27) and (28), the following solutions result:

$$V_x = \frac{\left[1.5 \left(\frac{\Theta}{6Q}\right)^{\frac{2}{3}}\right]}{\left[\lambda(dt) \left(\frac{2}{D_F}\right)^{-1} x\right]^{1/3}} \cdot \operatorname{sech}^2 \cdot \frac{\left[(0.5y) \left(\frac{\Theta}{6Q}\right)^{\frac{1}{3}}\right]}{\left[\lambda(dt) \left(\frac{2}{D_F}\right)^{-1} x\right]^{2/3}} \quad (29)$$

$$V_y = \frac{\left[4.5 \left(\frac{\Theta}{6Q}\right)^{\frac{2}{3}}\right]}{\left[3\lambda(dt) \left(\frac{2}{D_F}\right)^{-1} x\right]^{1/3}} \cdot \left[\frac{\left[y \left(\frac{\Theta}{6Q}\right)^{\frac{1}{3}}\right]}{\left[\lambda(dt) \left(\frac{2}{D_F}\right)^{-1} x\right]^{\frac{2}{3}}} \operatorname{sech}^2 \cdot \frac{\left[(0.5y) \left(\frac{\Theta}{6Q}\right)^{\frac{1}{3}}\right]}{\left[\lambda(dt) \left(\frac{2}{D_F}\right)^{-1} x\right]^{\frac{2}{3}}} - \tanh \frac{\left[(0.5y) \left(\frac{\Theta}{6Q}\right)^{\frac{1}{3}}\right]}{\left[\lambda(dt) \left(\frac{2}{D_F}\right)^{-1} x\right]^{\frac{2}{3}}} \right] \quad (30)$$

Relations (29) and (30) suggest that the bio-structure velocity field is highly non-linear through topological solitons of kink type (\tanh), various non-topological solitons of breather type (sech^2), and through topological – non-topological soliton mixtures of kink-breather type ($\text{sech}^2\text{-tanh}$). Given the structural complexity of the bio-structure (which is given by its various structural units, that retains their own velocity field) an accurate way of writing relations (29) and (30) will be the one in which we assign indexes for each component.

For $y = 0$, we obtain in relation (29) the flow critical velocity of the bio-structure in the form

$$V_x(x, y = 0) = V_c = \frac{\left[1.5 \left(\frac{\Theta}{6Q} \right)^{\frac{2}{3}} \right]}{\left[\lambda(dt) \left(\frac{2}{D_F} \right)^{-1} x \right]^{1/3}} \quad (31)$$

while taking into account (31), relation (28) becomes

$$\Theta = \rho \int_{-\infty}^{+\infty} V_x^2(x, y) dy = \int_{-d_c}^{+d_c} V_c^2(x, 0) dy, \quad (32)$$

so that the critical cross section of the strains lines tube of the bio-structure is given by:

$$d_c(x, y = 0) = \frac{\Theta}{2\rho V_c^2} = 2.42 \left[\lambda(dt) \left(\frac{2}{D_F} \right)^{-1} x \right]^{\frac{2}{3}} \left(\frac{\rho}{\Theta} \right)^{1/3} \quad (33)$$

Relations (29) and (30) can be strongly simplified if we introduce the normalized quantities

$$\zeta = \frac{x}{x_0}, \eta = \frac{y}{y_0}, u = \frac{V_x}{w_0}, v = \frac{V_y}{w_0}, \Omega = \frac{\left(\frac{\Theta}{6Q} \right)^{\frac{2}{3}}}{w_0 \left[\lambda(dt) \left(\frac{2}{D_F} \right)^{-1} x_0 \right]^{1/3}}, \omega = \frac{\left(\frac{\Theta}{6Q} \right)^{\frac{2}{3}} y_0}{\left[\lambda(dt) \left(\frac{2}{D_F} \right)^{-1} x_0 \right]^{2/3}} \quad (34)$$

where x_0, y_0, w_0 are specific lengths and the specific velocity, respectively, of the laminar flow of the bio-structure. It results that

$$u(\zeta, \eta) = \frac{1.5\Omega}{\zeta^{\frac{1}{3}}} \text{sech}^2 \left(\frac{0.5\Omega\omega\eta}{\zeta^{\frac{2}{3}}} \right), \quad (35)$$

$$v(\zeta, \eta) = \frac{4.5^{2/3}}{3^{\frac{1}{3}}} \cdot \frac{\Omega}{\zeta^{\frac{1}{3}}} \left[\frac{\omega\eta}{\zeta^{\frac{2}{3}}} \text{sech}^2 \left(\frac{0.5\Omega\omega\eta}{\zeta^{\frac{2}{3}}} \right) - \tanh \left(\frac{0.5\Omega\omega\eta}{\zeta^{\frac{2}{3}}} \right) \right] \quad (36)$$

We present in **Figures 1a, b** and **2a, b** the dependence of the normalized velocity field u on the normalized spatial coordinates ξ, η for various nonlinearity degrees ($\omega = 0.3; 6$). The results

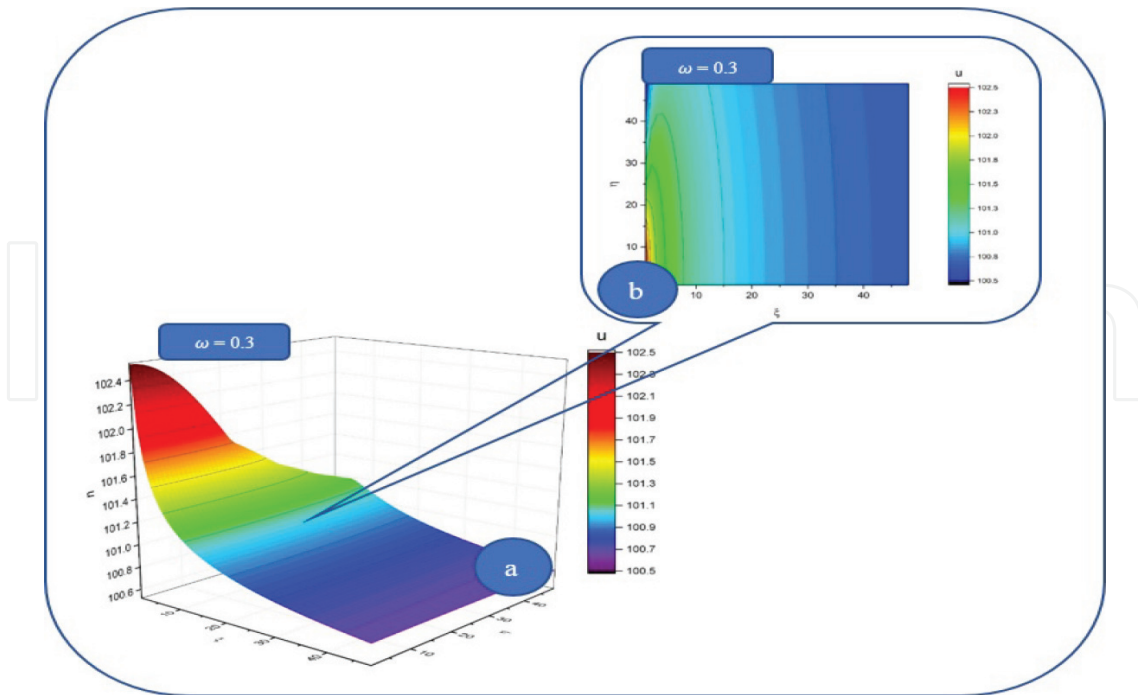


Figure 1. The dependence of the normalized velocity field u on the normalized spatial coordinates ξ, η for the nonlinearity degree $\omega = 0.3$: (a) 3D representation; contour plot (b).

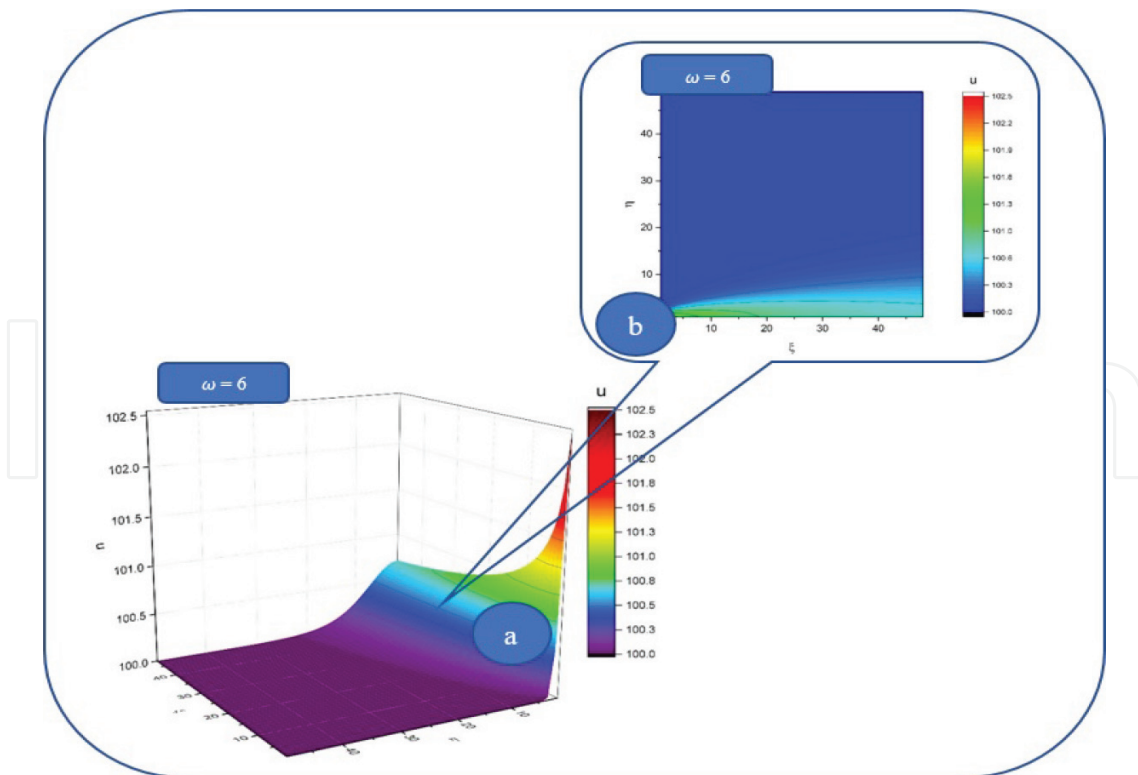


Figure 2. The dependence of the normalized velocity field u on the normalized spatial coordinates ξ, η for the nonlinearity degree $\omega = 6$: (a) 3D representation; contour plot (b).

showcase that the velocity field on the bio-structure flow direction (ξ) is affected in a weak manner by the nonlinearity degree (the velocity always decreases on the flow axes regardless of the nonlinearity degree). Also, the bio-structure flow direction (η) is strongly affected. Bio-structure flow starts from constant values on the η axes and with the increase of ω , preferential bio-structure flow direction can be identified.

In **Figures 3a, b** and **4a, b** the dependences of the normalized velocity field v on the normalized spatial coordinates ξ, η for various non-linearity degrees ($\omega = 0.3; 6$) are represented. For small non-linearity degrees, the variations (increase/decrease) of the velocity field have similar behaviors on both directions (ξ, η), while for higher values of the non-linearity degree these variations are only focused on a single direction (ξ).

Taking the above into account, the force that the bio-structure will exercise to the walls of the flow vessels is of great importance for the understanding of arterial occlusion and other circulatory system diseases.

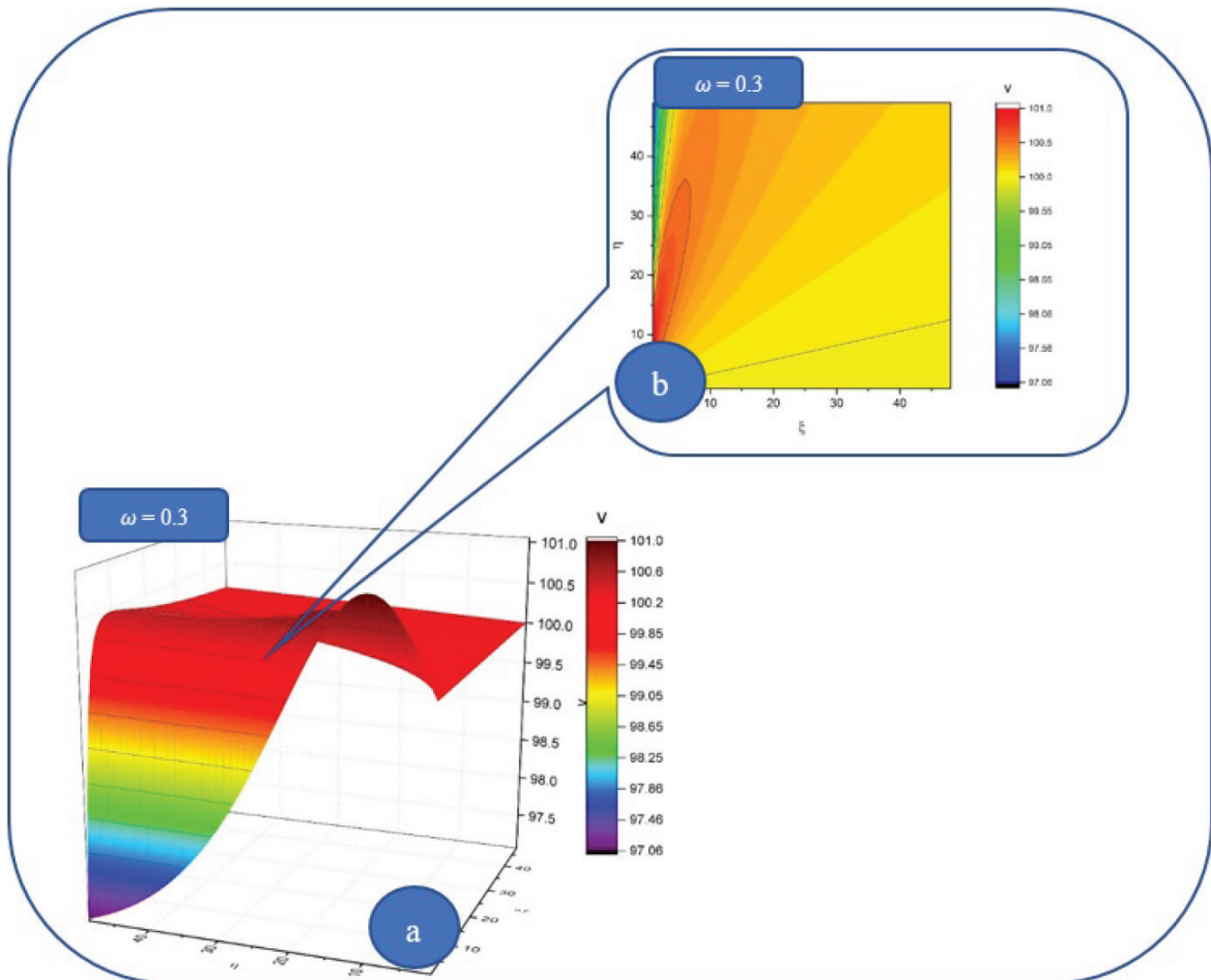


Figure 3. The 3D representation (a) and the contour plot (b) of the normalized velocity field v on the normalized spatial coordinates ξ, η for the nonlinearity degree $\omega = 0.3$.

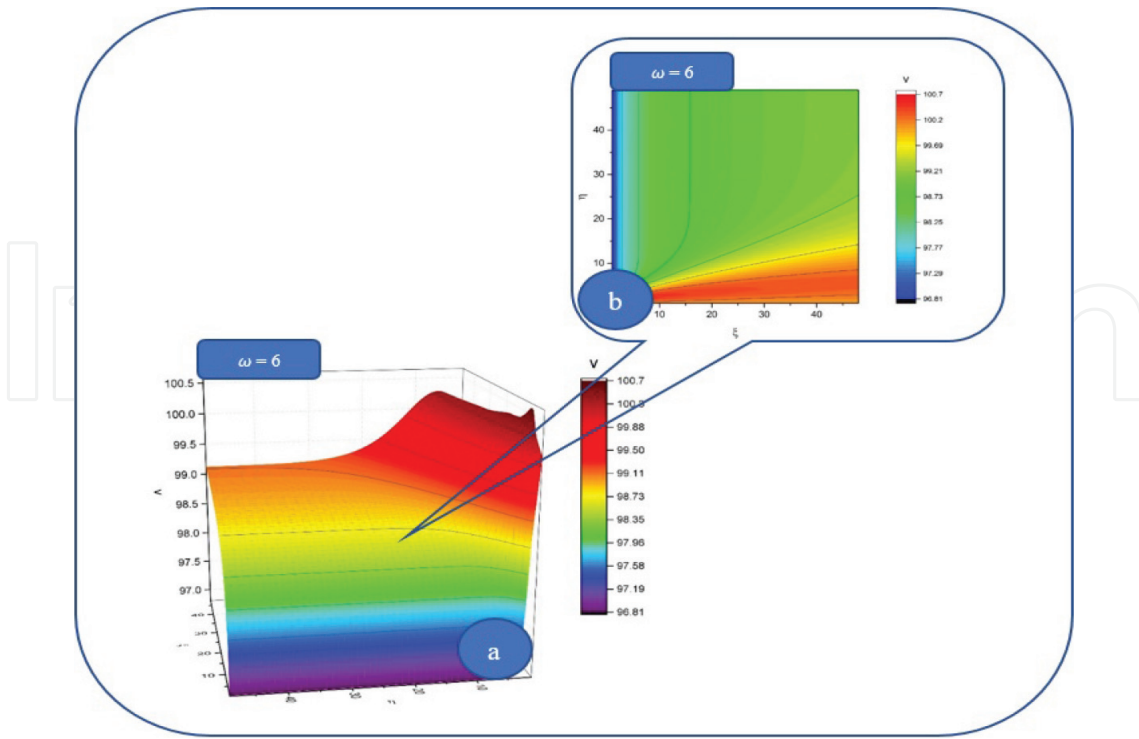


Figure 4. The 3D representation (a) and the contour plot (b) of the normalized velocity field v on the normalized spatial coordinates ξ, η for the non-linearity degree $\omega = 6$.

In our case the normalized force is given by the relation:

$$\begin{aligned}
 f(\zeta, \eta) &= \partial_{\eta} u - \partial_{\zeta} v \\
 &= -1.5 \frac{\Omega \operatorname{sech}^2\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right) \tanh\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right) \omega}{\zeta} - \frac{1}{\zeta^{\frac{1}{3}}} \\
 &\cdot \left(0.9\Omega \left(-\frac{2}{3} \frac{\omega\eta \operatorname{sech}^2\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right)}{\zeta^{\frac{2}{3}}} + \frac{0.66\omega^2\eta^2 \operatorname{sech}^2\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right) \tanh\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right) \omega}{\zeta^{\frac{7}{3}}} \right) \right. \\
 &\left. + \frac{0.33 \left[\left(1 - \tanh^2\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right) \right) \omega\eta \right]}{\zeta^{\frac{5}{3}}} \right) \\
 &+ \frac{0.3\Omega \left(\frac{\omega\eta \operatorname{sech}^2\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right)}{\zeta^{\frac{2}{3}}} - \tanh\left(\frac{0.5\omega\eta}{\zeta^{\frac{2}{3}}}\right) \right)}{\zeta^{\frac{4}{3}}} 3^{3/2}
 \end{aligned} \tag{37}$$

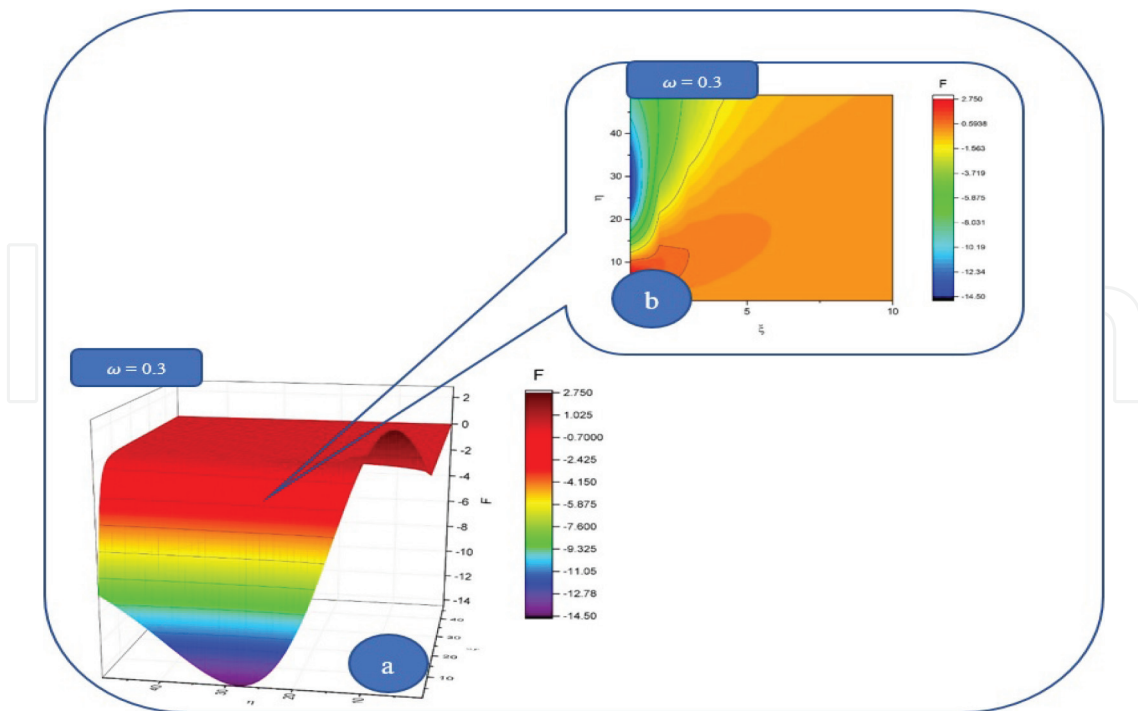


Figure 5. The dependence of the normalized force field F of a bio-structure flow on the vessels, on the normalized spatial coordinates ξ, η for two resolution scales: 3D representation (a); contour plot (b) for the non-linearity degree $\omega = 0.3$.

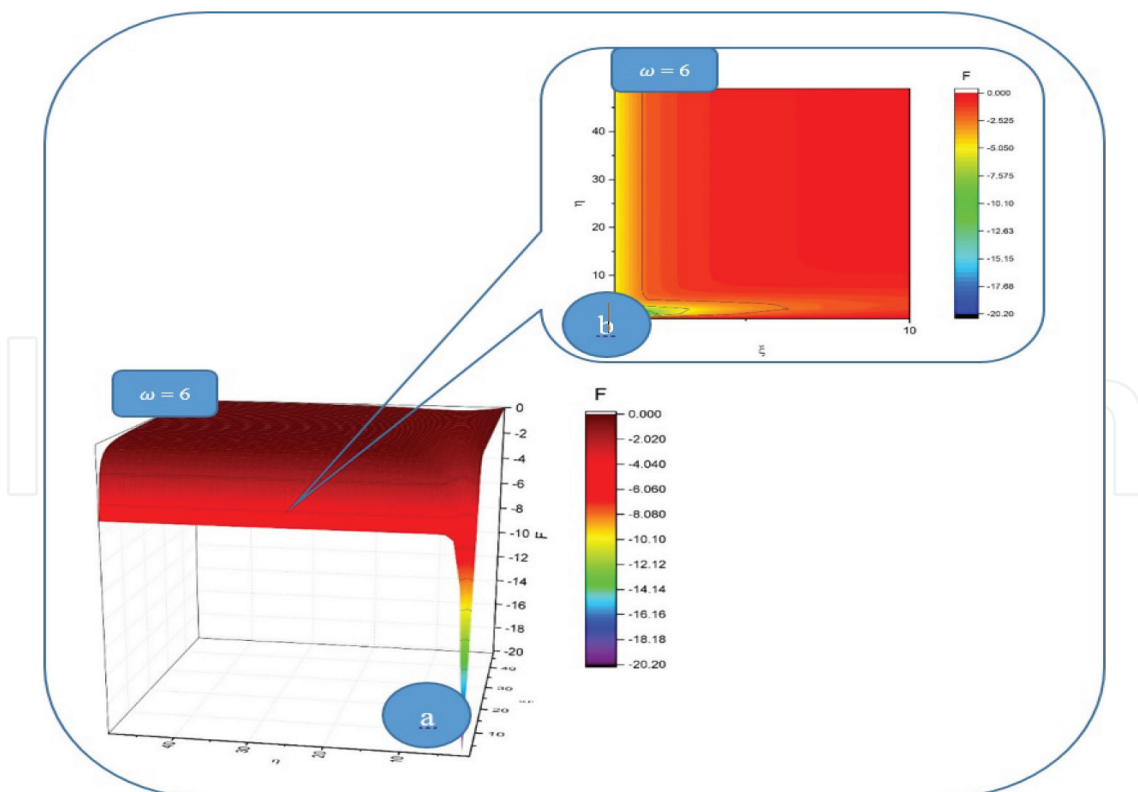


Figure 6. The dependence of the normalized force field F of a bio-structure flow on the vessels, on the normalized spatial coordinates ξ, η for two resolution scales: 3D representation (a) and contour plot (b), for the non-linearity degree $\omega = 6$.

In **Figures 5a, b** and **6a, b** the normalized force field evolution on the two-flow direction (ξ, η) for various non-linear degrees is represented. It results that with the increase of the non-linearity of the bio-structure the force toward the walls increases.

3. Discussions

The theory proposed in this chapter explains from a fractal viewpoint the atherogenesis process [21], basically “molding” to the classical anatomical and histopathological descriptions and completely respecting the process they postulate. In consequence, the fractal physics model sustains already accumulated morpho-pathological information and research. There are plenty of electronic and optical microscopy images that describe the spatial-temporal hologram of the phenomenon; we can thus discuss about the non-fractal – fractal and microscopic – macroscopic translation through holographically reproducible auto-similarity [21]. In this way, we affirm that fractality is the mathematical and semantic quintessence for defining atherogenesis, a process that can be physically characterized by fractal physics. This physics becomes in this situation more of a component rather than an explanation for the complex biological system represented by the atheroma plaque [22, 23].

In what concerns the recovery of such biological diseases, there are a huge number of techniques. We recall that external electrical stimulation can cause changes in the bio-structure vessels. Although atherosclerosis cause vasodilatation in the affected area and bio-structure flow remains unchanged for an extended period of time, the vascular wall stiffness will increase the pulse pressure. The purpose of the study developed in [24, 25] was to measure the effects of electrical stimulation (ES) on bio-structure flow and bio-structure pressure. All subjects received electrical stimulation at intensity sufficient to produce torque equal to 15% of the predetermined maximal voluntary contraction of their right quadriceps femor is muscle. The conclusions were that the increase in bio-structure flow occurred within 5 min after the onset of ES and dropped to resting levels within 1 min after a 10-min period of ES [25]. Kinesiotherapy or Kinesitherapy or kinesiatics, is the therapeutic treatment of disease by passive and active muscular movements (as by massage) and of exercise [26]. From the physiotherapeutic viewpoint, an efficient treatment is directed toward improving bio-structure flow and also decreasing the disparity between the demand for bio-structure and its supply [22]. An effective vascular rehabilitation training program for improving walking efficiency and vascular remodeling in patients with diabetic atherosclerosis suffering from intermittent claudication could be a supervised treadmill walking exercise combined with Allen-Burger exercises [23].

4. Conclusions

The present chapter proposes a fractal model for the dynamics analysis of bio-structure flows. The fractal hydrodynamic equations were obtained and applied for the laminar flow of bio-structure.

A second application was proposed for bio-structure flow, and of cholesterol deposition on the vessel walls. The results revealed the directional flow toward the walls. This could explain in our opinion the thickening effect which is one of the sources of arteriosclerosis. Moreover, our model imposes redefinition of “good” and “bad” cholesterol (which are traditionally associated with HDL and LDL respectively); instead they should be replaced by the following notions: specific cholesterol entities, associated with a certain non-differentiable curve, that have a major endothelial impact and specific cholesterol entities which have no or low endothelial impact.

There is currently a great number of works describing matter organization and behavior in all of its variations, from which we mention [27]. We consider that our bio-structure flow model could also be used to further development in the study of other complex systems dynamics (such as pulmonary and metabolic diseases or environmental systems). Moreover, possible therapeutic treatments can be developed, e.g., new drug release mechanisms.

Conflict of interest

The authors declare no conflict of interest.

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