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# High-Gain Observer–Based Sliding Mode Control of Multimotor Drive Systems

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Additional information is available at the end of the chapter

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## Abstract

Multimotor drive systems have been widely used in many modern industries. It is a nonlinear, multi-input, multi-output (MIMO) and strong-coupling complicated system, including the effect of friction, elastic, and backlash. The control law for this drive system much depends on the determining of the tension. However, it is hard to obtain this tension in practice by using a load cell or a pressure meter due to the accuracy of sensors or external disturbance. In order to solve this problem, a high-gain observer is proposed to estimate the state variables in this drive system, such as speeds and tension. An emerging proposed technique in the control law is the use of high-gain observers together with adaptive sliding mode control scheme to obtain a separation principle for the stabilization of whole system. The theory analysis and simulation results point out the good effectiveness of the proposed output feedback for the drive system.

**Keywords:** high-gain observer, multimotor drive systems, sliding mode control, tension, output feedback controller

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## 1. Introduction

Multimotor drive systems have been researched by many researchers in the recent times. The control law based on neural network technique has been proposed by Yaojie Mi et al. (see [1–4], for examples). However, it is hard to find the corresponding networks as well as learning rules. Besides, the model of this system is approximately described as a linear system to use the transfer function to design the control law. Furthermore, the tracking ability or the stabilization of the whole system is not still solved under the effects of observer using neural network technique. In the multimotor drive control systems, it is necessary to obtain the belt tension to design the suitable state feedback control law. However, it is hard to measure this belt tension by using sensors, and the observer based on high-gain technique is proposed in our work.

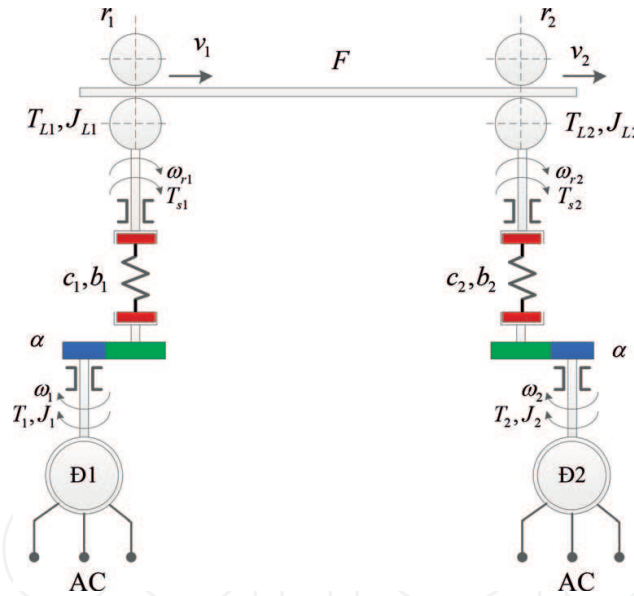
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Besides, the state feedback control design based on sliding mode control technique enables to remove efficient disturbances and uncertainties. Therefore, a high-gain observer is proposed to estimate the tension in this system and combine with the state feedback controller to obtain the output feedback control law satisfying the separation principle. The stability of whole system is obtained by the output feedback control law and verified by theory analysis and simulations.

This work is composed of 7 sections. In Section 2, the problem statements are shown and the dynamic equations of the two-motor system are described by the effect of friction, backlash, and elastic. Sections 3–5 describe the output feedback control design. Then, the high-gain observer for multimotor system is explained. Next, the sliding mode control of this system and the ability to satisfy the separation principle of output feedback controller are discussed. In Section 6, simulation results are shown. The conclusion is summarized in Section 7.

## 2. Problem statements

In [1], the multimotor system (in **Figure 1**) using two induction motor is described by the following dynamic Eq. (1), and the nomenclatures used in these equations are summarized in **Table 1**:



**Figure 1.** The two-motor drive system.

$$\begin{cases} \frac{dx'_1}{dt} = \frac{n_{p1}}{J_1} \left[ (u_1 - x'_1) \frac{n_{p1} T_{r1}}{L_{r1}} \varphi_{r1}^2 - (T_{L1} + r_1 x'_3) \right] \\ \frac{dx'_2}{dt} = \frac{n_{p2}}{J_2} \left[ (u_2 - x'_2) \frac{n_{p2} T_{r2}}{L_{r2}} \varphi_{r2}^2 - (T_{L2} - r_2 x'_3) \right] \\ \frac{dx'_3}{dt} = \frac{K}{T} \left( \frac{1}{n_{p1}} r_1 k_1 x'_1 - \frac{1}{n_{p2}} r_2 k_2 x'_2 \right) - \frac{x'_3}{T} \end{cases} \quad (1)$$

$K = E/V$	Transfer function
$E$	Young's Modulus of belt
$V$	Expected line velocity
$T = \frac{L_0}{AV}$	Time constant of tension variation
$L_0, A$	Distance between racks, section area (m <sup>2</sup> )
$n_{pi}$	Number of pole-pairs in the $i$ th motor
$J_1, J_2, J_{L1}, J_{L2}$	Inertia moment of motors and loads (kgm <sup>2</sup> )
$T, T_L, \varphi_r$	Motor, load torque (Nm), flux of rotor (Wb)
$L_r$	Self-induction of rotor (H)
$r, k, \omega_r, \omega, F$ (in Eq. (1))	Radius of roller, velocity ratio, electric angle velocity of rotor, angle velocity of stator, belt tension
$\omega_1, \omega_2, \omega_{r1}, \omega_{r2}$ (in Eqs. (2) and (3))	The angle velocity of motor and load
$c_1, c_2, b_1, b_2$	Stiffness and friction coefficient
$\Delta\omega_1, \Delta\omega_2$	The errors of angle speed in presence of backlash and elastic

**Table 1.** Dynamic parameters.

where

$x' = [x'_1, x'_2, x'_3]^T = [\omega_{r1}, \omega_{r2}, F]^T \in \mathbb{R}^3$  is state variable and  $u = (u_1, u_2) = (\omega_1, \omega_2) \in \mathbb{R}^2$  is a control variable.

However, due to the effects by backlash and elastic (**Figure 1**), we extend this model to obtain the equivalent diagram (**Figure 2**) and the following dynamic Eqs. (2) and (3):

$$\begin{cases} \Delta\dot{\varphi}_1 = \frac{1}{T}(\omega_1 - \omega_{r1}) \\ \Delta\dot{\varphi}_2 = \frac{1}{T}(\omega_2 - \omega_{r2}) \\ \dot{\omega}_{r1} = J_{L1} \left[ \frac{1}{K_{TC1}} f_{11}(\Delta\varphi_1) + K_{C1} \Delta\omega_1 f_{12}(\Delta\varphi_1) - (T_{L1} + r_1 F_{21}) \right] \\ \dot{\omega}_{r2} = J_{L2} \left[ \frac{1}{K_{TC2}} f_{21}(\Delta\varphi_2) + K_{C2} \Delta\omega_2 f_{22}(\Delta\varphi_2) - (T_{L2} + r_2 F_{12}) \right] \\ \dot{F} = C_{12} \left[ r_1 \omega_{r1} - r_2 \omega_{r2} \left( 1 + \frac{1}{C_{12} \cdot l} F \right) \right] \end{cases} \quad (2)$$

We denote  $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} \Delta\varphi_1 \\ \Delta\varphi_2 \end{bmatrix} \in \mathbb{R}^2$ ;  $x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} \omega_{r1} \\ \omega_{r2} \end{bmatrix} \in \mathbb{R}^2$ ;  $x_3 = \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix} = \begin{bmatrix} F_{21} \\ F_{12} \end{bmatrix} \in \mathbb{R}^2$

to obtain the following dynamic equation described in state-space representation:

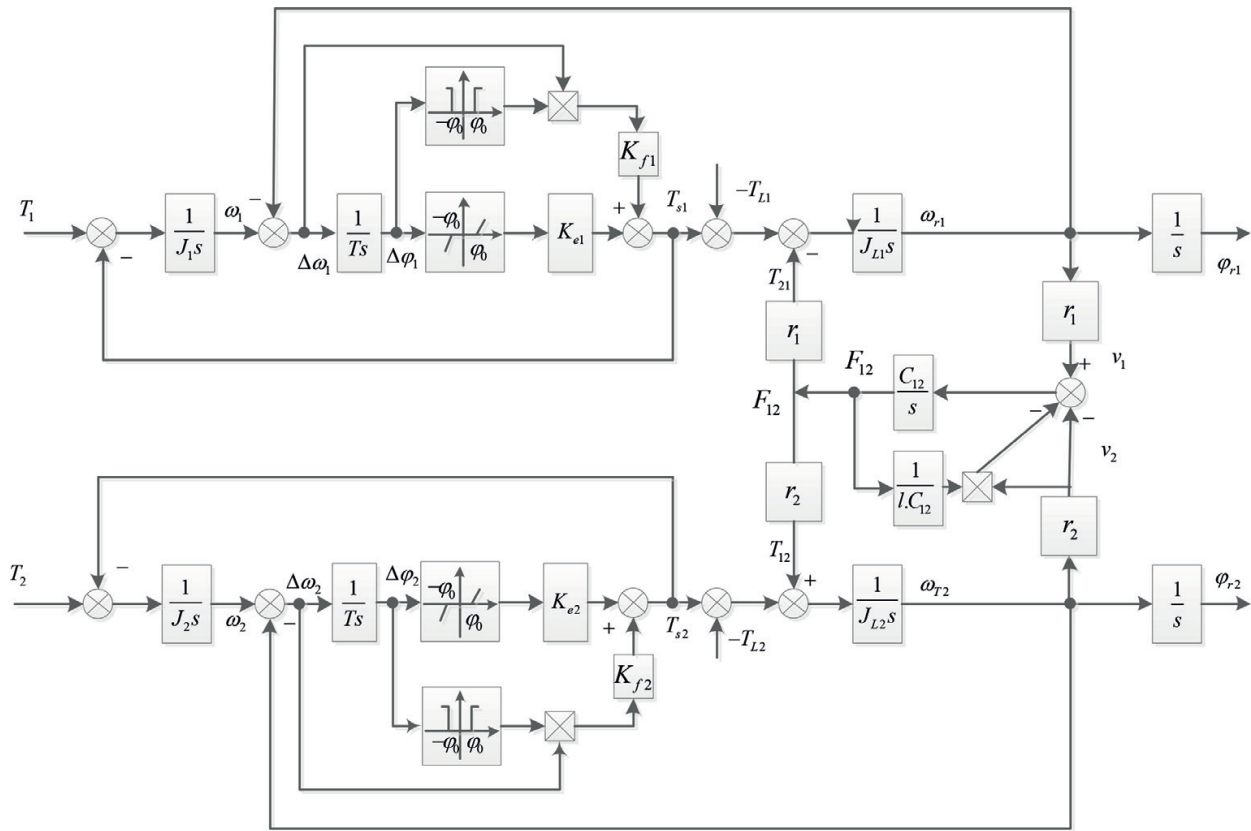


Figure 2. The equivalent diagram of the two-motor drive system.

$$\begin{cases} \dot{x}_1 = \frac{1}{T}(u - x_2) \\ \dot{x}_2 = J_L \left[ \frac{1}{K_{TC}} f_1(x_1) + K_C(u - x_2) f_2(x_1) - (T_L + r \cdot x_3) \right] \\ \dot{x}_3 = C_{12} \left[ r_1 x_{21} - r_2 x_{22} \left( 1 + \frac{1}{C_{12} \cdot l} x_3 \right) \right] \\ y = x_3 \end{cases} \tag{3}$$

**Remark 1:**

The dynamic Eqs. (2) and (3) and **Figures 1** and **2** are described by the effect of friction, backlash, and elastic and pointed out the nonlinear property of multimotor systems.

The control objective is to find the synchronous speeds  $u = (u_1, u_2) = (\omega_1, \omega_2) \in \mathbb{R}^2$  to obtain that the desired value are tracked by tensions in the presence of friction and elastic. In order to implement this work, a new scheme is proposed to design an output feedback controller involving a high-gain observer and a sliding mode control law. Moreover, the effectiveness to satisfy the separation principle is pointed out in multimotor control system.

### 3. Observer design

As mentioned above, the main motivation of the work is to find an equivalent high-gain observer for the class of multimotor systems. In the following, one will present the proposed high-gain observer to estimate the tension in this system and provide a full analysis of observation error convergence.

MISO systems are described as follows:

$$\begin{cases} \frac{d}{dt}x = Ax + \gamma(x, u, y) + \varphi(u, y) \\ y = c^T x + \xi(u) \end{cases} \quad (4)$$

where  $\gamma(x, u, y)$  satisfy the global Lipschitz condition  $|\gamma(x, u) - \gamma(\hat{x}, u)| \leq \alpha|x - \hat{x}|$  and

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & & 0 \\ & & & \dots & \vdots \\ & & & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}, c^T = [1, 0, \dots, 0].$$

**Lemma 1** [5]: The classical high-gain observer is pointed out by the following equations:

$$\frac{d}{dt}\hat{x} = A\hat{x} + L(y - c^T\hat{x}) + \gamma(\hat{x}, u) \quad (5)$$

where  $L = \begin{bmatrix} h_1\varepsilon^{-1} \\ \vdots \\ h_n\varepsilon^{-n} \end{bmatrix}$  and  $\varepsilon$  is a small enough positive number and  $h_n, h_{n-1}, \dots, h_1$  are coefficients of a Hurwitz polynomial (6)

$$P(s) = h_n + h_{n-1}s + \dots + h_1s^{n-1} + s^n \quad (6)$$

**Remark 1:**

The classical high-gain observer is the next development of Lipschitz observer with the additional contents of the coefficient  $\varepsilon$  to obtain  $a < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}$  without solving the LMIs problem.

However, the previous observer (5) is only suitable to systems with one output. In order to design for multi-output systems, Farza et al. develop many observers for a class of MIMO nonlinear systems [6–9]. Based on the proposed high-gain observer that is pointed out in (7) [4], we obtain the observer (8) for multimotor systems (3):

$$\dot{\hat{x}} = f(u, s, \hat{x}) - \begin{pmatrix} \theta C_1^q I_{n_1} \\ \theta^2 C_2^q \left[ \frac{\partial f_1}{\partial x^2}(u, s, x) \right]^+ \\ \vdots \\ \theta^q C_q^q \left[ \prod_{i=1}^{q-1} \frac{\partial f_k}{\partial x^{k+1}}(u, s, x) \right]^+ \end{pmatrix} \bar{C}(\hat{x} - x) \quad (7)$$

$$\begin{cases} \dot{\hat{x}}_1 = \frac{1}{T}(u - \hat{x}_2) - 3\theta(\hat{x}_3 - x_3) \\ \dot{\hat{x}}_2 = J_L \left[ \frac{1}{K_{TC}} f_1(\hat{x}_1) + K_C(u - \hat{x}_2) f_2(\hat{x}_1) - (T_L + r \cdot \hat{x}_3) \right] + \frac{\theta^2}{T}(\hat{x}_3 - x_3) \\ \dot{\hat{x}}_3 = C_{12} \left[ r_1 \hat{x}_{21} - r_2 \hat{x}_{22} \left( 1 + \frac{1}{C_{12} \cdot l} \hat{x}_3 \right) \right] + r J_L \theta^3 (\hat{x}_3 - x_3) \\ y = x_3 \end{cases} \quad (8)$$

**Remark 2:** The convergence of observer error based on the high-gain observer (8) is pointed out in [3, 4].

#### 4. Sliding mode control

In this section, the main work is to find a state feedback control law based on the sliding mode control technique for the class of multimotor systems.

Nonlinear systems are described as follows:

$$\frac{d}{dt}x = Ax + B(u + u_d(x, t)) \quad (9)$$

where  $u_d(x, t)$  is the nonlinear term in system.

**Lemma 2** [10]: The sliding mode controller is described as follows

$$u = -[S \cdot Ax + \beta \text{sgn}(\sigma)] \quad (10)$$

based on the sliding surface:

$$\begin{cases} \{x : \sigma = Sx = 0\}, \\ S = (B^T X^{-1} B)^{-1} B^T X^{-1} X \end{cases} \quad (11)$$

with  $X$  is satisfied, the LMI problem as follows:

$$II^T (AX + XA^T) II < 0, X > 0, II = [1 \ 0 \ 0]^T \quad (12)$$

**Remark 3.** We obtain the sliding mode control for multimotor systems (2) based on Lemma 3 because it belongs to the class of systems (9).

## 5. Observer-integrated sliding mode control

After we obtain the output feedback control law combined between sliding mode controller and high-gain observer, the main work is to point out the ability to obtain the separation principle of the proposed solution.

Consider the nonlinear systems:

$$\begin{cases} \frac{d}{dt}x = Ax + f(x, u, t) \\ y = Cx \end{cases} \quad (13)$$

with  $f(x, u, t)$  satisfying the global Lipschitz condition

$$|f(x, u, t) - f(x', u, t)| \leq a|x - x'| \quad (\forall x, x', u) \quad (14)$$

**Lemma 3** [5]: If there exists a control Lyapunov function  $V(x)$  and the corresponding control input  $u = r(x)$  satisfy

$$\left| \frac{\partial V}{\partial x} [f(x, u, t) - f(x', u, t)] \right| \leq b|x - x'|^2, \quad \forall x, x' > 0 \quad (15)$$

Then the output feedback control law using the observer (16) and (17) and the state feedback controller  $u = r(x)$  is described as above:

$$\frac{d\hat{x}}{dt} = A\hat{x} + f(\hat{x}, u, t) + L(y - C\hat{x}) \quad (16)$$

where  $L$  is the matrix is satisfied all the real parts of eigenvalues of  $(A - LC)$  that is negative and matrices  $P, Q$  satisfy the Lyapunov equation

$$(A - LC)^T P + P(A - LC) = -Q \quad (17)$$

and

$$a < \frac{\lambda_{\min}(Q) - b}{2\lambda_{\max}(P)} \quad (18)$$

**Theorem 1.** The whole system (**Figure 1**) is asymptotically stable by the output feedback control law with the high-gain observer (8) and the nonlinear state feedback controller (10).



**Proof:** Using the Lyapunov candidate function  $V(x) = x^T P x$ , we obtain the inequality (15) based on  $x$  being the state trajectory of multimotor system (5).

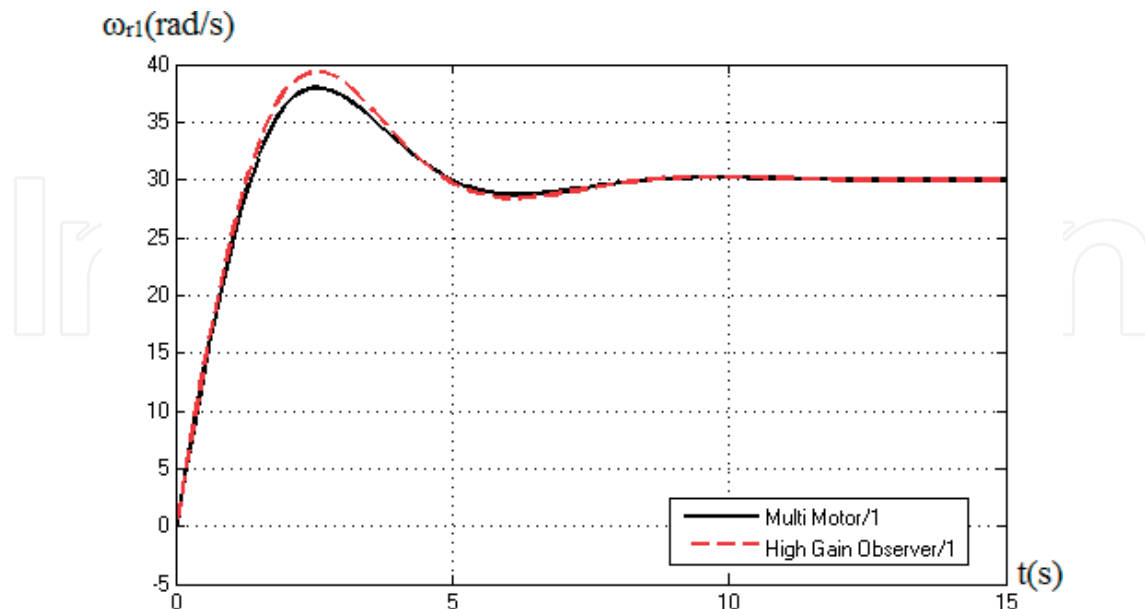
**Remark 4.** This result is a development from the results in [1–4], because the separation principle of output feedback controller has not been implemented in previous researches.

## 6. Simulation results

In this section, we consider several simulation results to demonstrate the effectiveness of the proposed output feedback control law based on the two-motor system as shown in **Table 2**. **Figures 3** and **4** show the high-performance behavior of velocity based on the proposed high-gain

$n_{p1}$	4
$J_1$	50 kgm <sup>2</sup>
$L_{r1}$	0.2 H
$T_{L1}$	30 Nm
$n_{p2}$	4
$J_2$	55 kgm <sup>2</sup>
$L_{r2}$	0.3 H
$T_{L2}$	25 Nm

**Table 2.** Multimotor system parameters.



**Figure 3.** The velocity of motor 1 and estimation of it.

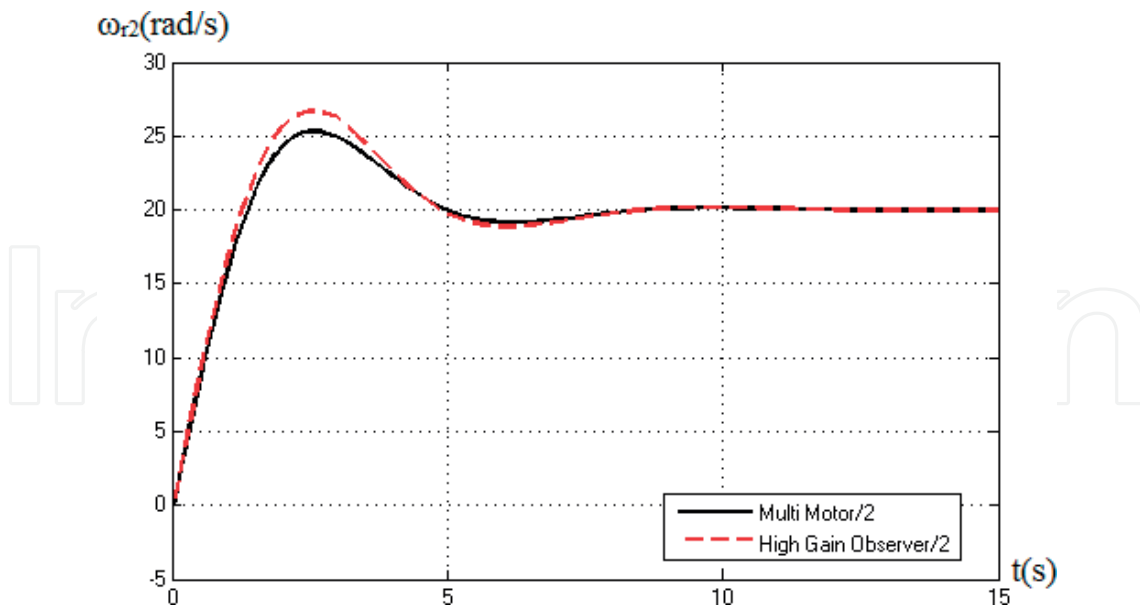


Figure 4. The velocity of motor 2 and estimation of it.

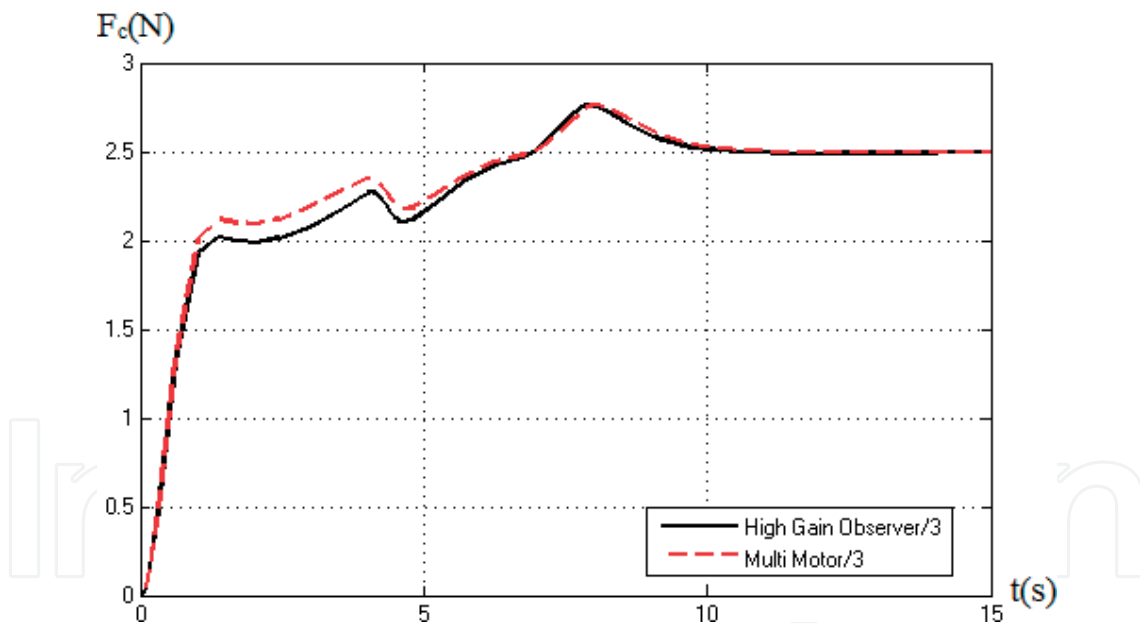


Figure 5. The tension of system and estimation of it.

observer. Moreover, we obtain the high tracking performance of tension in the presence of friction and elastic (Figure 5). Furthermore, Figures 6 and 7 show the tracking performance behavior of velocity based on adaptive sliding mode control law in the presence of disturbance (Figure 9). Figures 8 and 10 show the high tracking performance behavior of velocity based on adaptive sliding mode control law without disturbance.

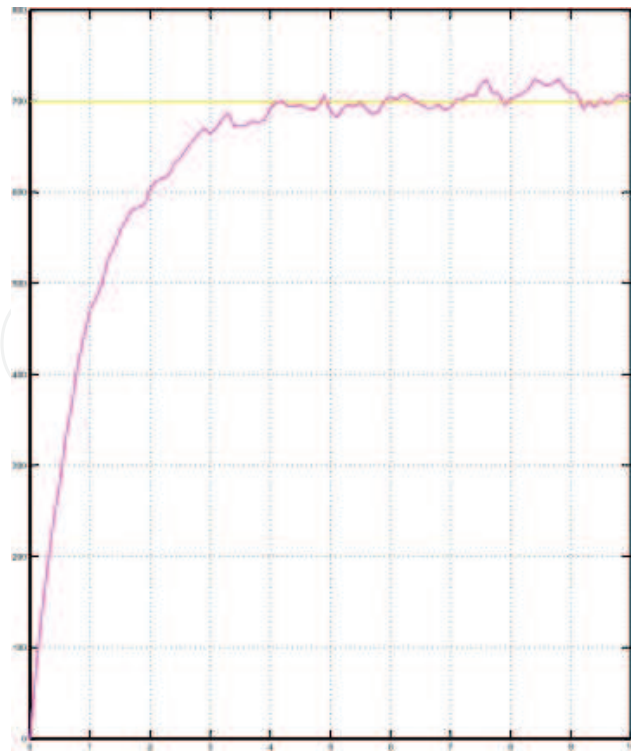


Figure 6. The behavior of the first motor's speed in the presence of disturbance.

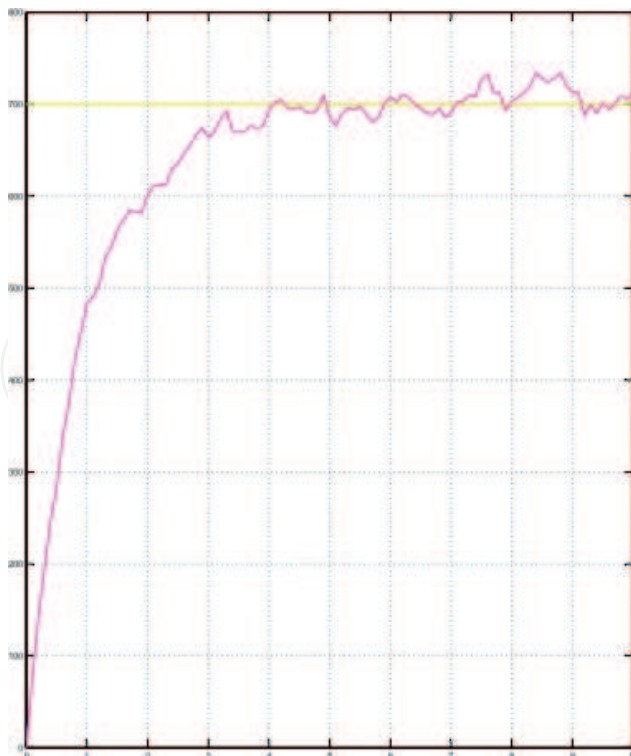
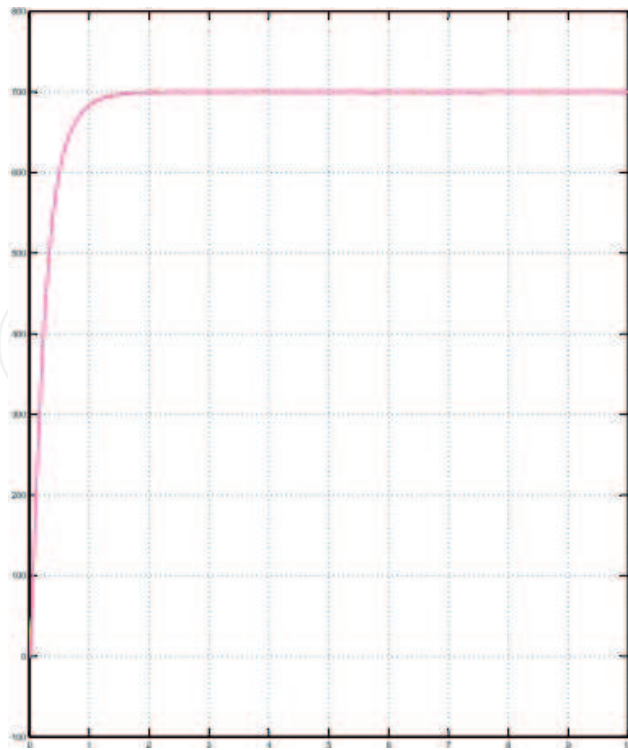
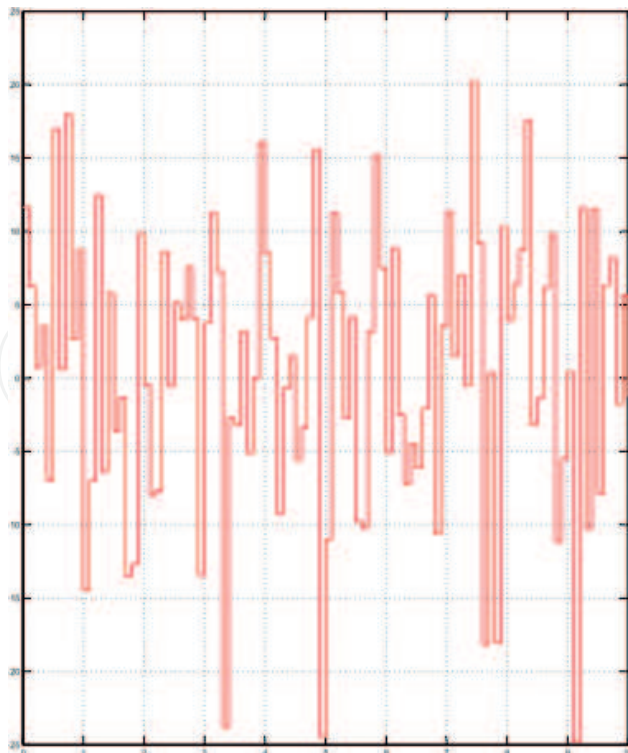


Figure 7. The behavior of the second motor's speed in the presence of disturbance.



**Figure 8.** The behavior of the first motor's speed without disturbance.



**Figure 9.** The behavior of the second motor's speed without disturbance.



Figure 10. The system being affected by disturbance.

## 7. Conclusions

This chapter described an output feedback control law based on the combination between high-gain observer and sliding mode control for the two-motor system in the presence of elastic, backlash, and friction. The proposed control law allows to obtain the separation principle in the presence of friction and elastic due to the tuning of parameter in proposed high-gain observer. The effectiveness of the proposed control scheme was pointed out by theory analysis and simulation results.

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