

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

## 4,800

Open access books available

## 122,000

International authors and editors

## 135M

Downloads

Our authors are among the

## 154

Countries delivered to

## TOP 1%

most cited scientists

## 12.2%

Contributors from top 500 universities

**WEB OF SCIENCE™**Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.

For more information visit [www.intechopen.com](http://www.intechopen.com)

---

# Nonlinear Control of Flexible Two-Dimensional Overhead Cranes

---

Tung Lam Nguyen and Minh Duc Duong

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/intechopen.71657>

---

## Abstract

Considering gantry cable as an elastic string having a distributed mass, we constitute a dynamic model for coupled flexural overhead cranes by using the extended Hamilton principle. Two kinds of nonlinear controllers are proposed based on the Lyapunov stability and its improved version entitled barrier Lyapunov candidate to maintain payload motion in a certain defined range. With such a continuously distributed model, the finite difference method is utilized to numerically simulate the control system. The results show that the controllers work well and the crane system is stabilized.

**Keywords:** overhead cranes, finite difference method, Lyapunov stability, distributed modeling

---

## 1. Introduction

Nowadays, cargo transportation plays an important role in many industrial fields. For carrying the cargo in short distance or small area, such as in automotive factories and shipyards, the overhead cranes are naturally applied. To increase productivity, the overhead cranes today are required in high-speed operation. However, the fast motion of overhead cranes usually leads to the large swings of cargo and non-precise movements of trolley and bridge. The faster the cargo transport is, the larger the cargo swings. This makes dangerous and unsafe situation during the operating process. The crane itself and the concerning equipment in the factory can be damaged without proper control strategies.

In recent decades, the control problems of overhead cranes in both theory and practice have attracted many researchers. Various kinds of crane control techniques have been applied from classical methods such as linear control [1], nonlinear control [2, 5, 6], optimal approach [7], adaptive algorithms [8, 9] to modern techniques such as fuzzy logic [3, 4, 10], neural network [11], command shaping [12], and so on.

---

The abovementioned researches deal with crane motion modeled as pendulum or multi-section pendulum systems. As a result, their dynamics are described as an ordinary differential equation or a system of ordinary differential equations. In practice, the crane rope exhibits a certain degree of flexibility; hence, the equation of motions of the gantry crane with flexible rope is represented by a set of partial differential and ordinary differential equations. In [13–15], the authors successfully design a controller that can stabilize the system with the rope flexibility. Flexible rope also is considered in [16, 17] where coupled longitudinal-transverse motion and 3D model are investigated.

This chapter accesses the modeling and control of overhead cranes according to the other research direction. We construct a distributed model of overhead cranes in which the mass and the flexibility of payload suspending cable are fully taken into account. We utilize the analytical mechanics including Hamilton principle for constructing such the mathematical model. With the received model, we analyze and design two nonlinear control algorithms based on two versions of Lyapunov stability: one is the so-called traditional Lyapunov function and the other is the so-called barrier Lyapunov. Dissimilar to the preceding study [18, 19] whereas the problem of actuated payload positioning system is considered, the proposed controllers track the trolley to destination precisely while keeping the payload swing small during the transport process and absolutely suppressed at the payload destination with control forces exerted at the trolley end of the system. The quality of control system is investigated by numerical simulation. Since the system dynamics is characterized by a distributed mass model, the finite difference method is applied to simulate the system responses in MATLAB® environment.

The chapter content is structured as follows. Section 2 constructs a distributed mass model of overhead cranes. Section 3 analyzes and designs two nonlinear controllers based on Lyapunov direct theory. The analysis of system stability is included. Section 4 numerically simulates the system responses and analyzes the received results. Finally, the remarks and conclusions are shown in Section 5.

## 2. Distributed mass modeling of overhead cranes

Let us constitute a mathematical model for overhead cranes fully considering the flexibility and mass of cable. In other words, payload handling cable with length  $L$  is considered as a distributed mass string with density  $\rho$  (kg/m). An overhead crane with its physical features is depicted in **Figure 1**. The trolley with mass  $M$  (kg) handling the payload  $m$  (kg) moves along  $Ox$  which can induce the payload swing. The force  $F_x$  (N) of motor is created to push the trolley but guaranteeing the payload oscillation as small as possible. The other parameters can be seen in **Figures 1** and **2**.

Before carrying system modeling, we assume that:

1. Moving masses at the trolley end are symmetrical in  $X$  and  $Y$  directions.
2. The gantry moving in  $XY$  plane and the rope length are unchanged.

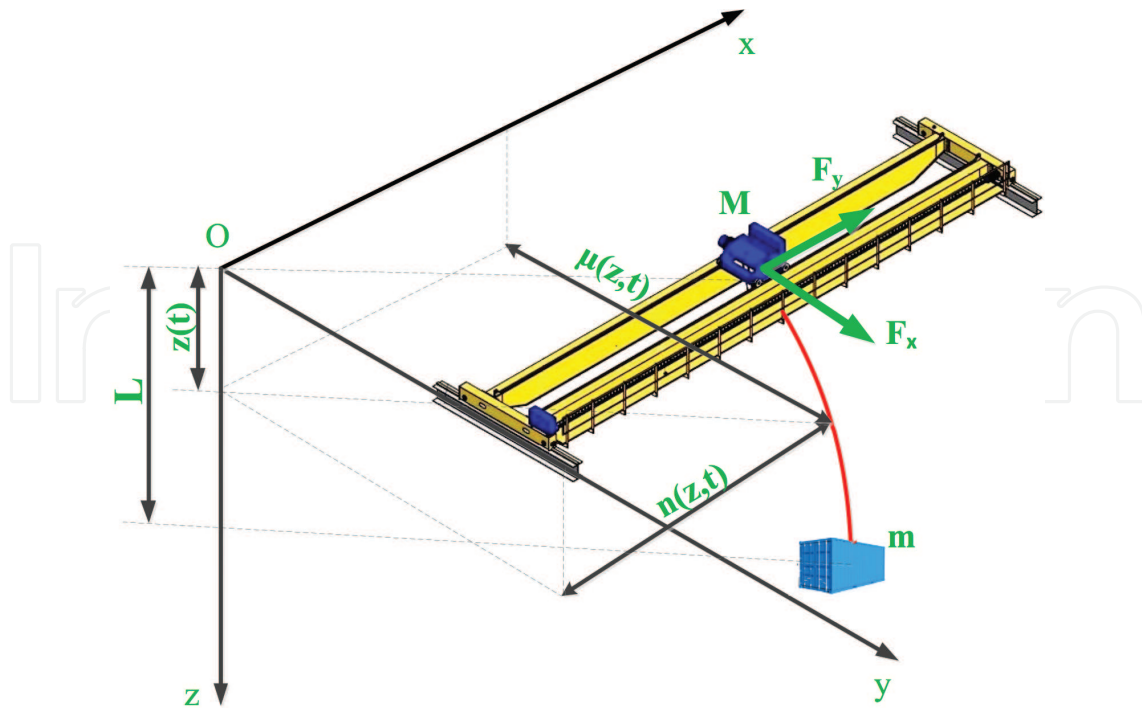


Figure 1. A practical overhead crane.

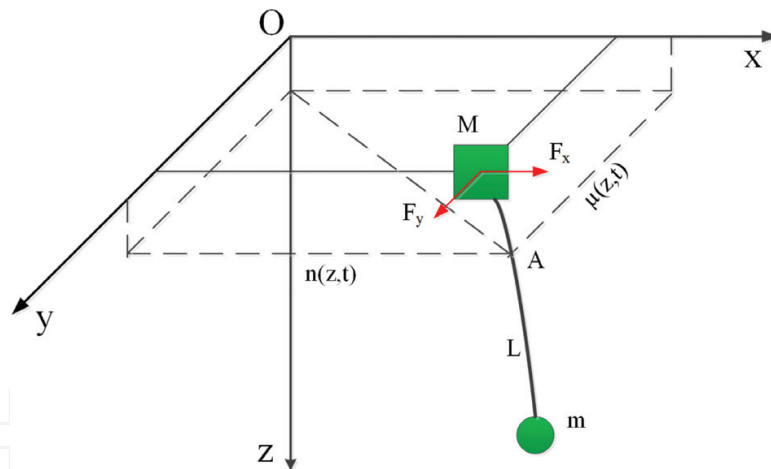


Figure 2. Physical modeling of overhead crane in OXYZ.

3. Friction and external distributed forces are neglected.
4. Longitudinal deformation of the crane rope is negligible.

From this point onward, the argument  $(z, t)$  is dropped whenever it is not confusing and  $(\bullet)_{tr}$ ,  $(\bullet)_{tt}$ ,  $(\bullet)_{tz}$  and  $(\bullet)_{zz}$  are used to denote the first and second time and spatial derivatives of  $(\bullet)$ , respectively. We consider the physical model of an overhead crane as shown in Figure 2. The tension of the handing cable is of the form

$$P = g[\rho(L - z) + m] \quad (1)$$

With the differential derivation along the cable length  $L$ , the potential energy due to the elasticity of cable and gravity is determined by

$$U = \frac{1}{2} \int_0^L P(n_z^2 + \mu_z^2) dz + \frac{1}{2} EA \int_0^L \left[ \frac{1}{2} (n_z^2 + \mu_z^2) \right]^2 + P_0 \quad (2)$$

where  $\frac{1}{2} EA \int_0^L \left[ \frac{1}{2} (n_z^2 + \mu_z^2) \right]^2$  is a potential component due to the axial deformation of the cable. The kinetic energy of system includes those of the trolley, payload, and cable motion described by

$$T = \frac{1}{2} \int_0^L \rho(n_t^2 + \mu_t^2) dz + \frac{1}{2} M(n_t^2(0, t) + \mu_t^2(0, t)) + \frac{1}{2} m(n_t^2(L, t) + \mu_t^2(L, t)) \quad (3)$$

With two force components to move trolley and bridge  $F_x$  and  $F_y$ , the total virtual works of system are in the form of

$$W = F_x n(0) + F_y \mu(0) \quad (4)$$

Using the generalized form of Hamilton principle, one has the following equation:

$$H = \int_{t_1}^{t_2} (\delta T - \delta U + \delta W) dt = 0 \quad (5)$$

in which the small variations of kinematic and potential energies, respectively, are described by

$$\delta T = \delta \left[ \frac{1}{2} \int_0^L \rho(n_t^2 + \mu_t^2) dz + \frac{1}{2} M(n_t^2(0, t) + \mu_t^2(0, t)) + \frac{1}{2} m(n_t^2(L, t) + \mu_t^2(L, t)) \right] \quad (6)$$

$$\delta U = \delta \left[ \frac{1}{2} \int_0^L P(n_z^2 + \mu_z^2) dz \right] + \delta \left\{ \frac{1}{2} EA \int_0^L \left[ \frac{1}{2} (n_z^2 + \mu_z^2) \right]^2 dz \right\} \quad (7)$$

and the small derivation of virtual work is written as

$$\delta W = F_x \delta n(0, t) + F_y \delta \mu(0, t) \quad (8)$$

First, one obtains

$$\delta \left[ \frac{1}{2} \int_0^L \rho(n_t^2 + \mu_t^2) dz \right] - \delta \left[ \frac{1}{2} \int_0^L P(n_z^2 + \mu_z^2) dz \right] - \delta \left\{ \frac{1}{2} EA \int_0^L \left[ \frac{1}{2} (n_z^2 + \mu_z^2) \right]^2 dz \right\}$$

We define  $L_c$  as a multivariable function

$$L_c = \frac{1}{2}\rho(n_t^2 + \mu_t^2) - \frac{1}{2}P(n_z^2 + \mu_z^2) - \frac{1}{2}EA \cdot \frac{1}{4}(n_z^2 + \mu_z^2)^2 = L_c(t : n_t, \mu_t, n_z, \mu_z) \quad (9)$$

and apply the following property:

$$\delta \int_0^L L_c dz = \int_0^L \delta L_c dz$$

with

$$\int_0^L \delta L_c dz = \int_0^L \left( \frac{\partial L_c}{\partial n_t} \delta(n_t) + \frac{\partial L_c}{\partial \mu_t} \delta(\mu_t) + \frac{\partial L_c}{\partial n_z} \delta(n_z) + \frac{\partial L_c}{\partial \mu_z} \delta(\mu_z) \right) dz \quad (10)$$

We calculate the components of (10) using the expressions of partial integration as follows:

$$\int_0^L \frac{\partial L_c}{\partial n_z} \delta(n_z) dz = \frac{\partial L_c}{\partial n_z} \delta(n) \Big|_0^L - \int_0^L \left( \frac{\partial L_c}{\partial n_z} \right)_z \delta(n) dz \quad (11)$$

$$\int_0^L \frac{\partial L_c}{\partial \mu_z} \delta(\mu_z) dz = \frac{\partial L_c}{\partial \mu_z} \delta(\mu) \Big|_0^L - \int_0^L \left( \frac{\partial L_c}{\partial \mu_z} \right)_z \delta(\mu) dz \quad (12)$$

Inserting (11) and (12) into (10) leads to

$$\begin{aligned} \int_0^L \delta L_c dz = & \int_0^L \left[ \frac{\partial L_c}{\partial n_t} \delta(n_t) + \frac{\partial L_c}{\partial \mu_t} \delta(\mu_t) - \left( \frac{\partial L_c}{\partial n_z} \right)_z \delta(n) - \left( \frac{\partial L_c}{\partial \mu_z} \right)_z \delta(\mu) \right] dz \\ & + \frac{\partial L_c}{\partial n_z} \delta(n) \Big|_0^L + \frac{\partial L_c}{\partial \mu_z} \delta(\mu) \Big|_0^L \end{aligned}$$

Integrating the abovementioned equation in term of time side by side, one has  $\int_{t_1}^{t_2} \left( \int_0^L \delta L_c dz \right) dt =$

$$\int_{t_1}^{t_2} \left\{ \int_0^L \left[ \frac{\partial L_c}{\partial n_t} \delta(n_t) + \frac{\partial L_c}{\partial \mu_t} \delta(\mu_t) - \left( \frac{\partial L_c}{\partial n_z} \right)_z \delta(n) - \left( \frac{\partial L_c}{\partial \mu_z} \right)_z \delta(\mu) \right] dz + \frac{\partial L_c}{\partial n_z} \delta(n) \Big|_0^L + \frac{\partial L_c}{\partial \mu_z} \delta(\mu) \Big|_0^L \right\} dt$$

due to  $\frac{\partial L_c}{\partial n_z} \delta(n_t) \Big|_{t_1}^{t_2} = 0$ . Similarly, one has the following results after a series of calculation

$$\int_{t_1}^{t_2} \left[ \int_0^L \frac{\partial L_c}{\partial \mu_t} \delta(\mu_t) dz \right] dt = - \int_{t_1}^{t_2} \left[ \int_0^L \left( \frac{\partial L_c}{\partial \mu_t} \right)_t \delta(\mu) dz \right] dt$$

which yields

$$\begin{aligned} \int_{t_1}^{t_2} \int_0^L \delta L_c dz dt = & \int_{t_1}^{t_2} \left\{ \int_0^L \left[ - \left( \frac{\partial L_c}{\partial n_t} \right)_t \delta n - \left( \frac{\partial L_c}{\partial \mu_t} \right)_t \delta \mu - \left( \frac{\partial L_c}{\partial n_z} \right)_z \delta n - \left( \frac{\partial L_c}{\partial \mu_z} \right)_z \delta \mu \right] dz \right. \\ & \left. + \frac{\partial L_c}{\partial n_z} \delta n \Big|_0^L + \frac{\partial L_c}{\partial \mu_z} \delta \mu \Big|_0^L \right\} dt \quad (13) \end{aligned}$$

Next, let us calculate

$$\delta \left[ \frac{1}{2} M(n_t^2(0, t) + \mu_t^2(0, t)) \right] + \delta \left[ \frac{1}{2} m(n_t^2(L, t) + \mu_t^2(L, t)) \right]$$

with the below notations

$$\delta \left[ \frac{1}{2} M(n_t^2(0, t) + \mu_t^2(0, t)) \right] = Mn_t(0, t)\delta(n_t(0, t)) + M\mu_t(0, t)\delta(\mu_t(0, t)) \quad (14)$$

and

$$\delta \left[ \frac{1}{2} m(n_t^2(L, t) + \mu_t^2(L, t)) \right] = mn_t(L, t)\delta(n_t(L, t)) + m\mu_t(L, t)\delta(\mu_t(L, t)) \quad (15)$$

Substituting (8), (13), (14), and (15) into (5), one obtains

$$\begin{aligned} & \int_{t_1}^{t_2} \left\{ \int_0^L \left[ -\left(\frac{\partial L_c}{\partial n_t}\right)_t \delta(n) - \left(\frac{\partial L_c}{\partial \mu_t}\right)_t \delta(\mu) - \left(\frac{\partial L_c}{\partial n_z}\right)_z \delta(n) - \left(\frac{\partial L_c}{\partial \mu_z}\right)_z \delta(\mu) \right] dz \right. \\ & + \left. \frac{\partial L_c}{\partial n_z} \delta(n) \Big|_0^L + \frac{\partial L_c}{\partial \mu_z} \delta(\mu) \Big|_0^L + Mn_t(0, t)\delta(n_t(0, t)) + M\mu_t(0, t)\delta(\mu_t(0, t)) \right. \\ & \left. + mn_t(L, t)\delta(n_t(L, t)) + m\mu_t(L, t)\delta(\mu_t(L, t)) + F_x \delta n(0, t) + F_y \delta \mu(0, t) \right\} dt = 0 \end{aligned} \quad (16)$$

which is simplified as

$$\left( \int_{t_1}^{t_2} \left\{ \int_0^L \left\{ \left[ -\left(\frac{\partial L_c}{\partial n_t}\right)_t - \left(\frac{\partial L_c}{\partial n_z}\right)_z \right] \delta(n) + \left[ -\left(\frac{\partial L_c}{\partial \mu_t}\right)_t - \left(\frac{\partial L_c}{\partial \mu_z}\right)_z \right] \delta(\mu) \right\} dz \right. \right. \\ \left. \left. + \frac{\partial L_c}{\partial n_z} \delta n(L, t) - \frac{\partial L_c}{\partial n_z} \delta n(0, t) + \frac{\partial L_c}{\partial \mu_z} \delta \mu(L, t) - \frac{\partial L_c}{\partial \mu_z} \delta \mu(0, t) \right. \right. \\ \left. \left. - M \delta n(0, t) n_{tt}(0, t) - M \delta \mu(0, t) \mu_{tt}(0, t) - m \delta n(L, t) n_{tt}(L, t) \right. \right. \\ \left. \left. - m \delta \mu(L, t) \mu_{tt}(L, t) + F_x \delta n(0, t) + F_y \delta \mu(0, t) \right\} dt \right) = 0 \quad (17)$$

Consider the following boundaries at  $x = 0$  and  $x = L$ :

$$\begin{aligned} & \left(\frac{\partial L_c}{\partial n_t}\right)_t + \left(\frac{\partial L_c}{\partial n_z}\right)_z = 0; \quad \left(\frac{\partial L_c}{\partial \mu_t}\right)_t + \left(\frac{\partial L_c}{\partial \mu_z}\right)_z = 0; \quad \frac{\partial L_c}{\partial n_z} - mn_{tt}(L, t) = 0; \\ & \frac{\partial L_c}{\partial \mu_z} - m\mu_{tt}(L, t) = 0; \quad \frac{\partial L_c}{\partial n_z} - Mn_{tt}(0, t) + F_x = 0; \quad \frac{\partial L_c}{\partial \mu_z} - M\mu_{tt}(0, t) + F_y = 0; \end{aligned} \quad (18)$$

which leads to

$$\left(\frac{\partial L_c}{\partial n_t}\right)_t = \rho n_{tt} \quad (19a)$$

and

$$\frac{\partial L_c}{\partial n_z} = -Pn_z - \frac{1}{8}EA(4n_z^3 + 2.2n_z\mu_z^2) \quad (19b)$$

Submitting (18) into (19a) and (19b) in the interval [0, L] of z, one has

$$\rho n_{tt} - (Pn_z)_z - \frac{1}{2}EA(3n_z^2 n_{zz} + n_{zz}\mu_z^2 + 2n_z\mu_z\mu_{zz}) = 0 \quad (20)$$

and

$$\rho\mu_{tt} - \left[ (P\mu_z)_z + \frac{1}{2}EA(3\mu_z^2\mu_{zz} + \mu_{zz}n_z^2 + 2n_z\mu_z n_{zz}) \right] = 0 \quad (21)$$

At boundary condition z = L, one obtains

$$Pn_z(L, t) + \frac{1}{2}EA(n_z^3(L, t) + n_z(L, t)\mu_z^2(L, t)) + mn_{tt}(L, t) = 0 \quad (22)$$

and

$$P\mu_z(L, t) + \frac{1}{2}EA(\mu_z^3(L, t) + \mu_z(L, t)n_z^2(L, t)) + m\mu_{tt}(L, t) = 0 \quad (23)$$

At boundary condition z = 0, one has

$$Pn_z(0, t) + \frac{1}{2}EA(n_z^3(0, t) + n_z\mu_z^2(0, t)) - Mn_{tt}(0, t) + F_x = 0 \quad (24)$$

and

$$P\mu_z(0, t) + \frac{1}{2}EA(\mu_z^3(0, t) + \mu_z(0, t)n_z^2(0, t)) - M\mu_{tt}(0, t) + F_y = 0 \quad (25)$$

In summary, the dynamic behavior of overhead crane governed a set of six nonlinear partial differential Eqs. (20), (21), (22), (23), (24), and (25), as follows:

$$\left\{ \begin{array}{l} \rho n_{tt} - (Pn_z)_z - \frac{1}{2}EA(3n_z^2 n_{zz} + n_{zz}\mu_z^2 + 2n_z\mu_z\mu_{zz}) = 0 \\ \rho\mu_{tt} - (P\mu_z)_z - \frac{1}{2}EA(3\mu_z^2\mu_{zz} + \mu_{zz}n_z^2 + 2n_z\mu_z n_{zz}) = 0 \\ -Pn_z(L, t) - \frac{1}{2}EA(n_z^3(L, t) + n_z(L, t)\mu_z^2(L, t)) - mn_{tt}(L, t) = 0 \\ -P\mu_z(L, t) - \frac{1}{2}EA(\mu_z^3(L, t) + \mu_z(L, t)n_z^2(L, t)) - m\mu_{tt}(L, t) = 0 \\ Pn_z(0, t) + \frac{1}{2}EA(n_z^3(0, t) + n_z(0, t)\mu_z^2(0, t)) - Mn_{tt}(0, t) + F_x = 0 \\ P\mu_z(0, t) + \frac{1}{2}EA(\mu_z^3(0, t) + \mu_z(0, t)n_z^2(0, t)) - M\mu_{tt}(0, t) + F_y = 0 \end{array} \right.$$



The first and the second equations of the above system of equation represent dynamics of the gantry rope. Boundary conditions at load and trolley ends are given in the third, fourth, fifth, and sixth equations, respectively.

### 3. Lyapunov-based control design

Let us construct two nonlinear controllers using a traditional Lyapunov stability and its advanced version. In the first method, the control law is referred from the negative condition of a Lyapunov candidate  $\dot{V} \leq 0$ . In the second method, the Lyapunov function is determined so that it satisfies  $0 < V \leq b$  with  $b > 0$ .

#### 3.1. Conventional Lyapunov controller

The following theorem points out a nonlinear controller designed based on the second method of Lyapunov stability. The proposed control scheme tracks the outputs of a crane system approach to references asymptotically.

**Theorem.** Consider a mass distributed model of overhead crane that is described by six partial differential equations: (20) to (25). The following control law composed of two inputs:

$$F_x = K_a \left[ n_z(0, t) + \frac{EA}{2P(0)} (n_z^3(0, t) + n_z(0, t)\mu_z^2(0, t)) \right] - K_p \left( n(0, t) - \frac{q_d n(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) - K_d n_t(0, t) \quad (26)$$

and

$$F_y = K_a \left[ \mu_z(0, t) + \frac{EA}{2P(0)} (\mu_z^3(0, t) + \mu_z(0, t)n_z^2(0, t)) \right] - K_p \left( \mu(0, t) - \frac{q_d \mu(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) - K_d \mu_t(0, t) \quad (27)$$

pushes all state outputs of dynamic model (20)–(25) to reference  $q_d$  exponentially.

**Proof.** Define a positive Lyapunov candidate as follows:

$$V = \frac{1}{2} \int_0^L \left\{ \rho(n_t^2 + \mu_t^2) + P(n_z^2 + \mu_z^2) + EA \left( \frac{1}{2} (n_z^2 + \mu_z^2) \right)^2 \right\} dz + \frac{MP(0)}{2(P(0) + K_a)} (n_t^2(0, t) + \mu_t^2(0, t)) + \frac{1}{2} m(n_t^2(L, t) + \mu_t^2(L, t)) + \frac{P(0)K_p}{2(P(0) + K_a)} \left( \sqrt{n^2(0, t) + \mu^2(0, t)} - q_d \right)^2 \quad (28)$$

where  $P(0)$  is the tension force of cable at boundary  $x = 0$ .  $K_p$  and  $K_a$  are positive gains.

With the notations that  $\|w\|^2 = \int_0^L \left\{ (n_t^2 + \mu_t^2) + (n_z^2 + \mu_z^2) + (n_z^2 + \mu_z^2)^2 \right\} dz + (n_t^2(0, t) + \mu_t^2(0, t)) + (n_t^2(L, t) + \mu_t^2(L, t)) + (\sqrt{n^2(0, t) + \mu^2(0, t)} - q_d)^2, .$

one has

$$K_{\min} \|w\|^2 \leq V(t) \leq K_{\max} \|w\|^2$$

with

$$K_{\min} = \frac{1}{2} \min \left( \rho, P, \frac{EA}{4}, \frac{MP(0)}{P(0) + K_a}, m, \frac{P(0)K_p}{P(0) + K_a} \right)$$

and

$$K_{\max} = \frac{1}{2} \max \left( \rho, P, \frac{EA}{4}, \frac{MP(0)}{P(0) + K_a}, m, \frac{P(0)K_p}{P(0) + K_a} \right)$$

Differentiating Lyapunov function (28) with respect to time, one obtains

$$\dot{V} = \int_0^L \left[ \rho (n_t n_{tt} + \mu_t \mu_{tt}) + P (n_z n_{zt} + \mu_z \mu_{zt}) + \frac{EA}{2} (n_z^3 n_{tz} + \mu_z^3 \mu_{zt} + n_z n_{zt} \mu_z^2 + \mu_z \mu_{zt} \mu_z^2) \right] dz + \frac{MP(0)}{P(0) + K_a} \left\{ \begin{array}{l} n_t(0, t) n_{tt}(0, t) + \mu_t(0, t) \mu_{tt}(0, t) \\ + \mu(0, t) \mu_t(0, t) + n(0, t) n_t(0, t) \\ + m [n_t(L, t) n_{tt}(L, t) + \mu_t(L, t) \mu_{tt}(L, t)] \\ - \left( \frac{q_d (\mu(0, t) \mu_t(0, t) + n(0, t) n_t(0, t))}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) \end{array} \right\} \quad (29)$$

Let us calculate the components of Lyapunov derivative (29). We refer from (20) and (21) that

$$\int_0^L \rho (n_t n_{tt} + \mu_t \mu_{tt}) dz = \int_0^L \left\{ n_t \left[ (P n_z)_z + \frac{1}{2} EA (3 n_z^2 n_{zz} + n_{zz} \mu_z^2 + 2 n_z \mu_z \mu_{zz}) \right] + \mu_t \left[ (P \mu_z)_z + \frac{1}{2} EA (3 \mu_z^2 \mu_{zz} + \mu_{zz} n_z^2 + 2 n_z \mu_z n_{zz}) \right] \right\} dz \quad (30)$$

Using partial integration

$$\int_0^L n_t (P n_z)_z dz = n_t P n_z \Big|_0^L - \int_0^L P n_z n_{tz} dz$$

and

$$\int_0^L \mu_t (P\mu_z)_z dz = \mu_t P\mu_z \Big|_0^L - \int_0^L P\mu_z \mu_{tz} dz,$$

one obtains the following components of (30) as follows:

$$\int_0^L \frac{EA}{2} n_z^3 n_{tz} dz = \int_0^L \frac{EA}{2} n_z^3 d(n_t) = \frac{EA}{2} n_z^3 n_t \Big|_0^L - \int_0^L n_t \frac{EA}{2} 3n_z^2 n_{zz} dz$$

and

$$\int_0^L \frac{EA}{2} \mu_z^3 \mu_{tz} dz = \frac{EA}{2} \mu_z^3 \mu_t \Big|_0^L - \int_0^L \mu_t \frac{EA}{2} 3\mu_z^2 \mu_{zz} dz$$

Then,

$$\int_0^L \frac{EA}{2} (n_z n_{zt} \mu_z^2) dz = \frac{EA}{2} n_z \mu_z^2 n_t \Big|_0^L - \frac{EA}{2} \int_0^L n_t (n_{zz} \mu_z^2 + 2n_z \mu_z \mu_{zz}) dz$$

and

$$\int_0^L \frac{EA}{2} (\mu_z \mu_{zt} n_z^2) dz = \frac{EA}{2} \mu_z n_z^2 \mu_t \Big|_0^L - \frac{EA}{2} \int_0^L \mu_t (\mu_{zz} n_z^2 + 2n_z \mu_z n_{zz}) dz$$

The Lyapunov derivative (29) now becomes

$$\begin{aligned} \dot{V} = & n_t P n_z \Big|_0^L + \mu_t P \mu_z \Big|_0^L + \frac{EA}{2} n_z^3 n_t \Big|_0^L + \frac{EA}{2} \mu_z^3 \mu_t \Big|_0^L + \frac{EA}{2} n_z \mu_z^2 n_t \Big|_0^L + \frac{EA}{2} \mu_z n_z^2 \mu_t \Big|_0^L \\ & + \frac{MP(0)}{P(0) + K_a} (n_t(0, t) n_{tt}(0, t) + \mu_t(0, t) \mu_{tt}(0, t)) + m (n_t(L, t) n_{tt}(L, t) + \mu_t(L, t) \mu_{tt}(L, t)) \\ & + \frac{P(0)K_p}{P(0) + K_a} (\mu(0, t) \mu_t(0, t) + n(0, t) n_t(0, t)) - \frac{P(0)K_p}{P(0) + K_a} \frac{q_d (\mu(0, t) \mu_t(0, t) + n(0, t) n_t(0, t))}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \end{aligned} \quad (31)$$

Additionally, modification of (24) and (25) yields

$$\begin{aligned} & \frac{MP(0)}{P(0) + K_a} (n_t(0, t)n_{tt}(0, t) + \mu_t(0, t)\mu_{tt}(0, t)) \\ &= \frac{P(0)}{P(0) + K_a} n_t(0, t) \left\{ F_x + \left[ P(0)n_z(0, t) + \frac{EA}{2} (n_z^3(0, t) + n_z(0, t)\mu_z^2(0, t)) \right] \right\} \\ &+ \frac{P(0)}{P(0) + K_a} \mu_t(0, t) \left\{ F_y + \left[ P(0)\mu_z(0, t) + \frac{EA}{2} (\mu_z^3(0, t) + \mu_z(0, t)n_z^2(0, t)) \right] \right\} \end{aligned} \quad (32)$$

Submitting (32) into (31) with a series of calculation, we obtain

$$\begin{aligned} \dot{V} &= \frac{P(0)}{P(0) + K_a} n_t(0, t) \left\{ -K_a \left[ n_z(0, t) + \frac{EA}{2P(0)} (n_z^3(0, t) + n_z(0, t)\mu_z^2(0, t)) \right] \right. \\ &\quad \left. + K_p \left( n(0, t) - \frac{q_d n(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) + F_x \right\} \\ &+ \frac{P(0)}{P(0) + K_a} \mu_t(0, t) \left\{ -K_a \left[ \mu_z(0, t) + \frac{EA}{2P(0)} (\mu_z^3(0, t) + \mu_z(0, t)n_z^2(0, t)) \right] \right. \\ &\quad \left. + K_p \left( \mu(0, t) - \frac{q_d \mu(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) + F_y \right\} \end{aligned} \quad (33)$$

Substituting the control law (26) and (27) into (33) leads the Lyapunov function to

$$\dot{V} = -\frac{P(0)K_d}{P(0) + K_a} n_t^2(0, t) - \frac{P(0)K_d}{P(0) + K_a} \mu_t^2(0, t) \leq 0 \quad (34)$$

With the negative definition of expression (34), we can conclude that the system is now exponential stability.

### 3.2. Barrier Lyapunov controller

We utilize an improved version of Lyapunov stability to design a control law for overhead cranes. The Lyapunov function is chosen so that its derivative is smaller than a positive constant. By this way, the Lyapunov candidate is selected similar to Eq. (28) but supplementing derivation of payload position  $\frac{1}{2} \frac{P(0)}{P(0)+K_a} \ln \left( \frac{k_{b1}^2}{k_{b1}^2 - z_1^2} \right)$ . A modified version of Lyapunov candidate is the so-called barrier Lyapunov  $V_1(t)$  being in the form of

$$\begin{aligned} V_1 &= \frac{1}{2} \int_0^L \left\{ \rho(n_t^2 + \mu_t^2) + P(n_z^2 + \mu_z^2) + EA \left( \frac{1}{2} (n_z^2 + \mu_z^2) \right)^2 \right\} dz \\ &+ \frac{MP(0)}{2(P(0) + K_a)} (n_t^2(0, t) + \mu_t^2(0, t)) + \frac{1}{2} m(n_t^2(L, t) + \mu_t^2(L, t)) \\ &+ \frac{P(0)K_p}{2(P(0) + K_a)} \left( \sqrt{n^2(0, t) + \mu^2(0, t)} - q_d \right)^2 + \frac{1}{2} \frac{P(0)}{P(0) + K_a} \ln \left( \frac{k_{b1}^2}{k_{b1}^2 - z_1^2} \right) \end{aligned} \quad (35)$$

where  $z_1 = \sqrt{n^2(L, t) + \mu^2(L, t)} - \sqrt{n^2(0, t) + \mu^2(0, t)}$  is relative position of payload in comparison with that of trolley.  $k_{b1}$  is a positive gain satisfying condition  $k_{b1} > |z_1|$ . The modification of (35) leads to

$$\begin{aligned} \dot{V}_1 = & \frac{P(0)}{P(0) + K_a} n_t(0, t) \left\{ F_x - K_a \left[ n_z(0, t) + \frac{EA}{2.P(0)} (n_z^3(0, t) + n_z(0, t)\mu_z^2(0, t)) \right] \right. \\ & \left. + K_p \left( n(0, t) - \frac{q_d n(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) \right\} \\ & + \frac{P(0)}{P(0) + K_a} \mu_t(0, t) \left\{ F_y - K_a \left[ \mu_z(0, t) + \frac{EA}{2.P(0)} (\mu_z^3(0, t) + \mu_z(0, t)n_z^2(0, t)) \right] \right. \\ & \left. + K_p \left( \mu(0, t) - \frac{q_d \mu(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) \right\} + \frac{P(0)}{P(0) + K_a} \frac{z_1(z_1)_t}{k_{b1}^2 - z_1^2} \end{aligned} \quad (36)$$

Applying the following inequality

$$|(z_1)_t| \leq K \sqrt{n_t^2(0, t) + \mu_t^2(0, t)}$$

or

$$z_1(z_1)_t \leq |z_1(z_1)_t| = |z_1| |(z_1)_t| \leq k_{b1} K \sqrt{n_t^2(0, t) + \mu_t^2(0, t)}$$

with K being positive constant leads to

$$\frac{P(0)}{P(0) + K_a} \frac{z_1(z_1)_t}{k_{b1}^2 - z_1^2} \leq \frac{P(0)}{P(0) + K_a} \frac{1}{k_{b1}^2 - z_1^2} k_{b1} K \sqrt{n_t^2(0, t) + \mu_t^2(0, t)} \quad (37)$$

Inserting (37) into (36) yields

$$\begin{aligned} \dot{V}_1 \leq & \frac{P(0)n_t(0, t)}{P(0) + K_a} \left\{ F_x - K_a \left[ n_z(0, t) + \frac{EA}{2.P(0)} (n_z^3(0, t) + n_z(0, t)\mu_z^2(0, t)) \right] \right. \\ & \left. + K_p \left( n(0, t) - \frac{q_d n(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) \right\} + K_p \left( \mu(0, t) - \frac{q_d \mu(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) \\ & + \frac{P(0)\mu_t(0, t)}{P(0) + K_a} \left\{ F_y - K_a \left[ \mu_z(0, t) + \frac{EA}{2.P(0)} (\mu_z^3(0, t) + \mu_z(0, t)n_z^2(0, t)) \right] \right. \\ & \left. + \frac{P(0)k_{b1}K}{(P(0) + K_a)(k_{b1}^2 - z_1^2)} \sqrt{n_t^2(0, t) + \mu_t^2(0, t)} \right\} \end{aligned} \quad (38)$$

Inserting the following inequality

$$\sqrt{n_t^2(0, t) + \mu_t^2(0, t)} \leq |n_t(0, t)| + |\mu_t(0, t)|$$

or

$$\sqrt{n_t^2(0, t) + \mu_t^2(0, t)} \leq n_t(0, t) \operatorname{sgn}(n_t(0, t)) + \mu_t(0, t) \operatorname{sgn}(\mu_t(0, t))$$

into (38), one obtains

$$\begin{aligned} \dot{V}_1 \leq & \frac{P(0)}{P(0) + K_a} n_t(0, t) \left\{ F_x - K_a \left[ n_z(0, t) + \frac{EA}{2P(0)} (n_z^3(0, t) + n_z(0, t) \mu_z^2(0, t)) \right] \right. \\ & \left. + K_p \left( n(0, t) - \frac{q_d n(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) + \frac{1}{k_{b1}^2 - z_1^2} k_{b1} K \operatorname{sgn}(n_t(0, t)) \right\} \\ & + \frac{P(0)}{P(0) + K_a} \mu_t(0, t) \left\{ F_y - K_a \left[ \mu_z(0, t) + \frac{EA}{2.P(0)} (\mu_z^3(0, t) + \mu_z(0, t) n_z^2(0, t)) \right] \right. \\ & \left. + K_p \left( \mu(0, t) - \frac{q_d \mu(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) + \frac{1}{k_{b1}^2 - z_1^2} k_{b1} K \operatorname{sgn}(\mu_t(0, t)) \right\} \end{aligned} \quad (39)$$

To force the Lyapunov differentiation being negative, the control law with two components is structured as

$$\begin{aligned} F_x = & K_a \left[ n_z(0, t) + \frac{EA}{2.P(0)} (n_z^3(0, t) + n_z(0, t) \mu_z^2(0, t)) \right] - K_d n_t(0, t) \\ & - K_p \left( n(0, t) - \frac{q_d n(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) - \frac{1}{k_{b1}^2 - z_1^2} k_{b1} K \operatorname{sgn}(n_t(0, t)) \end{aligned} \quad (40)$$

and

$$\begin{aligned} F_y = & K_a \left[ \mu_z(0, t) + \frac{EA}{2.P(0)} (\mu_z^3(0, t) + \mu_z(0, t) n_z^2(0, t)) \right] - K_d \mu_t(0, t) \\ & - K_p \left( \mu(0, t) - \frac{q_d \mu(0, t)}{\sqrt{\mu^2(0, t) + n^2(0, t)}} \right) - \frac{1}{k_{b1}^2 - z_1^2} k_{b1} K \operatorname{sgn}(\mu_t(0, t)) \end{aligned} \quad (41)$$

which leads the Eq. (31) to

$$\dot{V}_1 \leq -\frac{P(0)K_d}{P(0) + K_a} (n_t^2(0, t) + \mu_t^2(0, t)) \leq 0 \quad (42)$$

for every positive gains  $K_d > 0$  and  $K_a > 0$ . This implies that  $V \leq V(0)$ . Hence, the system is now asymptotical stability.

## 4. Simulation and results

Consider the case that only the trolley motion is activated, we numerically simulate the distributed system dynamics (20)–(25) driven by either conventional Lyapunov-based input or barrier Lyapunov-based law. The finite difference method is applied for programming the control system in MATLAB environment. The system parameters used in simulation are composed of

$$m = 5 \text{ kg}; M = 1 \text{ kg}; L = 3, 6, 9 \text{ m}; K_d = 200; K_p = 5; K_d = 42;$$

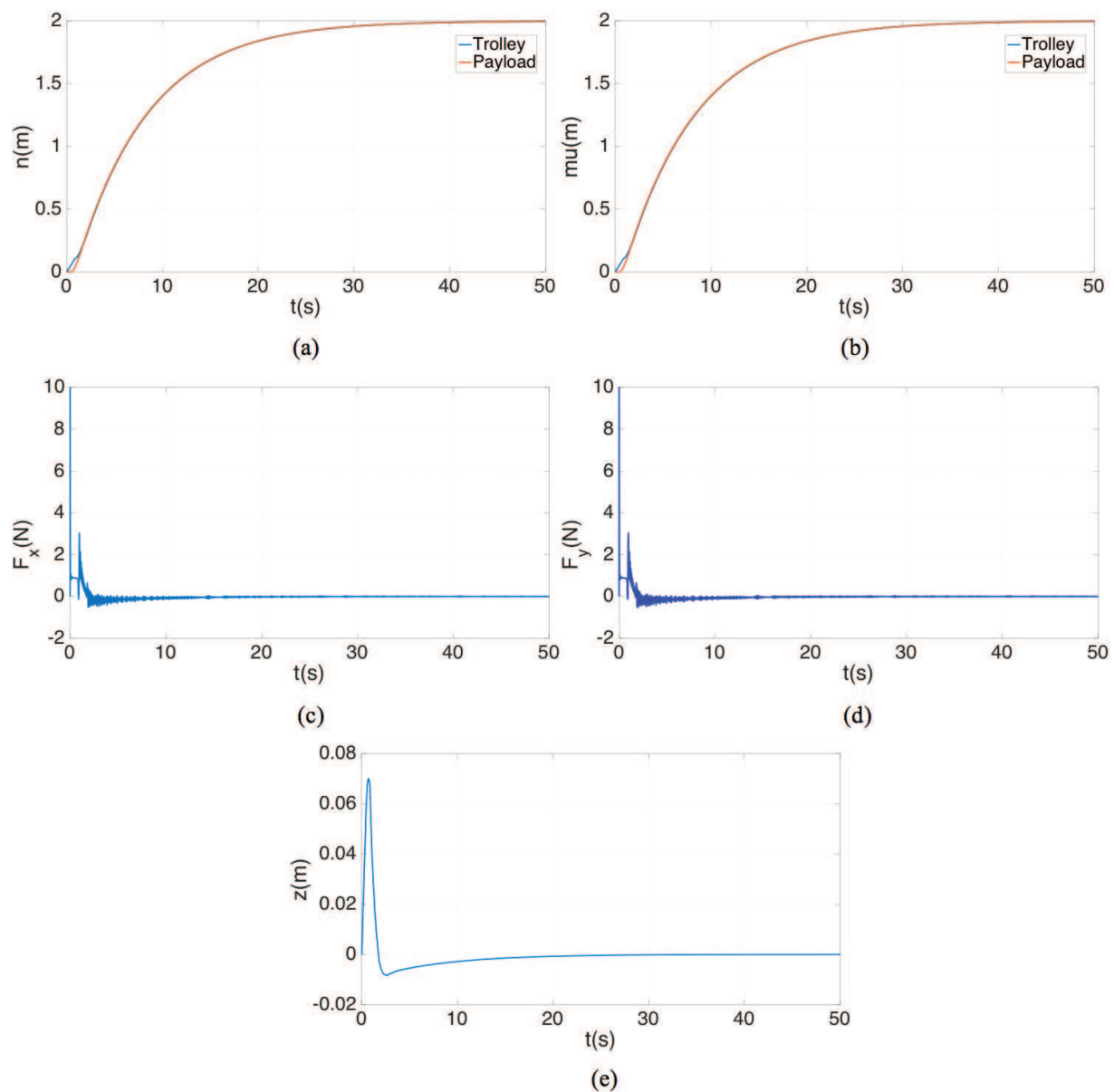
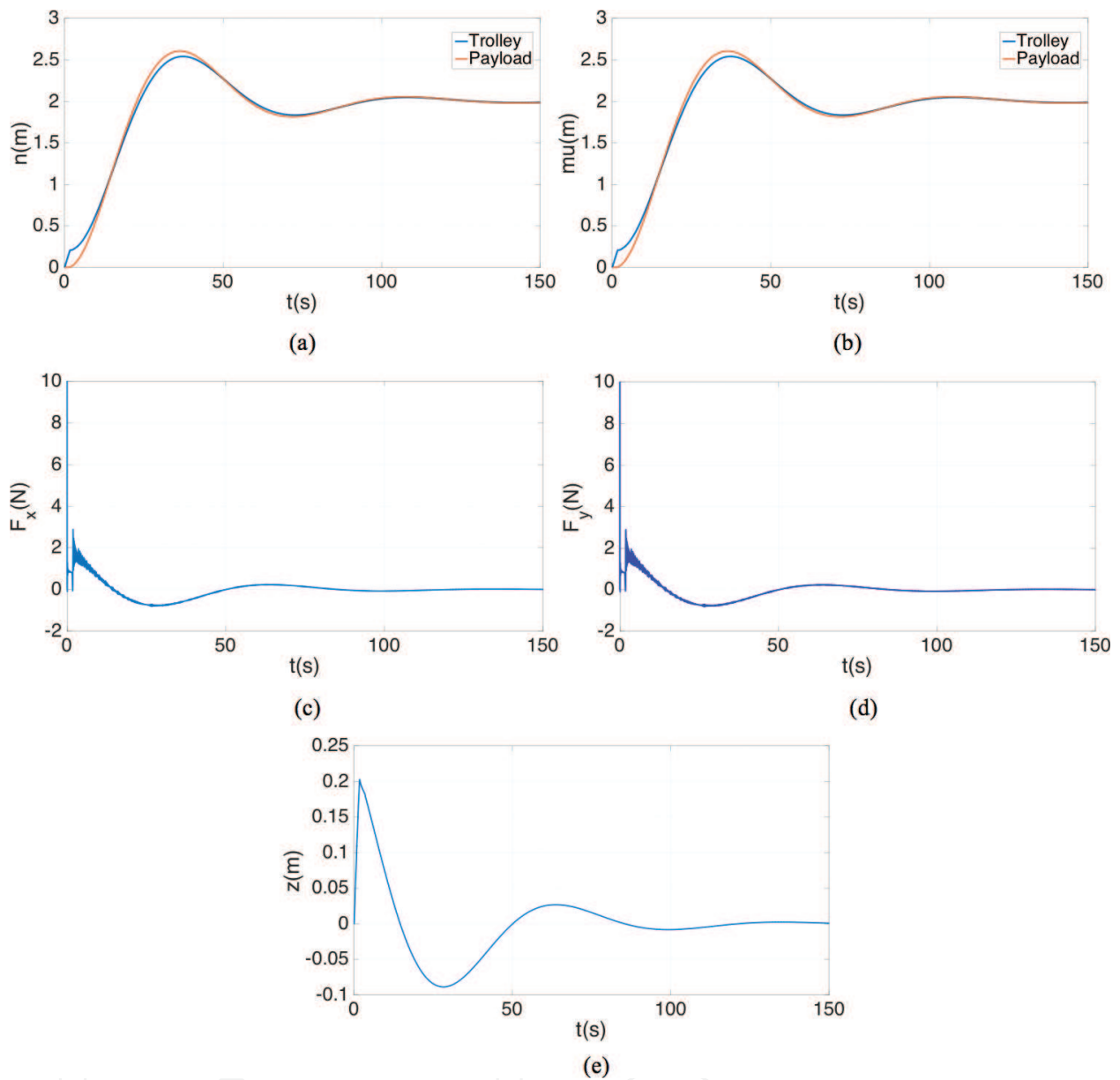


Figure 3. System responses in the case of  $L = 3$  m and  $m = 3$  kg.



**Figure 4.** System responses in the case of  $L = 6$  m and  $m = 6$  kg.

The simulation results are depicted in **Figures 3–6**. Trolley and payload approach to destination  $q_d = 2$  m precisely and speedy without maximum overshoots. The payload swing stays in a small region during the transient state and absolutely suppressed at steady state (or payload destination). However, the longer length of cable is, the larger the payload swings. The system responses show the robustness in the face of parametric uncertainty. Despite the large variation of cable length, the system responses still kept consistency as shown in **Figures 3–5**. It can be seen from **Figure 6** that with the application of the barrier Lyapunov function, payload fluctuation is controlled in an area defined by  $k_b$ . Because the motion of the trolley in X and Y directions is forced to travel the same distance to reach the desired location, system responses in X and Y directions are similar.



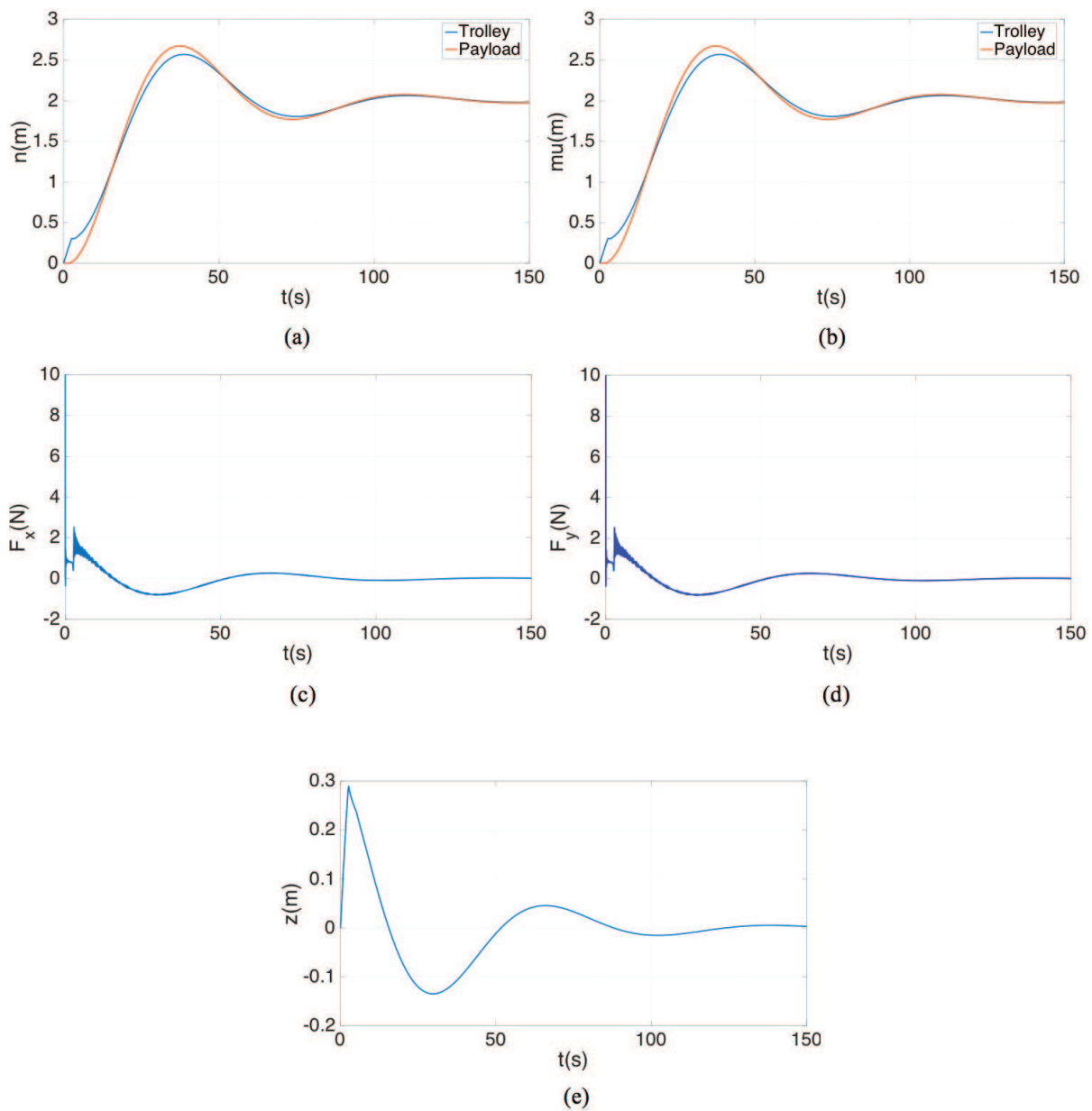
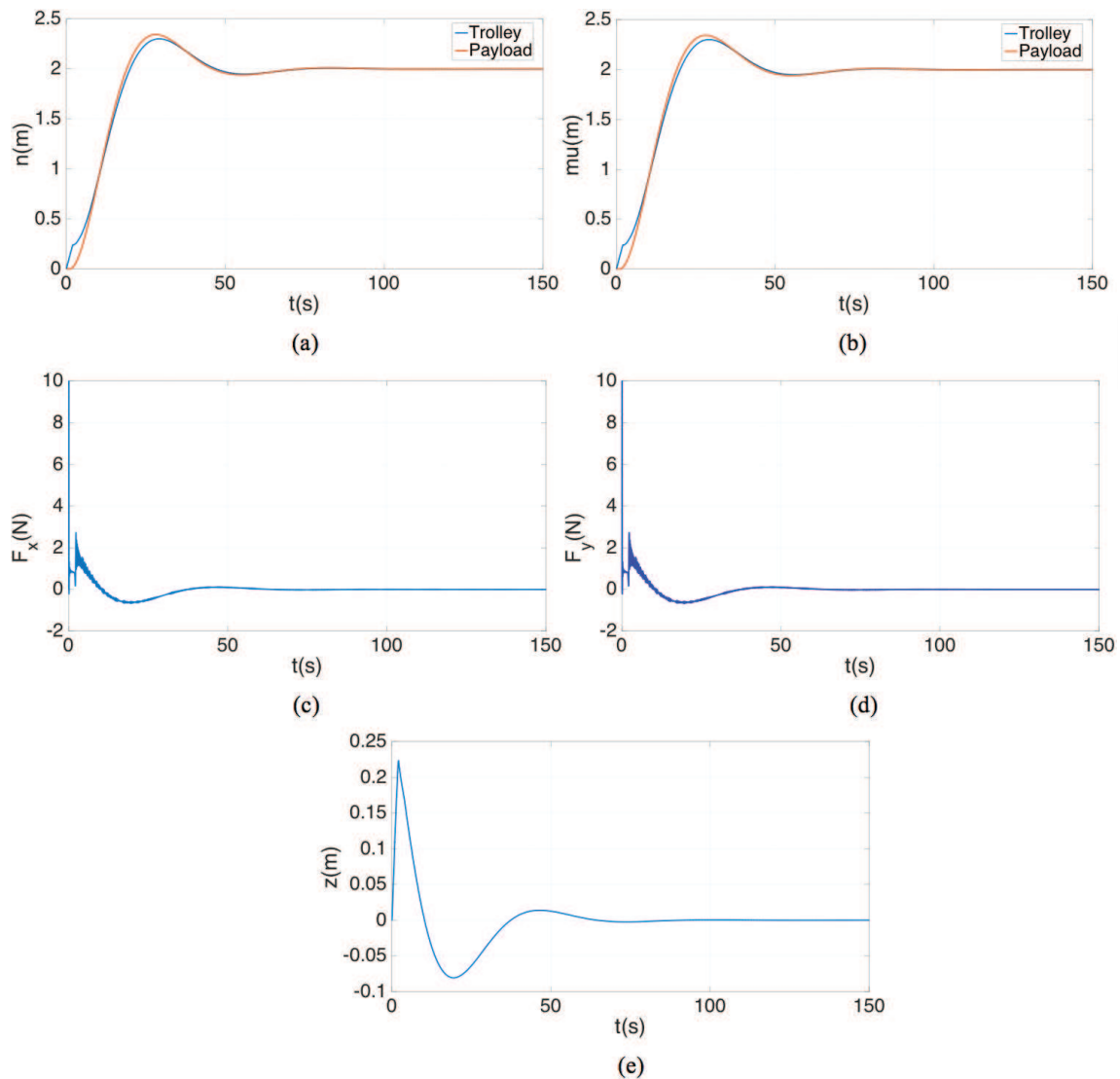


Figure 5. System responses in the case of  $L = 9$  m and  $m = 9$  kg with conventional Lyapunov function approach.

## 5. Conclusions

The dynamic model of overhead crane with distributed mass and elasticity of handling cable is formulated using the extended Hamilton's principle. Based on the model, we successfully analyzed and designed two nonlinear robust controllers using two versions of Lyapunov candidate functions. The first can steer the payload to the desired location, while the second can maintain payload fluctuation in a defined span. The proposed controllers well stabilize all



**Figure 6.** System responses in the case of  $L = 9$  m and  $m = 9$  kg with barrier Lyapunov function approach.

system responses despite the large variation of cable length and payload weight. Enhancing for 3D motion with carrying rope length will be proposed in the future studies.

## Author details

Tung Lam Nguyen\* and Minh Duc Duong

\*Address all correspondence to: lam.nguyentung@hust.edu.vn

Department of Industrial Automation, School of Electrical Engineering, Hanoi University of Science and Technology, Hanoi, Vietnam

## References

- [1] Sakawa Y, Sano H. Nonlinear model and linear robust control of overhead travelling cranes. *Nonlinear Analysis, Theory, Methods & Applications*. 1997;**30**(4):2197-2207
- [2] Su SW, Nguyen HT, Jarman R, Zhu J, Lowe DB, McLean PB, Weng K. Model predictive control of gantry crane with input nonlinearity compensation. In: *International Conference on Control, Automation and Systems Engineering*. 2009. pp. 312-316
- [3] Park MS, Chwa D, Hong SK. Antisway tracking control of overhead cranes with system uncertainty and actuator nonlinearity using an adaptive fuzzy sliding-mode control. *IEEE Transactions on Industrial Electronics*. 2008;**55**(11):3972-3984
- [4] Almutairi NB, Zribi M. Sliding mode control of a three-dimensional overhead crane. *Journal of Vibration and Control*. 2009;**15**(11):1679-1730
- [5] Chwa D. Nonlinear tracking control of 3-D overhead cranes against the initial swing angle and the variation of payload weight. *IEEE Transactions on Control Systems Technology*. 2009;**17**(4):876-883
- [6] Moustafa KAF. Reference trajectory tracking of overhead cranes. *Journal of Dynamic Systems, Measurement, and Control*. 2001;**123**(1):139-141
- [7] Sakawa Y, Shindo Y. Optimal control of container cranes. *Automatica*;**18**(3):257-266
- [8] Hua YJ, Shine YK. Adaptive coupling control for overhead crane systems. *Mechatronics*. 2007;**17**(2-3):143-152
- [9] Mizumoto I, Chen T, Ohdaira S, Kumon M, Iwai Z. Adaptive output feedback control of general MIMO systems using multi-rate sampling and its application to a cart-crane system. *Automatica*. 2007;**43**(12):2077-2085
- [10] Benhidjeb A, Gissinger GL. Fuzzy control of an overhead crane performance comparison with classic control. *Control Engineering Practice*. 1995;**3**(12):1687-1696
- [11] Suh JH, Lee JW, Lee YJ, Lee KS. Anti-sway position control of an automated transfer crane based on neural network predictive PID controller. *Journal of Mechanical Science and Technology*. 2005;**19**(2):505-519
- [12] Singhose W, Perter L, Kenison M, Krrikk E. Effects of hoisting on the input shaping control of gantry cranes. *Control Engineering Practice*. 2000;**8**(10):1159-1165
- [13] Joshi S. Position control of a flexible cable gantry crane: Theory and experiment. In: *Proceedings of the American Control Conference*. Vol. June. 1995. pp. 2820-2824
- [14] D'Andréa-Novel B, Coron JM. Exponential stabilization of an overhead crane with flexible cable via a back-stepping approach. *Automatica*. 2000;**36**(4):587-593
- [15] Lodewijks G. Anti-sway control of container cranes as a flexible cable system. In: *Proceedings of the 2004 IEEE International Conference on Control Applications*. 2004. pp. 1564-1569

- [16] Ge SS, Zhang S, He W. Vibration control of a coupled nonlinear string system in transverse and longitudinal directions. *Proceedings of the IEEE Conference on Decision and Control*. 2011;(ii):3742-3747
- [17] Liu J-K, Qin H, He W. Modelling and vibration control for a flexible string system in three-dimensional space. *IET Control Theory & Applications*. 2015;9(16):2387-2394
- [18] He W, Zhang S, Ge SS. Adaptive control of a flexible crane system with the boundary output constraint. *IEEE Transactions on Industrial Electronics*. 2014;61(8):4126-4133
- [19] Tee KP, Ge SS, Tay EH. Barrier Lyapunov functions for the control of output constrained nonlinear systems. *Automatica*. 2009;45(4):918-927

