Chapter 3

Dispersion Properties of TM and TE Modes of Gyrotropic Magnetophotonic Crystals

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Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/intechopen.71273

Abstract

This chapter discusses the propagation of TM and TE waves in the one-dimensional gyrotropic magnetophotonic crystals with ferrite and plasma-like layers. Elements of the transfer matrix are calculated in closed analytical form on the base of electrodynamic problem rigorous solution for arbitrary location of the gyrotropic elements on the structure period. Dispersion equation of the layered periodic structure with gyrotropic elements is obtained. Dispersion properties of the structure for TE and TM modes are analyzed for different configurations of magnetophotonic crystals (ferrite and plasma-like layers). Existence areas of transmission bands for surface and bulk waves are obtained. The effect of problem parameters on the dispersion properties of magnetophotonic crystals for TM and TE modes is investigated. Regimes of complete transmission of wave through limited magnetophotonic crystal are analyzed for bulk and surface waves.

Keywords: magnetophotonic crystal, gyrotropic media, dispersion diagrams, TE and TM modes, bulk and surface waves

1. Introduction

Photonic crystals (PCs) are artificial periodic structures with spatially modulated refractive index in one or more coordinates [1, 2]. Their outstanding optical properties are due to the existence of frequency band gaps where the propagation of electromagnetic waves is impossible. Application of these structures became very attractive for modern optoelectronics which uses the various waveguides, resonators, sensors, and other devices on the basis of PC [3, 4]. Moreover, the control of the PC structure characteristics is the important problem that is usually solved using external electric or magnetic fields. These methods of providing controllability are based on the variation of refractive index of special materials such as liquid crystals

and magnetic materials [5, 6]. Since these sensitive materials are anisotropic, then theoretical analysis of their properties is more complicated.

When at least one of the PCs' unit cell components is a magnetically sensitive (gyrotropic) material, they exhibit unique magneto-optical properties and identified as magnetophotonic crystals (MPCs). Investigations of the MPCs are begun for simplest one-dimensional structures [7, 8]. However, one-dimensional MPCs are the basis elements for various active field-controlling applications so far [9, 10]. Changing of the permeability by external magnetic field is one of the main phenomena that allow developing electronically tuned devices in different frequency bands: filters, circulators and so on [11–13].

Along with the properties inherent in conventional PCs, these structures have additional optical and magneto-optical properties which considerably expand their functionality. Kerr effect, Faraday rotation and optical nonlinearity can be enhanced in MPC due to light localization within magnetic multilayer. Magneto-optical system with large Faraday or Kerr rotation can be used for effective optical isolators [14, 15], spatial light-phase modulators [16] and magnetic field and current sensor [17] development. Furthermore, one can obtain stronger enhancement of the magneto-optical phenomena due to resonant effects in the MPCs [18], which characterized by specific polarization properties. Using PCs with magneto-optical layers provides possibility of control of optical bistability threshold in structure based on graphene layer [19]. It should be noted that not only magnetic materials are suitable for MPC. Namely, one-dimensional PC with plasma layers can be tunable by external magnetic field [20].

A number of applications of the MPCs are inspired by their nonreciprocal properties. For example, special spatial structure of the MPC layers provides the asymmetry of dispersion characteristics and, as a result, the effect of unidirectional wave propagation [21]. This phenomenon allows enhancing field amplitude in the MPC without any periodicity defects. In this case, the so-called frozen mode regime occurs instead the defect mode one.

One of the unique properties of gyrotropic materials is the possibility of negative values of material parameters under the certain conditions. Usually, these are so-called single-negative media that are divided into epsilon-negative media (plasma) and mu-negative ones (gyrotropic magnetic materials). The term "double-negative media" or "left-handed materials" is used for media with negative values of both permittivity and permeability and often replaced by term "metamaterials." Application of metamaterials in one-dimensional PC systems results in unusual regularities of bulk and surface wave propagation and is the subject of experimental and theoretical research [22, 23].

Theoretical description of the various types of one-dimensional PCs is usually based on the transfer-matrix method of Abelès [24] that was applied by Yeh et al. to periodic layered media [25]. This method cannot be applied in general case for anisotropic multilayer structures because of mode coupling. However, this is possible in special cases, namely, in two-dimensional model of wave propagation in periodic layered media [26]. This case is considered in this chapter. Such an approach makes it possible to simplify significantly the analysis of physical phenomena in complex layered media with various combinations of gyrotropic and isotropic elements. Moreover using well-known permutation duality principle of Maxwell's

equations results in a reduction of unique combinations number. In turn, this allows better understanding of regularities of bulk and surface wave propagation in one-dimensional MPC and finding new modes for applications in modern microwave, terahertz and optical devices.

2. Formulation and solution of the problem for modes of gyrotropic periodic structures

2.1. Basic relationships

We study electromagnetic wave propagation in periodic structure in general case with bigyrotropic layers (one-dimensional MPC) (**Figure 1**). Each of two layers on the structure period $L=a+b$ is an anisotropic medium (plasma or ferrite or their combinations). Their permittivity and permeability are characterized by tensor values of standard form [26]:

$$\overleftrightarrow{\varepsilon}_j = \begin{Vmatrix} \varepsilon_j & -i\varepsilon_{aj} & 0 \\ i\varepsilon_{aj} & \varepsilon_j & 0 \\ 0 & 0 & \varepsilon_{\|j} \end{Vmatrix}, \quad \overleftrightarrow{\mu}_j = \begin{Vmatrix} \mu_j & -i\mu_{aj} & 0 \\ i\mu_{aj} & \mu_j & 0 \\ 0 & 0 & \mu_{\|j} \end{Vmatrix}. \tag{1}$$

For plasma media, the value of the permittivity is tensor, whereas the value of the permeability is scalar. Such media are called electrically gyrotropic. It is opposite for the ferrite media; the permeability is tensor, whereas permittivity is scalar. Such media are usually called magnetically gyrotropic. If the permittivity and permeability are simultaneously described by the tensors (Eq. (1)), such media are called gyrotropic or bigyrotropic. The material parameters included in tensors $\overleftrightarrow{\varepsilon}_j$ and $\overleftrightarrow{\mu}_j$ are defined by the value of the external bias magnetic field $\vec{H}_0 = \vec{z}_0 H_0$, which is directed along Oz axis.

The study of the general case of gyrotropic media with material parameters of form (Eq. (1)) is reasonable, primarily because it allows using the permutation duality principle when obtaining main equations for fields and characteristic Eqs. [26]. According to this principle generalized for gyrotropic media, namely, when simultaneously the substitution of fields

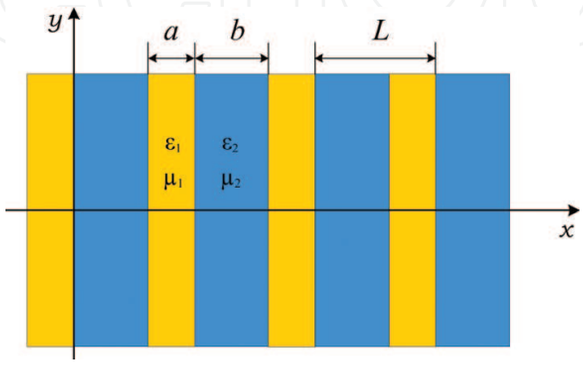

Figure 1: Schematic of the periodic structure.

$\vec{E} \leftrightarrow \vec{H}$ and material parameters $\overleftrightarrow{\varepsilon} \leftrightarrow -\overleftrightarrow{\mu}$ is done, the receiving of general equations, from which the equations for magneto-gyrotropic media (ferrite), electro-gyrotropic media (plasma), and gyrotropic media (bianisotropic media) can be obtained easily, turns to be more simple than in each of the mentioned particular cases separately.

Indeed, it follows from Maxwell equations:

$$rot\ \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad rot\ \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}, \tag{2}$$

where $\vec{D} = \overleftrightarrow{\varepsilon}\vec{E}$ is inductance vector of electric field and $\vec{B} = \overleftrightarrow{\mu}\vec{H}$ is inductance vector of magnetic field. Components of these vectors can be written as:

$$D_x = \left(\overleftrightarrow{\varepsilon}\vec{E}\right)_x = \varepsilon E_x - i\varepsilon_a E_y, \quad D_y = \left(\overleftrightarrow{\varepsilon}\vec{E}\right)_y = i\varepsilon_a E_x + \varepsilon E_y, \quad D_z = \left(\overleftrightarrow{\varepsilon}\vec{E}\right)_z = \varepsilon_{\|} E_z,$$

$$B_x = \left(\overleftrightarrow{\mu}\vec{H}\right)_x = \mu H_x - i\mu_a H_y, \quad B_y = \left(\overleftrightarrow{\mu}\vec{H}\right)_y = i\mu_a H_x + \mu H_y, \quad B_z = \left(\overleftrightarrow{\mu}\vec{H}\right)_z = \mu_{\|} H_z.$$

In general case, one can obtain two connected differential equations for longitudinal components of electromagnetic fields E_z and H_z (along Oz axis and the direction of bias magnetic field $\vec{H}_0 = \vec{z}_0 H_0$). In two-dimensional case ($\partial/\partial z = 0$), these equations can be broken up into two independent Helmholtz equations with respect to the selected longitudinal field components E_z and H_z (TM and TE waves) [26]. Indeed, we can show it for two-dimensional case and harmonic dependence $\exp(-i\omega t)$ of the fields on time t.

Using the relation between field components E_z and H_z via transverse field components, one can obtain the Helmholtz equation for two polarizations E_z and H_z, respectively:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k^2 \mu_{\perp} \varepsilon_{\|} E_z = 0, \quad \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k^2 \varepsilon_{\perp} \mu_{\|} H_z = 0. \tag{3}$$

Here:

$$\mu_{\perp j} = \mu_j \left(1 - \frac{\mu_{aj}^2}{\mu_j^2}\right), \quad \varepsilon_{\perp j} = \varepsilon_j \left(1 - \frac{\varepsilon_{aj}^2}{\varepsilon_j^2}\right).$$

Eq. (3) describes TM waves (H_x, H_y, E_z), E_z-polarization (s-polarization) and TE waves (E_x, E_y, H_z), H_z-polarization (p-polarization). Therefore, the vector of the electric field for s-polarization is directed perpendicular to the xy plane and the vector of the electric field for the p-polarization is parallel to this plane. It is necessary to use the boundary conditions for the tangential components of the electric and magnetic fields at the media interfaces to solve the boundary electrodynamic problem for the eigenvalues and the eigenfunctions of the Laplace operator. We use the conditions of continuity of the components E_z and H_y for TM waves and H_z and E_y for TE waves.

This analysis shows the important conclusion that in the general case of a gyrotropic medium, the fields of TM and TE waves with respect to the direction of the bias magnetic field $\vec{H}_0$ in the two-dimensional case are divided into two independent solutions of the Maxwell equations. Moreover, Eq. (3) and the expressions for the tangential components of fields E_y and H_y yield the permutation duality principle for the TM and TE waves. These equations are transformed each other after replacing the component of field E_z by H_z, and simultaneously, the effective permeability $\mu_\perp$ should be replaced by the effective permittivity $(-\varepsilon_\perp)$ and $\varepsilon_\parallel$ by $(-\mu_\parallel)$. The boundary conditions for the components E_y and H_y also satisfy the permutation duality principle. Hence, it is possible to simplify this general electrodynamic problem, and we can consider only one type of TM or TE waves for any kind of media in order to obtain a solution for another type of wave. Below, we consider the propagation of E_z-polarized (TM) waves in a gyrotropic PC structure.

To determine the eigenvalues and the corresponding eigenfunctions of the two-layer gyromagnetic MPC, we consider Helmholtz equation for the E_z-polarization with the corresponding boundary conditions for the tangential components of the fields E_z and H_y at the interfaces of the periodic structure layers. For the case of H_z-polarization (TE waves), it is necessary to use the permutation duality principle in the solution for TM waves.

The solution of the Helmholtz equation for TM waves for both tangential components of the fields E_z and H_y can be written in the following form:

$$\begin{aligned} E_z^1(x,y) &= \left(a_{n-1}e^{i\xi_1(x-(n-1)L)} + b_{n-1}e^{-i\xi_1(x-(n-1)L)}\right)e^{i\beta y}, \quad b < x-(n-1)L < L \\ H_y^1(x,y) &= \frac{\xi_1}{-k\mu_{\perp 1}}\left(a_{n-1}g_1^+ e^{i\xi_1(x-(n-1)L)} - b_{n-1}g_1^- e^{-i\xi_1(x-(n-1)L)}\right)e^{i\beta y}, \end{aligned} \tag{4}$$

$$\begin{aligned} E_z^2(x,y) &= \left(c_n e^{i\xi_2(x-nL)} + d_n e^{-i\xi_2(x-nL)}\right)e^{i\beta y}, \quad 0 < x-nL < b \\ H_y^2(x,y) &= \frac{\xi_2}{-k\mu_{\perp 2}}\left(c_n g_2^+ e^{i\xi_2(x-nL)} - d_n g_2^- e^{-i\xi_2(x-nL)}\right)e^{i\beta y}. \end{aligned} \tag{5}$$

Here, $n = 1, 2, \ldots$ is a number of the period; $\xi_j = \sqrt{k^2\mu_{\perp j}\varepsilon_{\parallel j} - \beta^2}$ are the transverse wave numbers in gyrotropic layers in the direction of the Ox axis; β is the longitudinal wave number of the MPC; a_n, b_n, c_n, and d_n are the wave amplitudes in the layers; and $g_j^\pm = 1 \pm i\frac{\mu_{aj}}{\mu_j\xi_j}\beta$.

Let us note one feature in the expressions for the electromagnetic fields (Eqs. (4) and (5)). The presence of gyrotropy in the layers ($\mu_{aj} \neq 0$) leads to the fact that the distributions of the amplitude of the fields in the layers differ by factors $g_j^\pm$ for the forward and backward waves, which propagating in the layers along the direction of periodicity.

To find the dispersion equation that relates the longitudinal wave number β with the structure parameters for a given frequency ω, it is necessary to use the boundary conditions on the interfaces of the layers and the Floquet-Bloch theorem [27, 28] for the periodic structure. Using the conditions of continuity of the E_z and H_y components of the fields at the interfaces $x - (n-1)L = 0$ and $x = nL$, we get matrix equations:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} c_n \\ d_n \end{pmatrix}, \qquad \begin{pmatrix} c_n \\ d_n \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}. \tag{6}$$

where

$$a_{11} = \frac{1}{2}\left(g_1^- + \frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} g_2^+\right) e^{-i\xi_2 L}, \quad a_{12} = \frac{1}{2}\left(g_1^- - \frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} g_2^-\right) e^{i\xi_2 L}, \tag{7}$$

$$a_{21} = \frac{1}{2}\left(g_1^+ - \frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} g_2^+\right) e^{-i\xi_2 L}, \quad a_{22} = \frac{1}{2}\left(g_1^+ + \frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} g_2^-\right) e^{i\xi_2 L}. \tag{8}$$

$$b_{11} = \frac{1}{2}\left(g_2^- + \frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}} g_1^+\right) e^{i\xi_2 a} e^{-i\xi_1 a}, \quad b_{12} = \frac{1}{2}\left(g_2^- - \frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}} g_1^-\right) e^{i\xi_2 a} e^{i\xi_1 a}, \tag{9}$$

$$b_{21} = \frac{1}{2}\left(g_2^+ - \frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}} g_1^+\right) e^{-i\xi_2 a} e^{-i\xi_1 a}, \quad b_{22} = \frac{1}{2}\left(g_2^+ + \frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}} g_1^-\right) e^{-i\xi_2 a} e^{i\xi_1 a}. \tag{10}$$

Eliminating the coefficients c_n, d_n in the matrix equations (Eq. (6)), we obtain the relation for the coefficients in identical layers for two neighboring periods of the structure:

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} c_n \\ d_n \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}. \tag{11}$$

The elements of the ABCD matrix are calculated by the rule of multiplying two matrices. Using the Eqs. (7)–(10), we find the elements of the given transfer matrix, namely:

$$A = \left\{ \cos\xi_2 b - i\frac{1}{2}\left[\frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}} + \frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} + \frac{\beta^2}{\xi_1\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left(\frac{\mu_{a1}}{\mu_1} - \frac{\mu_{\perp 1}}{\mu_{\perp 2}}\frac{\mu_{a2}}{\mu_2}\right)^2\right] \sin\xi_2 b \right\} e^{-i\xi_1 a}, \tag{12}$$

$$D = \left\{ \cos\xi_2 b + i\frac{1}{2}\left[\frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}} + \frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} + \frac{\beta^2}{\xi_1\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left(\frac{\mu_{a1}}{\mu_1} - \frac{\mu_{\perp 1}}{\mu_{\perp 2}}\frac{\mu_{a2}}{\mu_2}\right)^2\right] \sin\xi_2 b \right\} e^{i\xi_1 a}, \tag{13}$$

$$B = i\frac{1}{2}\sin\xi_2 b \left\{ -\frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} + \frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left[1 - i\frac{\beta}{\xi_1}\left(\frac{\mu_{a1}}{\mu_1} - \frac{\mu_{a2}}{\mu_2}\frac{\mu_{\perp 1}}{\mu_{\perp 2}}\right)\right]^2 \right\} e^{i\xi_1 a}, \tag{14}$$

$$C = -i\frac{1}{2}\sin\xi_2 b \left\{ -\frac{\xi_2}{\xi_1}\frac{\mu_{\perp 1}}{\mu_{\perp 2}} + \frac{\xi_1}{\xi_2}\frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left[1 + i\frac{\beta}{\xi_1}\left(\frac{\mu_{a1}}{\mu_1} - \frac{\mu_{a2}}{\mu_2}\frac{\mu_{\perp 1}}{\mu_{\perp 2}}\right)\right]^2 \right\} e^{-i\xi_1 a}. \tag{15}$$

An important property of the *ABCD* matrix is the unimodularity property, when the ratio between the elements of the matrix is fulfilled: $AD - BC = 1$. Using the expressions for the elements of the matrix *ABCD*, one can show that this condition is satisfied. We note that when ξ_1 is a real number, then the elements of the matrix $A = D^*$ и $B = C^*$ are pairwise conjugate.

The resulting matrix equation (Eq. (11)), which determines the relationship of the unknown coefficients in two identical layers of different periods of the periodic structure and the

Floquet-Bloch theorem, allows us to find the characteristic (dispersion) equation for determining the previously introduced unknown longitudinal wave number β for a wave propagating along gyrotropic layers (along the *Oy* axis) and the Floquet-Bloch wave number *K*. According to the Floquet-Bloch theorem in its matrix formulation, one can obtain

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-iKL} \begin{pmatrix} a_n \\ b_n \end{pmatrix}. \tag{16}$$

The phase factor e^{-iKL} is the eigenvalue of the transfer-matrix ABCD, which is determined from the characteristic equation:

$$e^{-iKL} = \frac{1}{2}(A+D) \pm i\sqrt{1-\left[\frac{1}{2}(A+D)\right]^2}. \tag{17}$$

The unknown real values of the roots of the characteristic equation have the form:

$$K_{TM}(\beta) = \frac{1}{L}\arccos\left\{ \cos\xi_2 b \cos\xi_1 a - \frac{1}{2}\left[\begin{array}{l} \dfrac{\xi_1}{\xi_2}\dfrac{\mu_{\perp 2}}{\mu_{\perp 1}} + \dfrac{\xi_2}{\xi_1}\dfrac{\mu_{\perp 1}}{\mu_{\perp 2}} + \\ + \dfrac{\beta^2}{\xi_1\xi_2}\dfrac{\mu_{\perp 2}}{\mu_{\perp 1}}\left(\dfrac{\mu_{a1}}{\mu_1} - \dfrac{\mu_{\perp 1}}{\mu_{\perp 2}}\dfrac{\mu_{a2}}{\mu_2}\right)^2 \end{array} \right] \sin\xi_2 b \sin\xi_1 a \right\}. \tag{18}$$

It is easy to show that this expression is transformed to the well-known solution of the dispersion equation for the case of two magnetodielectric layers ($\mu_{a1} = \mu_{a2} = 0$) [25].

Note that the longitudinal wave number enters into equation as β squared. This indicates that its absolute value is the same for opposite directions of wave propagation along the *Oy* axis. That is, the dispersion is the same for forward and backward waves propagating along the layers. However, the field distributions for the case of gyrotropic media in the direction of periodicity for forward and backward waves are different.

Using the permutation duality principle, we found the solutions of the electrodynamic problem for TE wave propagation in the gyrotropic MPC. For this case, we change the material parameters according to the rule $\overleftrightarrow{\mu} \leftrightarrow -\overleftrightarrow{\varepsilon}$ in the transfer-matrix elements (Eqs. (12)–(15)), in the dispersion relation, and in the solution (Eq. (18)). Then, we obtain

$$A_{TE} = \left\{ \cos\xi_2 b - i\frac{1}{2}\left[\frac{\xi_1}{\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}} + \frac{\xi_2}{\xi_1}\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}} + \frac{\beta^2}{\xi_1\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}\left(\frac{\varepsilon_{a1}}{\varepsilon_1} - \frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}\frac{\varepsilon_{a2}}{\varepsilon_2}\right)^2\right] \sin\xi_2 b \right\} e^{-i\xi_1 a}, \tag{19}$$

$$D_{TE} = \left\{ \cos\xi_2 b + i\frac{1}{2}\left[\frac{\xi_1}{\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}} + \frac{\xi_2}{\xi_1}\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}} + \frac{\beta^2}{\xi_1\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}\left(\frac{\varepsilon_{a1}}{\varepsilon_1} - \frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}\frac{\varepsilon_{a2}}{\varepsilon_2}\right)^2\right] \sin\xi_2 b \right\} e^{i\xi_1 a}, \tag{20}$$

$$B_{TE} = i\frac{1}{2}\sin\xi_2 b \left\{ -\frac{\xi_2}{\xi_1}\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}} + \frac{\xi_1}{\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}\left[1 - i\frac{\beta}{\xi_1}\left(\frac{\varepsilon_{a1}}{\varepsilon_1} - \frac{\varepsilon_{a2}}{\varepsilon_2}\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}\right)\right]^2 \right\} e^{i\xi_1 a}, \tag{21}$$

$$C_{TE} = -i\frac{1}{2}\sin\xi_2 b\left\{-\frac{\xi_2}{\xi_1}\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}+\frac{\xi_1}{\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}\left[1+i\frac{\beta}{\xi_1}\left(\frac{\varepsilon_{a1}}{\varepsilon_1}-\frac{\varepsilon_{a2}}{\varepsilon_2}\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}\right)\right]^2\right\}e^{-i\xi_1 a}. \quad (22)$$

Here $\xi_1 = \sqrt{k^2\varepsilon_{\perp 1}\mu_{\parallel 1}-\beta^2}$ and $\xi_2 = \sqrt{k^2\varepsilon_{\perp 2}\mu_{\parallel 2}-\beta^2}$.

It is apparent that in this case, one can write

$$K_{TE}(\beta) = \frac{1}{L}\arccos\left\{\cos\xi_2 b\cos\xi_1 a-\frac{1}{2}\left[\begin{array}{l}\frac{\xi_1}{\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}+\frac{\xi_2}{\xi_1}\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}+\\ +\frac{\beta^2}{\xi_1\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_{\perp 1}}\left(\frac{\varepsilon_{a1}}{\varepsilon_1}-\frac{\varepsilon_{\perp 1}}{\varepsilon_{\perp 2}}\frac{\varepsilon_{a2}}{\varepsilon_2}\right)^2\end{array}\right]\sin\xi_2 b\sin\xi_1 a\right\}. \quad (23)$$

In the absence of gyrotropy ($\varepsilon_{aj}=0$), this solution is transformed into the well-known expression for the Bloch wave number for TE waves for the magnetodielectric PC [25].

The given elements of the transfer-matrix *ABCD* and the solutions of the dispersion equations (Eqs. (18) and (23)) for both *s*- and *p*-polarizations (TM and TE waves) are suitable for analysis of wide variety of MPCs with different material parameters: two isotropic layers on the crystal period (dielectric, magnetic, magnetodielectric), one isotropic layer with another anisotropic layer, two gyroelectric layers or gyromagnetic ones or a combination of them, and two gyrotropic layers. Such an abundance of variants makes Eq. (17) universal in terms of analyzing the dispersion characteristics of TE and TM waves and establishing the features of their propagation in various one-dimensional MPCs.

2.2. Eigen regimes of MPCs

We perform the analysis of the features for the propagation of the electromagnetic waves in different MPCs for various eigenmodes. We identified 10 variants of such regimes.

1. *The crystal contains two layers of magnetodielectric*: $\varepsilon_{a1}=\varepsilon_{a2}=\mu_{a1}=\mu_{a2}=0$.

2. *The crystal contains magnetodielectric layer,* $\varepsilon_{a1}=\mu_{a1}=0$, *and the layer of a semiconductor plasma*: $\varepsilon_{a2}\neq 0$ and $\mu_{a2}=0$.

3. *The crystal contains magnetodielectric layer,* $\varepsilon_{a1}=\mu_{a1}=0$, *and the ferrite layer:* $\mu_{a2}\neq 0$ and $\varepsilon_{a2}=0$.

Taking into account the permutation duality principle, we obtain equations analogous to variant 2.

4. *The crystal contains magnetodielectric layer and gyrotropic one*: $\varepsilon_{a1}=\mu_{a1}=0$, $\varepsilon_{a2}\neq 0$, and $\mu_{a2}\neq 0$.

5. *The crystal contains layer of semiconductor plasma:* $\varepsilon_{a1}\neq 0$ *and* $\mu_{a1}=0$ *and the ferrite layer*: $\varepsilon_{a2}=0$ and $\mu_{a2}\neq 0$.

6. *The crystal contains two layers of semiconductor plasma*: $\varepsilon_{a1}\neq 0$, $\varepsilon_{a2}\neq 0$ and $\mu_{a1}=0$, $\mu_{a2}=0$.

7. *The crystal contains two ferrite layers*: $\mu_{a1}\neq 0$, $\mu_{a2}\neq 0$ and $\varepsilon_{a1}=0$, $\varepsilon_{a2}=0$.

Taking into account the permutation duality principle, we obtain equations analogous to variant 6.

8. *The crystal contains the plasma layer and gyrotropic layer*: $\varepsilon_{a1} \neq 0$, $\varepsilon_{a2} \neq 0$ and $\mu_{a1} = 0$, $\mu_{a2} \neq 0$.

9. *The crystal contains the ferrite layer and gyrotropic layer*: $\varepsilon_{a1} = 0$, $\varepsilon_{a2} \neq 0$ and $\mu_{a1} \neq 0$, $\mu_{a2} \neq 0$.

Taking into account the permutation duality principle, we obtain equations analogous to variant 8.

10. *The crystal contains two gyrotropic layers* $\mu_{a1} \neq 0$, $\mu_{a2} \neq 0$ and $\varepsilon_{a1} \neq 0$, $\varepsilon_{a2} \neq 0$

In this case, one can use solutions Eqs. (18) and (23) for TM and TE waves, respectively.

3. Analysis of the propagation of TE and TM waves in MPCs

3.1. General aspects of wave propagation in MPCs

Let us do a physical analysis of the obtained results and determine general rules of electromagnetic wave propagation in MPC. If the wave number $K(\beta)$ is real, then the electromagnetic wave propagates in a MPC without attenuation. These particular Floquet-Bloch wave numbers $K(\beta)$ correspond to the transmission bands and satisfy the condition $|\cos K(\beta)L| \leq 1$. On the other hand, a different behavior is observed when the wave number is complex: $K(\beta) = K'(\beta) + iK''(\beta)$. Such waves cannot propagate in a MPC and thus decay along the direction of periodicity. As a result, the forbidden zones are formed that satisfied the condition $|\cos K(\beta)L| > 1$. This condition allows easy determination of the imaginary part $K''(\beta)$ of the wave number. When $K''(\beta) \neq 0$, then $\sin K'(\beta)L = 0$ and $K'(\beta)L = \pi m$ (where $m = 0, 1, 2\ldots$ are forbidden zone numbers). As a result, we can obtain the equation for $K''(\beta)$: $chK''(\beta)L = (-1)^m \frac{1}{2}(A + D)$.

The value $m = 0$ corresponds to the zeroth forbidden zone where the wave number is purely imaginary. It is important to distinguish two cases: the absence ($\varepsilon_{aj} = 0$, $\mu_{aj} = 0$) and presence ($\varepsilon_{aj} \neq 0$, $\mu_{aj} \neq 0$) of gyrotropy in a MPC. Besides of that, if in j^{th} layer the conditions $k^2\varepsilon_{\perp j}\mu_{\parallel j} - \beta^2 < 0$ or $k^2\mu_{\perp j}\varepsilon_{\parallel j} - \beta^2 < 0$ are satisfied, and in one layer $\varepsilon_{\perp 1} < 0$ (or$\mu_{\perp 1} < 0$) and in another layer $\varepsilon_{\perp 2} > 0 (\mu_{\perp 2} > 0)$, then the wave in such layer would decay in amplitude along the x-axis. This wave is a surface wave and delayed in respect to the speed of light.

3.2. Analysis of dispersion characteristics

Let us consider first the case of a MPC in the absence of gyrotropy ($\varepsilon_{aj} = 0$ and $\mu_{aj} = 0$). This structure is a periodic sequence of magnetodielectric layers. Dispersion equations for these structures were previously considered in a simplified version by many authors. However, the first investigation of such characteristic (dispersion) equations was carried out as early as the nineteenth century by Rayleigh [29] in the solution of the one-dimensional Hill equation. If the medium is periodic, then the latter equation becomes the traditional one-dimensional Helmholtz equation. Further investigations were performed by various researchers, for example, Brillouin [30], Yeh et al. [25], and Bass [31].

The solutions of the dispersion equations for the magnetodielectric periodic structures written out in this section include all possible combinations of the signs and magnitudes of the material

parameters. As an example, let us consider dispersion characteristics of one-dimensional magnetodielectric PCs for several combinations of the parameters.

Figure 2 shows dispersion curves for the value of the longitudinal wave number $\beta = 0.9$. In this case, the periodic structure consists of two dielectrics ($\mu_1 = \mu_2 = \mu_{\|} = 1$) with positive values of permittivity $\varepsilon_1 = \varepsilon_{\|1} = 2$ and $\varepsilon_2 = \varepsilon_{\|2} = 9$. The normalized width of the layers is $a/L = 0.8$ and $b/L = 0.2$. Solid and dotted curves denote the real and imaginary parts of the Floquet-Bloch wave number, respectively. The imaginary parts of the Floquet-Bloch wave number characterize the degree of wave decay in the forbidden bands. The wave decay is different in forbidden zones. Moreover, the maximum attenuation is observed for the zeroth zone.

If we consider a MPC consisting of two different magnetodielectrics, then there are no fundamental differences.

Figure 3 shows the dispersion diagrams for PC with two dielectric layers ($\mu_1 = \mu_2 = 1$) on the structure period and with different signs of the permittivities ($\varepsilon_1 > 0$ and $\varepsilon_2 < 0$), namely, $\varepsilon_1 = \varepsilon_{\|1} = 2$ and $\varepsilon_2 = \varepsilon_{\|2} = -6$. First the problem of wave propagation at the boundary of two half-spaces from dielectrics with opposite signs of permittivity values was considered by Sommerfeld [32].

The existence of surface waves at the boundaries of PC layers is illustrated in the dispersion diagram (**Figure 3**) for TE waves (*p*-polarization). The region of these waves existence is located below the light line $k = \beta(\varepsilon_1)^{-1/2}$ (the solid line on the diagram). Taking into account the identical physical nature of the surface wave existence in PC and Zenneck-Sommerfeld wave for two media [33], this regime can be classified as a modified surface Zenneck-Sommerfeld wave. For this wave, the condition $\beta = k\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$ is satisfied approximately, as follows from the dispersion equation. It should be noted that if we consider PC with magnetic layers, then a surface wave will exist for *s*-polarization.

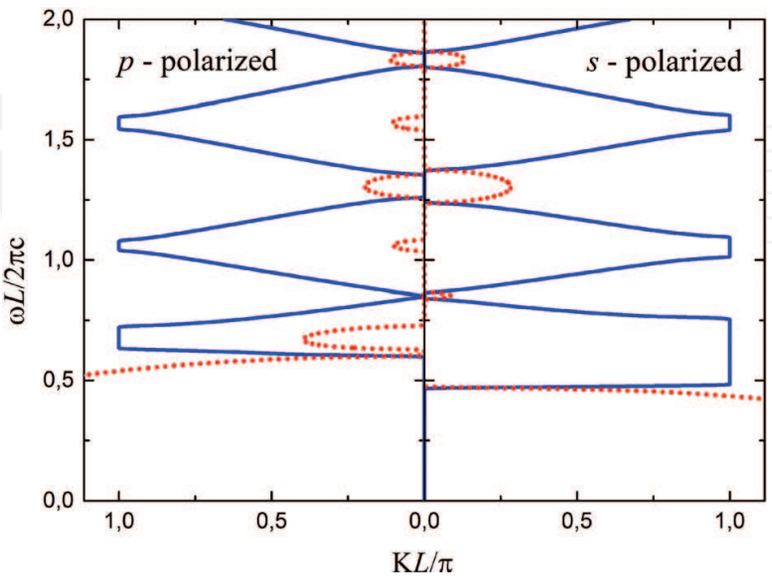

Figure 2: Dispersion characteristics of the one-dimensional photonic crystal.

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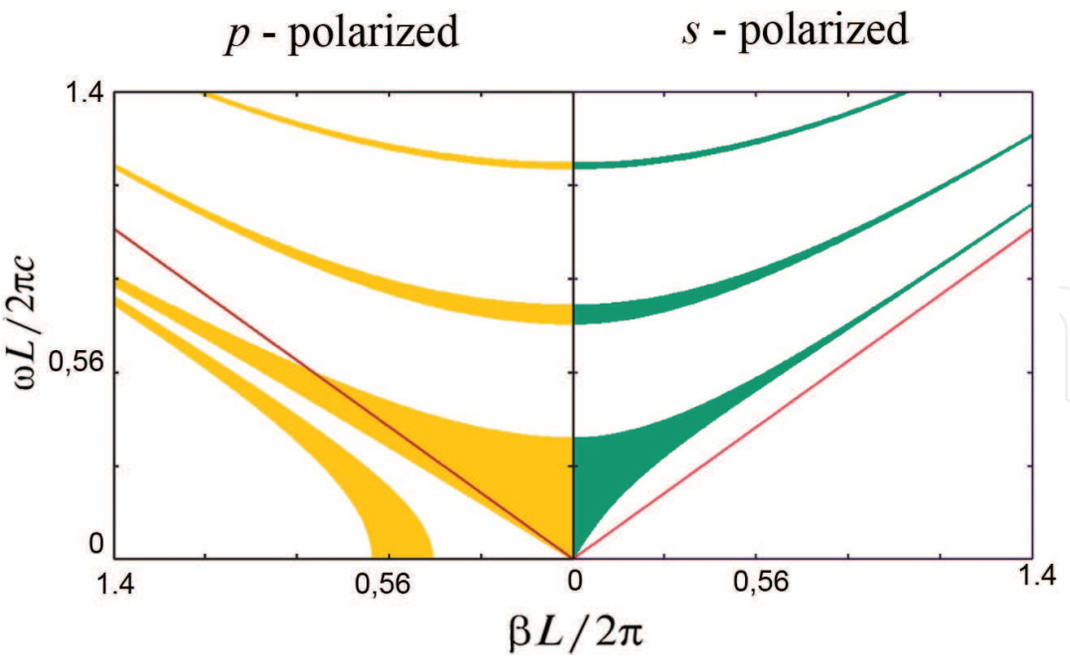

Figure 3. Dispersion diagrams for both polarizations.

With the advent of new artificial media (metamaterials) for which the permittivity and permeability are simultaneously negative, optoelectronics devices with new functionalities are developed. In this connection, it is expedient to consider a MPC, one of whose layers on the structure period is a metamaterial (e.g., $\varepsilon_2 < 0$, $\mu_2 < 0$).

In **Figure 4**, dispersion diagrams are calculated for the one-dimensional PC with alternative layers of magnetodielectric and metamaterial ($\varepsilon_2 < 0$, $\mu_2 < 0$). The following parameters were

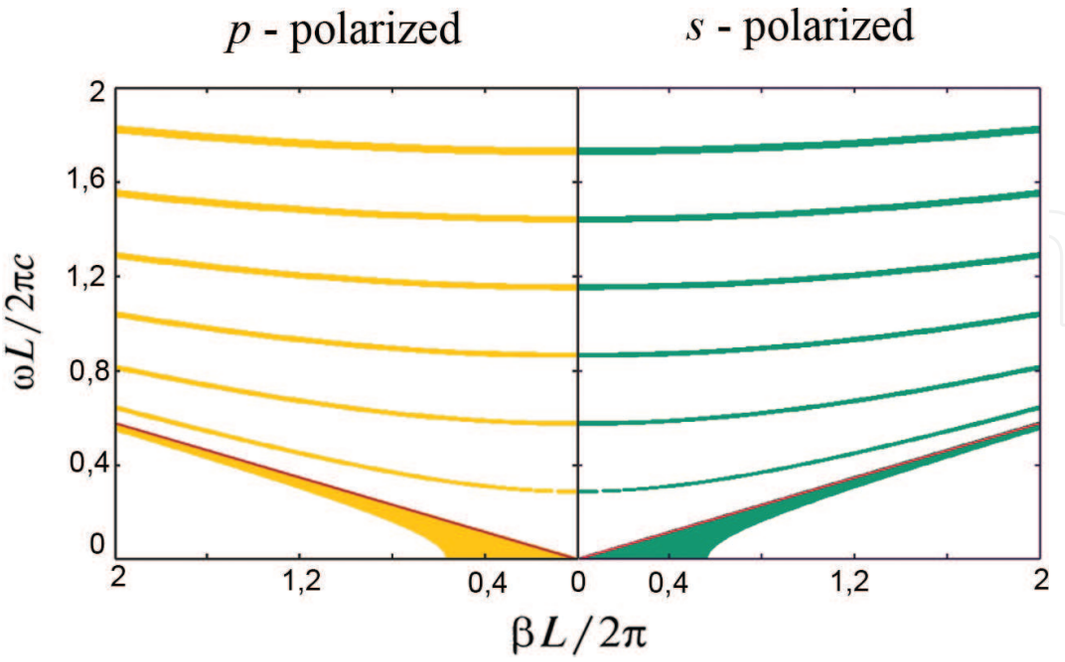

Figure 4. Dispersion diagram of the photonic crystal with metamaterial layer.

selected: $\varepsilon_1 = 4$, $\mu_1 = 3$, $\varepsilon_2 = -2$, $\mu_2 = -6$, and $a/L = 0.5$. **Figure 5a** and **b** shows the dispersion characteristics for such MPC at the values $\beta = 0$ and $\beta = 2\pi/L$, respectively. On the presented graphs, identical dispersion diagrams for both polarizations are seen. Therefore, it is a case of polarization indifference for the forbidden zones and transmission ones. For the presented case, the refractive indices of the layers are the same in absolute value and have phase difference of π, namely, $n_1 = \sqrt{\varepsilon_1 \mu_1} = \sqrt{12}$ and $n_2 = \sqrt{|\varepsilon_2| e^{-i\pi} |\mu_2| e^{-i\pi}} = e^{-i\pi}\sqrt{12}$. For this case, the complete Floquet-Bloch wave transmission in PC for two polarizations is possible.

It is clear that there are transmission bands only for surface slow waves (modified Zenneck-Sommerfeld waves) that are located below the light line $\omega\sqrt{12} = \beta c$. For bulk waves ($k^2 \varepsilon_j \mu_j > \beta^2$), the transmission bands degenerate into curves. These curves are described by equation $(2\pi m)^2 = (k^2 |\varepsilon_j \mu_j| - \beta^2) L^2$ ($m = 1, 2, \ldots$) for both polarizations.

Dispersion curves for real K' and imaginary K'' values (**Figure 5a** and **b**) show that the wave decay for both polarizations is the same. The propagation of waves is observed at discrete points on which the conditions $K'' = 0$ and $|\cos K(\beta) L| = 1$ are satisfied. These points are located outside the transmission bands of surface waves. The phenomenon of polarization indifference makes it possible to develop devices with a finite number of periods of a MPC in which a complete narrow-band propagation of the wave is possible simultaneously for a whole spectrum of discrete frequencies and both polarizations [34].

We now turn to an analysis of the propagation of waves in MPC consisting of a magnetodielectric and a semiconductor plasma layer [35]. It is advisable to consider two cases in the presence of gyrotropy of the medium ($\varepsilon_{a2} \neq 0$), when $\varepsilon_{\perp 2} > 0$ and $\varepsilon_{\perp 2} < 0$ for TE waves. We note that the dispersion characteristics for TM waves are the same as for a conventional magnetodielectric. Let us first consider the features of TE wave propagation in MPC when the plasma layer gyrotropy is such that condition $\varepsilon_{\perp 2} > 0$ ($\varepsilon_{a2} < \varepsilon_2$) is fulfilled. From an analysis of the solutions of the dispersion equation, it follows that in the periodic structure at such a value of the effective permittivity of a plasma medium, there can exist two modes of TE wave propagation. The first

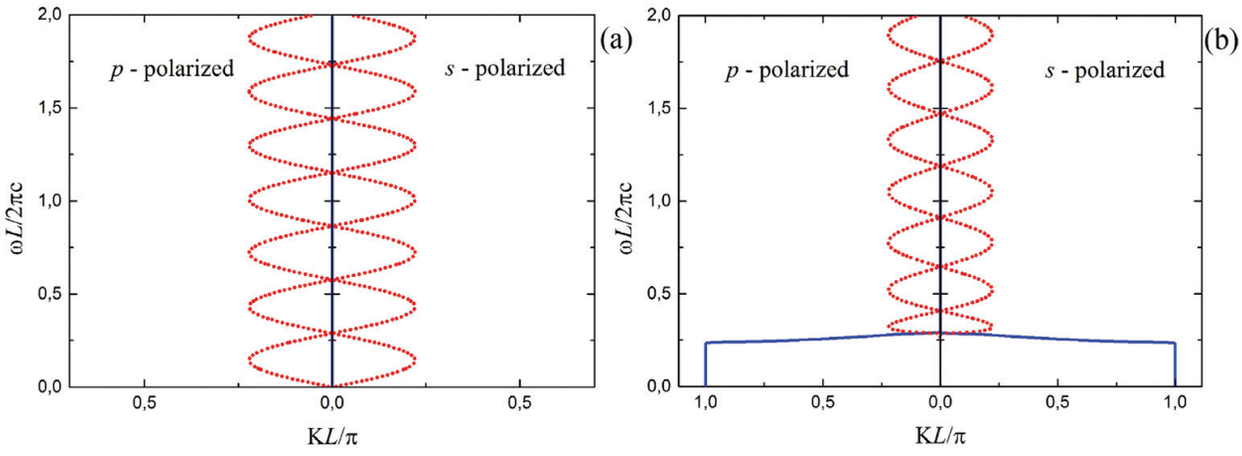

Figure 5. Dispersion characteristics for different values of longitudinal wavenumber: (a) $\beta = 0$; (b) $\beta = 2\pi/L$.

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mode is bulk waves whose domain of existence is determined by the simultaneous fulfillment of two conditions: $k^2\varepsilon_{\perp 1}\mu_{\parallel 1}-\beta^2>0$ and $k^2\varepsilon_{\perp 2}\mu_{\parallel 2}-\beta^2>0$. The second mode is the propagation mode of gyrotropic surface waves. This mode is observed when three conditions are fulfilled simultaneously: $k^2\varepsilon_{\perp 1}\mu_{\parallel 1}-\beta^2<0$, $k^2\varepsilon_{\perp 2}\mu_{\parallel 2}-\beta^2<0$, and $\frac{\xi_1}{\xi_2}\frac{\varepsilon_{\perp 2}}{\varepsilon_1}+\frac{\xi_2}{\xi_1}\frac{\varepsilon_1}{\varepsilon_{\perp 2}}+\frac{\beta^2}{\xi_1\xi_2}\frac{\varepsilon_1}{\varepsilon_{\perp 2}}\left(\frac{\varepsilon_{a2}}{\varepsilon_2}\right)^2<0$.

If the above conditions are satisfied, then the existence of real values of the Floquet-Bloch wave number other than zero is possible when the condition $|\cos K(\beta)L|\le 1$ is fulfilled for pure imaginary values of transverse wave numbers ξ_1 and ξ_2.

The results of calculations of the dispersion diagram are shown in **Figure 6**. The calculation was carried out with the following parameters: $a/L=0.8$, $\mu_1=\mu_2=\mu_{\parallel 1}=\mu_{\parallel 2}=1$, $\varepsilon_1=2$, $\varepsilon_2=4$, $\varepsilon_{a2}=3.2$, and $\varepsilon_{\perp 2}=1.44$. The solid line shows the light line for the first layer on the period of the structure. It can be seen from the figure that when the conditions $k\sqrt{\varepsilon_{\perp 2}\mu_{\parallel 2}}<\beta$, $k\sqrt{\varepsilon_1\mu_{\parallel 1}}<\beta$, and $\frac{\varepsilon_{\perp 2}}{|\varepsilon_{\perp 2}|}\left[\frac{\beta^2}{|\xi_1||\xi_2|}\frac{\varepsilon_1}{|\varepsilon_{\perp 2}|}\left(\frac{\varepsilon_{a2}}{\varepsilon_2}\right)^2-\left(\left|\frac{\xi_1}{\xi_2}\right|\frac{|\varepsilon_{\perp 2}|}{\varepsilon_1}+\left|\frac{\xi_2}{\xi_1}\right|\frac{\varepsilon_1}{|\varepsilon_{\perp 2}|}\right)\right]>0$ are satisfied, there exists a region of real values of the Floquet-Bloch wave number for which the gyrotropic surface wave regime is observed (more shaded area of the transmission zone).

In the case $\varepsilon_{\perp 2}<0$, as for PC with dielectric layers, the regime of a modified Zenneck-Sommerfeld surface wave can exist in MPC. However, in addition to that wave, there is also a gyrotropic surface wave for other parameters of the problem. Indeed, such a wave exists when both transverse wave numbers in two layers are pure imaginary $(\xi_1^2<0,\xi_2^2<0)$. Also under these conditions and $\varepsilon_{\perp 2}<0$, the condition $\frac{\beta^2}{|\xi_1||\xi_2|}\frac{\varepsilon_1}{|\varepsilon_{\perp 2}|}\left(\frac{\varepsilon_{a2}}{\varepsilon_2}\right)^2<\left|\frac{\xi_1}{\xi_2}\right|\frac{|\varepsilon_{\perp 2}|}{\varepsilon_1}+\left|\frac{\xi_2}{\xi_1}\right|\frac{\varepsilon_1}{|\varepsilon_{\perp 2}|}$ must be fulfilled, that is, opposite to the condition for positive values of the second-layer effective permittivity.

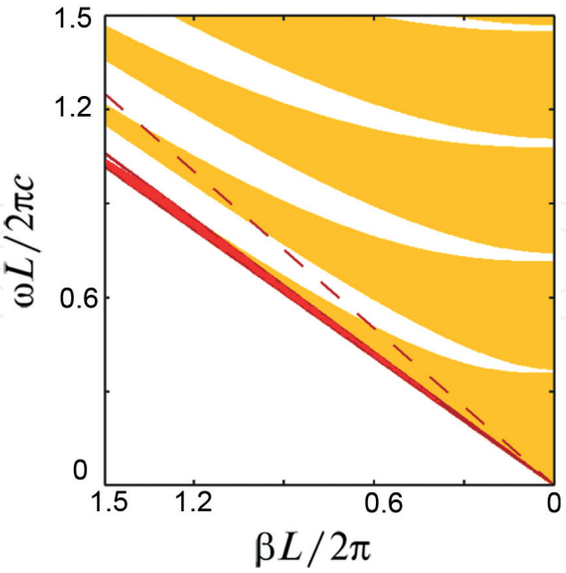

Figure 6. Band structure for TE polarization.

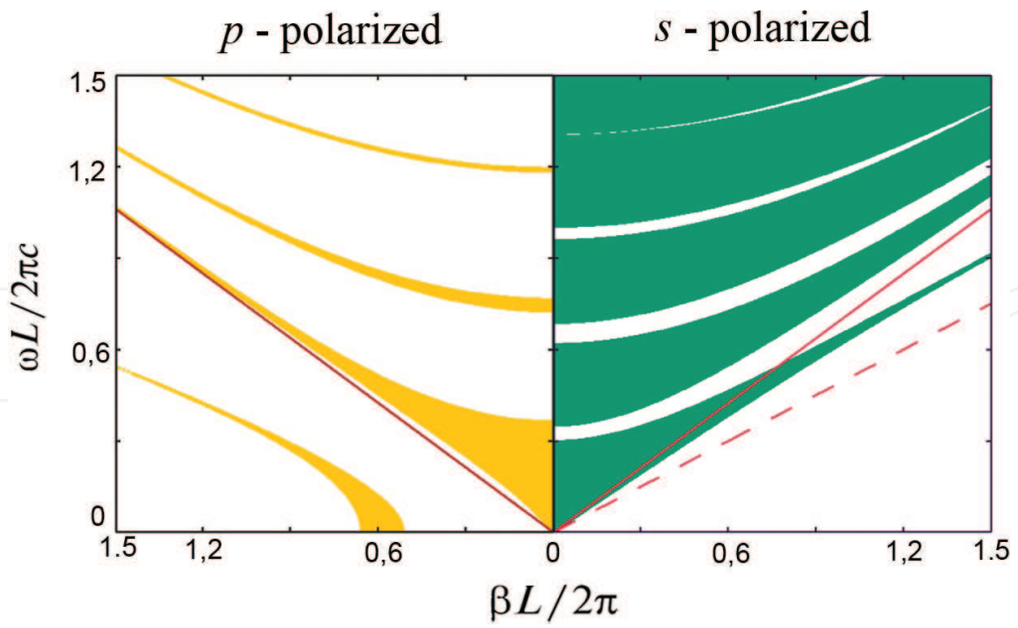

Figure 7. Dispersion diagrams of the magnetophotonic crystal containing plasma layers.

Figure 7 shows the dispersion diagrams for both polarizations in the case $\varepsilon_{\perp 2} < 0$. Parameters of the problem were chosen as follows: $a/L = 0.8$, $\mu_1 = \mu_2 = \mu_{\parallel 1} = \mu_{\parallel 2} = 1$, $\varepsilon_1 = 2$, $\varepsilon_2 = 4$, $\varepsilon_{a2} = 6.7$, and $\varepsilon_{\perp 2} = -7.2$. The solid line $k = \beta(\varepsilon_{\perp 1}\mu_{\parallel 1})^{-1/2}$ separates the areas of the existence of bulk waves and surface waves for p-polarization. One area relates to the modified Zenneck-Sommerfeld waves, and the other one relates to the gyrotropic surface wave which also exists for positive values of the effective permittivity, as shown above. We note that in the transmission bands of TE waves in dispersion diagrams, there are two regions in which surface waves propagate.

The case of MPC with a magnetodielectric and ferrite layer is analogous to that considered earlier by virtue of the permutation duality principle. Let us now consider the following case, when both layers on the structure period are ferrite with different material parameters. In this case, modes of bulk wave's existence in the transmission bands can be observed when two conditions $\xi_1^2 = k^2\mu_{\perp 1}\varepsilon_{\parallel 1} - \beta^2 > 0$ and $\xi_2^2 = k^2\mu_{2\perp}\varepsilon_{\parallel 2} - \beta^2 > 0$ are fulfilled. Regime of surface waves is realized when opposite conditions ($\xi_1^2 < 0$, $\xi_2^2 < 0$) and also the additional condition are fulfilled:

$$\frac{\mu_{\perp 1}}{|\mu_{\perp 1}|}\frac{\mu_{\perp 2}}{|\mu_{\perp 2}|}\left[\left|\frac{\xi_1}{\xi_2}\right|\left|\frac{\mu_{\perp 2}}{\mu_{\perp 1}}\right| + \left|\frac{\xi_2}{\xi_1}\right|\left|\frac{\mu_{\perp 1}}{\mu_{\perp 2}}\right| - \frac{\beta^2}{|\xi_1\xi_2|}\left|\frac{\mu_{\perp 2}}{\mu_{\perp 1}}\right|\left(\frac{\mu_{a1}}{\mu_1} - \frac{\mu_{\perp 1}}{\mu_{\perp 2}}\frac{\mu_{a2}}{\mu_2}\right)^2\right] < 0 \tag{24}$$

Note that this condition is equally suitable for both positive and negative values of the effective magnetic permeability of ferrite $\mu_{\perp j}$. Here, as in the case of the plasma semiconductor layer, the existence of gyrotropic surface waves is possible [36].

In **Figure 8**, we represent the dispersion diagrams of TM waves for the considered above MPC with two ferrite layers at the period of the structure. In the calculation, the following task parameters were chosen: $a/L = 0.5$, $\mu_1 = \mu_{\parallel 1} = 2$, $\mu_2 = \mu_{\parallel 2} = 3$, $\varepsilon_1 = 2$, $\varepsilon_2 = 4$, $\mu_{a2} = 2.9$,$\mu_{\perp 2} = 0.197\mu_{a1} = 0.1$, and $\mu_{\perp 1} = 1.995$.

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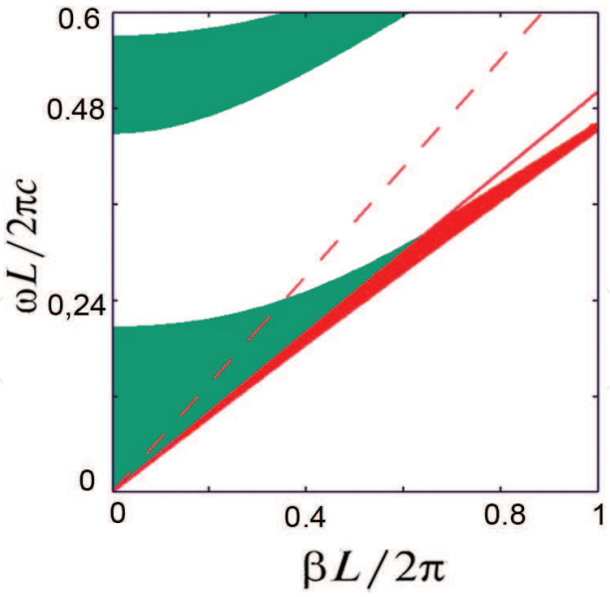

Figure 8. Band structure of the ferrite magnetophotonic crystal.

The solid line in the figure, below which the solutions of the dispersion equation are in the regime of surface waves, is determined by the equation $k=\beta(\mu_{\perp 1}\varepsilon_{\|1})^{-1/2}$. Analysis of the dispersion equation solutions and the conditions for the existence of surface waves show that for the chosen values of the material parameters in the ferrite MPC, there exists a range of parameters on the dispersion diagram that corresponds to the regime of propagation of the surface wave at the boundaries of the layers (in this figure it is marked as more shaded area in transmission zone). We emphasize that the regime of the surface wave is realized with positive values of the both magnetic permeabilities of the ferrite layers.

Figure 9 illustrates the case of the existence of a modified Zenneck-Sommerfeld wave for the case when the effective magnetic permeability of one of the layers is negative. Here, the dispersion diagrams are calculated with the following parameters of the problem: $a/L=0.85$, $\mu_1=\mu_{\|1}=2$, $\mu_2=\mu_{\|2}=3$, $\varepsilon_1=1.5$, $\varepsilon_2=1.8$, $\mu_{a2}=7.9$, $\mu_{\perp 2}=-17.8$, $\mu_{a1}=0.7$, and $\mu_{\perp 1}=1.755$.

Figure 10 illustrates the evolution of dispersion diagrams at change of ferrite effective magnetic permeability $\mu_{\perp 2}(\mu_{a2}=5.6;5.8;6.1)$ for $a/L=0.83$, $\mu_1=\mu_{\|1}=2$, $\mu_2=\mu_{\|2}=3$, $\varepsilon_1=4$, $\varepsilon_2=2$, and $\mu_{a1}=0.4$. A darker shade shows the transmission bands in which the conditions of existence of surface waves are fulfilled. The change of the bias magnetic field value leads to a significant change of the parameters of these areas. An increase in the value of the effective magnetic permeability $\mu_{\perp 2}$ owing to the magnitude μ_{a2} results in a change in the width and location of the transmission band of the modified Zenneck-Sommerfeld surface wave.

Figure 11 shows dispersion diagrams for the modified Zenneck-Sommerfeld surface wave of MPC with two ferrite layers on the structure period at change of width of the layer b ($b/L=0.3;0.5;0.7;0.8$) with $\mu_{\perp 2}<0$ and for the following parameters of the problem: $\mu_1=\mu_{\|1}=2$, $\mu_2=\mu_{\|2}=3$, $\varepsilon_1=4$, $\varepsilon_2=2$, $\mu_{a1}=0.4$, $\mu_{\perp 1}=1.92$, $\mu_{a1}=5.6$, and $\mu_{\perp 2}=-7.45$.

Increase of the second-layer width b leads to an expansion of existence area of surface waves with a simultaneous shift of the bandwidth toward the value $\beta=0$ (**Figure 11a** and **b**). The

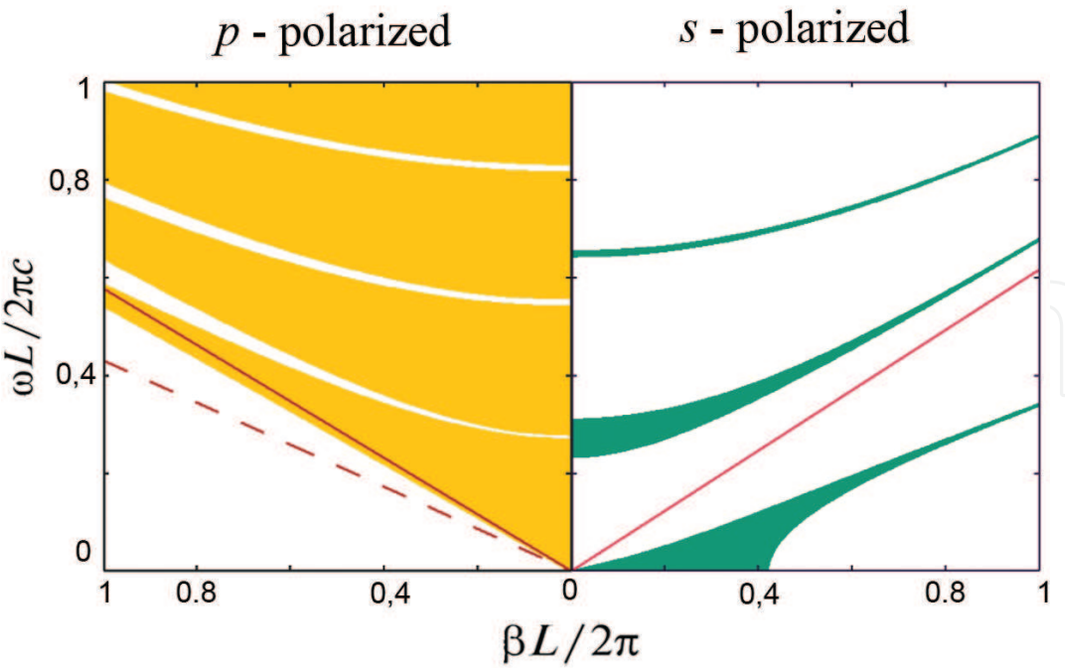

Figure 9. Dispersion diagrams of the ferrite magnetophotonic crystal for both polarizations.

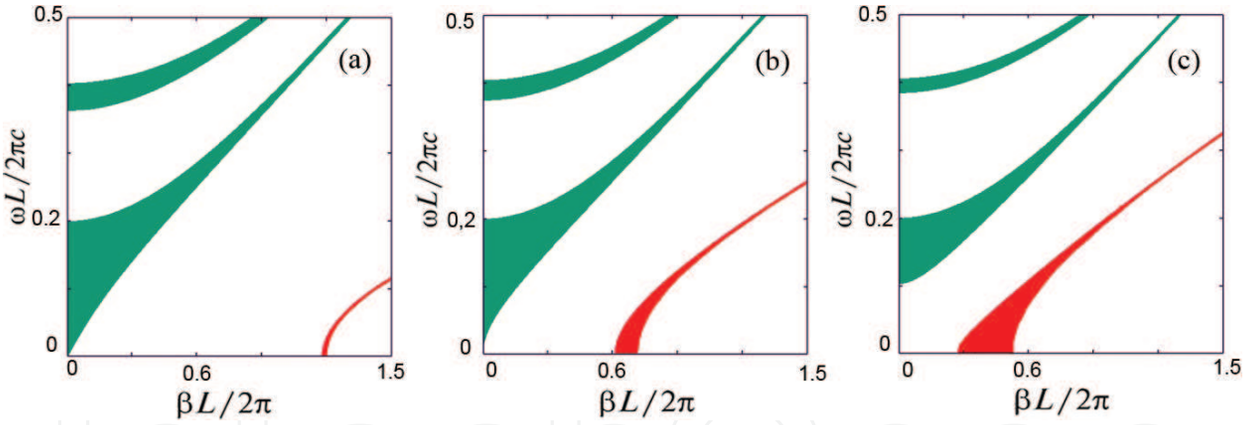

Figure 10. Dispersion diagrams for different values of bias magnetic field: (a) $\mu_{a2}=5.6$; (b) $\mu_{a2}=5.8$; (c) $\mu_{a2}=6.1$.

largest existence area of surface waves is realized in dispersion diagram for approximately equal values of the layer thicknesses. Further increase of the parameter *b* is accompanied by a displacement of this area to the opposite direction (**Figure 11c** and **d**). Thus the surface wave modes in a ferrite MPC are determined both by the material parameters of the system and by the width of the layers on the period of the structure.

Let us move on the dispersion diagrams for two bigyrotropic layers on the structure period. Taking into account the complete symmetry of the dispersion for TE and TM waves, the case of polarization indifference can be realized in this case. We will show this by example.

Figure 12 shows the dispersion diagrams for TE and TM waves for MPC consisting of two bigyrotropic layers on the period of the structure. **Figure 12a** corresponds to the following

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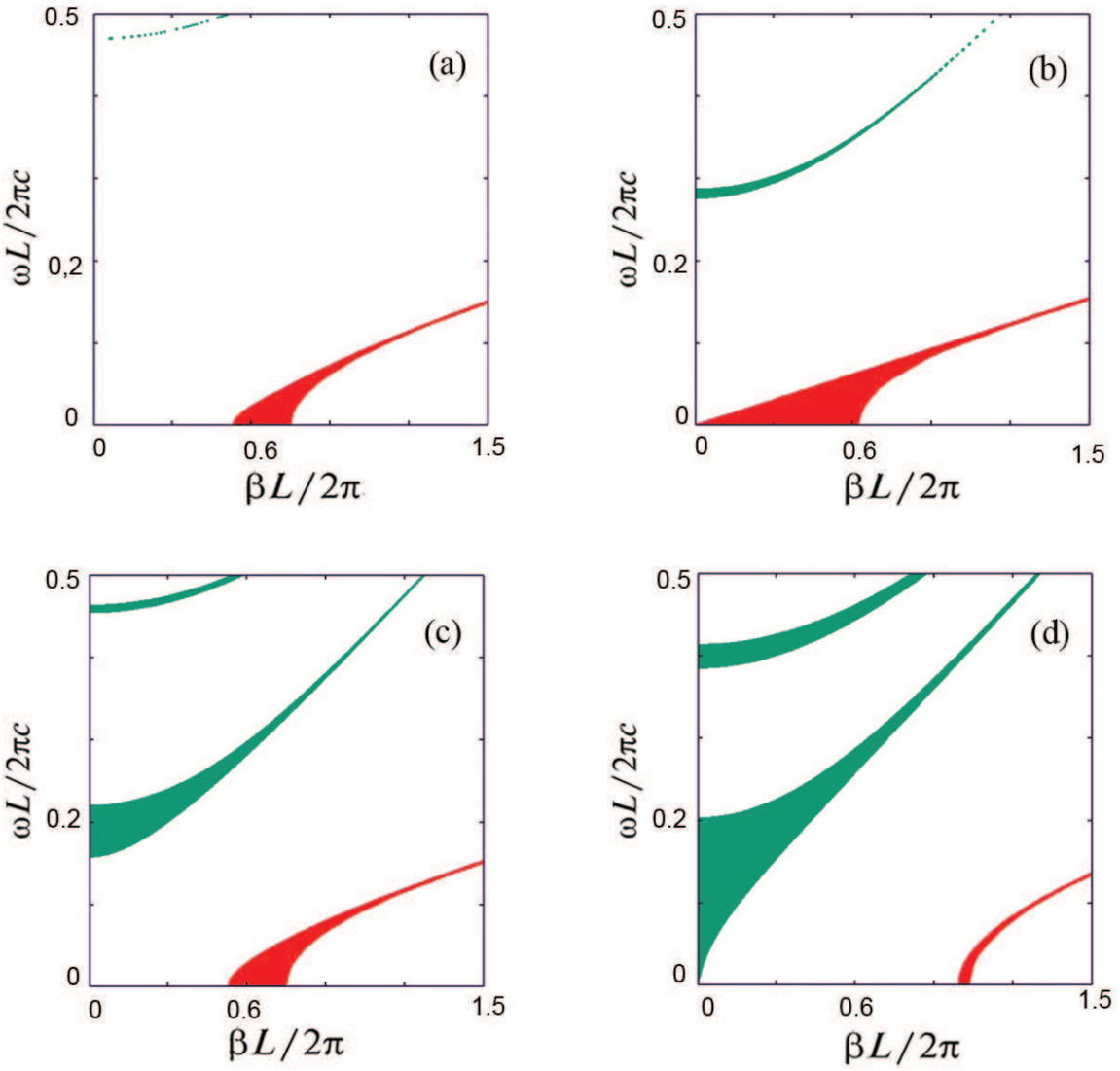

Figure 11. Dispersion diagrams for different values of second layer thickness: (a) $b/L = 0.3$; (b) $b/L = 0.5$; (c) $b/L = 0.7$; (d) $b/L = 0.8$.

values of the parameters: $a/L = 0.85$, $\mu_j = \mu_{\|j} = 2$, $\varepsilon_j = \varepsilon_{\|j} = 2$, $\varepsilon_{a1} = \mu_{a1} = 0.5$, $\varepsilon_{a2} = \mu_{a2} = 5.2$, $\mu_{\perp 1} = 1.875$, and $\mu_{\perp 2} = -11.52$. In **Figure 12b**, only the values of the effective permittivity and permeability of the second layer differ: $\varepsilon_{a2} = \mu_{a2} = 6.8$.

The solid lines in the figures distinguish the area of fast (upper part of figures) and slow wave (the lower part of figures). The complete identity of the dispersion diagrams for the transmission bands of both surface and bulk waves follows from the figures and formulas (18) and (23). By changing the bias magnetic field, it is possible to control the width and location of transmission bands for both polarizations.

Dispersion diagrams in **Figure 13** correspond to the case when only the surface wave transmission bands for both polarizations are realized. The parameters of the problem were chosen as follows: $a/L = 0.28$, $\mu_1 = \mu_{\|1} = 1.5$, $\mu_2 = \mu_{\|2} = 2$, $\varepsilon_1 = \varepsilon_{\|1} = 1.2$, $\varepsilon_2 = \varepsilon_{\|2} = 1.8$, $\varepsilon_{a1} = 4.95$, $\varepsilon_{a2} = 1.7$, $\mu_{a1} = 0.7$, and $\mu_{a2} = 5$. The polarization sensitivity of the MPC is realized in this case. Only the surface waves with certain polarization can propagate through the periodic structure for defined values of parameters k and β.

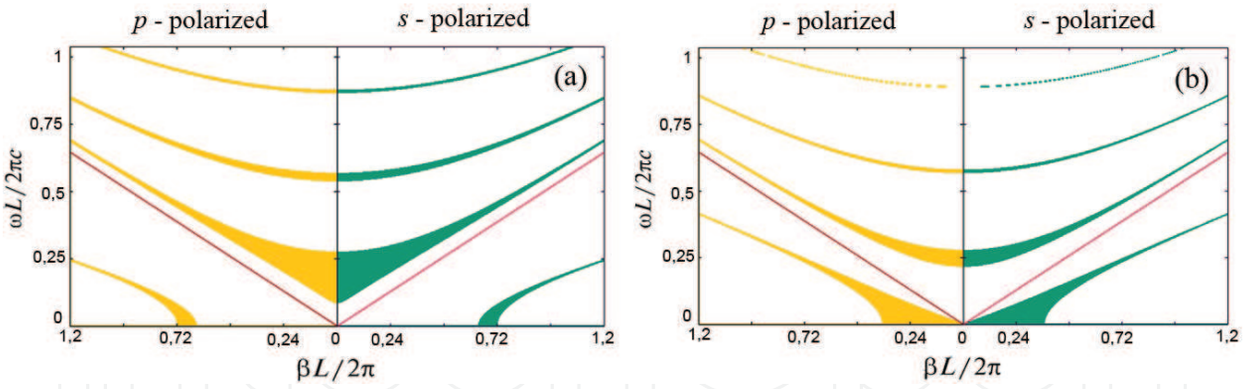

Figure 12. Dispersion diagrams of the bigyrotropic magnetophotonic crystal: (a) $\varepsilon_{a2} = \mu_{a2} = 5.2$; (b) $\varepsilon_{a2} = \mu_{a2} = 6.8$.

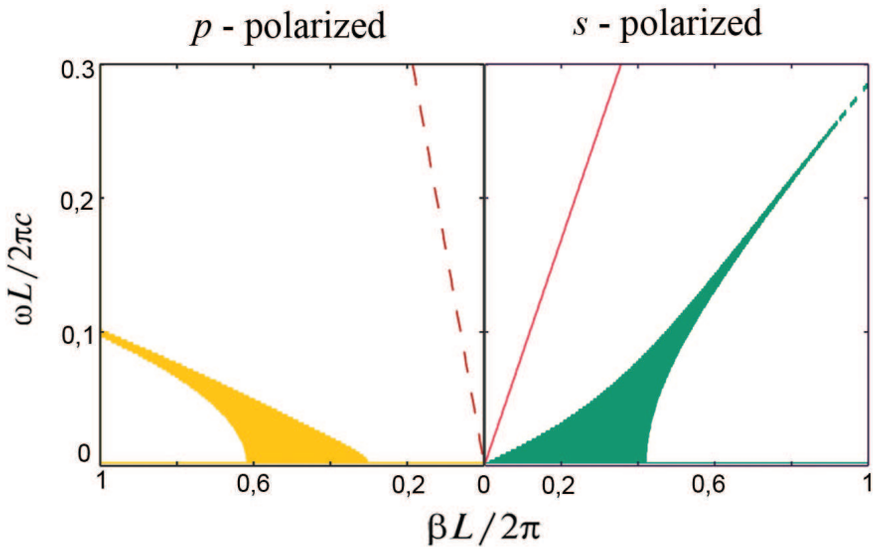

Figure 13. Transmission bands for surface waves in the bigyrotropic magnetophotonic crystal.

Therefore, we have considered the main features of the propagation of TE and TM waves in various magnetophotonic gyrotropic crystals. Important application of this one-dimensional PC theory is the problem of the electromagnetic waves scattering by a structures with limited number of periods.

4. Scattering of a plane wave by a MPC

In this section, the scattering of the plane wave on gyromagnetic MPC with N periods is considered. When a p-polarized plane wave is scattered on MPC with a limited number of periods, the problem is divided into three stages. At the first stage, the problem of scattering of a plane wave on the first gyromagnetic layer of MPC is solved. In the second stage, the coupling between the field coefficients of the first and last layers of the MPC is used in the problem for the MPC modes. And, finally, in the third stage, the problem of wave transmission from the last layer of the structure to the surrounding area is considered. Following

[37], we write out the final expression for the reflection and transmission coefficients for gyrotropic MPC:

$$R = \frac{[(n_{21}M_{11} + n_{22}M_{21})k_{11} + (n_{21}M_{12} + n_{22}M_{22})k_{21}]}{[(n_{11}M_{11} + n_{12}M_{21})k_{11} + (n_{11}M_{12} + n_{12}M_{22})k_{21}]} \tag{25}$$

$$T = \frac{1}{[(n_{11}M_{11} + n_{12}M_{21})k_{11} + (n_{11}M_{12} + n_{12}M_{22})k_{21}]} \tag{26}$$

Here, matrix M is the Nth power of an ABCD matrix. Other notations correspond to Ref. [37]. **Figure 14a** shows the dependences of the transmission coefficient modulus on the normalized frequency in the case of normal wave incidence ($\beta = 0$) on MPC with 20 periods in the regime of bulk waves. Calculation parameters are the same as in **Figure 7**.

There are three transmission bands in the frequency range under consideration. Each of these bands contains resonances which observed with respect to the frequency of the complete transmission of the wave (**Figure 14b** and **c**). Frequency resonances correspond to different modes of the periodic structure. The number of modes is determined by the number of periods of the structure ($N - 1$).

Figure 15 depicts the frequency dependences of the transmission coefficient modulus in the regime of the surface waves. In this case the incident angle of wave is greater than the angle of total internal reflection. We can see one transmission band in this case. Inset in **Figure 15** shows enlarged frequency dependence within this band. Complete propagation is observed for each resonant frequency of modes of the limited periodic structure.

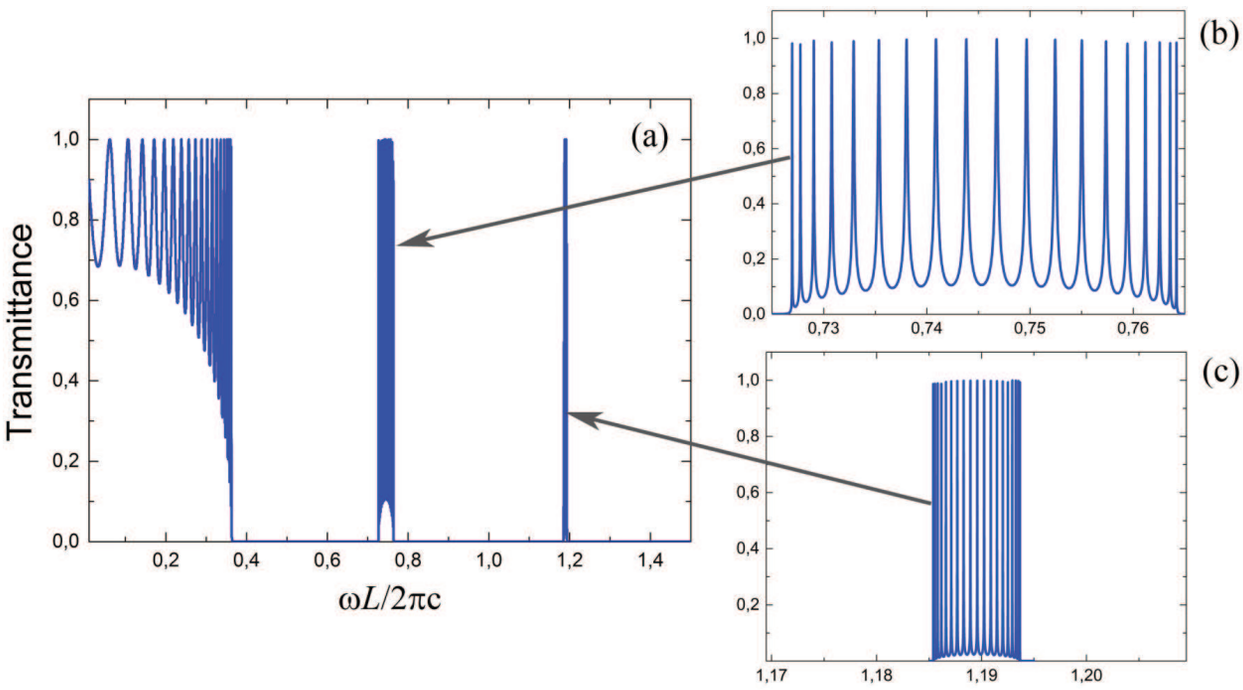

Figure 14. The transmittance vs frequencies for the case of normal incidence of wave: (a) spectral characteristic; (b) fine structure of spectral characteristic in second transmission band; (c) fine structure of spectral characteristic in third transmission band.

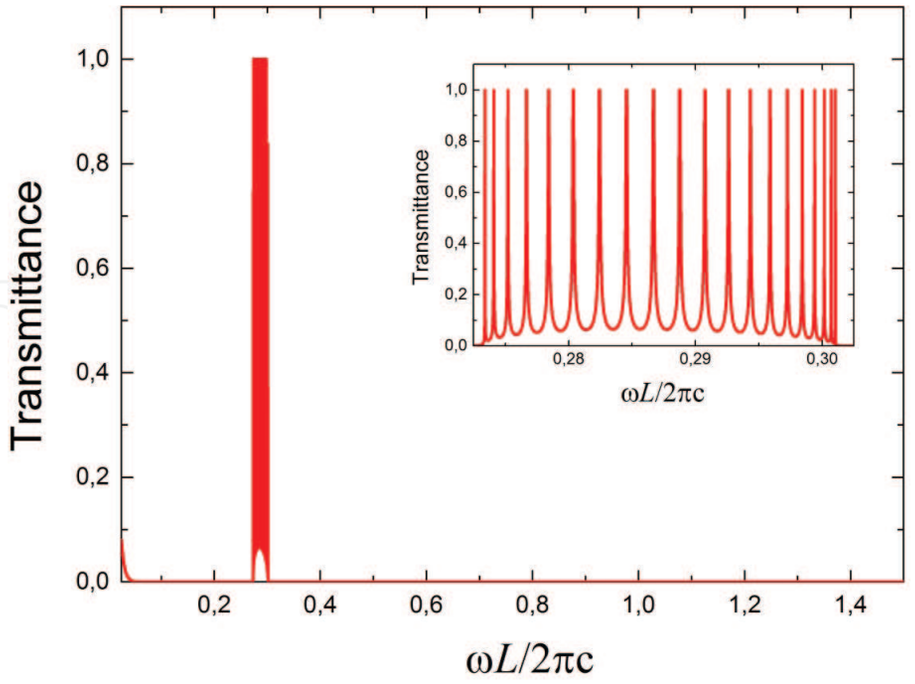

Figure 15. The transmittance vs. frequencies for the case of surface wave mode.

5. Conclusions

The electrodynamic problem is solved for the proper TE and TM waves of a MPC with two gyrotropic layers. The elements of the transmission matrix, the dispersion equation, and its solution are obtained analytically. An analysis of the dispersion properties of TE and TM waves for MPC is carried out, and features of the existence of fast and slow waves are revealed. Different regimes of gyrotropic surface waves are found. The conditions for the existence of surface waves are established for positive and negative values of the permittivity and permeability. Analytic expressions for the reflection and transmission coefficients for a limited MPC are obtained, and their analysis is performed for the regime of bulk and surface waves. Complete transmission of the wave through this structure is realized at resonant frequencies that correspond to different spatial distributions of the mode field in limited MPC.

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