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# Self-Organization, Coherence and Turbulence in Laser Optics

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## Abstract

In the last decades, rapid progress in modern nonlinear science was marked by the development of the concept of dissipative soliton (DS). This concept is highly useful in many different fields of science ranging from field theory, optics, and condensed matter physics to biology, medicine, and even sociology. This chapter aims to present a DS appearance from random fluctuations, development, and growth, the formation of the nontrivial internal structure of mature DS and its breakup, in other words, a full life cycle of DS as a self-organized object. Our extensive numerical simulations of the generalized cubic-quintic nonlinear Ginzburg-Landau equation, which models, in particular, dynamics of mode-locked fiber lasers, demonstrate a close analogy between the properties of DS and the general properties of turbulent and chaotic systems. In particular, we show a disintegration of DS into a noncoherent (or partially coherent) multisoliton complex. Thus, a DS can be interpreted as a complex of nonlinearly coupled coherent “internal modes” that allows developing the kinetic and thermodynamic theory of the nonequilibrium dissipative phenomena. Also, we demonstrate an improvement of DS integrity and, as a result, its disintegration suppression due to noninstantaneous nonlinearity caused by the stimulated Raman scattering. This effect leads to an appearance of a new coherent structure, namely, a dissipative Raman soliton.

**Keywords:** optical turbulence, dissipative solitons, chaos in nonlinear optical systems, generalized cubic-quintic nonlinear Ginzburg-Landau equation, dissipative Raman soliton

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## 1. Introduction

Coherent and partially coherent structures emerging in nonlinear systems far from the thermodynamic equilibrium play an important role in different research areas ranging from hydrodynamics and plasma physics to biophysics and sociology. Nontrivial dynamics of such

structures including chaos and turbulence is a challenge for modern nonlinear science and one may assume that “the problem of turbulence is one of the central problems in theoretical physics” [1]. The reasonable approach to this issue, which can translate some contra-intuitive and obscure ideas in this area into explicit and verifiable concepts, is a realization of simpler dynamics in quite different material context. Such an approach can be named *metaphorical or analog* modeling [2], and a rapid progress of modern laser technology provides an *ideal playground* for such enterprise due to high controllability, relative simplicity, and unique potential of statistic gathering. Such progress was marked by the development of the concept of a dissipative soliton (DS) [3]. The existence of DS under nonequilibrium conditions requires a well-organized energy exchange with an environment so that this energy flow forms a nontrivial internal structure of DS, which provides the energy redistribution inside it and can distort the soliton coherence. Such a DS with nontrivial internal structure can develop in lasers, and the DS dynamics can become chaotic and turbulent [3–5]. For instance, such emergent structures can be considered as a classical analog of Bose-Einstein condensate in low dissipative limit and, contrariwise, as a primitive analog of cell in the case of extensive and well-structured energy exchange with an environment. Formally, these inherently nonHamiltonian entities mimic some features of Hamiltonian systems that remain an obscure and insufficiently explored topic regarding the fundamental properties of coherent dissipative structures. The range of turbulence, noise, and rogue wave phenomena emulated by the optical DS is so broad that it turns them into a universal testbed for studies in the fields of nonlinear dynamical systems and nonequilibrium thermodynamics.

In this work, we conjecture a spectacular analogy between the spectral structures of DS and strong Langmuir turbulence. Such close relation leads to chaotization of DS dynamics with the energy growth. This analogy is deepened by analysis of energy flows inside DS so that a DS can be represented as a “glass of boiling water” or, mathematically, as an ensemble of interacting quasi-particles or “nonlinear modes.” The phase decoupling of these “modes” leads to turbulence or DS dissolving. Such a representation open the door for building the *kinetic theory* of open (dissipative) semi-coherent structures which mimics, in particular, a quantum Bose-Einstein condensate in a dissipative environment. Moreover, our preliminary investigations demonstrated a mechanism of *turbulence control* provided by noninstantaneous nonlinearity (stimulated Raman scattering in optical case) [6]. This phenomenon is especially interesting because an inherently noisy process (Raman scattering) suppresses a turbulence under some conditions that is the manifestation of *stochastic resonance*, which can be significant for a dissipation control in coherent quantum systems (particularly, a quantum computer and a quantum cryptography device).

## 2. Analogy between DS and turbulence

The phenomenon of turbulence appears in many areas of our experience ranging from atmospheric and oceanic rogue events, aero- and hydrodynamics, optics to cardiology and neurophysiology [1, 5, 7–13]. Such a broad class of phenomena cannot be grasped by some single and simple model. However, there are some comparatively simple equations which allow

describing an extremely broad class of phenomena. It is possible that the most known one is the famous nonlinear Schrödinger equation (NSE) which describes an evolution of slowly varying wave in a nonlinear medium and can be considered as a “metaphoric” simulation tool for a study of nonlinear phenomena far from equilibrium [14, 15]:

$$\frac{\partial \Psi}{\partial T} + \sum_{j=1}^d \frac{\partial \omega}{\partial k_j} \frac{\partial \Psi}{\partial x_j} - \frac{i}{2} \sum_{j,l=1}^d \frac{\partial^2 \omega}{\partial k_j \partial k_l} \frac{\partial^2 \Psi}{\partial x_j \partial x_l} + i \left( \frac{\partial \omega}{\partial |\Psi|^2} \right) |\Psi|^2 \Psi = 0. \quad (1)$$

The dimensionality of this equation is relative: the evolutionary coordinate can be a time  $T$  or a propagation distance  $z$  ( $T \leftrightarrow z$ ), the transverse coordinate can be transverse multidimensional spatial  $x_j$  ( $j=1\dots d$ ) one or a local time  $t$  ( $x \leftrightarrow t$ ,  $d=1$ ). The Fourier representations of a “field” slowly varying envelope  $\Psi$  are interchangeable between frequency and momentum domains ( $\omega \leftrightarrow k$ ,  $d=1$ ). Eq. (1) may describe the propagation of optical pulses in a nonlinear medium (then  $\Psi$  is a complex field amplitude and  $|\Psi|^2$  is proportional to a field power), the capillary waves on a fluid surface, the Langmuir waves in plasma, or the weakly nonlinear Bose-gas in classic limit (in the last case Eq. (1) represents the famous time-dependent Gross-Pitaevskii equation [16]).

Here  $\Psi(x, t)$  is a slowly varying amplitude of wave propagating in dispersive ( $\sum_{j,l=1}^d \frac{\partial^2 \omega}{\partial k_j \partial k_l} \frac{\partial^2 \Psi}{\partial x_j \partial x_l}$  - term; let us  $\beta \equiv \frac{\partial^2 \omega}{\partial k_j \partial k_l}$ ) and nonlinear ( $\left( \frac{\partial \omega}{\partial |\Psi|^2} \right) |\Psi|^2 \Psi$  - term; let us  $\gamma \equiv \frac{\partial \omega}{\partial |\Psi|^2}$ ) medium. The nonlinear term in Eq. (1) can have the different forms; in particular, a nonlinear response can be non-instantaneous.

The notion of turbulence is fuzzy in some sense. Here, the turbulence will be treated as a phenomenon related to the excitation of a sufficiently large number of degrees of freedom that causes a loss of their mutual phase information [15]. As a consequence, a wave package decouples into a set of individual modes (“particles”) which interaction can be described in the framework of kinetic theory as many-particle collisions in Bose-gas. In other words, as some *degrees* of freedom become very large for sufficiently large energies, phase information becomes irrelevant, and the waves *decohere* [8, 15]. Thus, a wave can be considered as a set of decoupled “modes”  $n_k$  in a spectral (or wave-number) space:

$$\langle \Psi(k) \Psi(k') \rangle = n_k \delta(k - k'). \quad (2)$$

Thus, we come to a “kinetic” theory of turbulence, for example, to a model of four-boson interaction described by the nonlinear Schrödinger equation:

$$\frac{\partial n_k}{\partial t} \propto \int (n_{k_1} n_{k_2} n_{k_3} + n_k n_{k_2} n_{k_3} - n_k n_{k_1} n_{k_2} - n_k n_{k_1} n_{k_3}) \times \delta(\vec{k} + \vec{k}_1 - \vec{k}_2 - \vec{k}_3) \delta(\omega + \omega_1 - \omega_2 - \omega_3) d \vec{k}_1 d \vec{k}_2 d \vec{k}_3. \quad (3)$$

Such an equation becomes nontrivial in a dissipative environment [17, 18]. A simple generalization of NSE (1) taking into account the dissipative effects includes a saturable gain (energy “source”)  $\sigma$ , dissipative nonlinearity (self-amplitude modulation, SAM)  $F(|\Psi|^2)$ , and spectral dissipation (spectral in the sense of dissipation in the Fourier space)  $\sum_{j,l=1}^d \alpha(x_j, x_l) \frac{\partial^2 \Psi}{\partial x_j \partial x_l}$ :

$$\text{RHS of Eq.(1)} = \sigma\Psi + F(|\Psi|^2)\Psi + \sum_{j,l=1}^d \alpha(x_j, x_l) \frac{\partial^2 \Psi}{\partial x_j \partial x_l}. \tag{4}$$

Eqs. (1) and (4) called the generalized complex nonlinear Ginzburg-Landau equation have the strongly localized (in  $x$ -space) steady-state (in  $T$ -space) solutions which are named *dissipative solitons* (DS) [3]. A classical (nondissipative) soliton, which possesses the quite specific mathematical properties [18–20], develops due to mutual compensation of dispersive spreading and self-compression caused by the phase nonlinearity under the condition of  $\frac{\partial^2 \omega}{\partial k_y \partial k_x} \times \frac{\partial \omega}{\partial |\Psi|^2} > 0$  and is stable in a (1 + 1)-dimensional (i.e.,  $T$  plus  $d=1$  in Eq. (1)) case.<sup>1</sup> The parameters of such soliton are not fixed but only interrelated. One may say that a soliton “lives in solitude” (“pratyekabuddha,” **Figure 1**).

Dissipation adds new bounds on the soliton parameters and fixes them so that one may say that the DS lives in “the heart of nonlinear world” (“bodhisattva,” **Figure 2**).

The mutual balance of dispersion and phase nonlinearity remains a crucial factor for DS formation, but its physical meaning differs substantially from that for nondissipative soliton. The crucial factor here is a *resonance* between dispersive (linear) waves and DS: equality of their wave-numbers defines the frequency window where DS can exist. Indeed, a wave-number of DS is  $q = \gamma P_0$  ( $P_0 \equiv \max(|\Psi|^2)$ ) [23]. The dispersion relation providing the resonance with linear waves is  $k(\omega) = \beta \omega^2 / 2$ . To be stable (i.e., nonradiating), the DS spectrum has to be localized within a frequency window  $\pm \Delta$ :  $k(\pm \Delta) = q$ , where  $\Delta = \sqrt{2\gamma P_0 / \beta}$  (**Figure 3**).<sup>2</sup> Formation of these “domain walls” [24–26] due to phase effects in a dissipative system results in natural frequency cut-off, which is essential for inherent analogy between DS and a turbulent entity.

However, sole dispersive balance is not sufficient for the DS stability. The spectral dissipation  $\sim \alpha \Delta^2$  plays a crucial role cutting the spectrum and defining the DS width (**Figure 4**). As



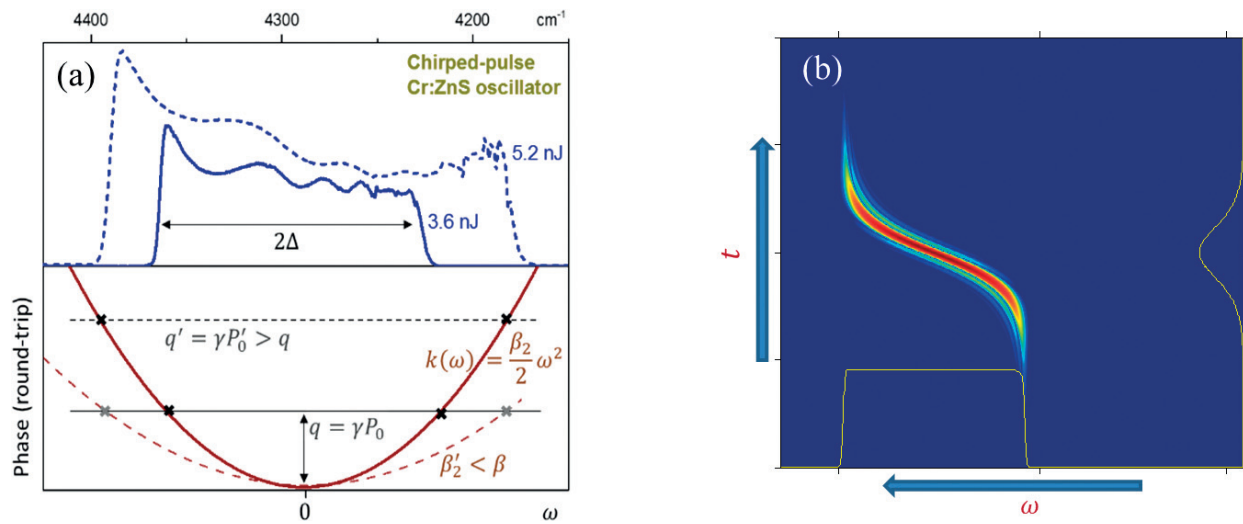
**Figure 1.** Soliton exists under a balance between phase nonlinearity and dispersion [21, 22].



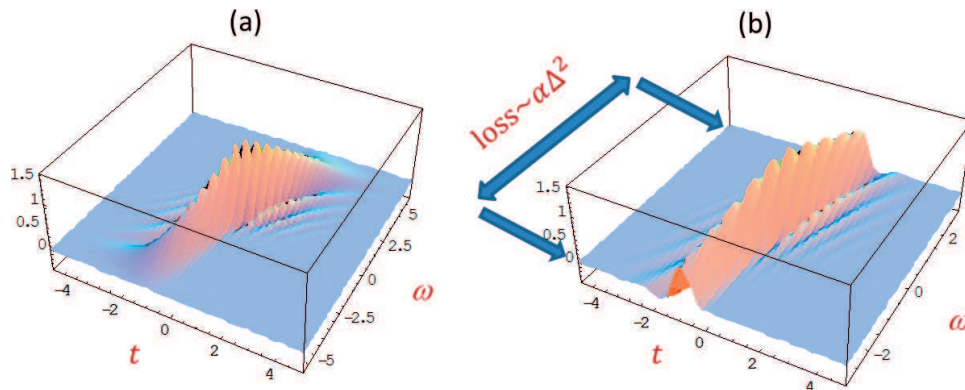
**Figure 2.** DS parameters are fixed by both nondissipative and dissipative factors [21, 22].

<sup>1</sup>Further, namely one-dimensional ( $d=1$ ) systems will be under consideration that is a quite precise approximation for solid-state and fiber laser dynamics [21].

<sup>2</sup>One has to remind the  $x \leftrightarrow t$  and  $k \leftrightarrow \omega$  dualities in Eq. (1).



**Figure 3.** Resonance conditions for DS and linear waves and DS spectra in dependence on DS energy (a); and the Wigner (time-spectral) diagram of DS (b) [23].



**Figure 4.** Wigner representation of DS (a) and cut-off due to spectral dissipation defining the DS width (b).

will be seen, this factor is crucial for dissipative soliton turbulence. A spectral dissipation must be balanced by a nonlinear gain  $\sim \kappa P_0$  (we assume  $F(|\Psi|^2) \approx \kappa |\Psi|^2 - \zeta |\Psi|^4 + \dots$ , where the first term is leading) that results in the additional relation between soliton spectral width and its peak power:  $\Delta = \sqrt{\kappa P_0 / \alpha}$ . In combination with the dispersive relation, it gives the condition for the soliton existence which combines the dissipative and nondissipative factors:  $\frac{\alpha \gamma}{\beta \kappa} \leq 1/2$ .<sup>3</sup> One has to note, that both considered mechanisms of DS formation act in the spectral domain and, as was shown in [28], a transition to spectral domain is fruitful for developing a DS theory.

The key feature of DS is its nontrivial internal structure revealing itself in the phase inhomogeneity<sup>4</sup> and the internal energy redistribution ( $E$  is an energy flow):

<sup>3</sup>More precise analysis [27] gives the conditions of asymptotical stability:  $\frac{\alpha \gamma}{\beta \kappa} \leq \begin{cases} 1/3 & \text{if } E \rightarrow \infty \\ 1/2 & \text{if } E \rightarrow 0 \end{cases}$ , where  $E$  is a DS energy.

<sup>4</sup>The measure of this inhomogeneity is a so-called *chirp*  $\Theta \propto \frac{\partial^2 \arg(\Psi)}{\partial t^2}$ .

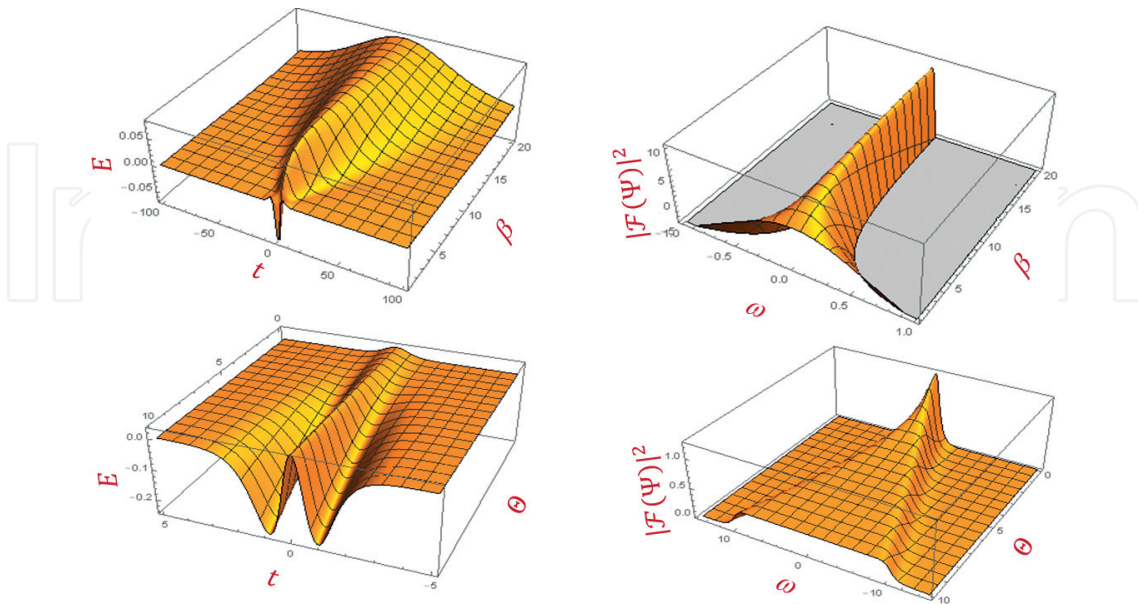
$$E \equiv \frac{i}{2} \frac{\partial}{\partial t} \left( \Psi \frac{\partial \Psi^*}{\partial t} - \Psi^* \frac{\partial \Psi}{\partial t} \right) = 2\sigma |\Psi|^2 + 2\kappa |\Psi|^4 - 2\alpha \left| \frac{\partial \Psi}{\partial t} \right| + \alpha \frac{\partial^2 |\Psi|^2}{\partial t^2}. \quad (5)$$

The third term in RHS of Eq. (5) is phase-sensitive and, thus, there is an energy flow from DS center, where spectral components with minimal relative frequencies are located, to the DS wings, where frequency components with maximum relative frequencies are located (**Figure 5**). Here, energy dissipates. Such nontrivial internal “life” of DS intensifies with the growth of phase inhomogeneity  $\Theta$ . Simultaneously, DS becomes an energy scalable coherent concentrate with the energy (“concentrate mass”)  $\propto \Theta$  [5].

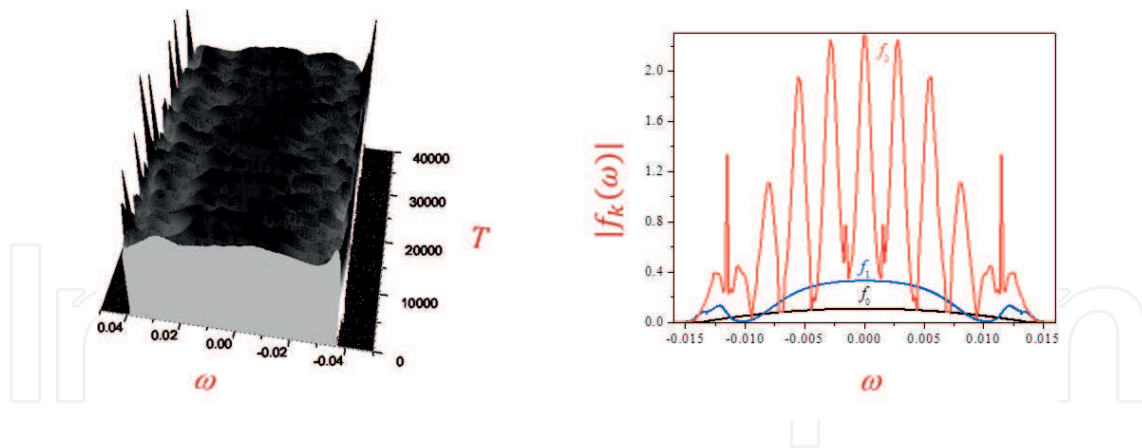
As a result of phase inhomogeneity and intensive internal energy flows, the internal coherence of DS can become partially broken. Then, DS splits into partially coherent “internal modes” which interact with each other as the independent “sub-solitons.” [29–31] Thus, DS becomes a strongly localized “cloud” of interacting “quasi-particles” or “glass of boiling water” (**Figure 6**).

These figures demonstrate an affinity between the structures of DS and turbulence [8, 23]. Both spectral structures are defined by dispersion relations: between soliton and dispersive waves for the former and Langmuir dispersion relation for the latter (**Figure 7**). Secondly, both high-energy DS and turbulence are characterized by spectral condensation at zero frequency (wavenumber) with subsequent scattering to higher frequencies confined by cut-off at  $\pm \Delta$ .

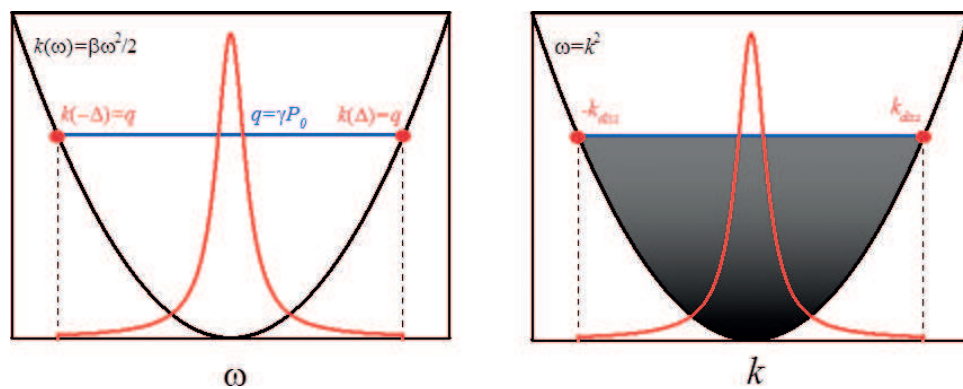
Such an analogy between DS and turbulence opens a door for building the kinetic and quantum [5, 32–34] theory of open (dissipative) semi-coherent structures which mimics, in particular, a quantum Bose-Einstein condensate in a dissipative environment.



**Figure 5.** Energy flows (left column) and corresponding spectral profiles (right column,  $\mathcal{F}$  is a Fourier image of  $\Psi$ ) in dependence on dispersion  $\beta$  and chirp  $\Theta$  for a DS with the profile  $\Psi \propto \text{sech}(t)^{1+i\Theta}$  [4].



**Figure 6.** DS spectrum as a “glass of boiling water” (left) [30] demonstrating the dynamics of internal perturbation modes (right) [31]. The last picture is obtained by a perturbation analysis in spectral domain with the Neumann series expansion.



**Figure 7.** DS spectrum (central red curve in the left picture) where the cut-off frequency  $\Delta$  is defined by the resonance condition between linear waves with a wave-number  $k$  and DS with a wave-number  $q$  (parabolic black curve in the left picture;  $P_0$  is a DS peak power) [23]. The turbulence in the wave-number space is defined by the Langmuir dispersion relation (parabolic black curve in the right picture). Spectral condensation at  $k = 0$  is illustrated by shading and forms a characteristic turbulence spectrum (central red curve; right picture) with the cut-off wave-number  $\pm k_{diss}$  defined by a dissipation (adapted from [8]).

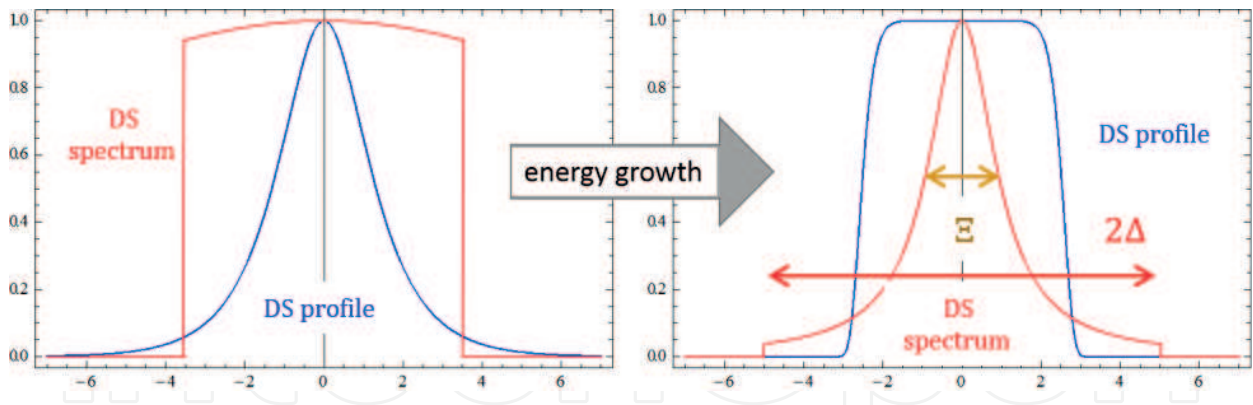
### 3. Transition to a DS turbulence

The mechanism of transition to turbulence for DS can be associated with the time/spectral duality (**Figure 8**). When the energy increases (i.e.,  $E \rightarrow \infty$  that corresponds to a system with “infinite capacity” [36]), the spectrum condensates around  $\omega = 0$  within a diapason of  $\Xi \rightarrow 0$  (**Figure 7**). Simultaneously, DS broadens in time domain  $\propto 1/\Xi$  by analogy with a growth of Bose-Einstein condensate “mass.”<sup>5</sup> The DS peak power tends to some constant value  $P_0 \propto 1/\zeta$  defined by a saturation of dissipative nonlinearity (see above), and, thereby, the cut-off frequency  $\Delta = \sqrt{\kappa P_0/\alpha}$  tends to be constant. The last value defines the coherence scale  $\propto 1/\Delta$  (few picoseconds for a typical DS).<sup>6</sup> As a result, DS becomes “decoupled,” and even small perturbations can

<sup>5</sup>The value  $\propto 1/\Xi$  can be treated as a measure of “long-range” correlation scale.

<sup>6</sup>The value  $\propto 1/\Delta$  can be treated as a measure of “short-range” correlation scale.





**Figure 8.** DS spectrum (red curves) which cut-off frequency  $\Delta$  defines the correlation scale  $\propto 1/\Delta$ . It tends to some constant value with the DS energy growth. Simultaneously, DS broadens in time domain  $\propto 1/\Xi$  (blue curves) in parallel with the concentration of energy around  $\omega=0$  in spectral domain [35].

destroy its internal coherence and split (“nucleate”) it into a set of “internal modes” shown in **Figure 6**.

More close insight into this mechanism can be provided by the adiabatic theory of DS in spectral domain presented in [28]. As was shown, the DS spectrum can be expressed as follows:

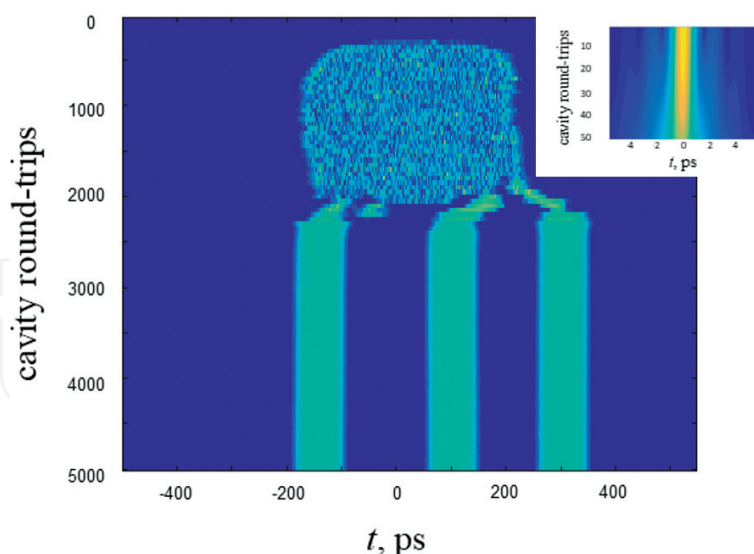
$$p(\omega) = \frac{\Upsilon}{\omega^2 + \Xi^2} H(\Delta^2 - \omega^2), \quad (6)$$

where  $p(\omega)$  is a DS spectral power, and  $H$  is the Heaviside function. Eq. (6) represents the spectra shown in **Figures 7 and 8**, and can be interpreted by analogy with the Rayleigh-Jeans distribution, so that  $\Xi^2$  plays a role of negative “chemical potential” [8, 36, 37]. The parameter  $\Upsilon = 6\pi\gamma/\kappa\zeta$  is an analog of “temperature” and is defined by both dissipative and nondissipative nonlinear parameters.

Since the “chemical potential”  $\Xi^2$  decays with the energy growth (**Figure 8**), a system tends to the state of “soliton gas” [38] with the characteristic “soliton size”  $\propto 1/\Delta$ . Thereby, a coherent “condensate” with minimum entropy becomes a state of the decomposed “quasi-particles” with the chaotically modulated powers because the required entropy growth is provided by such modulation<sup>7</sup> [40]. Thus, the energy growth (i.e., the growth of “condensate mass”  $\propto 1/\Xi$ ) leads to extra-sensitivity to *quantum-level* noises [40, 41] that urges the quantum theory of coherent and semi-coherent dissipative structures, which would weave largest and smallest scales in the DS dynamics.

The example of such “DS decomposition” through a turbulence is shown in **Figure 9**. This figure is obtained by numerical simulation of Eqs. (1) and (4) with taking into account of the gain saturation in the form of  $\sigma = \delta(1 - E/E_s)$  [27]. **Figure 9** demonstrates clearly two stages of DS evolution. The first stage corresponds to an incoherent and strongly turbulent DS, which is

<sup>7</sup>Here, one may draw an analogy with Hamiltonian systems, where the gradient of field is a measure of the amount of fluctuations [39].



**Figure 9.** Contour plot of the DS power evolution and the corresponding autocorrelation function on the turbulent stage. The evolution coordinate is measured in the laser cavity round-trips, and the transverse coordinate corresponds to the local time measured in picoseconds. The energy parameter  $E_s$  normalized on  $\gamma/\Delta t$  is of  $3 \times 10^5$ , where  $\gamma$  corresponds to a fused silica nonlinear coefficient and  $\Delta t=1$  femtosecond is a time discretization step. Other parameters are:  $\kappa=0.1\gamma$ ,  $\zeta=0.05\gamma$ ,  $\delta=0.05$ , and  $\alpha=40$  nm (Yb-fiber laser) [6].

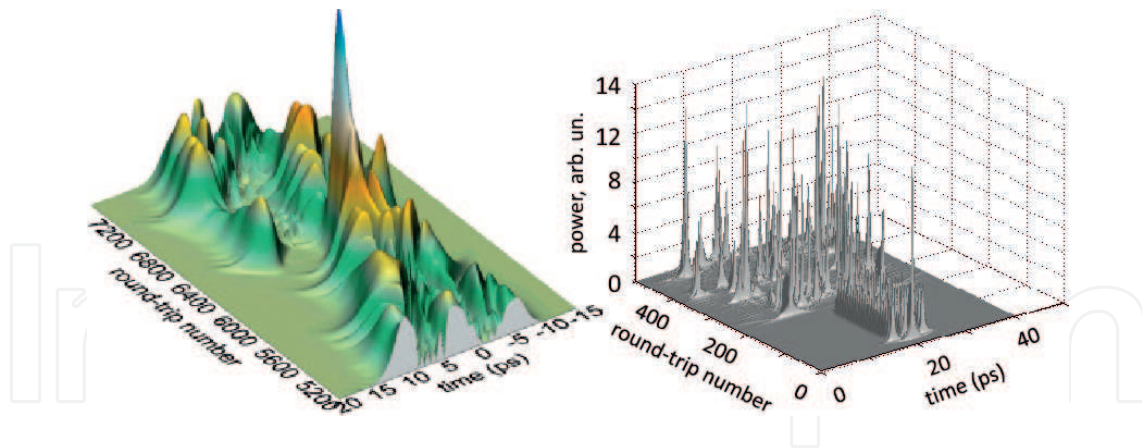
characterized by the short-range correlation time about of 1 ps. In the process of evolution, an incoherent DS splits into three almost identical coherent solitons, which widths are lower substantially and the corresponding long-range/short-range correlation times become smaller/larger, respectively. Small long-range correlation time prevents the DSs merging and larger short-range correlation time provides DS coherence.

An analysis of turbulent DS demonstrates its complicate internal structure which can be interpreted as the complex of strongly interacting bright, dark, and gray DSs on a finite but strongly self-localized background concentrating almost the entire part of the energy.<sup>8</sup> In some sense, an appearance of DS turbulence resembles the laminar-turbulent transition in a fiber laser when a macroscopically coherent field ( $\Xi \rightarrow 0$ ) becomes chaotically self-modulated [45].

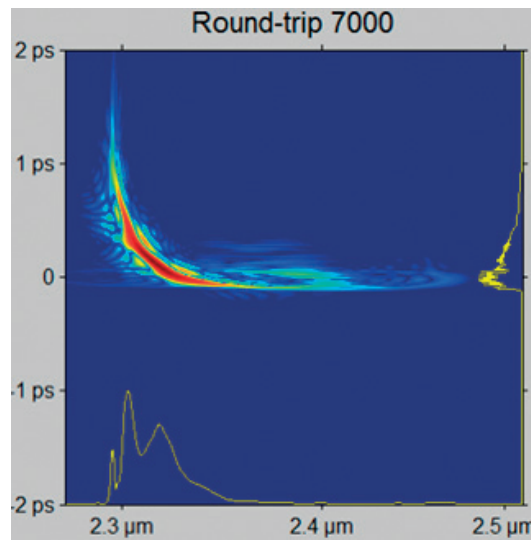
Nevertheless, such a scenario is not unique. The turbulent dynamics can result from the strong interaction between “individual” DSs forming a “soliton gas” or turbulent “soliton cluster” (**Figure 10**) [1, 5, 47]. Interaction of such cluster with a low-intensity background field can result in permanent radiation or absorption of DSs in the form of so-called “soliton rains” [48, 49].

Separately, one may note the chaotization of DS dynamics caused by resonant interaction with the dispersive waves in the presence of higher-order corrections to the dispersion term in Eq. (1). In this case, the collisions between DS and dispersive wave, which radiates by it, results in a chaotic dynamic preserving, nevertheless the DS integrity (**Figure 11**) [23].

<sup>8</sup>One has to distinguish such a structure from the breather-like structures on a continuous-wave background. Such structures can demonstrate chaotic and rogue waves dynamics, as well (e.g., see [43, 44]).



**Figure 10.** DS clusters in the form of a “persistent and coherent quasi-soliton” (left) [46] and a “sporadic rogue waves events that emerge from turbulent fluctuations” (right) [41].



**Figure 11.** Wigner (time-spectral) diagram of the chaotic DS in the presence of third-order dispersion [23]. DS (dark-red region around 2.3 μm) radiates a dispersive wave (blue tail around 2.4 μm). As a result of the difference between their group velocities, DS collides permanently with a dispersive wave that causes chaotization of dynamics and modulation of both DS spectrum and time-profile.

#### 4. Coherence of DS in the presence of nonlinearity with nonlocal/noninstantaneous response

A nonlinearity with the nonlocal/noninstantaneous response, which is of interest in optical context, can be taken into account by inclusion in Eq. (1) of the following term [50]

$$-i\gamma\Psi \int U(x - x')|\Psi|^2(T, x')dx'. \tag{7}$$

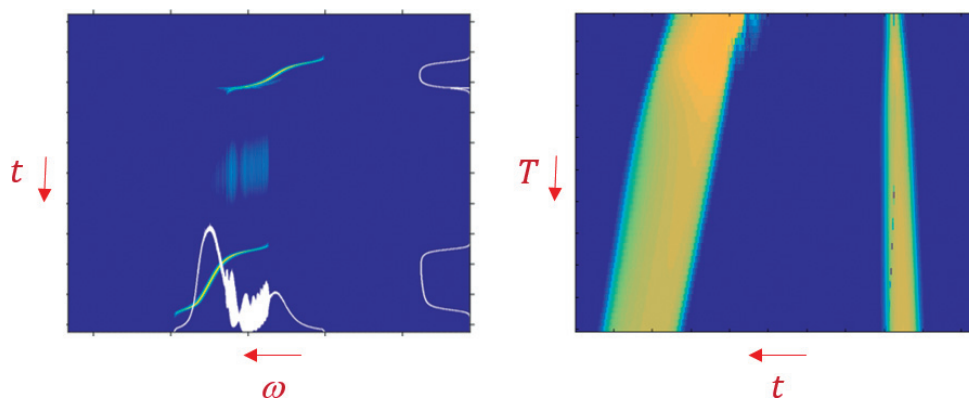
In the case of nonstationarity (i.e.,  $x \rightarrow t$  replacement), this equation describes the stimulated Raman scattering (SRS), for instance. Then, the response function is [6, 51]:

$$U(t) = \frac{T_1^2 + T_2^2}{T_1 T_2^2} \exp\left(-\frac{t}{T_2}\right) \sin\left(\frac{t}{T_1}\right), \quad (8)$$

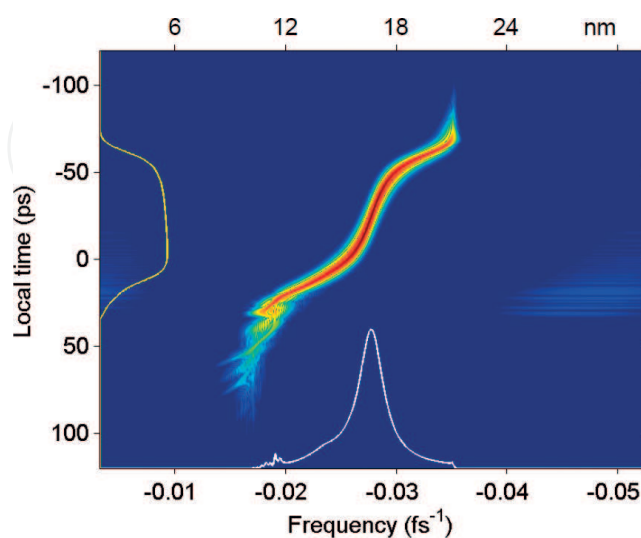
Where  $T_2$  and  $T_1$  define the effective relaxation time and resonant frequency of phonons in a nonlinear medium.

The simulations demonstrate [6, 35] that SRS suppresses the DS turbulence for the sufficiently large dispersions  $\beta$ . The first scenario is formation of uncoupled complex of DS and the dissipative Raman soliton (DRS) [6, 35, 42, 52] (Figure 12). One may assume, that such “energy discharging” is like the turbulence decay shown in Figure 9.

DRS is characterized by large chirp  $\Theta$  and frequency down-shift. The last results from intra-pulse SRS which is possible due to a broad spectrum, which is a common characteristic of DS and results from its large  $\Theta$ . A sole DRS develops with growing  $\beta$  (Figure 13) [6, 35]. It is



**Figure 12.** Wigner (time-spectral) diagram of the DS + DRS complex (left) and its evolution (contour plot of the field power; right) [35].



**Figure 13.** Wigner (time-spectral) diagram of a sole DRS developing for large  $\beta$  [6].

turbulence-free and characterized by perturbed anti-Stokes component, which is clearly visible on the Wigner diagram. Such perturbation induces a chaotic vibration of the DRS power [35]. Nevertheless, DRS exists within the parametric range, where an ordinary DS cannot develop in the turbulence-free regime. One may assume that the DRS stability results from passive negative feed-back based on interplay between nonlinear down-frequency shift due to SRS and spectral dissipation.

Another spectacular manifestation of the effect of a noninstantaneous nonlinearity on an incoherent field is an appearance of the spectral incoherent solitons (SIS) [53]. The spectrally localized soliton-like structures appear without any time-localization due to the causality property inherent to SRS so that a field cannot reach thermal equilibrium [54]. Formally, the corresponding evolution equation in the Langmuir turbulence limit has soliton-like solutions in the spectral domain [55, 56]. As a result, such structure localized in spectral domain possesses main properties of solitons including the property of elastic scattering.

## 5. Conclusions

The problem of DS coherence, chaotic, and turbulent dynamics has been outlined. A nontrivial internal structure of DS caused by its intensive energy exchange with dissipative environment allows conjecturing a close analogy with turbulent structure forming far from equilibrium. The existence of long-range correlation scale provides the DS energy scaling (or mass scaling for Bose-Einstein condensate). However, such “macroscopic” scaling is provided by strong phase inhomogeneity (chirp) so that internal coherence of DS defined by short-range correlation scale breaks and DS becomes a “cloud” of interacting “quasi-particles” or “glass of boiling water.” Such structure is very sensitive to perturbation of even quantum level. Such extra-sensitivity combines macro- and micro-scales that raises an issue of the quantum theory of the macroscopic coherent, partially, and incoherent dissipative structures.

In the context of this work, such DS “decomposition” leads to turbulent dynamics and DS fragmentation. In particular, interactions inside such “soliton cluster” can result in the rogue waves’ formation. An additional source of soliton destabilization is resonant interaction with a dispersive wave that results in chaotization of dynamics and formation of “soliton rains.”

Nonlinearity with noninstantaneous response (e.g., SRS) leads to new interesting effect. In particular, SRS suppresses the DS turbulence due to the formation of DS + DRS pairs or sole DRS. Although DRS is turbulence-free within a broad parametric range, it has a perturbed anti-Stokes component, which causes chaotic vibrations of DRS parameters.

Another and spectacular manifestation of the noninstantaneous response of nonlinearity is the formation of SIS. This structure is a soliton in the spectral domain but incoherent and delocalized in the time domain.

The above-considered phenomena and conjectures are of interest in the context of the development of approaches to the self-consistent theory of nonequilibrium dissipative structures in classical and quantum aspects, which would use the optical DSs as a testbed.

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