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# Optimization of Double-Well Bistable Stochastic Resonance Systems and Its Applications in Cognitive Radio Networks

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## Abstract

In this chapter, the optimization method of double-well bistable stochastic resonance (SR) system and one of its applications in cognitive radio networks are introduced, especially in the energy detection problem. The chapter will be divided into five sections. Firstly, the conventional double-well bistable stochastic resonance system is introduced with its special properties. Then based on the conventional discrete overdamped double-well bistable SR oscillator, the optimization method and the analyses results are given especially under low signal-to-noise ratio (SNR). In the applications, a novel spectrum sensing approach used in the cognitive radio networks (CRN) based on SR is proposed. The detection probability is also derived theoretically under a constant false-alarm rate (CFAR). Moreover, a cooperative spectrum sensing technique in CRN based on the data fusion of various SR energy detectors is proposed. Finally the whole chapter is summarized.

**Keywords:** stochastic resonance, optimization, cognitive radio networks, spectrum sensing, energy detection, cooperative spectrum sensing

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## 1. Introduction of conventional double-well bistable stochastic resonance system

In many different dynamic systems, it can be found that the stochastic resonance (SR) is a kind of complex nonlinear phenomenon with many applications [1, 2]. In this kind of dynamic system, it possesses some good performances, while it can help to increase the periodic driving signal power under some special conditions. A lot of researches have demonstrated that

the SR system may help to convert some power of the state variable signal in the SR system into the spectral power of the single-frequency driving signal in the SR system [1–3]. So, the SR system has been widely used in many applications, such as the weak target identification, weak signal detection and estimation, and so on [4–6].

In the dynamic SR processing, according to the SR noise power influence to the SR system, it can be found that the improvement effects, which include the signal power amplification and the signal-to-noise ratio (SNR) enhancement, have great relationships between the SR driving sinusoidal signal power and the SR noise power [3].

Mathematically, an SR system in a continuous form can be written as [3]

$$d\mathbf{x}(t)/dt = f[\mathbf{x}(t), r(t)], \quad (1)$$

while in the above equation,  $f[\cdot]$  is the dynamic SR system,  $\mathbf{x}(t)$  is the state vector, and  $r(t)$  is the driving signal of the SR system.

In many SR systems, it can be found that the quartic double-well bistable system is a widely used SR system with many researches and discussions, and it has been applied in many applications. It can be expressed as

$$dx(t)/dt = ax(t) - bx^3(t) + k \cdot r(t). \quad (2)$$

In the expression above,  $x(t)$  is the state variable of the SR system, parameters  $a$  and  $b$  determine the properties of the SR system, and the driving parameter  $k$  influences the effect of driving signal  $r(t)$  seriously. In many studies,  $r(t)$  is set as a single-frequency sinusoidal signal, which is also influenced by some additive noise  $n(t)$ , which is

$$r(t) = \varepsilon \sin \omega_s t + n(t), \quad (3)$$

while in the above equation, the parameters  $\varepsilon$  and  $\omega_s$  are the corresponding signal amplitude and signal angular frequency of the driving signal;  $n(t)$  is the additive noise. To simplify the analyses,  $n(t)$  can always be supposed to obey the Gaussian distribution, which possesses mean 0 and variance  $\sigma_n^2$ . So, the SNR of the driving signal  $r(t)$  (or the SNR of the input SR system) can be expressed by

$$SNR_i = \varepsilon^2 / 2 \sigma_n^2. \quad (4)$$

According to the linear response theory of SR system [3], the output of the SR system state variable  $x(t)$  can be expressed as a sum of two components as

$$x(t) = \varepsilon_o \sin(\omega_s t + \varphi_o) + n_o(t), \quad (5)$$

where  $\varepsilon_o$  is amplitude of the output signal,  $\varphi_o$  is the phase of the output signal at the input frequency point  $\omega_s$ , and  $n_o(t)$  is the additive noise in the output signal.

Based on the above assumptions, when  $\omega_s \rightarrow 0$  or even  $\omega_s = 0$ , the output SNR of the SR system may be estimated by [2]

$$SNR_o \approx \left[ \frac{\sqrt{2} a \varepsilon^2 c^2}{k^3 \sigma_n^4} e^{-\frac{2U_0}{k^2 \sigma_n^2}} \right] \cdot \left[ 1 - \left( \frac{4 a^2 \varepsilon^2 c^2}{\pi^2 k^3 \sigma_n^4} e^{-\frac{4U_0}{k^2 \sigma_n^2}} \right) \left/ \left( \frac{2 a^2}{\pi^2} e^{-\frac{4U_0}{k^2 \sigma_n^2}} + \omega_s^2 \right) \right. \right]^{-1}, \quad (6)$$

where  $c = \sqrt{a/b}$  and  $U_0 = a^2/4b$  are constants corresponding to the selection of parameters  $a$  and  $b$  in (2). It can also be found in (6) that in many real applications, the parameter  $k$  is the only parameter which can be adjusted, and also it cannot influence the parameter  $SNR_i$  in (4), so it is a very important factor which can determine the SR phenomenon [2].

## 2. Optimization of double-well bistable stochastic resonance system

### 2.1. System optimization and performance analyses

As described in last section, to make the SR system more applicable to the weak target identification or detection problems, we investigate a kind of optimization method to the quartic double-well bistable SR system in (2), and the target is to guarantee the enhancement of the signal SNR and also reach a maximal output SNR at the same time.

Although the result in (6) is based on the assumption  $\omega_s \rightarrow 0$ , when under some conditions that  $\omega_s \rightarrow 0$  cannot be guaranteed, some traditional down-conversion methods can be applied if the frequency of the sinusoidal signal cannot fulfill  $\omega_s \rightarrow 0$ . Without loss of generality, an additive SR noise  $n_1(t)$  is also introduced into the SR system, which possesses mean 0 and variance 1; then the quartic double-well bistable system in (2) can be rewritten as

$$\begin{aligned} dx(t)/dt &= ax(t) - bx^3(t) + k_1 \cdot r(t)/\|r(t)\| + k_2 n_1(t) \\ &= ax(t) - bx^3(t) + k_1 \varepsilon \sin \omega_s t / \|r(t)\| + k_1 n(t)/\|r(t)\| + k_2 n_1(t), \end{aligned} \quad (7)$$

where  $k_1$  and  $k_2$  are the positive driving parameters corresponding to  $r(t)$  and  $n_1(t)$ , respectively.  $r(t)$  is normalized by  $\|r(t)\|$  to simplify the analyses, which is defined by

$$\|r(t)\| \stackrel{def}{=} \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N r^2(t) = \frac{1}{2} \varepsilon^2 + \sigma_n^2, \quad (8)$$

where  $N$  is the sampling number. And when  $SNR_i$  is small enough, we have

$$\hat{\sigma}_n^2 \approx \|r(t)\|. \quad (9)$$

Based on the analyses in Ref. [4], if we want to reach an optimal result, it requires that the SR noise should be symmetric, and then  $n_1(t)$  can also be chosen as a kind of noise signal with Gaussian distribution. So, (7) can be rewritten as

$$dx(t)/dt = ax(t) - bx^3(t) + k_3 \varepsilon \sin \omega_s t + k_4 n_{SR}(t), \quad (10)$$

where the parameters are defined by

$$k_3 \stackrel{\text{def}}{=} k_1 / \|r(t)\|, \quad (11)$$

$$k_4 \stackrel{\text{def}}{=} \sqrt{k_1^2 \cdot \hat{\sigma}_n^2 / \|r(t)\|^2 + k_2^2}, \quad (12)$$

and  $n_{\text{SR}}(t)$  in (10) is a Gaussian noise with mean 0 and variance 1.

With the assumptions above, the  $\text{SNR}_o$  in (6) can be rewritten as

$$\text{SNR}_o \approx \left[ \sqrt{2} a k_3^2 \varepsilon^2 c^2 k_4^{-4} e^{-\frac{2U_0}{k_4^2}} \right] \cdot [1 - 2 k_3^2 \varepsilon^2 c^2 k_4^{-4}]^{-1} = \frac{\sqrt{2} a k_3^2 \varepsilon^2 c^2 e^{-\frac{2U_0}{k_4^2}}}{k_4^4 - 2 k_3^2 \varepsilon^2 c^2}. \quad (13)$$

Firstly, to ensure the SNR improvement effect of the SR system, it requires  $\text{SNR}_o > \text{SNR}_i$ , so

$$\frac{\sqrt{2} a k_3^2 \varepsilon^2 c^2 e^{-\frac{2U_0}{k_4^2}}}{k_4^4 - 2 k_3^2 \varepsilon^2 c^2} > \frac{\varepsilon^2}{2 \sigma_n^2}. \quad (14)$$

And when  $\text{SNR}_i$  is low enough, (14) can be simplified to

$$k_3^2 > k_4^4 e^{\frac{2U_0}{k_4^2}} / (2 \sqrt{2} U_0 \sigma_n^2). \quad (15)$$

When  $U_0$  and  $\sigma_n^2$  are fixed, it is obvious that  $k_4 = \sqrt{U_0}$  will lead to the maximal value of the right side expression of (15). So when we have:

$$k_3^2 > U_0 e^2 / (2 \sqrt{2} \hat{\sigma}_n^2), \quad (16)$$

the SNR enhancement can be achieved.

What is more, to reach the maximum output SNR of the system, we can set up an optimization problem, where we suppose (13) as the corresponding objective function and let  $k_1$  be fixed, and then we let

$$\partial \text{SNR}_o / \partial k_4^2 = 0. \quad (17)$$

And the result can be changed to

$$k_4^6 - U_0 k_4^4 + 2 U_0 k_3^2 \varepsilon^2 c^2 = 0, \quad (18)$$

or  $k_4$  is the solution of the above equation.

By calculating the discriminant  $\Delta$  of (18), we have

$$\Delta = U_0^2 k_3^4 \varepsilon^4 c^4 - \frac{2}{27} U_0^4 k_3^2 \varepsilon^2 c^2 = \frac{U_0^2 a k_3^2 \varepsilon^2 c^2}{216 b^2} (216 k_3^2 \varepsilon^2 b - a^3). \quad (19)$$

Then the optimization result can be decided by the power or the amplitude of the driving sinusoidal signal. It can be found that  $k_3$  should also satisfy (16), so we can choose a reasonable  $k_3$  big enough to fulfill  $\Delta > 0$  and guarantee that the optimization result of  $k_4$  can be

achieved. When substituting the optimal values of  $k_3$  and  $k_4$  into (11) and (12), the optimal driving parameters  $k_1$  and  $k_2$  can finally be achieved.

## 2.2. Computer simulations

To testify the effectiveness of the above proposed optimization method, we give out a testifying example and carry out corresponding computer simulation results based on the analyses in the last section.

To simplify the simulations, a single-frequency sinusoidal signal corrupted with additive white Gaussian noise (AWGN) is assumed as the signal  $r(t)$ , and the amplitude and angular frequency of the signal are chosen as  $\varepsilon = 1$ ,  $\omega_s = 0.01$ , respectively. The sampling number is  $N = 1 \times 10^5$ ; and the parameters are chosen as  $a = 1$  and  $b = 1$  in the SR system.

In the following computer simulations, the maximum likelihood estimate (MLE) method [7] is applied to estimate the amplitude of the signal as

$$\hat{\varepsilon} = \sqrt{\hat{\alpha}_1^2 + \hat{\alpha}_2^2}, \quad (20)$$

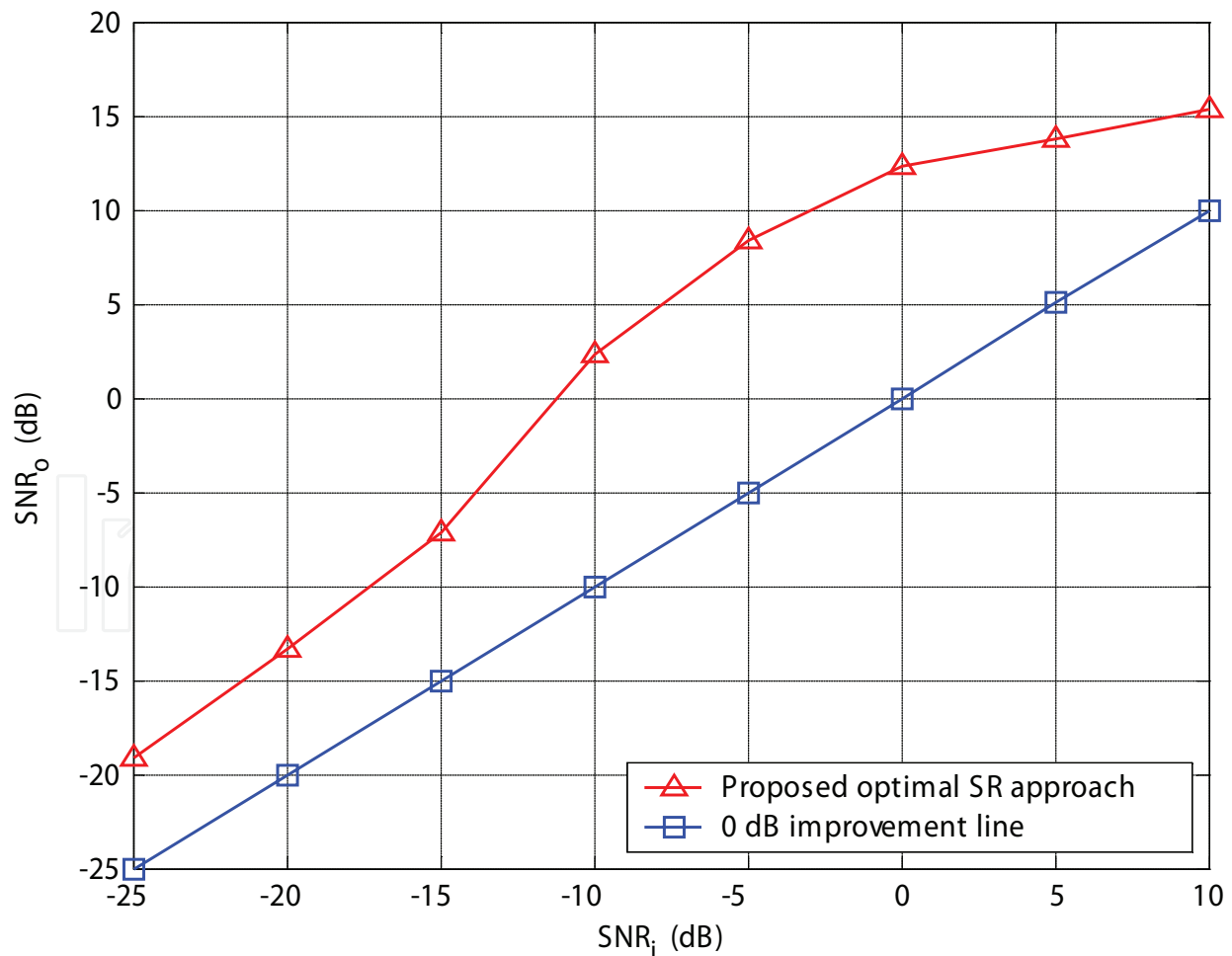


Figure 1. SNR<sub>o</sub> vs. SNR<sub>i</sub> by using the proposed optimal SR approach.

where we have

$$\hat{\alpha}_1 = \frac{2}{N} \sum_{t=0}^{N-1} r(t) \cos \omega_s t, \quad \hat{\alpha}_2 = \frac{2}{N} \sum_{t=0}^{N-1} r(t) \sin \omega_s t. \quad (21)$$

Eq. (7) is changed to the following difference equation for simulations [8]:

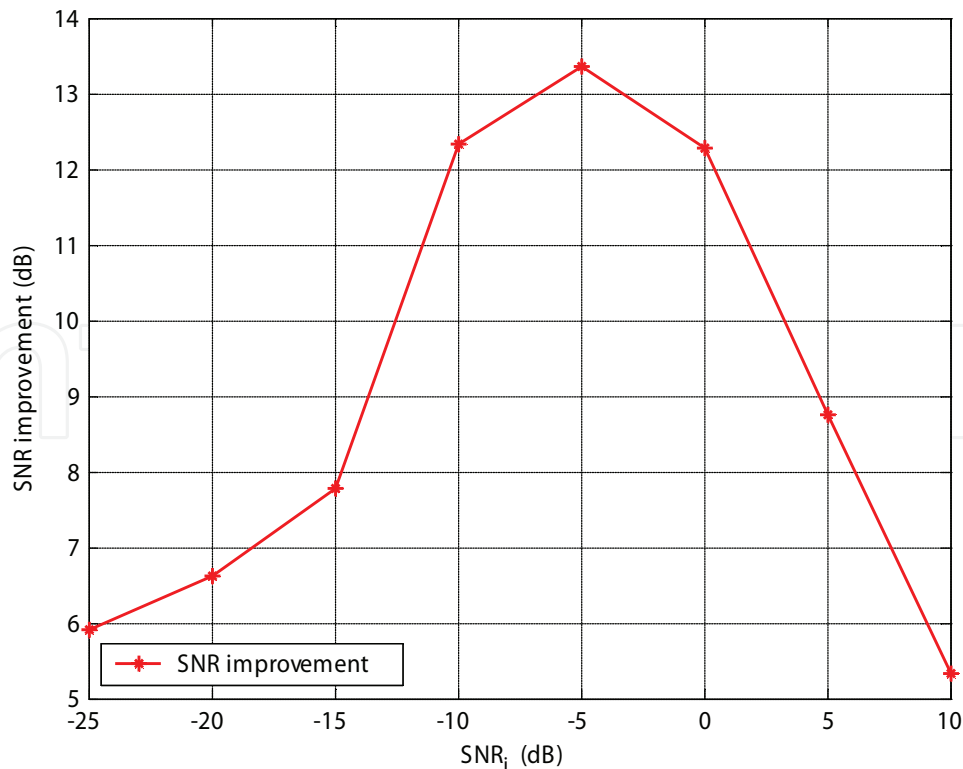
$$x(t+1) = x(t) + \Delta t \cdot [ax(t) - bx^3(t) + k_1 \cdot r(t)/\|r(t)\| + k_2 n_1(t)], \quad (t = 0, 1, \dots, N-2), \quad (22)$$

where the parameter  $\Delta t$  is chosen as 0.0195.

**Figure 1** gives the  $\text{SNR}_o$  vs.  $\text{SNR}_i$  comparison performance through the proposed method, while the range of  $\text{SNR}_i$  is between  $-25$  dB and  $10$  dB. From the result, it can be discovered that the SNR of  $r(t)$  has been enhanced especially under low SNR, for example,  $\text{SNR}_i < 0$  dB.

**Figure 2** shows a result regarding to the SNR enhancement through the proposed optimal SR method. The  $\text{SNR}_i$  also changes from  $-25$  dB to  $10$  dB. It can be found that the SNR enhancement can also be reached even under low SNR.

**Figure 3** shows the  $\text{SNR}_o$  performance when the parameters  $k_1$  and  $k_2$  are adjustable under  $\text{SNR}_i = -25$  dB. It can be discovered clearly that a maximal  $\text{SNR}_o$  can be reached with some certain optimal  $k_1$  and  $k_2$  values. **Figure 4** shows the performance of  $\text{SNR}_o$  vs.  $k_2$  under optimal  $k_1$  under the condition  $\text{SNR}_i = -25$  dB. **Figures 5** and **6** give the same computer simulation results under the condition  $\text{SNR}_i = -20$  dB, and they also verify the reliability of the proposed method.



**Figure 2.** SNR improvement by using the proposed optimal SR approach.



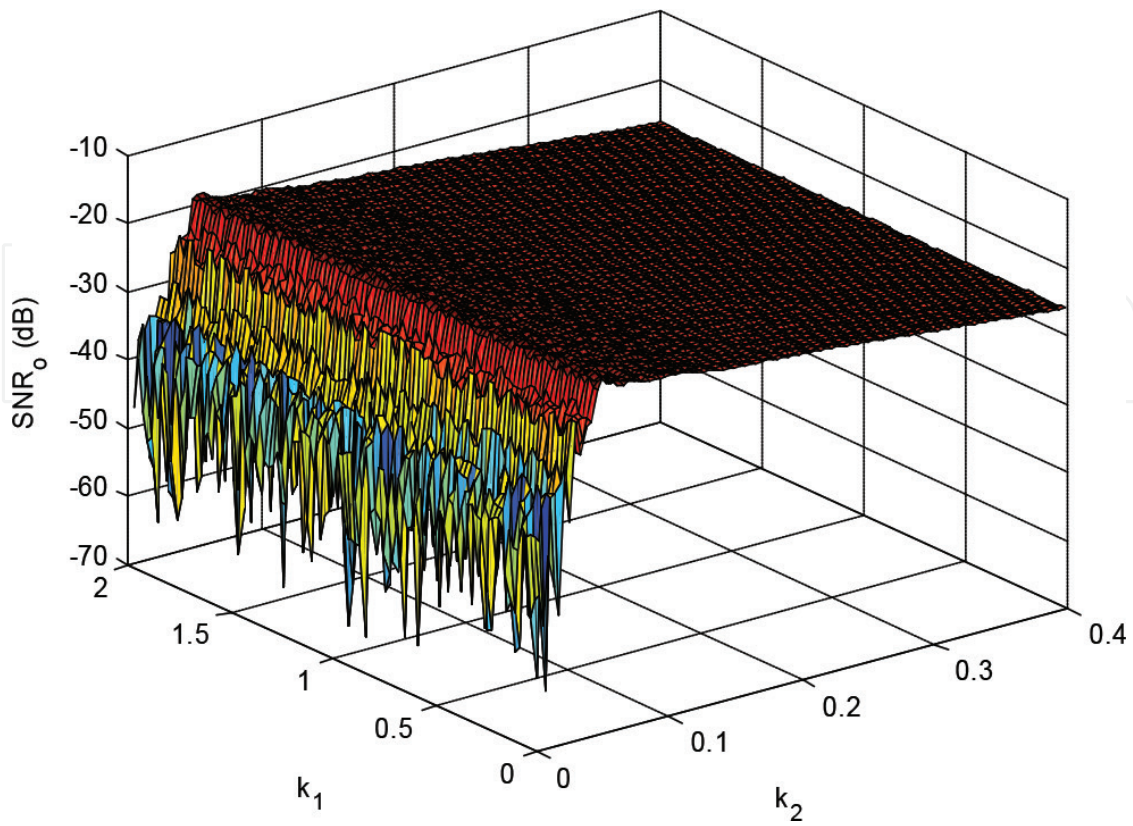


Figure 3. Performance of  $SNR_o$  under  $SNR_i = -25$  dB when  $k_1$  and  $k_2$  are adjustable.

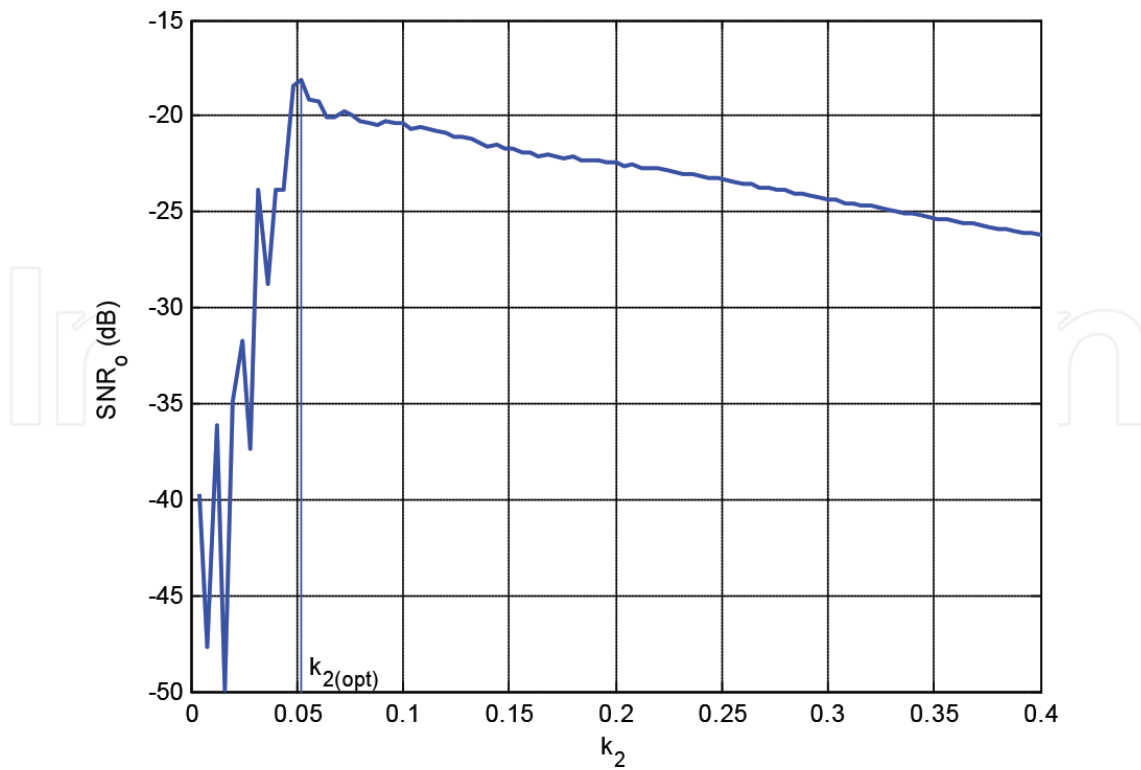


Figure 4.  $SNR_o$  vs.  $k_2$  under optimal  $k_1$  and  $SNR_i = -25$  dB.



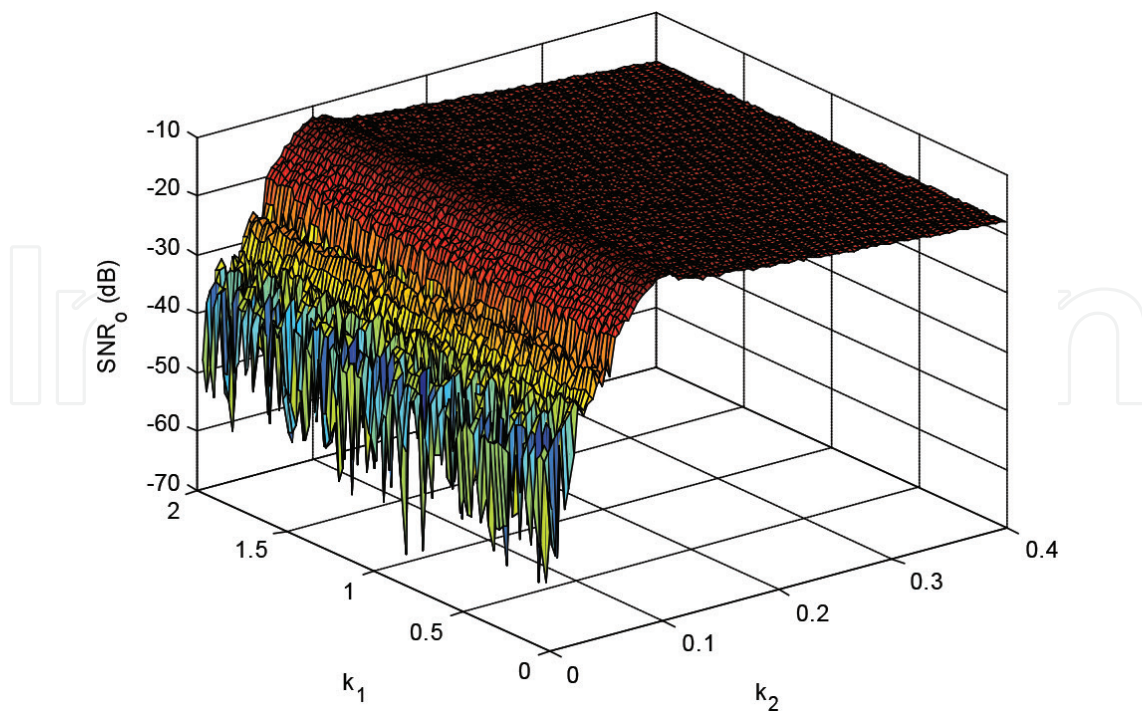


Figure 5. Performance of  $SNR_o$  under  $SNR_i = -20$  dB when  $k_1$  and  $k_2$  are adjustable.

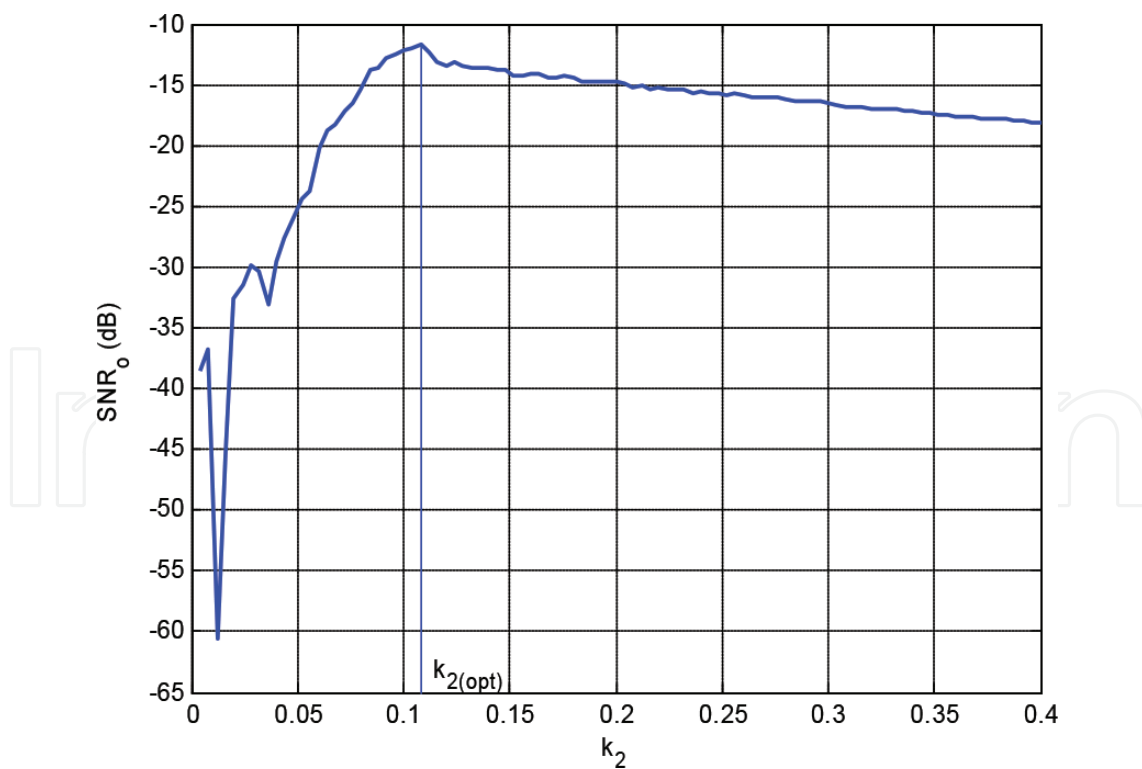


Figure 6.  $SNR_o$  vs.  $k_2$  under optimal  $k_1$  and  $SNR_i = -20$  dB.

### 3. Applications in the energy detection problem in cognitive radio networks

#### 3.1. Energy detection problem in cognitive radio networks

In the past years, the research works in the area of cognitive radio (CR) network have been widely reported with fast progress. A lot of novel research developments make the research topics in the related areas more and more attractive [9]. As is known, in CR, the spectrum sensing approaches play an important role in CR network because it helps the secondary or opportunistic users (SUs) to detect the existence of the primary users (PUs) and define whether they can transmit the information or not [10]. Without loss of generality, we suppose that only the overlay mode in CR networks is considered in this discussion. The main target of spectrum sensing is to define the presence of PUs under some unpredictable noisy wireless communications conditions. So when the PUs are detected to be absent, the SUs are permitted to use the spectrum holes on an opportunistic basis which are occupied by PUs before, so that it can enhance the spectrum utility significantly [11]. In other words, the spectrum sensing can be regarded as a base of CR networks seriously.

In the literatures, many approaches have been proposed to ensure the performance of spectrum sensing and minimize the interference to some other users, including the PUs [9]. With the previous studies, it is found that the energy detection is a very general spectrum sensing method which does not need any prior knowledge of PUs; and based on the traditional Neyman-Pearson criterion [7], the spectrum sensing problem can be converted to a detection problem as the following two hypotheses:

$$\begin{aligned} H_0: r(t) &= n(t), \quad (t = 0, 1, \dots, N-1) \\ H_1: r(t) &= h \cdot s(t) + n(t), \quad (t = 0, 1, \dots, N-1), \end{aligned} \quad (23)$$

where  $r(t)$  is the signal at the receiver,  $s(t)$  is the PU signal, and it is assumed that  $s(t)$  obeys the distribution with mean 0 and variance  $\sigma_s^2$  and  $h$  is the channel fading factor between the transmitter (PU) and the receiver (SU). In the wireless communications applications, it can always be assumed that it has Rayleigh distribution with the second-order moment  $E[h^2] = m_h^2$  independent to PU, and  $n(t)$  is the additive noise independent to  $s(t)$  and  $h$ . Simultaneously, sometimes the co-channel interference or multiuser interference of the PU signal can also be regarded as another additive part of  $n(t)$ . So to simplify the analyses, we suppose that  $h$  is predictable or can be estimated properly at the receiver and  $n(t)$  also obeys the additive white Gaussian noise (AWGN) distribution with mean 0 and variance  $\sigma_n^2$ .

For the traditional Neyman-Pearson detection, the assumption or decision  $H_1$  can be made when the likelihood ratio exceeds a certain threshold  $\gamma$ , as follows:

$$L(\mathbf{r}) = p(\mathbf{r}; H_1) / p(\mathbf{r}; H_0) > \gamma, \quad (24)$$

where  $\mathbf{r} = [r(0), r(1), \dots, r(N-1)]^T$  is the receiving signal vector and  $p(\mathbf{r}; H_0)$  and  $p(\mathbf{r}; H_1)$  represent the probability density functions (PDFs) of the receiving signal vector  $\mathbf{r}$  under  $H_0$  and  $H_1$ , respectively, while  $L(\mathbf{r})$  is the likelihood ratio to be calculated.

Based on the analyses above, under two different hypotheses, the receiving signal  $\mathbf{r}$  obeys Gaussian distribution with different variances, which can be expressed by

$$\mathbf{r} \sim N(\mathbf{0}, \sigma_n^2 \mathbf{I}) \quad \text{under } H_0 \quad (25)$$

$$\mathbf{r} \sim N(\mathbf{0}, (m_h^2 \sigma_s^2 + \sigma_n^2) \mathbf{I}) \quad \text{under } H_1. \quad (26)$$

Thus,  $H_1$  is decided when

$$T(\mathbf{r}) = \sum_{t=0}^{N-1} r^2(t) > \left[ 2 \ln \gamma - N \ln \left( \frac{\sigma_n^2}{m_h^2 \sigma_s^2 + \sigma_n^2} \right) \right] \left[ \sigma_n^2 (m_h^2 \sigma_s^2 + \sigma_n^2) \right] m_h^{-2} \sigma_s^{-2} = \gamma_{ED} \quad (27)$$

where  $T(\mathbf{r})$  is the statistic of the traditional energy detector and  $\gamma_{ED}$  is the threshold to satisfy  $P_{fa} = \alpha$  for a given CFAR  $\alpha$ . Because the Neyman-Pearson detector calculates the energy of the receiving signal  $r(t)$ , it is also called an energy detector.

In the following, the corresponding false alarm rate  $P_{fa(ED)}$  and the detection probability  $P_{d(ED)}$  of the above energy detector can be given as

$$P_{fa(ED)} = \Pr\{T(\mathbf{r}) > \gamma_{ED}; H_0\} = \Pr\left\{\frac{T(\mathbf{r})}{\sigma_n^2} > \frac{\gamma_{ED}}{\sigma_n^2}; H_0\right\} = Q_{\chi_N^2}\left(\frac{\gamma_{ED}}{\sigma_n^2}\right), \quad (28)$$

$$P_{d(ED)} = \Pr\{T(\mathbf{r}) > \gamma_{ED}; H_1\} = \Pr\left\{\frac{T(\mathbf{r})}{m_h^2 \sigma_s^2 + \sigma_n^2} > \frac{\gamma_{ED}}{m_h^2 \sigma_s^2 + \sigma_n^2}; H_1\right\} = Q_{\chi_N^2}\left(\frac{\gamma_{ED}}{m_h^2 \sigma_s^2 + \sigma_n^2}\right), \quad (29)$$

while  $Q_{\chi_N^2}(\cdot)$  is the right-tail probability function with  $N$  degrees of freedom.

It can be found in the researches that this kind of energy detection method could perform well under high SNR. But its performance degrades seriously when SNR is reduced, especially when  $\text{SNR} < -10$  dB. For example, the value of detection probability  $P_d$  under  $N = 10^3$  and  $P_{fa(ED)} = 0.1$  will decrease from 0.795 to 0.283 when the SNR changes from  $-10$  dB to  $-15$  dB, which may be a very general case in CR networks [12].

### 3.2. SR-based spectrum sensing approach

In this subsection, we propose a novel spectrum sensing method with the combination of traditional energy detector and the SR processing. First, let the receiving signal pass the SR system, and the amplified signal can be observed at the output of the SR system. Then the amplified signal goes through the conventional energy detector to get the final spectrum sensing decision.

In this proposed scheme based on SR, first, we set the normalized signal of  $r(t)$  in (23), say  $r_0(t)$ , as the input of an SR system  $f[\cdot]$ ; then we have

$$\dot{\mathbf{x}}(t) = f[\mathbf{x}(t), r_0(t) + n_0(t)], \quad (30)$$

$$r_0(t) = r(t) / \sqrt{\text{var}[r(t)]}, \quad (t = 0, 1, \dots, N-1) \quad (31)$$

where  $\mathbf{x}(t)$  is still the SR system status vector and  $n_0(t)$  is the SR noise with mean 0 and variance  $\sigma_{n_0}^2$ , so  $r_0(t) + n_0(t)$  can be taken as the drive signal of the SR system.

Based on the SR linear response theory [3], the status vector of SR system can be divided into two independent additive parts, say

$$\mathbf{x}(t) = \mathbf{s}_{SR}(t) + \mathbf{n}_{SR}(t), \quad (32)$$

where  $\mathbf{s}_{SR}(t)$  is the system response signal corresponding to the normalized PU signal  $h \cdot s(t) / \sqrt{\text{var}[r(t)]}$  and  $\mathbf{n}_{SR}(t)$  is the system response signal corresponding to the noise signal  $n(t) / \sqrt{\text{var}[r(t)]} + n_0(t)$ . It can be found that the additive channel noise  $n(t)$  also plays a part role of SR noise.

From the above analyses, to reach a maximal SNR<sub>o</sub>, the optimal variance of the introduced SR noise  $\sigma_{n_0(\text{opt})}^2$  can be calculated according to the derivations in the last section.

### 3.3. Experimental and comparison results

In the following, we present some experimental and comparison outcomes. In the computer simulations, the discrete overdamped bistable oscillator in (22) is used as the dynamic SR system model.

As is known QPSK and QAM are the mostly used modulation methods [13, 14] in the broadcasting systems. So in the computer simulations thereafter, a QPSK signal as the PU signal together with a co-channel interference QPSK signal with AWGN through the Rayleigh fading channel is utilized as the driving signal of the SR system, which can be expressed by

$$r(t) = h \cdot [A_P \cdot \sin(\omega_P t + \varphi_P) + A_M \cdot \sin(\omega_M t + \varphi_M)] + n(t), \quad (33)$$

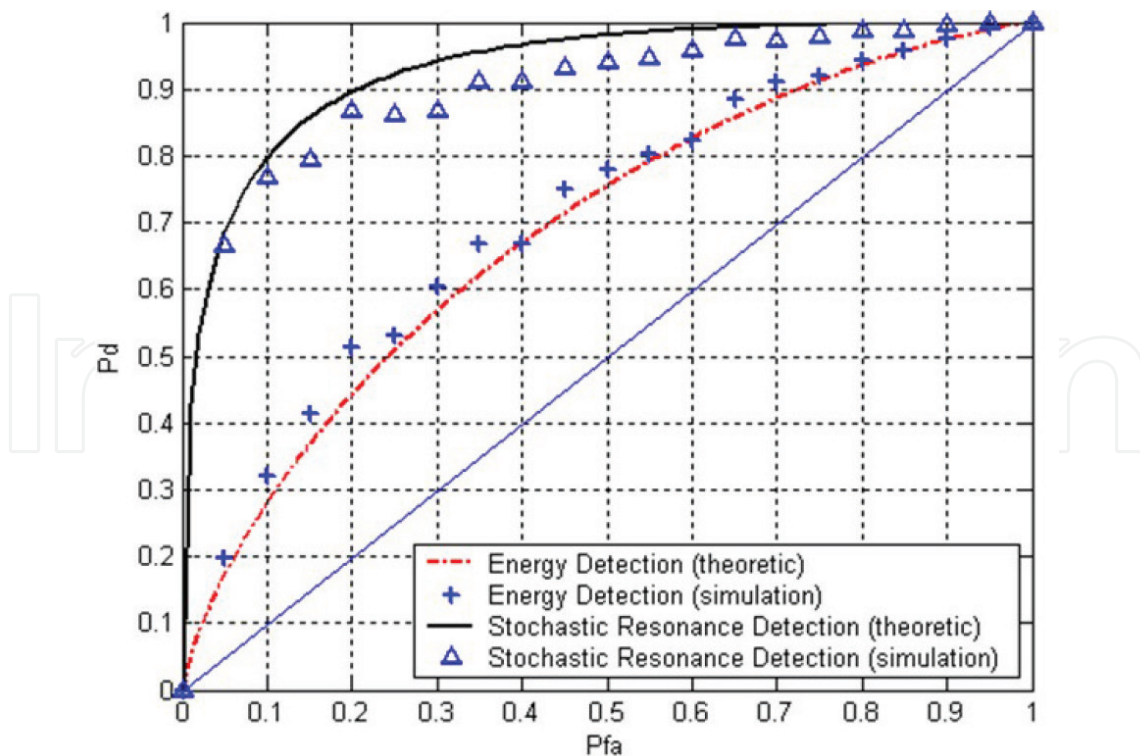
where  $h$  is the Rayleigh channel gain with mean 1;  $A_P$ ,  $\omega_P$  and  $\varphi_P$  are the amplitude, angular frequency, and phase of the PU sinusoidal carrier signal;  $A_M$ ,  $\omega_M$  and  $\varphi_M$  are the amplitude, angular frequency, and phase of the multiuser interference sinusoidal carrier signal, respectively. Here,  $\varphi_P, \varphi_M \in \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$  in QPSK. In this case, the input SNR can be calculated by [15]

$$SNR_i = \frac{1}{2} m_h^2 A_P^2 / \left( \frac{1}{2} m_h^2 A_M^2 + \sigma_n^2 \right). \quad (34)$$

In the following simulations, we choose  $\omega_P = 0.04\pi$ ,  $\omega_M = 0.2\pi$ , and  $\sigma_n^2 = 1$ . So the optimal variance of the introduced white Gaussian SR noise with mean 0 can be calculated through the analyses in the last section, and we can get  $\sigma_{n_0(\text{opt})}^2 = 1 - k^2$  which requires  $k \leq 1$ .

**Figures 7 and 8** give the performance comparison results of the receiver operating characteristic (ROC) plots between the traditional energy detector and the proposed SR-based energy detector under the conditions  $\text{SNR} = -15$  dB and  $\text{SNR} = -20$  dB. The total sampling number is  $N = 10^3$ . In the figures, both the theoretical results and the computer simulation results of the above two methods are given, and the theoretical results of detection probability of the proposed method are calculated based on (29). It can be discovered that the detection probabilities of the proposed approach are higher than the energy detector, especially under low SNR as  $\text{SNR} < -10$  dB, which is a good performance to the real applications; and it can also be discovered that even under  $\text{SNR} = -20$  dB which is also very common in CR networks, the proposed detection method can still perform better than the energy detection method with a significant detection probability enhancement.

Besides the ROC curve performance comparison, the results of the detection probability versus SNR under CFAR are also presented. **Figures 9 and 10** give the performance comparison results between the proposed detection method and the conventional energy detection method under  $P_{fa} = 0.05$  and  $P_{fa} = 0.1$ , respectively. The total sampling number is still selected as  $N = 10^3$ . In the following simulations, the input SNR changes from  $-20$  dB to  $0$  dB. And both the theoretical analyses results and the computer simulation results are given in **Figures 9 and 10**. It is obvious that the detection probability of the proposed SR-based method can be improved, especially under low SNR of  $\text{SNR} < -10$  dB, and also it can be discovered that a 5 dB SNR enhancement can be achieved. Based on the simulation results, the main problems of the conventional energy detection method can be solved.



**Figure 7.** ROC curves of different spectrum sensing approaches under  $\text{SNR} = -15$  dB.



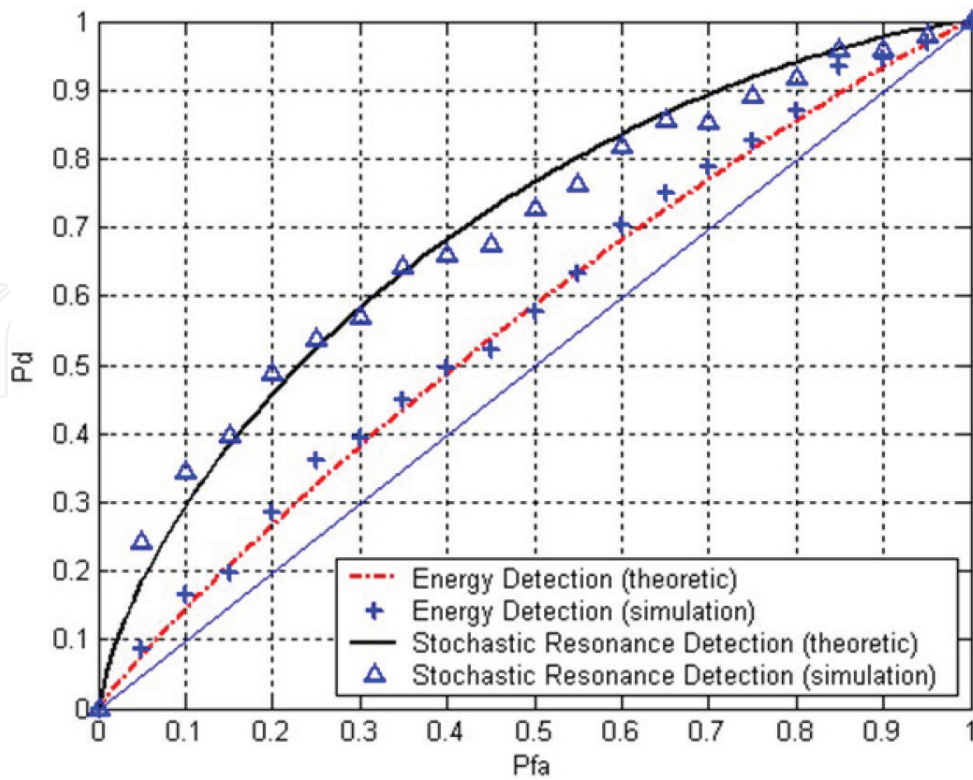


Figure 8. ROC curves of different spectrum sensing approaches under SNR = -20 dB.

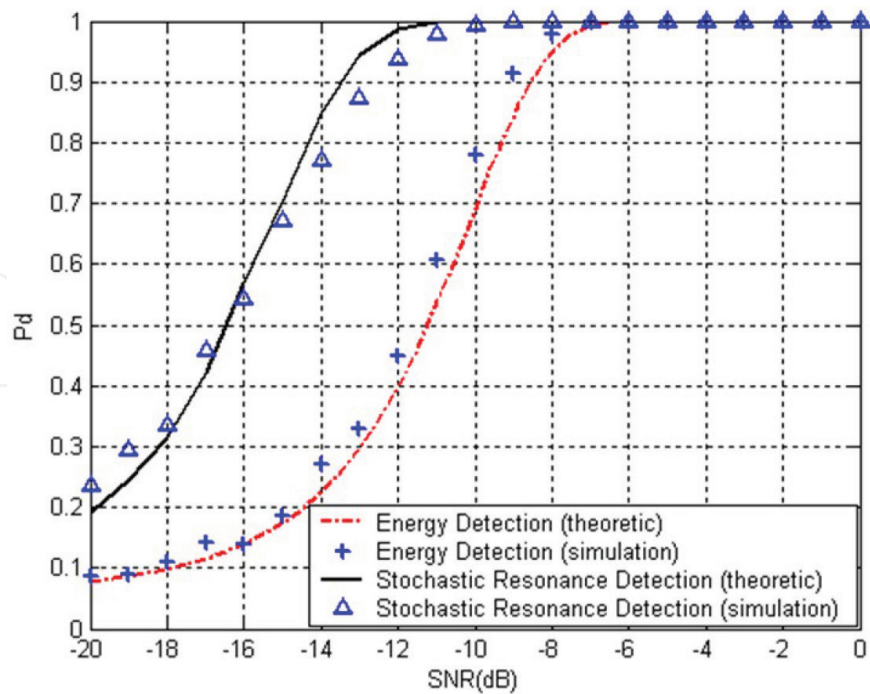


Figure 9. Detection probability versus SNR under  $P_{fa} = 0.05$ .

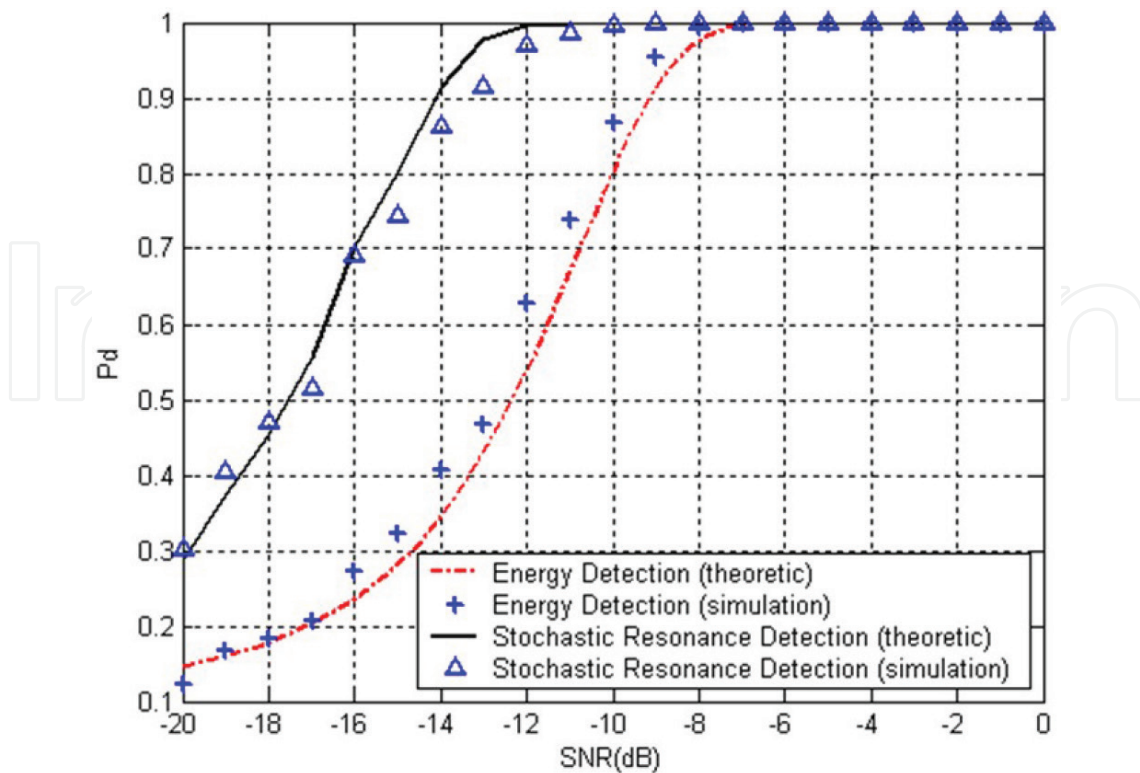


Figure 10. Detection probability versus SNR under  $P_{fa} = 0.1$ .

## 4. Application of cooperative stochastic resonance in the energy detection problem in cognitive radio networks

### 4.1. Cooperative SR-based spectrum sensing approach

In these years' studies, the spectrum sensing techniques in physical layer can be divided into two classes: noncooperative sensing techniques and cooperative sensing techniques. Recently it has become a new direction by introducing some cooperation methods into the spectrum sensing or PU signal detection procedure with the cooperation of different secondary user (SU) sensing results [16, 17]. So based on the results in the last two sections, here we introduce the chaotic stochastic resonance (CSR) system to improve the spectrum sensing performance especially under low SNR circumstances.

At the first step, we could randomly select  $K$  SUs to carry out the spectrum sensing process independently to the communication channel. To realize the data fusion, we carry out two kinds of sensing methods at each SU at the same time. One is the traditional energy detection method with the statistics  $A(x)$ , and another is the CSR energy detector with the statistics  $B(x)$ . While in the real applications, it is also very difficult to determine what kind of SR noise will be best or optimal, so we choose different types of CSR noise as the CSR noise signal candidate  $\beta_0(t)$  in each of the CSR systems, but the CSR systems in each SUs are all the same. To get some certain fusion result, some commonly used noise types in the wireless communication systems can be used, for example, AWGN signal, lognormal distribution noise,



Weibull distribution noise, etc. Here we list the noise-type candidates as  $\{\beta_1(t), \beta_2(t), \dots, \beta_K(t)\}$ . When some certain CSR system  $f[\cdot]$  and the noise types are fixed, the corresponding optimal parameters with these noise-type candidates can also be calculated.

Theoretically and without loss of generality, it can be assumed that all the receiving signal at different SU obey the same distribution, so (23) is suitable for each SU. To simplify the analyses thereafter, we suppose that  $h = 1$ . So we have

$$\begin{cases} H_0: E[A_1(\mathbf{x})] = E[A_2(\mathbf{x})] = \dots = E[A_K(\mathbf{x})] = E[A(\mathbf{x})] = \sigma_{n'}^2 \\ H_1: E[A_1(\mathbf{x})] = E[A_2(\mathbf{x})] = \dots = E[A_K(\mathbf{x})] = E[A(\mathbf{x})] = \sigma_s^2 + \sigma_{n'}^2 \end{cases} \quad (35)$$

while  $A_1(\mathbf{x}), A_2(\mathbf{x}), \dots,$  and  $A_K(\mathbf{x})$  are the statistics of SUs 1, 2, ..., K, respectively.

In the data fusion processing, we introduce the traditional Bayesian fusion method to realize the cooperative spectrum sensing. Simultaneously, if the same traditional energy detection method and the same threshold  $\gamma_{ED}$  are used at each SU detector, the expectation result  $E[A^{1,2,\dots,K}(\mathbf{x})]$  of the Bayesian fusion can be written as

$$\begin{aligned} E[A^{1,2,\dots,K}(\mathbf{x})] &= \frac{\prod_{k=1}^K E[A_k(\mathbf{x}) | A_k(\mathbf{x}) = \frac{1}{N} \sum_{t=1}^N x_k^2(t)]}{\prod_{k=1}^K E[A_k(\mathbf{x}) | A_k(\mathbf{x}) = \frac{1}{N-1} \sum_{t=1}^{N-1} x_k^2(t)]} \cdot E[A^{1,2,\dots,K}(\mathbf{x}) | A_k(\mathbf{x}) \\ &= \frac{1}{N-1} \sum_{t=1}^{N-1} x_k^2(t) = E[A(\mathbf{x})], \end{aligned} \quad (36)$$

where  $x_k(t)$  is the output of the  $k$ th SU's CSR system.

Let the receiving signal  $r(t)$  goes through the dynamic CSR system with different CSR noise  $\beta_1(t), \beta_2(t), \dots, \beta_K(t)$ , and we can denote the output of each SU's CSR energy detector to be  $B_1(\mathbf{x}), B_2(\mathbf{x}), \dots, B_K(\mathbf{x})$ , respectively. Introducing the conventional Bayesian fusion method to fuse all K SUs' statistical results  $\{A_1(\mathbf{x}), A_2(\mathbf{x}), \dots, A_K(\mathbf{x})\}, \{B_1(\mathbf{x}), B_2(\mathbf{x}), \dots, B_K(\mathbf{x})\}$ , and  $A_{1,2,\dots,K}(\mathbf{x})$ , then the following Theorem 1 [18] could verify the effectiveness of the proposed cooperative spectrum sensing method.

**Theorem 1.** The cooperative spectrum sensing approach proposed by using the Bayesian fusion to all K SUs' statistics  $\{A_1(\mathbf{x}), A_2(\mathbf{x}), \dots, A_K(\mathbf{x})\}, \{B_1(\mathbf{x}), B_2(\mathbf{x}), \dots, B_K(\mathbf{x})\}$ , and  $A_{1,2,\dots,K}(\mathbf{x})$  shown in **Figure 1** can improve the sensing performance of conventional energy detection method.

**Proof:** Please refer Theorem 1 in Ref. [18] for details.

#### 4.2. Computer simulation results

In the following, some computer simulations are carried out to certify the correctness of the proposed method. Here, a QPSK signal is selected as the PU signal, that is

$$s(t) = A_p \sin(\omega_p t + \varphi_p), \quad (37)$$

where  $A_p, \omega_p$  and  $\varphi_p$  are the amplitude, angular frequency, and phase of the PU signal and  $\varphi_p \in \{\pm\pi/4, \pm3\pi/4\}$  in QPSK. In the following simulations, we set  $A_p = 5$  and  $\omega_p = 0.02\pi$ .

Also in the simulations, a kind of conventional discrete overdamped bistable oscillator is utilized as the CSR system, that is [19]

$$x_i(t+1) = [g \cdot x_i(t) - x_i^3(t)] e^{-x_i^2(t)/h} + d \cdot r(t) + \beta_i(t). \tag{38}$$

In the equation above,  $x_i(t)$  is the state variable and  $g$  and  $h$  are the corresponding parameters which determine the performance of the system seriously. In the simulations, we choose  $g = 2.85$  and  $h = 10$ .  $d$  is the driving parameter of the CSR system.

The additive channel noise  $n(t)$  is supposed to be composed by a sinusoidal interference signal and an AWGN signal in the computer simulations as

$$n(t) = n_0(t) + \varepsilon \cdot \sin \omega_\varepsilon t, \tag{39}$$

while  $n_0(t)$  is the AWGN signal, and the amplitude and angular frequency of the sinusoidal signal are set as  $\varepsilon = 0.1$  and  $\omega_\varepsilon = 0.8\pi$ .

Simultaneously, we choose the following types of CSR noise: uniform distribution noise, Weibull distribution noise, and lognormal distribution noise. While the uniform distribution noise is evenly distributed within the range  $[-1,+1]$ .

The pdf of the Weibull distribution noise is

$$g(x; u, v) = uv^{-u} x^{u-1} e^{-(x/v)^u}, \tag{40}$$

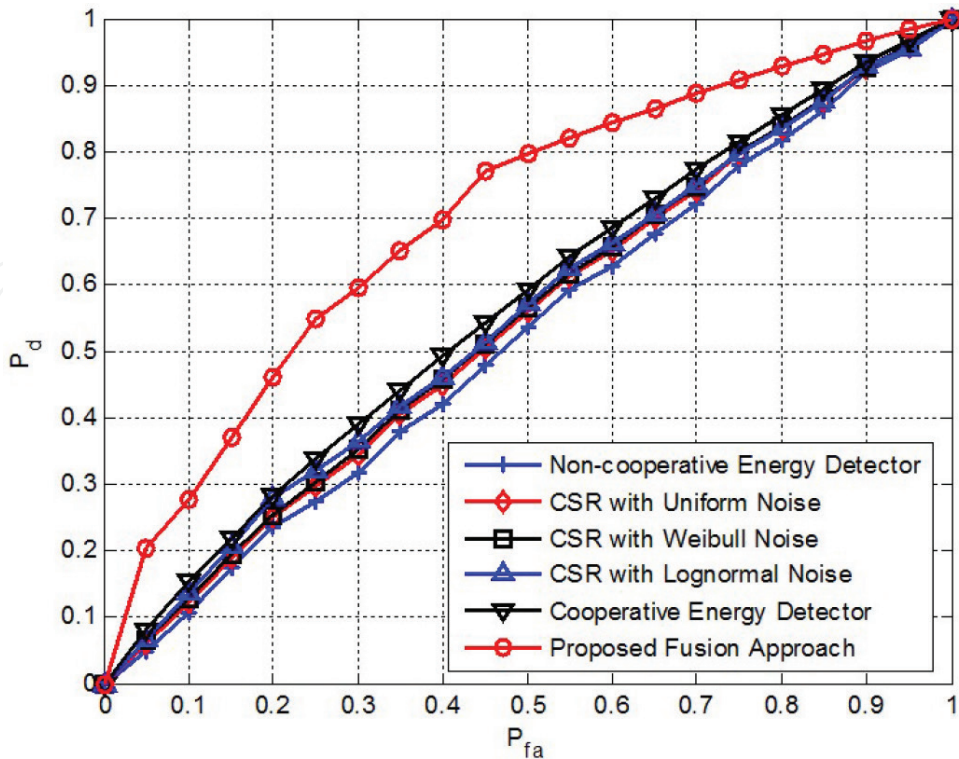


Figure 11. ROC curves of different spectrum sensing approach under SNR = -20 dB.

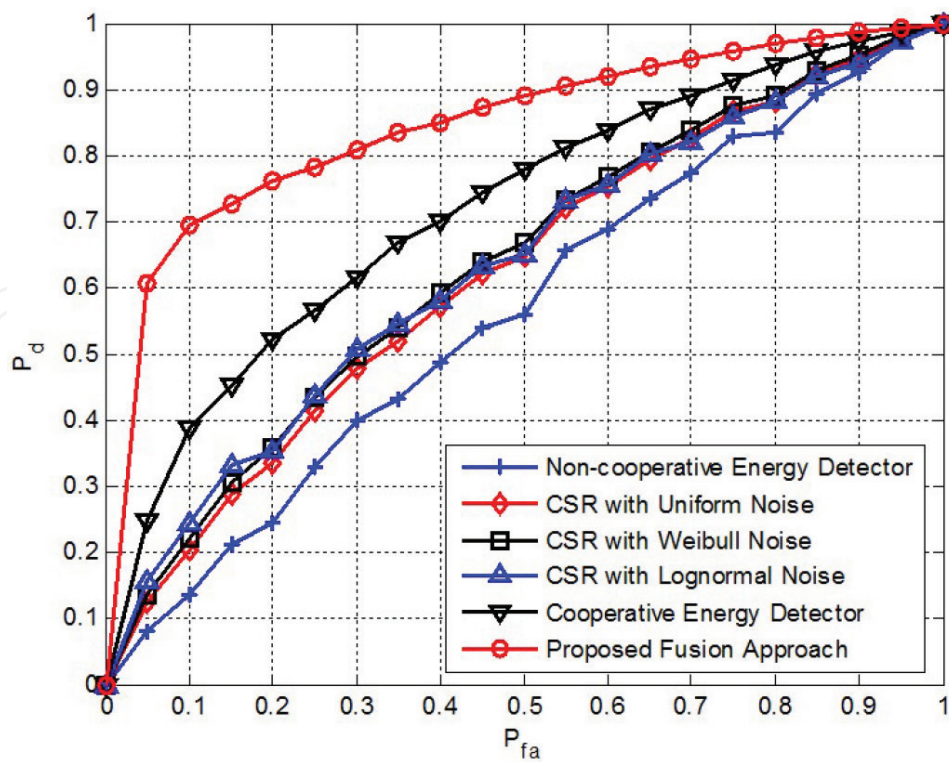


Figure 12. ROC curves of different spectrum sensing approach under SNR = -15 dB.

where  $u = 2$  and  $v = 1$ . The pdf of the Lognormal distribution noise is

$$g(x; \mu, \sigma) = e^{-(\ln x - \mu)^2 / 2\sigma^2} / (x\sigma\sqrt{2\pi}), \quad (41)$$

where the parameters of are fixed as  $\mu = 1$  and  $\sigma = 1$ .

In the computer simulations, the total sampling number is  $N = 10^6$ , and the Bayesian fusion process is performed under the CSR energy detection spectrum sensing driven by these three various kinds of noises, respectively.

Both Figures 11 and 12 give the ROC curves of different spectrum sensing results under SNR = -20 dB and -15 dB, respectively. It can be found obviously that the proposed cooperative approach can achieve some better performance than the conventional noncooperative spectrum sensing methods.

## 5. Summary

In this chapter, some conventional double-well bistable SR systems are introduced first. Then based on the conventional discrete overdamped double-well bistable SR oscillator, the optimization method and the corresponding analyses results are given especially under low SNR circumstances. Besides, a novel spectrum sensing approach used in CRN based on SR is proposed. And a cooperative spectrum sensing technique in CRN based on the data fusion technique is also proposed. The last section summarizes the whole chapter.

The optimization approach introduced is especially applicable under low SNR, which are familiar in the wireless communications. In the applications, the performance analyses and computer simulations show that the effectiveness of the proposed spectrum sensing approach is better than the traditional energy detection methods, and this methodology can be extended to some other problems with the same two-hypothesis decisions.

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