# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

4,800

122,000

International authors and editors

135M

Downloads

154
Countries delivered to

Our authors are among the

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



#### WEB OF SCIENCE

Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us? Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.

For more information visit www.intechopen.com



# **Resonances and Exceptional Broadcasting Conditions**

Juan Manuel Velázquez-Arcos, Alejandro Pérez-Ricardez, Ricardo Teodoro Páez-Hernández and Jaime Granados-Samaniego

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/intechopen.70520

#### **Abstract**

Many technologies have been developed to improve the quality of broadcasting, but persist with the problem that avoids the continuity of communications when the physical conditions of the media change. However, loss of signal propagation cannot be avoided because the refractive index of propagation media changes at the same time as magnetization, electromagnetic potential and other local parameters. That is, there is neither a device nor theories that take into account the effect of the sign of the refractive index under the broadcasting process. Simultaneously with the change of refractive index, conventional waves may find travel conditions inaccessible to the desired destination. In this chapter, we proposed that a sudden change in conditions is due to a resonant behavior of the media naturally described by a homogeneous integral equation of Fredholm. In addition, we propose a method to avoid the loss of the signal due to drastic changes in the broadcasting regime.

**Keywords:** resonances, broadcasting, evanescent waves, communications, negative refraction index

#### 1. Introduction

As we mentioned in the abstract, we propose the behavior of the electromagnetic waves propagating media—a model that consists in the division of the space in several portions and layers that eventually are considered as a superposition of thin layers of plasma. We must underline that only when exceptional conditions locally prevail in a particular portion of space, we can suppose the existence of these plasma layers. When an alternation of unmagnetized and magnetized layers occurs, we can observe that for some intervals of the magnetization and electric potential values, the refraction index of the set of alternating plasma layers becomes negative. That is, we have left-hand material conditions as we have called them. Because



Xiang-kun Kong et al. [1] found experimentally that they could change the refraction index sign as they wanted in a succession of magnetized and unmagnetized plasma layers (which they called as a plasma sandwich), we have applied the plasma sandwich model (PSM) to our proposal. The reason they assume for the change in the refraction index sign is very different to the explanation we present nowadays. Xiang-Kun Kong et al. [1], suppose is the coupling between the electromagnetic polarized waves and the evanescent waves. Instead this reasoning, we have shown in several papers that the homogeneous integral equation of Fredholm (HFE) and its Fourier transform (THFE) give us a simple reason, that is, the brake of confinement of the evanescent waves that turn to be traveling waves. In addition, last explanation is accompanied by the properties of the resonant solutions of the HFE and THFE equations. One of the most important resonance properties is the orthogonality that allows the possibility to send signals with little loss. Another important property of the resonances is the fact that the resonances cannot live on the original sites where the evanescent waves lived. The generation of propagation modes from the evanescent ones is due to a resonant behavior mechanism. We also preserved the term precursor for the evanescent waves that become traveling waves. With this definition, the traveling resonant waves cannot live where the precursors lived. One of the advantages of the PSM is the fact that we can model the resonant broadcasting regime from a little set of PSM parameters. Also, instead of the formalism employed by Xiang-kun Kong et al. [1], we used our own formalism, the vector matrix formalism (VMF) [2-4]. The most decisive variables are the electrical potential and the magnetization intensities.

# 2. Resonances and the Fredholm's eigenvalue

We remember that we can represent the broadcasting process through a Fourier transform of a generalized inhomogeneous Fredholm equation (TGIFE) [5,6] for the electric and magnetic fields that is by the Fourier transform of the equation:

$$E_{j}^{m}(t) = E_{j}^{m(0)}(t) + \int_{-\infty}^{\infty} K_{j,k}^{m,n(0)}(t,t') E_{k}^{n}(t') dt'$$
 (1)

In Eq. (1),  $E_j^{m}(\omega)$  represents any of the electric or the magnetic field components and the kernel is

$$K_{j,k}^{m,n(\circ)}(t,T') = G^{m,n(\circ)}(\mathbf{r}_j,t;\mathbf{r}_k,t')A_k^{m,n}$$
(2)

In Eq. (2),  $G^{m,n(\circ)}(\mathbf{r}_j,t;\mathbf{r}_k,t')$  is the free Green function and  $A_k^{m,n}$  is the interaction.

Then, the Fourier transform of Eq. (1) can be written as [2–4]:

$$\mathbf{f}^{m(\circ)}(\omega) = \left[\mathbf{1} - \mathbf{K}^{(\circ)}(\omega)\right]_{n}^{m} \mathbf{f}^{n}(\omega)$$
(3)

In Eq. (3),  $f^n(\omega)$  represents the Fourier transform of any of the electric or magnetic fields due to the source  $f^{m(^\circ)}(\omega)$ , but as we can see from Eq. (1), both are vectors whose components are also

vectors. Eq. (3) is an example of which we have called the vector-matrix formalism that avoids a more complicated treatment in terms of integral equations.

From Eq. (3), we can input the condition for the existence of a resonance, which implies that the source term vanishes; in other words, we are imposing the left-hand material conditions [5–10]. Simultaneously, for a purpose of mathematical clearance, we let the discrete indexes J and K in Eq. (1) take continuum values, so we now have a spatial dependence on  $\mathbf{r}$  and  $\mathbf{r}'$ ; so Eq. (3) with the left term equal to zero yields

$$W_{R}^{m}(\mathbf{r};\omega) = \eta_{R}(\omega) \int_{0}^{\infty} K_{n}^{m(0)}(\omega;\mathbf{r},\mathbf{r}') w_{R}^{n}(\mathbf{r}';\omega) d\mathbf{r}'$$
(4)

In this equation,  $w_R^m(\mathbf{r};\omega)$  are the resonances, and we have introduced the Fredholm eigenvalue [2]  $\eta_R(\omega)$ , corresponding to the R resonance. The introduced parameter  $\eta_R(\omega)$  allows for asking about nontrivial solutions for Eq. (4) by means of Fredholm theory of integral equations. We have shown that the structure of  $\eta_R(\omega)$  can be chosen in the same way as a phase factor [6]:

$$\eta_{R}(\omega) = e^{ih(\omega_{R})} \tag{5}$$

For the resonant frequency  $\omega_{R'}$  where in general it is given as:

$$\omega_{R} = K_{R} - i\Lambda_{R} \tag{6}$$

So, we must ask for  $h(\omega_R)$  to be a real number even when refraction index can be complex and dependent of arbitrary magnetization or ionization conditions.

Resonances analyzed in the present chapter are electromagnetic traveling waves that comes from the so-called precursors or evanescent waves, but they share very close mathematical properties with the quantum mechanics resonances; i.e., it fulfills the following theorem we have tested elsewhere [4]:

Theorem I

Suppose that  $\mathbf{w}_{\iota}(\omega)$  and  $\mathbf{w}_{\iota}(\omega)$  are solutions of Eq. (4), then,

$$\mathbf{w}_{l}^{t}(\boldsymbol{\omega})\mathbf{A}\mathbf{w}_{u}(\boldsymbol{\omega})\left[\lambda_{u}^{-1}-\lambda_{l}^{-1}\right]=0$$
(7)

We must remember the relation:

$$w_R^m(\mathbf{r};\omega) \to \mathbf{w}_R(\omega)$$
 (8)

On the other hand, the resonances  $w_{\mathbb{R}}^m(\mathbf{r};\omega)$  comply with the important orthogonality condition between the eigenvalue function  $\frac{1}{\eta_{\mathbb{R}}(\omega)}$  and resonance on the site of a punctual antenna located at  $\mathbf{r}_{\mathbb{A}}$  [5]:

$$\frac{1}{\eta_{R}(\omega)} w_{R}^{m}(\mathbf{r}_{A};\omega) = 0 \tag{9}$$

That implies that the resonances vanish on the sites of the antennas that generate this precursor signals, but we underline that not on the sites that generates the precursor signals of distinct resonances  $w_{R}^{m}(\mathbf{r};\omega)$ .

### 3. The VMF formalism

Now, we can return to our discrete proposal where we can put the parameters appeared in the PSM [2, 3, 5] into the VMF model [2–4]. To this end, let us recall that Eq. (1) can be written as:

$$\left[\mathbf{1} - \eta_{R}(\omega)\mathbf{K}^{(\circ)}(\omega)\right]_{R}^{m}\mathbf{w}_{R}^{n}(\omega) = 0 \tag{10}$$

where the kernel  $\mathbf{K}^{(\circ)}(\omega)$  is the product of the free Green function  $\mathbf{G}^{(\circ)}(\omega)$  with the interaction  $\mathbf{A}$  so explicitly,

$$\left[\mathbf{1} - \eta_{R}(\omega)\mathbf{G}^{(\circ)}(\omega)\mathbf{A}\right]_{R}^{m}\mathbf{w}_{R}^{n}(\omega) = 0 \tag{11}$$

Now, we can find the resonant frequencies in an academic example. To this end, we choose a convenient kernel  $\mathbf{K}^{(\cdot)}(\omega)$ ; for simplicity, we do not take into account the three components of the electromagnetic field. Supposing that we only have one component of the field, but we have two emitting antennas, a possible kernel is [2]:

$$\mathbf{K}^{(\circ)}(\omega) = \begin{pmatrix} \frac{\sin(\omega - \omega_p)\delta}{(\omega - \omega_p)\delta} & -i\frac{\cos(\omega - \omega_p)\delta}{(\omega - \omega_p)\delta} \\ i\frac{\cos(\omega - \omega_p)\delta}{(\omega - \omega_p)\delta} & \frac{\sin(\omega - \omega_p)\delta}{(\omega - \omega_p)\delta} \end{pmatrix}$$
(12)

In Eq. (12), we have introduced the PSM parameter  $\delta$ . This parameter is defined as:

$$\delta = \kappa d_{M} \tag{13}$$

where  $d_M$  is the average thickness of the plasma-magnetized layer involved in the change of sign of the refraction index;  $\kappa$  is the wave number of an incident beam interacting with the electric and magnetic fields in a way that the whole kernel is expressed in Eq. (12). The parameter  $\omega_p$  is an average value for the plasma frequency over the referred layer and can be expressed in terms of the local electron concentration in the layer as:

$$\omega_p = \frac{1}{2\pi} \left( \frac{Ne^2}{m\varepsilon_0} \right)^{\frac{1}{2}} \tag{14}$$

In Eq. (14),  $\mathcal{E}_0$  is the permittivity of vacuum, N is the electron concentration and e is the electronic charge.

Different broadcasting regimes occur when these parameters change, that is the refraction index sign changes. The PSM also considers a dynamical condition in the sense that we have a series of sets of iterated layers changing with time in a random manner and therefore with different effects for distinct frequencies.

Let us remember that the equation we must solve is Eq. (10) where the kernel is

$$\mathbf{K}_{i,j,m}^{n(\circ)}(\omega) = \begin{cases} 0 & \text{if } i = j \\ A_j^{n,m} G_{\omega}^{n,m(\circ)}(\mathbf{r}_i, \mathbf{r}_j) & \text{if } i \neq j \end{cases}$$
 (15)

The conditions for resonances are that Fredholm's determinant for Eq. (10) equals zero and that Fredholm's eigenvalue equals to one Eq. (16).

These last two conditions allow us to obtain the resonant frequencies for the system constituted by these two antennas but dependent on the parameters of plasma sandwich model. As their similar quantum mechanics case, the wave number or the resonant frequency has an imaginary part; that is, a resonant frequency can be represented by a complex frequency:

$$\omega = K - i\Lambda \tag{16}$$

The transformation of the evanescent waves into traveling waves is a consequence of the imaginary part  $\Lambda$  that avoids the electromagnetic field to be confined. In addition, we have the relation between  $\omega$  and the wave number  $\kappa$ , that is,

$$\kappa = \sqrt{\mu \varepsilon} \omega$$
(17)

By substituting Eq. (12) into Eq. (10), we have that one of the resonance conditions is that the Fredholm determinant  $\Delta$  must be zero, that is,

$$\Delta \left( \begin{array}{cc} A & -B \\ B & A \end{array} \right) = 0 \tag{18}$$

where

$$A = \frac{\sin(\omega - \omega_p)\delta}{(\omega - \omega_p)\delta} - \lambda \tag{19}$$

and

$$B = i \frac{\cos(\omega - \omega_{p})\delta}{(\omega - \omega_{p})\delta}$$
 (20)

In Eq. (19),  $\lambda^{-1}$  is the Fredholm eigenvalue.

We can put Eq. (16) into Eqs. (18)–(20) and express the Fredholm determinant as:

$$\Delta = 2\left[K - \omega_{p} - i\Lambda\right]^{-2} \delta^{-2} \left[\sin((K - \omega_{p})\delta)\cos(i\Lambda\delta) + \cos((K - \omega_{p})\delta)\sin(i\Lambda\delta)\right]^{2}$$

$$-2\left[K - \omega_{p} - i\Lambda\right]^{-1} \lambda \left[\sin((K - \omega_{p})\delta)\cos(i\Lambda\delta) + \cos((K - \omega_{p})\delta)\sin(i\Lambda\delta)\right]$$

$$+\lambda^{2}$$
(21)

We can explore some of the conditions for the existence of resonances (**Figure 1**); for example, if we take  $K = \omega_p + \frac{n\pi}{2\delta}$ ,  $\lambda = 1$ , and the condition  $\Delta = 0$ , we obtain the following equation for  $\Delta$ :

$$\frac{n\pi}{2}\cosh(\Lambda\delta) - ((\frac{n\pi}{2})^2 + \Lambda^2\delta^2)\lambda = 0$$
 (22)

or defining

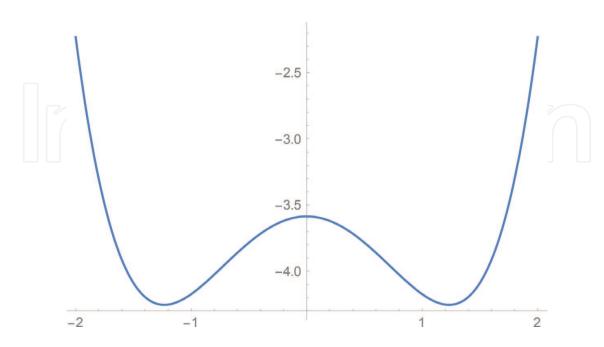
$$x \equiv \Lambda \delta \tag{23}$$

$$2\pi \cosh(x) - 4x^2 - \pi^2 = 0 \tag{24}$$

Then, the resonant frequencies will have the following form:

$$\omega_{res} = \omega_p + \frac{n\pi}{2\delta} - i\frac{x}{\delta} \tag{25}$$

Now, we can put realistic values for  $\delta$  and  $\omega_p$  taken from reference [1], that is,



**Figure 1.** Behavior of Eq. (22) with n = 1.

$$\delta = 1.68 \times 10^5 Hz \tag{26}$$

and

$$\omega_p = 300 \times 10^6 Hz. \tag{27}$$

So, the first two resonances are

$$\omega_{1,2} \equiv \omega_{\pm} \equiv (3005.1 \pm i(3.778)) \times 10^5 Hz,$$
 (28)

for  $x_{\pm} = \mp 2.2484$ .

# 4. Resonances on a broadcasting process

In the past section, we saw that resonances follow important orthogonal rules. But each resonance has only a unique associated frequency and not a complete band; indeed, the only way for using an individual frequency in a broadcasting process is to emit information in a telegraphic manner; that is, we must have a key and send in the same frequency a succession of intervals of signals with different lengths in time. Fortunately, communication theory (CT) brings us some clues about the problem for sending information [11–16]. First, we recall some statements from this theory, and then we use them. In accordance with these statements, suppose that f(t) is a function that is a member of a set defined in CT as an ensemble and suppose in addition that we are interested on functions that are limited to the band from 0 to W cycles per second, then we have the following theorem [11]:

Theorem II

Let f(t) contain no frequencies over W. Then,

$$f(t) = \sum_{-\infty}^{\infty} X_n \frac{\sin \pi (2Wt - n)}{\pi (2Wt - n)},$$
(29)

where

$$X_n = f\left(\frac{n}{2W}\right). \tag{30}$$

We can see that we have expanded f(t) in terms of orthogonal functions, and the respective coefficients  $X_n$  are coordinates in an infinite dimension space.

Theorem (21) can now be taken as a building stone for very special functions with very important properties in the broadcasting processes. We have called these functions as communication packs in previous works. First, we use the cut-off frequency W as a label for distinguishing different packs; second, we use each pack as a new component or coordinate of the signal message f(t) that is, to each  $W_q$  frequency corresponds a projection or coordinate

 $f_q(t)$ , and third, we choose each  $W_q$  as a resonant frequency that is  $W_q = \omega_R$ . Our proposal is that the complete signal f(t) can be recovered by adding its components  $f_q(t)$ .

## 5. Why to use communication packs?

We have shown how we can project a signal over different resonant dimensions, but why we must do this. The reason is that theoretically, each resonance is orthogonal to any other resonance, which means that there is no interference between signals traveling over different resonances. Then, we expect that communication packs do not interfere between them because we use different base functions in each pack but also because their defining frequency is a resonant one; that is, we have defined a new space for the broadcasting process and each pack carries a part of the signal over an orthogonal resonant dimension. In addition, we also expect that the infinite sum in Eq. (26) really have a relatively few dominant terms around the resonant frequencies in a manner that we do not need to sum an infinite number of terms for a good approximation. If we want to evaluate the relative broadcasting efficiency between one channel operating with a nonresonant situation and other channel operating with resonant conditions, it is necessary to take into account that a resonant wave cannot live where the precursors lived as we stated above. Therefore, as we have proposed in the abstract, we can provide a specific device, i.e., a pair of circuits, each one with a different response, by selecting the best circuit in any instant for a good reception and avoid the blocking effect in the conventional circuits. In other words, we must remember that resonant solutions vanish at the point sources. Let us take a simple example in which we have only two resonant frequencies and then we can build their respective communication packs with the recipe based on the theorem (29) and explicitly given in another previous work [2–4, 9]:

Suppose that P(t) is the specific signal

$$P(t) = \frac{\sin \pi (2Wt)}{\pi (2Wt)}. (31)$$

Following Theorem II and using the resonances, we get the two communication packs:

$$P_1(t) = \sum_{-\infty}^{\infty} X_{n,1} \frac{\sin \pi (2\omega_1 t - n)}{\pi (2\omega_1 t - n)}$$
(32)

$$P_2(t) = \sum_{-\infty}^{\infty} X_{m,2} \frac{\sin \pi (2\omega_2 t - m)}{\pi (2\omega_2 t - m)},$$
(33)

with  $\omega_1$  and  $\omega_2$  given by Eq. (28):

$$X_{n,1} = P\left(\frac{n}{2\omega_1}\right) \tag{34}$$

$$X_{m,2} = P\left(\frac{m}{2\omega_2}\right). \tag{35}$$

In example of Section 3, we have obtained two resonances so that the two packs are described by Eqs. (32)–(35), but with the numerical values obtained before:

$$X_{n,1} = \frac{\sin \pi \left(2W\left(\frac{n}{2\omega_1}\right)\right)}{\pi \left(2W\left(\frac{n}{2\omega_1}\right)\right)}$$
(36)

and

$$X_{m,2} = \frac{\sin \pi \left(2W\left(\frac{m}{2\omega_2}\right)\right)}{\pi \left(2W\left(\frac{m}{2\omega_2}\right)\right)},\tag{37}$$

That is,

$$X_{n,1} = \frac{\sin \pi \frac{Wn}{\omega_1}}{\pi \frac{Wn}{\omega_1}} \tag{38}$$

and

$$X_{m,2} = \frac{\sin \pi \frac{Wm}{\omega_2}}{\pi \frac{Wm}{\omega_2}} \tag{39}$$

So, the first CP is

$$P_1(t) = \sum_{-\infty}^{\infty} \left( \frac{\sin \pi \frac{W_n}{\omega_1}}{\pi \frac{W_n}{\omega_1}} \right) \frac{\sin \pi (2\omega_1 t - n)}{\pi (2\omega_1 t - n)},\tag{40}$$

and the second CP is

$$P_2(t) = \sum_{-\infty}^{\infty} \left( \frac{\sin \pi \frac{Wm}{\omega_2}}{\pi \frac{Wm}{\omega_2}} \right) \frac{\sin \pi (2\omega_2 t - m)}{\pi (2\omega_2 t - m)}. \tag{41}$$

Eqs. (40) and (41) can be considered the projections of the real signal (31) over the two dimensions of the resonance space.

# 6. Concluding remarks

We have shown how we can join several tools that we have developed for the purpose to enhance the broadcasting process; with this aim, we have incorporated the so-called PSM parameters into the algebraic equations (vector-matrix equations) of the VMF searching a way to make communications invulnerable to abrupt changes in the atmospheric conditions. This is very important particularly for high definition channels, which are more sensitive to

these abrupt changes, and the PSM (plasma sandwich model) predicts that the mathematical resonances are associated with the delivery of the so-called evanescent waves or to negative values of the refraction index. One of our fundamental proposals is that the atmosphere behaves like a collection of regions with changes from positive to negative (and vice versa) refraction index with unpredictable frequency, and then we can use the PSM to characterize them. On the other hand, we propose the use of the resonant frequencies to overcome the broadcasting barriers by defining a new resonance space created by using the resonances as a new dimension in which the communication packs are the projections of an arbitrary signal. In addition, we suppose that the conventional traveling waves change their regular trajectories when there is a local change in the refraction index sign, so the combined effect of the original paths and the prevalence of the resonant modes make the broadcasting process very difficult without the help of our proposals. By using the results of previous works, we also suggest the use of a device with the possibility for put on and put out of two internal independent circuits each one with a normal (positive refraction index) or resonant (negative refraction index) performance. We underline that communication packs can be constructed even when the current regime is not a resonant.

#### **Author details**

Juan Manuel Velázquez-Arcos\*, Alejandro Pérez-Ricardez, Ricardo Teodoro Páez-Hernández and Jaime Granados-Samaniego

\*Address all correspondence to: jmva@correo.azc.uam.mx

Universidad Autónoma Metropolitana, CDMX, México

#### References

- [1] Kong X-k, Liu S-b, Zhang H-f, Bian B-r, Li H-m, et al. Evanescent wave decomposition in a novel resonator comprising unmagnetized and magnetized plasma layers. Physics of Plasmas. 2013;**20**:043515. DOI: 10.1063/1.4802807
- [2] Velázquez-Arcos JM, Granados-Samaniego J. Wave propagation under confinement break. IOSR Journal of Electronics and Communication Engineering, Ver. 1. 2016;11(2): 42-48. DOI: 10.9790/2834-1102014248
- [3] Velázquez-Arcos JM, Granados-Samaniego J, Navarrete-González TD. Top efficiency on communication theory. IOSR Journal of Electronics and Communication Engineering. 2016;**11**(6):01-11. DOI: 10.9790/2834-1106020111.
- [4] Velázquez-Arcos JM. Fredholm's equations for subwavelength focusing. Journal of Mathematical Physics. 2012;53(10):103520. DOI: 10.1063/1.4759502

- [5] Velázquez-Arcos JM. Fredholm's alternative breaks the confinement of electromagnetic waves. AIP Advances. 2013;3:092114. DOI: 10.1063/1.4821336
- [6] Velázquez-Arcos JM, Granados-Samaniego J., Vargas CA. The confinement of electromagnetic waves and Fredholm's alternative. In: Electromagnetics in Advanced Applications (ICEAA) International Conference, 9–13 Sept. 2013. doi: 10.1109/ICEAA.2013.6632268; 2013. pp. 411–414
- [7] Velázquez-Arcos JM, Pérez-Martínez F, Rivera-Salamanca CA, Granados-Samaniego J. On the application of a recently discovered electromagnetic resonances to communication systems. IJETAE. 2013;3(1):466-471 www.ijetae.com, ISSN 2250-2459
- [8] Velázquez-Arcos JM, Granados-Samaniego J, Vargas CA. Communication theory and resonances on electromagnetic systems. Electromagnetics in Advanced Applications (ICEAA) 2012 International Conference, 2–7 Sept. 2012, IEEE Cape Town. pp. 392–395. DOI: 10.1109/ICEAA.2012.6328657.
- [9] Velázquez-Arcos JM, Granados-Samaniego J. Recent technologies and recording time detection on time reversal signals. International Journal on Recent and Innovation Trends in Computing and Communication. 2014;2(11):3447-3450
- [10] Hernández-Bautista F, Vargas CA, Velázquez-Arcos JM. Negative refractive index in split ring resonators. Revista Mexicana de Fisica. 2013;**59**(1):139-144 ISSN: 0035-00IX
- [11] Shannon CE. A mathematical theory of communication. The Bell System Technical Journal. 1948;**27**:379-423 623-656
- [12] Nyquist H. Certain factors affecting telegraph speed. Bell System Technical Journal. 1924; 47:324
- [13] Nyquist H. Certain topics in telegraph transmission theory. Transactions of the A.I.E.E. April 1928;47:617
- [14] Hartley RVL. Transmission of Information, Bell System Technical Journal, July 1928 pp. 535-563
- [15] Wiener N. The Ergodic theorem. Duke Mathematical Journal. 1939;5:1-20
- [16] Hopf E. On causality statistics and probability. Journal of Mathematical Physics. 1934;**13**(1): 51-102

# IntechOpen

IntechOpen