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# Non-Fragile Guaranteed Cost Control of Nonlinear Systems with Different State and Input Delays Based on T-S Fuzzy Local Bilinear Models

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## Abstract

This paper focuses on the non-fragile guaranteed cost control problem for a class of Takagi-Sugeno (T-S) fuzzy time-varying delay systems with local bilinear models and different state and input delays. A non-fragile guaranteed cost state-feedback controller is designed such that the closed-loop T-S fuzzy local bilinear control system is delay-dependent asymptotically stable, and the closed-loop fuzzy system performance is constrained to a certain upper bound when the additive controller gain perturbations exist. By employing the linear matrix inequality (LMI) technique, sufficient conditions are established for the existence of desired non-fragile guaranteed cost controllers. The simulation examples show that the proposed approach is effective and feasible.

**Keywords:** fuzzy control, non-fragile guaranteed cost control, delay-dependent, linear matrix inequality (LMI), T-S fuzzy bilinear model

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## 1. Introduction

In recent years, T-S (Takagi-Sugeno) model-based fuzzy control has attracted wide attention, essentially because the fuzzy model is an effective and flexible tool for the control of nonlinear systems [1–8]. Through the application of sector nonlinearity approach, local approximation in fuzzy partition spaces or other different approximation methods, T-S fuzzy models will be used to approximate or exactly represent a nonlinear system in a compact set of state variables. The merit of the model is that the consequent part of a fuzzy rule is a linear dynamic subsystem, which makes it possible to apply the classical and mature linear systems theory to nonlinear systems. Further, by using the fuzzy inference method, the overall fuzzy model will

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be obtained. A fuzzy controller is designed via the method titled 'parallel distributed compensation (PDC)' [3–6], the main idea of which is that for each linear subsystem, the corresponding linear controller is carried out. Finally, the overall nonlinear controller is obtained via fuzzy blending of each individual linear controller. Based on the above content, T-S fuzzy model has been widely studied, and many results have been obtained [1–8]. In practical applications, time delay often occurs in many dynamic systems such as biological systems, network systems, etc. It is shown that the existence of delays usually becomes the source of instability and deteriorating performance of systems [3–8]. In general, when delay-dependent results were calculated, the emergence of the inner product between two vectors often makes the process of calculation more complicated. In order to avoid it, some model transformations were utilized in many papers, unfortunately, which will arouse the generation of an inequality, resulting in possible conservatism. On the other hand, due to the influence of many factors such as finite word length, truncation errors in numerical computation and electronic component parameter change, the parameters of the controller in a certain degree will change, which lead to imprecision in controller implementation. In this case, some small perturbations of the controllers' coefficients will make the designed controllers sensitive, even worse, destabilize the closed-loop control system [9]. So the problem of non-fragile control has been important issues. Recently, the research of non-fragile control has been paid much attention, and a series of productions have been obtained [10–13].

As we know, bilinear models have been widely used in many physical systems, biotechnology, socioeconomics and dynamical processes in other engineering fields [14, 15]. Bilinear model is a special nonlinear model, the nonlinear part of which consists of the bilinear function of the state and input. Compared with a linear model, the bilinear models have two main advantages. One is that the bilinear model can better approximate a nonlinear system. Another is that because of nonlinearity of it, many real physical processes may be appropriately modeled as bilinear systems. A famous example of a bilinear system is the population of biological species, which can be showed by  $\frac{d\theta}{dt} = \theta v$ . In this equation,  $v$  is the birth rate minus death rate, and  $\theta$  denotes the population. Obviously, the equation cannot be approximated by a linear model [14].

Most of the existing results focus on the stability analysis and synthesis based on T-S fuzzy model with linear local model. However, when a nonlinear system has of complex nonlinearities, the constructed T-S model will consist of a number of fuzzy local models. This will lead to very heavy computational burden. According to the advantages of bilinear systems and T-S fuzzy control, so many researchers paid their attentions to the T-S fuzzy models with bilinear rule consequence [16–18]. From these papers, it is evident that the T-S fuzzy bilinear model may be suitable for some classes of nonlinear plants. In Ref. [16], a nonlinear system was transformed into a bilinear model via Taylor's series expansion, and the stability of T-S fuzzy bilinear model was studied. Moreover, the result was stretched into the complex fuzzy system with state time delay [17]. Ref. [18] presented robust stabilization for a class of discrete-time fuzzy bilinear system. Very recently, a class of nonlinear systems is described by T-S fuzzy models with nonlinear local models in Ref. [19], and in this paper, the scholars put forward a new fuzzy control scheme with local nonlinear feedbacks, the advantage of which over the

existing methods is that a fewer fuzzy rules and less computational burden. The non-fragile guaranteed cost controller was designed for a class of T-S discrete-time fuzzy bilinear systems in Ref. [20]. However, in Refs. [19, 20], the time-delay effects on the system is not considered. Ref. [17] is only considered the fuzzy system with the delay in the state and the derivatives of time-delay,  $\dot{d}(t) < 1$  is required. Refs. [21–23] dealt with the uncertain fuzzy systems with time-delay in different ways. It should be pointed out that all the aforementioned works did not take into account the effect of the control input delays on the systems. The results therein are not applicable to systems with input delay. Recently, some controller design approaches have been presented for systems with input delay, see [2, 3, 4, 18, 24–32] for fuzzy T-S systems and [8, 15, 33, 34] for non-fuzzy systems and the references therein. All of these results are required to know the exact delay values in the implementation. T-S fuzzy stochastic systems with state time-vary or distributed delays were studied in Refs. [35–39]. The researches of fractional order T-S fuzzy systems on robust stability, stability analysis about “ $0 < \alpha < 1$ ”, and decentralized stabilization in multiple time delays were presented in Refs. [40–42], respectively. For different delay types, the corresponding adaptive fuzzy controls for nonlinear systems were proposed in Refs. [33, 43, 44]. In Refs. [45, 46], to achieve small control amplitude, a new T-S fuzzy hyperbolic model was developed, moreover, Ref. [46] considered the input delay of the novel model. In Ref. [25, 47], the problems of observer-based fuzzy control design for T-S fuzzy systems were concerned.

So far, the problem of non-fragile guaranteed cost control for fuzzy system with local bilinear model with different time-varying state and input delays has not been discussed.

In this paper, the problem of delay-dependent non-fragile guaranteed cost control is studied for the fuzzy time-varying delay systems with local bilinear model and different state and input delays. Based on the PDC scheme, new delay-dependent stabilization conditions for the closed-loop fuzzy systems are derived. No model transformation is involved in the derivation. The merit of the proposed conditions lies in its reduced conservatism, which is achieved by circumventing the utilization of some bounding inequalities for the cross-product between two vectors as in Ref. [17]. The three main contributions of this paper are the following: (1) a non-fragile guaranteed cost controller is presented for the fuzzy system with time-varying delay in both state and input; (2) some free-weighting matrices are introduced in the derivation process, where the constraint of the derivatives of time-delay,  $\dot{d}(t) < 1$  and  $\dot{h}(t) < 1$ , is eliminated; and (3) the delay-dependent stability conditions for the fuzzy system are described by LMIs. Finally, simulation examples are given to illustrate the effectiveness of the obtained results.

The paper is organized as follows. Section 2 introduces the fuzzy delay system with local bilinear model, and non-fragile controller law for such system is designed based on the parallel distributed compensation approach in Section 3. Results of non-fragile guaranteed cost control are given in Section 4. Two simulation examples are used to illustrate the effectiveness of the proposed method in Section 5, which is followed by conclusions in Section 6.

Notation: Throughout this paper, the notation  $P > 0$  ( $P \geq 0$ ) stands for  $P$  being real symmetric and positive definite (or positive semi-definite). In symmetric block matrices, the asterisk (\*) refers to a term that is induced by symmetry, and  $\text{diag}\{\dots\}$  denotes a block-diagonal matrix.

The superscript  $T$  means matrix transposition. The notion  $\sum_{i,j=1}^s$  is an abbreviation of  $\sum_{i=1}^s \sum_{j=1}^s$ . Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

## 2. System description and assumptions

In this section, we introduce the T-S fuzzy time-delay system with local bilinear model. The  $i$ th rule of the fuzzy system is represented by the following form:

*Plant Rule  $i$  :*

*IF  $\vartheta_1(t)$  is  $F_{i1}$  and ... and  $\vartheta_v(t)$  is  $F_{iv}$ , THEN*

$$\dot{x}(t) = A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + B_{hi} u(t - h(t)) + N_i x(t) u(t) + N_{di} x(t - d(t)) u(t - h(t)) \quad (1)$$

$$x(t) = \phi(t), \quad t \in [-\tau_1, 0], \quad i = 1, 2, \dots, s$$

where  $F_{ij}$  is the fuzzy set,  $s$  is the number of fuzzy rules,  $x(t) \in R^n$  is the state vector, and  $u(t) \in R$  is the control input,  $\vartheta_1(t), \vartheta_2(t), \dots, \vartheta_v(t)$  are the premise variables. It is assumed that the premise variables do not depend on the input  $u(t)$ .  $A_i, A_{di}, N_i, N_{di} \in R^{n \times n}$ ,  $B_i, B_{hi} \in R^{n \times 1}$  denote the system matrices with appropriate dimensions.  $d(t)$  is a time-varying differentiable function that satisfies  $0 \leq d(t) \leq \tau_1$ ,  $0 \leq h(t) \leq \tau_2$ , where  $\tau_1, \tau_2$  are real positive constants as the upper bound of the time-varying delay. It is also assumed that  $\dot{d}(t) \leq \sigma_1$ ,  $\dot{h}(t) \leq \sigma_2$ , and  $\sigma_1, \sigma_2$  are known constants. The initial conditions  $\phi(t), \varphi(t)$  are continuous functions of  $t, t \in [-\tau, 0]$ ,  $\tau = \min(\tau_1, \tau_2)$ .

**Remark 1:** The fuzzy system with time-varying state and input delays will be investigated in this paper, which is different from the system in Ref. [17]. In Ref. [17], only state time-varying delay is considered. And also, here, we assume that the derivative of time-varying delay is less than or equal to a known constant that may be greater than 1; the assumption on time-varying delay in Ref. [17] is relaxed.

By using singleton fuzzifier, product inferred and weighted defuzzifier, the fuzzy system can be expressed by the following globe model:

$$\dot{x}(t) = \sum_{i=1}^s h_i(\vartheta(t)) [A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + B_{hi} u(t - h(t)) + N_i x(t) u(t) + N_{di} x(t - d(t)) u(t - h(t))] \quad (2)$$

where

$h_i(\vartheta(t)) = \omega_i(\vartheta(t)) / \sum_{i=1}^s \omega_i(\vartheta(t))$ ,  $\omega_i(\vartheta(t)) = \prod_{j=1}^v \mu_{ij}(\vartheta(t))$ ,  $\mu_{ij}(\vartheta(t))$  is the grade of membership of  $\vartheta_i(t)$  in  $F_{ij}$ . In this paper, it is assumed that  $\omega_i(\vartheta(t)) \geq 0$ ,  $\sum_{i=1}^s \omega_i(\vartheta(t)) > 0$  for all  $t$ . Then, we have the following conditions  $h_i(\vartheta(t)) \geq 0$ ,  $\sum_{i=1}^s h_i(\vartheta(t)) = 1$  for all  $t$ . In the consequent, we use abbreviation  $h_{iv}, h_{hi}, x_d(t), u_d(t), x_h(t), u_h(t)$ , to replace  $h_i(\vartheta(t)), h_i(\vartheta(t - h(t))), x(t - d(t)), u(t - d(t)), x(t - h(t)), u(t - h(t))$ , respectively, for convenience.

The objective of this paper is to design a state-feedback non-fragile guaranteed cost control law for the fuzzy system (2).

### 3. Non-fragile guaranteed cost controller design

Extending the design concept in Ref. [17], we give the following non-fragile fuzzy control law:

$$\begin{aligned} & \text{IF } \vartheta_1(t) \text{ is } F_1^i \text{ and } \dots \text{ and } \vartheta_v(t) \text{ is } F_v^i \\ & \text{THEN } u(t) = \frac{\rho(K_i + \Delta K_i)x(t)}{\sqrt{1 + x^T(K_i + \Delta K_i)^T(K_i + \Delta K_i)x}} = \rho \sin \theta_i = \rho \cos \theta_i(K_i + \Delta K_i)x(t) \end{aligned} \quad (3)$$

where  $\rho > 0$  is a scalar to be assigned, and  $K_i \in R^{l \times n}$  is a local controller gain to be determined.  $\Delta K_i$  represents the additive controller gain perturbations of the form  $\Delta K_i = H_i F_i(t) E_{ki}$  with  $H_i$  and  $E_{ki}$  being known constant matrices, and  $F_i(t)$  the uncertain parameter matrix satisfying  $F_i^T(t) F_i(t) \leq I$ .  $\sin \theta_i = \frac{\bar{K}_i x(t)}{\sqrt{1 + x^T \bar{K}_i^T \bar{K}_i x}}$ ,  $\cos \theta_i = \frac{1}{\sqrt{1 + x^T \bar{K}_i^T \bar{K}_i x}}$ ,  $\theta_i \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ ,  $\bar{K}_i = K_i + \Delta K_i(t) = K_i + H_i F_i(t) E_{ki}$ .

The overall fuzzy control law can be represented by

$$u(t) = \sum_{i=1}^s h_i \frac{\rho \bar{K}_i x(t)}{\sqrt{1 + x^T \bar{K}_i^T \bar{K}_i x}} = \sum_{i=1}^s h_i \rho \sin \theta_i = \sum_{i=1}^s h_i \rho \cos \theta_i \bar{K}_i x(t) \quad (4)$$

When there exists an input delay  $h(t)$ , we have that

$$u_h(t) = \sum_{l=1}^s h_{hl} \rho \sin \varphi_l = \sum_{l=1}^s h_{hl} \rho \cos \varphi_l \tilde{K}_l x_h(t) \quad (5)$$

where  $\sin \varphi_l = \frac{\tilde{K}_l x_h(t)}{\sqrt{1 + x_h^T \tilde{K}_l^T \tilde{K}_l x_h}}$ ,  $\cos \varphi_l = \frac{1}{\sqrt{1 + x_h^T \tilde{K}_l^T \tilde{K}_l x_h}}$ ,  $\varphi_l \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ ,  $\tilde{K}_l = K_l + \Delta K_l(t - h(t)) = K_l + H_l F_l(t - h(t)) E_{kl}$ .

So, it is natural and necessary to make an assumption that the functions  $h_i$  are well defined all  $t \in [-\tau_2, 0]$ , and satisfy the following properties:

$$h_i(\vartheta(t - h(t))) \geq 0, \text{ for } i = 1, 2, \dots, s, \text{ and } \sum_{i=1}^s h_i(\vartheta(t - h(t))) = 1.$$

By substituting Eq. (5) into Eq. (2), the closed-loop system can be given by

$$\dot{x}(t) = \sum_{i,j,l=1}^s h_i h_j h_{hl} (\Lambda_{ij} x(t) + \Lambda_{dij} x_d(t) + \Lambda_{hil} x_h(t)) \quad (6)$$

where

$$\Lambda_{ij} = A_i + \rho \sin \theta_j N_i + \rho \cos \theta_j B_i \bar{K}_j, \Lambda_{dil} = A_{di} + \rho \sin \varphi_l N_{di}, \Lambda_{hil} = \rho \cos \varphi_l B_{hi} \tilde{K}_l.$$

Given positive-definite symmetric matrices  $S \in R^{n \times n}$  and  $W \in R$ , we take the cost function

$$J = \int_0^\infty [x^T(t)Sx(t) + u^T(t)Wu(t)]dt \tag{7}$$

**Definition 1.** The fuzzy non-fragile control law  $u(t)$  is said to be non-fragile guaranteed cost if for the system (2), there exist control laws (4) and (5) and a scalar  $J_0$  such that the closed-loop system (6) is asymptotically stable and the closed-loop value of the cost function (7) satisfies  $J \leq J_0$ .

### 4. Analysis of stability for the closed-loop system

Firstly, the following lemmas are presented which will be used in the paper.

**Lemma 1** [20]: *Given any matrices  $M$  and  $N$  with appropriate dimensions such that  $\varepsilon > 0$ , we have  $M^T N + N^T M \leq \varepsilon M^T M + \varepsilon^{-1} N^T N$ .*

**Lemma 2** [21]: *Given constant matrices  $G, E$  and a symmetric constant matrix  $S$  of appropriate dimensions. The inequality  $S + GFE + E^T F^T G^T < 0$  holds, where  $F(t)$  satisfies  $F^T(t) F(t) \leq I$  if and only if, for some  $\varepsilon > 0$ ,  $S + \varepsilon GG^T + \varepsilon^{-1} E^T E < 0$ .*

The following theorem gives the sufficient conditions for the existence of the non-fragile guaranteed cost controller for system (6) with additive controller gain perturbations.

**Theorem 1.** *Consider system (6) associated with cost function (7). For given scalars  $\rho > 0, \tau_1 > 0, \tau_2 > 0, \sigma_1 > 0, \sigma_2 > 0$ , if there exist matrices  $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, K_i, i = 1, 2, \dots, s, X_1, X_2, X_3, X_4, Y_1, Y_2, Y_3, Y_4$ , and scalar  $\varepsilon > 0$  satisfying the inequalities (8), the system (6) is asymptotically stable and the control law (5) is a fuzzy non-fragile guaranteed cost control law, moreover,*

$$\begin{aligned}
 J \leq & x^T(0)Px(0) + \int_{-d(0)}^0 x^T(s)Q_1x(s)ds + \int_{-\tau_1}^0 \int_{\theta}^0 \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
 & + \int_{-h(0)}^0 x^T(s)Q_2x(s)ds + \int_{-\tau_2}^0 \int_{\theta}^0 \dot{x}^T(s)R_2\dot{x}(s)dsd\theta = J_0 \\
 & \begin{bmatrix} T_{ijl} & * & * \\ \tau_1 X^T & -\tau_1 R_1 & * \\ \tau_2 Z^T & 0 & -\tau_2 R_2 \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s \tag{8}
 \end{aligned}$$

where  $T_{ijl} = \begin{bmatrix} T_{11,ij} & * & * & * \\ T_{21,i} & T_{22,i} & * & * \\ T_{31,i} & T_{32,ij} & T_{33,il} & * \\ T_{41,i} & T_{42,i} & T_{43} & T_{44} \end{bmatrix},$

$$\begin{aligned}
 T_{11,ij} &= Q_1 + Q_2 + X_1 + X_1^T + Y_1 A_i + A_i^T Y_1^T + S + 2\varepsilon \rho^2 Y_1 Y_1^T + 4\varepsilon^{-1} (N_i^T N_i + (B_i \bar{K}_j)^T (B_i \bar{K}_j)) \\
 &\quad + Z_1^T + Z_1 + \rho^2 \bar{K}_i^T W \bar{K}_i, \\
 T_{21,i} &= -X_1^T + X_2 + Z_2 + Y_2 A_i + A_{di}^T Y_1^T, \quad T_{31,i} = Z_3 - Z_1 + X_3 + Y_3 A_i, \\
 T_{22,ij} &= -(1 - \sigma_1) Q_1 - X_2 - X_2^T + Y_2 A_{di} + A_{di}^T Y_2^T + 2\varepsilon \rho^2 Y_2 Y_2^T + 4\varepsilon^{-1} N_{di}^T N_{di}, \\
 T_{32,i} &= -X_3 + Y_3 A_{di} - Z_2^T, \quad T_{33,il} = -(1 - \sigma_2) Q_2 - Z_3 - Z_3^T + 2\varepsilon \rho^2 Y_3 Y_3^T + 4\varepsilon^{-1} (B_{hi} \tilde{K}_l)^T B_{hi} \tilde{K}_l, \\
 T_{41,i} &= P + X_4 + Z_4 + Y_4 A_i - Y_1^T, \quad T_{42,i} = -X_4 + Y_4 A_i - Y_2^T, \\
 T_{43} &= -Z_4 - Y_3^T, \quad T_{44} = \tau_1 R_1 + \tau_2 R_2 - Y_4 - Y_4^T + 2\varepsilon \rho^2 Y_4 Y_4^T.
 \end{aligned}$$

Proof: Take a Lyapunov function candidate as

$$\begin{aligned}
 V(x(t), t) &= x^T(t) P x(t) + \int_{t-d(t)}^t x^T(s) Q_1 x(s) ds + \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta \\
 &\quad + \int_{t-h(t)}^t x^T(s) Q_2 x(s) ds + \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta
 \end{aligned} \tag{9}$$

The time derivatives of  $V(x(t), t)$ , along the trajectory of the system (6), are given by

$$\begin{aligned}
 \dot{V}(x(t), t) &= 2x^T(t) P \dot{x}(t) + x^T(t) (Q_1 + Q_2) x(t) \\
 &\quad - (1 - \dot{d}(t)) x_d^T(t) Q_1 x_d(t) + \dot{x}^T(t) (\tau_1 R_1 + \tau_2 R_2) \dot{x}(t) \\
 &\quad - \int_{t-\tau_1}^t \dot{x}^T(s) R_1 \dot{x}(s) ds - (1 - \dot{h}(t)) x_h^T(t) Q_2 x_h(t) - \int_{t-\tau_2}^t \dot{x}^T(s) R_2 \dot{x}(s) ds
 \end{aligned} \tag{10}$$

Define the free-weighting matrices as  $X = [X_1^T \ X_2^T \ X_3^T \ X_4^T]^T$ ,  $Y = [Y_1^T \ Y_2^T \ Y_3^T \ Y_4^T]^T$ ,  $Z = [Z_1^T \ Z_2^T \ Z_3^T \ Z_4^T]^T$ , where  $X_k \in R^{n \times n}$ ,  $Y_k \in R^{n \times n}$ ,  $Z_k \in R^{n \times n}$ ,  $k = 1, 2, 3, 4$  will be determined later.

Using the Leibniz-Newton formula and system equation (6), we have the following identical equations:

$$\begin{aligned}
 [x^T(t) X_1 + x_d^T(t) X_2 + x_h^T(t) X_3 + \dot{x}^T(t) X_4] [x(t) - x_d(t) - \int_{t-d(t)}^t \dot{x}(s) ds] &\equiv 0, \\
 [x^T(t) Z_1 + x_d^T(t) Z_2 + x_h^T(t) Z_3 + \dot{x}^T(t) Z_4] [x(t) - x_h(t) - \int_{t-h(t)}^t \dot{x}(s) ds] &\equiv 0, \\
 \sum_{i,j=1}^s h_i h_j h_l [x^T(t) Y_1 + x_d^T(t) Y_2 + x_h^T(t) Y_4 + \dot{x}^T(t) Y_4] [\Lambda_{ij} x(t) + \Lambda_{dil} x_d(t) + \Lambda_{hil} x_h(t) - \dot{x}(t)] &\equiv 0
 \end{aligned} \tag{11}$$

Then, substituting Eq. (12) into Eq. (11) yields



$$\begin{aligned}
\dot{V}(x(t), t) &= 2x^T(t)P\dot{x}(t) + x^T(t)(Q_1 + Q_2)x(t) + \dot{x}^T(t)(\tau_1 R_1 + \tau_2 R_2)\dot{x}(t) \\
&\quad - (1 - \dot{d}(t))x_d^T(t)Q_1x_d(t) - (1 - \dot{h}(t))x_h^T(t)Q_2x_h(t) \\
&\quad - \int_{t-\tau_1}^t \dot{x}^T(s)R_1\dot{x}(s)ds + 2\eta^T(t)X[x(t) - x_d(t) - \int_{t-d(t)}^t \dot{x}(s)ds] \\
&\quad - \int_{t-\tau_2}^t \dot{x}^T(s)R_2\dot{x}(s)ds + 2\eta^T(t)Z[x(t) - x_h(t) - \int_{t-h(t)}^t \dot{x}(s)ds] \\
&\quad + 2\eta^T(t)Y \sum_{i,j,l=1}^s h_i h_j h_{hl} [\Lambda_{ij}x(t) + \Lambda_{dil}x_d(t) + \Lambda_{hil}x_h(t) - \dot{x}(t)] \\
&\leq 2x^T(t)P\dot{x}(t) + x^T(t)(Q_1 + Q_2)x(t) + \dot{x}^T(t)(\tau_1 R_1 + \tau_2 R_2)\dot{x}(t) \\
&\quad - (1 - \sigma_1)x_d^T(t)Q_1x_d(t) - (1 - \sigma_2)x_h^T(t)Q_2x_h(t) \\
&\quad - \int_{t-d(t)}^t \dot{x}^T(s)R_1(s)\dot{x}(s)ds + 2\eta^T(t)X[x(t) - x_d(t) - \int_{t-d(t)}^t \dot{x}(s)ds] \\
&\quad - \int_{t-h(t)}^t \dot{x}^T(s)R_2(s)\dot{x}(s)ds + 2\eta^T(t)Z[x(t) - x_h(t) - \int_{t-h(t)}^t \dot{x}(s)ds] \\
&\quad + 2\eta^T(t)Y \sum_{i,j,l=1}^s h_i h_j h_{hl} [\Lambda_{ij}x(t) + \Lambda_{dil}x_d(t) + \Lambda_{hil}x_h(t) - \dot{x}(t)] + x^T(t)Sx(t) \\
&\quad + \sum_{i,j=1}^s h_i h_j \rho^2 x^T(t) \bar{K}_i^T \cos \theta_i W \bar{K}_j \cos \theta_j x(t) - [x^T(t)Sx(t) + u^T(t)Wu(t)]
\end{aligned} \tag{12}$$

where  $\eta(t) = [x^T(t), x_d^T(t), x_h^T(t), \dot{x}^T(t)]^T$ .

Applying Lemma 1, we have the following inequalities:

$$\begin{aligned}
2x^T(t)Y_1\Lambda_{ij}x(t) &\leq 2x^T(t)Y_1A_ix(t) + \varepsilon\rho^2x^T(t)Y_1Y_1^Tx(t) + \varepsilon^{-1}x^T(t)(N_i^TN_i + (B_i\bar{K}_j)^T(B_i\bar{K}_j))x(t), \\
2x^T(t)Y_1\Lambda_{dil}x_d(t) &\leq 2x^T(t)Y_1A_{di}x_d(t) + \varepsilon\rho^2\sin^2\phi_lx^T(t)Y_1Y_1^Tx(t) + \varepsilon^{-1}x_d^T(t)N_{di}^TN_{di}x_d(t), \\
2x^T(t)Y_1\Lambda_{hil}x_h(t) &\leq \varepsilon\rho^2\cos^2\phi_lx^T(t)Y_1Y_1^Tx(t) + \varepsilon^{-1}x_h^T(t)(B_{hi}\tilde{K}_l)^T(B_{hi}\tilde{K}_l)x_h(t), \\
2x_d^T(t)Y_2\Lambda_{ij}x(t) &\leq 2x_d^T(t)Y_2A_ix(t) + \varepsilon\rho^2x_d^T(t)Y_2Y_2^Tx_d(t) + \varepsilon^{-1}x^T(t)(N_i^TN_i + (B_i\bar{K}_j)^T(B_i\bar{K}_j))x(t), \\
2x^T(t)Y_1\Lambda_{hil}x_h(t) &\leq \varepsilon\rho^2\cos^2\phi_lx^T(t)Y_1Y_1^Tx(t) + \varepsilon^{-1}x_h^T(t)(B_{hi}\tilde{K}_l)^T(B_{hi}\tilde{K}_l)x_h(t), \\
2x_d^T(t)Y_2\Lambda_{ij}x(t) &\leq 2x_d^T(t)Y_2A_ix(t) + \varepsilon\rho^2x_d^T(t)Y_2Y_2^Tx_d(t) + \varepsilon^{-1}x^T(t)(N_i^TN_i + (B_i\bar{K}_j)^T(B_i\bar{K}_j))x(t), \\
2x_d^T(t)Y_2\Lambda_{dil}x_d(t) &\leq 2x_d^T(t)Y_2A_{di}x_d(t) + \varepsilon\rho^2\sin^2\phi_lx_d^T(t)Y_2Y_2^Tx_d(t) + \varepsilon^{-1}x_d^T(t)N_{di}^TN_{di}x_d(t), \\
2x_d^T(t)Y_2\Lambda_{hil}x_h(t) &\leq \varepsilon\rho^2\cos^2\phi_lx_d^T(t)Y_2Y_2^Tx_d(t) + \varepsilon^{-1}x_h^T(t)(B_{hi}\tilde{K}_l)^T(B_{hi}\tilde{K}_l)x_h(t), \\
2x_h^T(t)Y_3\Lambda_{ij}x(t) &\leq 2x_h^T(t)Y_3A_ix(t) + \varepsilon\rho^2x_h^T(t)Y_3Y_3^Tx_h(t) + \varepsilon^{-1}x^T(t)(N_i^TN_i + (B_i\bar{K}_j)^T(B_i\bar{K}_j))x(t), \\
2x_h^T(t)Y_3\Lambda_{dil}x_d(t) &\leq 2x_d^T(t)Y_3A_{di}x_d(t) + \varepsilon\rho^2\sin^2\phi_lx_h^T(t)Y_3Y_3^Tx_h(t) + \varepsilon^{-1}x_d^T(t)N_{di}^TN_{di}x_d(t), \\
2x_h^T(t)Y_3\Lambda_{hil}x_h(t) &\leq \varepsilon\rho^2\cos^2\phi_lx_h^T(t)Y_3Y_3^Tx_h(t) + \varepsilon^{-1}x_h^T(t)(B_{hi}\tilde{K}_l)^T(B_{hi}\tilde{K}_l)x_h(t), \\
2\dot{x}^T(t)Y_4\Lambda_{ij}x(t) &\leq 2\dot{x}^T(t)Y_4A_ix(t) + \varepsilon\rho^2\dot{x}^T(t)Y_4Y_4^T\dot{x}(t) + \varepsilon^{-1}x^T(t)(N_i^TN_i + (B_i\bar{K}_j)^T(B_i\bar{K}_j))x(t), \\
2\dot{x}^T(t)Y_4\Lambda_{dil}x_d(t) &\leq 2\dot{x}^T(t)Y_4A_{di}x_d(t) + \varepsilon\rho^2\dot{x}^T(t)Y_4Y_4^T\dot{x}(t) + \varepsilon^{-1}x_d^T(t)N_{di}^TN_{di}x_d(t), \\
2\dot{x}^T(t)Y_4\Lambda_{hil}x_h(t) &\leq \varepsilon\rho^2\cos^2\phi_l\dot{x}^T(t)Y_4Y_4^T\dot{x}(t) + \varepsilon^{-1}x_h^T(t)(B_{hi}\tilde{K}_l)^T(B_{hi}\tilde{K}_l)x_h(t)
\end{aligned} \tag{13}$$

Substituting Eq. (13) into Eq. (12) results in

$$\begin{aligned}
 \dot{V}(x(t), t) &\leq \sum_{i,j,l=1}^s h_i h_j h_{hl} \eta^T(t) T_{ij} \eta(t) - \int_{t-d(t)}^t \dot{x}^T(s) R_1 \dot{x}(s) ds - \int_{t-h(t)}^t \dot{x}^T(s) R_2 \dot{x}(s) ds \\
 &\quad - 2\eta^T(t) X \int_{t-d(t)}^t \dot{x}(s) ds - 2\eta^T(t) Z \int_{t-h(t)}^t \dot{x}(s) ds - [x^T(t) S x(t) + u^T(t) W u(t)] \\
 &\leq \sum_{i,j,l=1}^s h_i h_j h_{hl} \eta^T(t) (T_{ijl} + \tau_1 X R_1^{-1} X^T + \tau_2 Z R_2^{-1} Z^T) \eta(t) \\
 &\quad - \int_{t-d(t)}^t (\eta^T(t) X + \dot{x}^T(s) R_1) R_1^{-1} (\eta^T(t) X + \dot{x}^T(s) R_1)^T ds \\
 &\quad - \int_{t-h(t)}^t (\eta^T(t) Z + \dot{x}^T(s) R_2) R_2^{-1} (\eta^T(t) Z + \dot{x}^T(s) R_2)^T ds - [x^T(t) S x(t) + u^T(t) W u(t)] \\
 &\leq \sum_{i,j,l=1}^s h_i h_j h_{hl} \eta^T(t) (\tilde{T}_{ijl} + \tau_1 X R_1^{-1} X^T + \tau_2 Z R_2^{-1} Z^T) \eta(t) - [x^T(t) S x(t) + u^T(t) W u(t)]
 \end{aligned} \tag{14}$$

where

$$\tilde{T}_{ijl} = \begin{bmatrix} \tilde{T}_{11,ij} & * & * & * \\ T_{21,i} & T_{22,i} & * & * \\ T_{31,i} & T_{32,ij} & T_{33,il} & * \\ T_{41,i} & T_{42,i} & T_{43} & T_{44} \end{bmatrix}, \quad \tilde{T}_{11,ij} = T_{11,ij} + \rho^2 \bar{K}_i^T \cos \theta_i W \bar{K}_j \cos \theta_j - \rho^2 \bar{K}_i^T W \bar{K}_j.$$

In light of the inequality  $\bar{K}_i^T W \bar{K}_j + \bar{K}_j^T W \bar{K}_i \leq \bar{K}_i^T W \bar{K}_i + \bar{K}_j^T W \bar{K}_j$ , we have

$$\dot{V}(x(t), t) \leq \sum_{i,j,l=1}^s h_i h_j h_{hl} \eta^T(t) (T_{ijl} + \tau_1 X R_1^{-1} X^T + \tau_2 Z R_2^{-1} Z^T) \eta(t) - [x^T(t) S x(t) + u^T(t) W u(t)] \tag{15}$$

Applying the Schur complement to Eq. (8) yields

$$T_{ii} + \tau_1 X R_1^{-1} X^T + \tau_2 Z R_2^{-1} Z^T < 0, \quad T_{ij} + T_{ji} + 2\tau_1 X R_1^{-1} X^T + 2\tau_2 Z R_2^{-1} Z^T < 0.$$

Therefore, it follows from Eq. (15) that

$$\dot{V}(x(t), t) \leq - [x^T(t) S x(t) + u^T(t) W u(t)] < 0 \tag{16}$$

which implies that the system (6) is asymptotically stable.

Integrating Eq. (16) from 0 to  $T$  produces

$$\int_0^T [x^T(t) S x(t) + u^T(t) W u(t)] dt \leq -V(x(T), T) + V(x(0), 0) < V(x(0), 0)$$

Because of  $V(x(t), t) \geq 0$  and  $\dot{V}(x(t), t) < 0$ , thus  $\lim_{T \rightarrow \infty} V(x(T), T) = c$ , where  $c$  is a nonnegative constant. Therefore, the following inequality can be obtained:

$$\begin{aligned}
 J \leq & x^T(0)Px(0) + \int_{-d(0)}^0 x^T(s)Q_1x(s)ds + \int_{-\tau_1}^0 \int_{\theta}^0 \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{-h(0)}^0 x^T(s)Q_2x(s)ds \\
 & + \int_{-\tau_2}^0 \int_{\theta}^0 \dot{x}^T(s)R_2\dot{x}(s)dsd\theta = J_0
 \end{aligned} \tag{17}$$

This completes the proof.

**Remark 2:** In the derivation of Theorem 1, the free-weighting matrices  $X_k \in R^{n \times n}$ ,  $Y_k \in R^{n \times n}$ ,  $k = 1, 2, 3, 4$  are introduced, the purpose of which is to reduce conservatism in the existing delay-dependent stabilization conditions, see Ref. [17].

In the following section, we shall turn the conditions given in Theorem 1 into linear matrix inequalities (LMIs). Under the assumptions that  $Y_1, Y_2, Y_3, Y_4$  are non-singular, we can define the matrix  $Y_i^{-T} = \lambda Z, i = 1, 2, 3, 4, Z = P^{-1}, \lambda > 0$ .

Pre- and post-multiply (8) and (9) with  $\Theta = \text{diag}\{Y_1^{-1}, Y_2^{-1}, Y_3^{-1}, Y_4^{-1}, Y_4^{-1}, Y_4^{-1}\}$  and  $\Theta^T = \text{diag}\{Y_1^{-T}, Y_2^{-T}, Y_3^{-T}, Y_4^{-T}, Y_4^{-T}, Y_4^{-T}\}$ , respectively, and letting  $\bar{Q}_1 = Y_1^{-1}Q_1Y_1^{-T}$ ,  $\bar{Q}_2 = Y_1^{-1}Q_2Y_1^{-T}$ ,  $\bar{R}_k = Y_4^{-1}R_kY_4^{-T}, k = 1, 2, \bar{X}_i = Y_i^{-1}X_iY_i^{-T}, \bar{Z}_i = Y_i^{-1}Z_iY_i^{-T}, i = 1, 2, 3, 4$ , we obtain the following inequality (18), which is equivalent to (8):

$$\begin{bmatrix}
 \bar{T}_{11,ij} & * & * & * & * & * \\
 \bar{T}_{21,i} & \bar{T}_{22,i} & * & * & * & * \\
 \bar{T}_{31,i} & \bar{T}_{32,i} & \bar{T}_{33,il} & * & * & * \\
 \bar{T}_{41,i} & \bar{T}_{42,i} & \bar{T}_{43} & \bar{T}_{44} & * & * \\
 \tau_1 \bar{X}_1 & \tau_1 \bar{X}_2 & \tau_1 \bar{X}_3 & \tau_1 \bar{X}_4 & -\tau_1 \bar{R}_1 & * \\
 \tau_2 \bar{Z}_1 & \tau_2 \bar{Z}_2 & \tau_2 \bar{Z}_3 & \tau_2 \bar{Z}_4 & 0 & -\tau_2 \bar{R}_2
 \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s \tag{18}$$

where

$$\begin{aligned}
 \bar{T}_{11,ij} = & \bar{Q}_1 + \bar{Q}_2 + \bar{X}_1 + \bar{X}_1^T + \lambda A_i Z + \lambda Z A_i^T + \lambda^2 Z S Z + 2\varepsilon \rho^2 I + 4\varepsilon^{-1} \lambda^2 Z N_i^T N_i Z \\
 & + Z_1 + Z_1^T + 4\varepsilon^{-1} \lambda^2 (B_i \bar{K}_i Z)^T (B_i \bar{K}_i Z) + \rho^2 \lambda^2 Z \bar{K}_i^T W \bar{K}_i Z, \\
 \bar{T}_{21,i} = & -\bar{X}_1^T + \bar{X}_2 + \bar{Z}_2 + \lambda A_i Z + \lambda Z A_{di}^T, \quad \bar{T}_{31,i} = \bar{Z}_3 - \bar{Z}_1 + \bar{X}_3 + \lambda A_i Z, \\
 \bar{T}_{41,i} = & \lambda^2 Z + \lambda A_i Z - \lambda Z + \bar{X}_4 + \bar{Z}_4, \\
 \bar{T}_{22,i} = & -(1 - \sigma_1) \bar{Q}_1 - \bar{X}_2 - \bar{X}_2^T + \lambda A_{di} Z + \lambda Z A_{di}^T + 2\varepsilon \rho^2 I + 4\varepsilon^{-1} \lambda^2 Z N_{di}^T N_{di} Z, \\
 \bar{T}_{32,i} = & -\bar{X}_3 - \bar{Z}_2 + \lambda A_{di} Z - \lambda Z A_{di}^T, \quad \bar{T}_{42,i} = -\bar{X}_4 + \lambda A_i Z - \lambda Z, \\
 \bar{T}_{33,il} = & -(1 - \sigma_2) \bar{Q}_2 - \bar{Z}_3 - \bar{Z}_3^T + 4\varepsilon^{-1} \lambda^2 (B_{hi} \tilde{K}_i Z)^T B_{hi} \tilde{K}_i Z + 2\varepsilon \rho^2 I, \\
 \bar{T}_{43} = & -\bar{Z}_4 - \lambda Z, \quad \bar{T}_{44} = \tau_1 R_1 + \tau_2 R_2 - \lambda Z - \lambda Z^T + 2\varepsilon \rho^2 I.
 \end{aligned}$$

Applying the Schur complement to Eq. (18) results in

$$\Gamma_{ijl} = \begin{bmatrix} \Phi_{11,i} & * & * \\ \Phi_{21,ij} & \Phi_{22} & * \\ \Phi_{31,il} & 0 & \Phi_{33} \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s \quad (19)$$

where

$$\Phi_{11,i} = \begin{bmatrix} \bar{T}_{11,i} & * & * & * & * & * \\ \bar{T}_{21,i} & \bar{T}_{22,i} & * & * & * & * \\ \bar{T}_{31,i} & \bar{T}_{32,i} & \bar{T}_{33,il} & * & * & * \\ \bar{T}_{41,i} & \bar{T}_{42,i} & \bar{T}_{43} & \bar{T}_{44} & * & * \\ \tau_1 X_1 & \tau_1 X_2 & \tau_1 X_3 & \tau_1 X_4 & -\tau_1 \bar{R}_1 & * \\ \tau_2 \bar{Z}_1 & \tau_2 \bar{Z}_2 & \tau_2 \bar{Z}_3 & \tau_2 \bar{Z}_4 & 0 & -\tau_2 \bar{R}_2 \end{bmatrix}$$

With

$$\begin{aligned} \bar{T}_{11,i} &= \bar{Q}_1 + \bar{Q}_2 + \bar{X}_1 + \bar{X}_1^T + \lambda A_i Z + \lambda Z A_i^T + Z_1 + Z_1^T + 2\varepsilon \rho^2 I, \\ \bar{T}_{22,i} &= -(1 - \sigma_1) \bar{Q}_1 - \bar{X}_2 - \bar{X}_2^T + \lambda A_{di} Z + \lambda Z A_{di}^T + 2\varepsilon \rho^2 I, \\ \bar{T}_{33} &= -(1 - \sigma_2) \bar{Q}_2 - \bar{Z}_3 - \bar{Z}_3^T + 2\varepsilon \rho^2 I. \end{aligned}$$

$$\Phi_{21,ij} = \begin{bmatrix} \lambda Z & 0 & 0 & 0 \\ \lambda N_i Z & 0 & 0 & 0 \\ \lambda B_i \bar{K}_j Z & 0 & 0 & 0 \\ \rho \lambda \bar{K}_j Z & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_{31,il} = \begin{bmatrix} 0 & \lambda B_{hi} \tilde{K}_l Z & 0 & 0 \\ 0 & 0 & \lambda N_{di} Z & 0 \end{bmatrix}$$

$$\Phi_{22} = \begin{bmatrix} -S^{-1} & 0 & 0 & 0 \\ 0 & -\frac{\varepsilon}{4} I & 0 & 0 \\ 0 & 0 & -\frac{\varepsilon}{4} I & 0 \\ 0 & 0 & 0 & -W^{-1} \end{bmatrix}$$

$$\Phi_{33} = \begin{bmatrix} -\frac{\varepsilon}{4} I & 0 \\ 0 & -\frac{\varepsilon}{4} I \end{bmatrix}$$

Obviously, the closed-loop fuzzy system (6) is asymptotically stable, if for some scalars  $\lambda > 0$ , there exist matrices  $Z > 0, \bar{Q} > 0, \bar{R} > 0$  and  $\bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{K}_i, i = 1, 2, \dots, s$  satisfying the inequality (19).

**Theorem 2.** Consider the system (6) associated with cost function (7). For given scalars  $\rho > 0, \tau_1 > 0, \tau_2 > 0, \sigma_1 > 0, \sigma_2 > 0$  and  $\lambda > 0, \delta > 0$ , if there exist matrices  $Z > 0, \bar{Q}_1 > 0, \bar{R}_1 > 0, \bar{Q}_2 > 0, \bar{R}_2 > 0$  and  $\bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{X}_4, M_i, i = 1, 2, \dots, s$  and scalar  $\varepsilon > 0$  satisfying the following LMI (20), the system (6) is asymptotically stable and the control law (5) is a fuzzy non-fragile guaranteed cost control law

$$\begin{bmatrix} \Theta_{1,ijl} & * \\ \Theta_{2,ijl} & \Theta_3 \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s \quad (20)$$

Moreover, the feedback gains are given by



$$+\delta^{-1} \begin{bmatrix} (\lambda E_{kj}Z)^T & (\lambda E_{ki}Z)^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\lambda E_{kl}Z)^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda E_{kj}Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda E_{ki}Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda E_{kl}Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \tag{21}$$

Using Lemma 2 and noting  $M_i = K_iZ$ , by the condition (21), the following inequality holds:

$$\Phi_{1,ijl} + \begin{bmatrix} 0 & * & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\ \lambda B_i \Delta \bar{K}_j Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * \\ \rho \lambda \Delta \bar{K}_i Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & \lambda B_{hi} \Delta \tilde{K}_l Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \tag{22}$$

where  $\Delta \tilde{K}_i = \Delta K_i(t - d(t))$ .

Therefore, it follows from Theorem 1 that the system (6) is asymptotically stable and the control law (5) is a fuzzy non-fragile guaranteed cost control law. Thus, we complete the proof.

Now consider the cost bound of

$$J \leq x^T(0)Px(0) + \int_{-d(0)}^0 x^T(s)Q_1x(s)ds + \int_{-\tau_1}^0 \int_{\theta}^0 \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{-h(0)}^0 x^T(s)Q_2x(s)ds \\
 + \int_{-\tau_2}^0 \int_{\theta}^0 \dot{x}^T(s)R_2\dot{x}(s)dsd\theta = J_0$$

Similar to Ref. [23], we supposed that there exist positive scalars  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ , such that  $Z^{-1} \leq \alpha_1 I, \frac{1}{\lambda^2} P \bar{Q}_1 P \leq \alpha_2 I, \frac{1}{\lambda^2} P \bar{Q}_2 P \leq \alpha_3 I, \frac{1}{\lambda^2} P \bar{R}_1 P \leq \alpha_4 I, \frac{1}{\lambda^2} P \bar{R}_2 P \leq \alpha_5 I$ .

Then, define  $S_{Q1} = \bar{Q}_1^{-1}, S_{Q2} = \bar{Q}_2^{-1}, S_{R1} = \bar{R}_1^{-1}, S_{R2} = \bar{R}_2^{-1}$ , by Schur complement lemma, we have the following inequalities:

$$\begin{aligned}
& \begin{bmatrix} -\alpha_1 I & I \\ I & -Z \end{bmatrix} \leq 0, \begin{bmatrix} -\alpha_2 I & \frac{1}{\lambda} P \\ \frac{1}{\lambda} P & S_{Q1} \end{bmatrix} \leq 0, \begin{bmatrix} -\alpha_3 I & \frac{1}{\lambda} P \\ \frac{1}{\lambda} P & S_{Q2} \end{bmatrix} \leq 0, \begin{bmatrix} -\alpha_4 I & \frac{1}{\lambda} P \\ \frac{1}{\lambda} P & S_{R1} \end{bmatrix} \leq 0, \\
& \begin{bmatrix} -\alpha_5 I & \frac{1}{\lambda} P \\ \frac{1}{\lambda} P & S_{R2} \end{bmatrix} \leq 0, \begin{bmatrix} Z & I \\ I & Z \end{bmatrix} \geq 0, \begin{bmatrix} S_{Q1} & I \\ I & \bar{Q}_1 \end{bmatrix} \geq 0, \begin{bmatrix} S_{Q2} & I \\ I & \bar{Q}_2 \end{bmatrix} \geq 0, \\
& \begin{bmatrix} S_{R1} & I \\ I & \bar{R}_1 \end{bmatrix} \geq 0, \begin{bmatrix} S_{R2} & I \\ I & \bar{R}_2 \end{bmatrix} \geq 0,
\end{aligned} \tag{23}$$

Using the idea of the cone complement linear algorithm in Ref. [24], we can obtain the solution of the minimization problem of upper bound of the value of the cost function as follows:

$$\begin{aligned}
& \text{minimize} \{ \text{trace}(PZ + S_{Q1}\bar{Q}_1 + S_{Q2}\bar{Q}_2 + S_{R1}\bar{R}_1 + S_{R2}\bar{R}_2 + \alpha_1 x^T(0)x(0) + \alpha_2 \int_{-d(0)}^0 x^T(s)x(s)ds \\
& + \alpha_4 \int_{-\tau_1}^0 \int_{\theta}^0 \dot{x}^T(s)\dot{x}(s)dsd\theta + \alpha_3 \int_{-h(0)}^0 x^T(s)x(s)ds + \alpha_5 \int_{-\tau_2}^0 \int_{\theta}^0 \dot{x}^T(s)\dot{x}(s)dsd\theta \} \\
& \text{subject to (20), (23), } \varepsilon > 0, \bar{Q}_1 > 0, \bar{Q}_2 > 0, \bar{R}_1 > 0, \bar{R}_2 > 0, Z > 0, \alpha_i > 0, i = 1, \dots, 5
\end{aligned} \tag{24}$$

Using the following cone complement linearization (CCL) algorithm [24] can iteratively solve the minimization problem (24).  $\square$

## 5. Simulation examples

In this section, the proposed approach is applied to the Van de Vusse system to verify its effectiveness.

Example: Consider the dynamics of an isothermal continuous stirred tank reactor for the Van de Vusse

$$\begin{aligned}
\dot{x}_1 &= -50x_1 - 10x_1^3 + u(10 - x_1) + u(t - h) + u(t - h)(0.5x_1(t - d) + 0.2x_2(t - d)) + 5x_2(t - d) \\
\dot{x}_2 &= 50x_1 - 100x_2 - u(t - h) + u(t - h)(0.3x_1(t - d) - 0.2x_2(t - d)) + 10x_2(t - d) - 5x_1(t - d)
\end{aligned} \tag{25}$$

From the system equation (25), some equilibrium points are tabulated in **Table 1**. According to these equilibrium points,  $[x_e \ u_e]$ , which are also chosen as the desired operating points,  $[x'_e \ u'_e]$ , we can use the similar modeling method that is described in Ref. [16].

$x_e^T$	$x_{de}^T$	$u_e$	$u_{de}$
[2.0422 1.2178]	[2.0422 1.2178]	20.3077	20.3077
[3.6626 2.5443]	[3.6626 2.5443]	77.7272	77.7272
[5.9543 5.5403]	[5.9543 5.5403]	296.2414	296.2414

**Table 1.** Data for equilibrium points.

Thus, the system (25) can be represented by

$R^1$  : if  $x_1$  is about 2.0422 then

$$\dot{x}_\delta(t) = A_1x_\delta(t) + A_{d1}x_{d\delta}(t) + B_1u_\delta(t) + B_{h1}u_{h\delta}(t) + N_1x_\delta(t)u_\delta(t) + N_{d1}x_{d\delta}(t)u_{h\delta}(t)$$

$R^2$  : if  $x_1$  is about 3.6626, then

$$\dot{x}_\delta(t) = A_2x_\delta(t) + A_{d2}x_{d\delta}(t) + B_2u_\delta(t) + B_{d2}u_{h\delta}(t) + N_2x_\delta(t)u_\delta(t) + N_{d2}x_{d\delta}(t)u_{h\delta}(t) \quad (26)$$

$R^3$  : if  $x_1$  is about 5.9543, then

$$\dot{x}_\delta(t) = A_3x_\delta(t) + A_{d3}x_{d\delta}(t) + B_3u_\delta(t) + B_{d3}u_{h\delta}(t) + N_3x_\delta(t)u_\delta(t) + N_{d3}x_{d\delta}(t)u_{h\delta}(t)$$

where

$$A_1 = \begin{bmatrix} -75.2383 & 7.7946 \\ 50 & -100 \end{bmatrix}, A_2 = \begin{bmatrix} -98.3005 & 11.7315 \\ 50 & -100 \end{bmatrix}, A_3 = \begin{bmatrix} -122.1228 & 8.8577 \\ 50 & -100 \end{bmatrix},$$

$$N_1 = N_2 = N_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; B_1 = B_2 = B_3 = \begin{bmatrix} 10 \\ 0 \end{bmatrix}; A_{d1} = A_{d2} = A_{d3} = \begin{bmatrix} 0 & 5 \\ 10 & -5 \end{bmatrix},$$

$$N_{d1} = N_{d2} = N_{d3} = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & -0.2 \end{bmatrix}, B_{h1} = B_{h2} = B_{h3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, x_\delta = x(t) - x'_e,$$

$$u_\delta = u(t) - u'_e, x_{d\delta} = x(t-d) - x'_{de}, u_{h\delta} = u(t-d) - u'_{he}.$$

The cost function associated with this system is given with  $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W = 1$ . The controller gain perturbation  $\Delta K$  of the additive form is give with  $H_1 = H_2 = H_3 = 0.1, E_{k1} = [0.05 \quad -0.01], E_{k2} = [0.02 \quad 0.01], E_{k3} = [-0.01 \quad 0]$ .

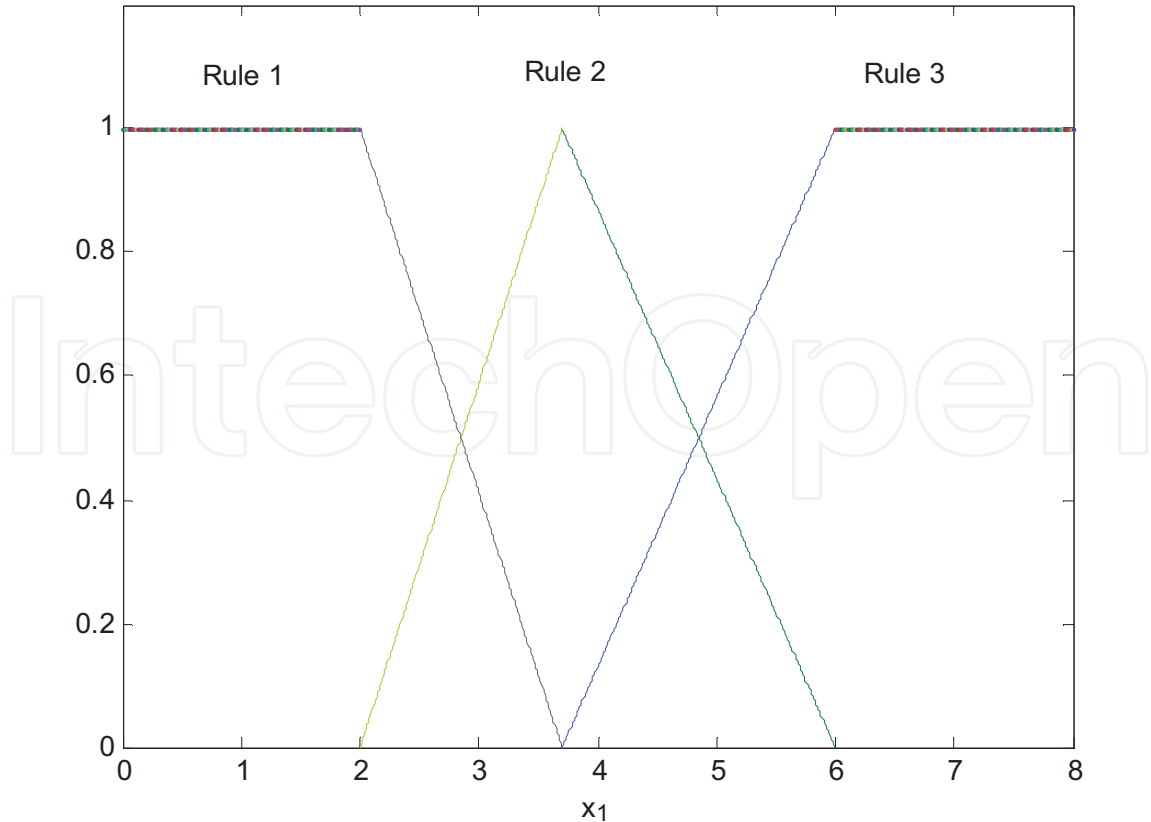


Figure 1. Membership functions.



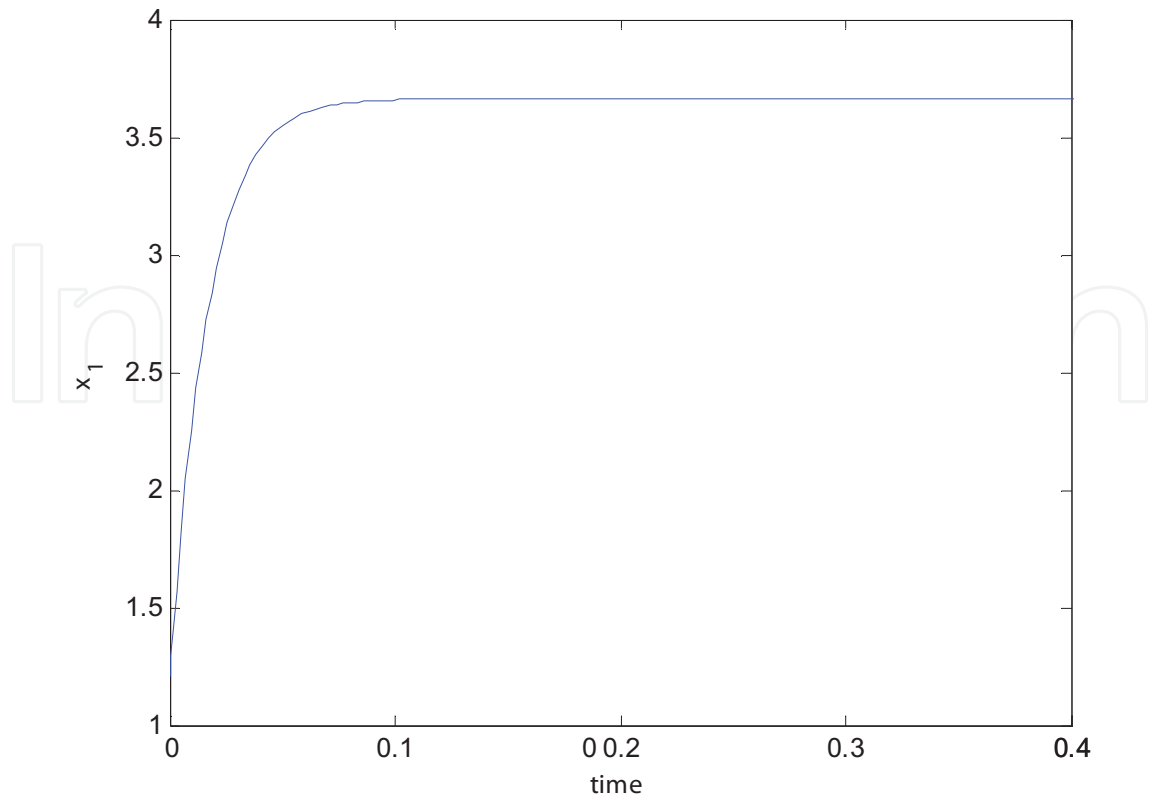


Figure 2. State responses of  $x_1(t)$ .

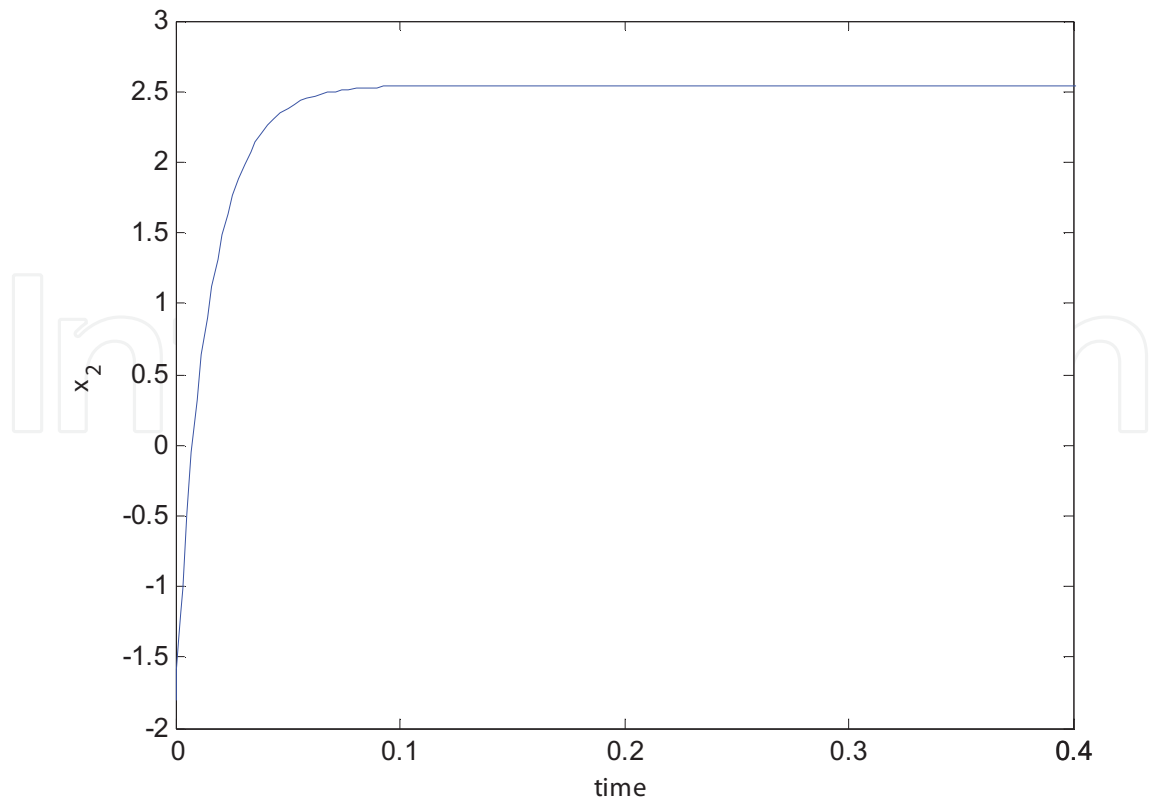
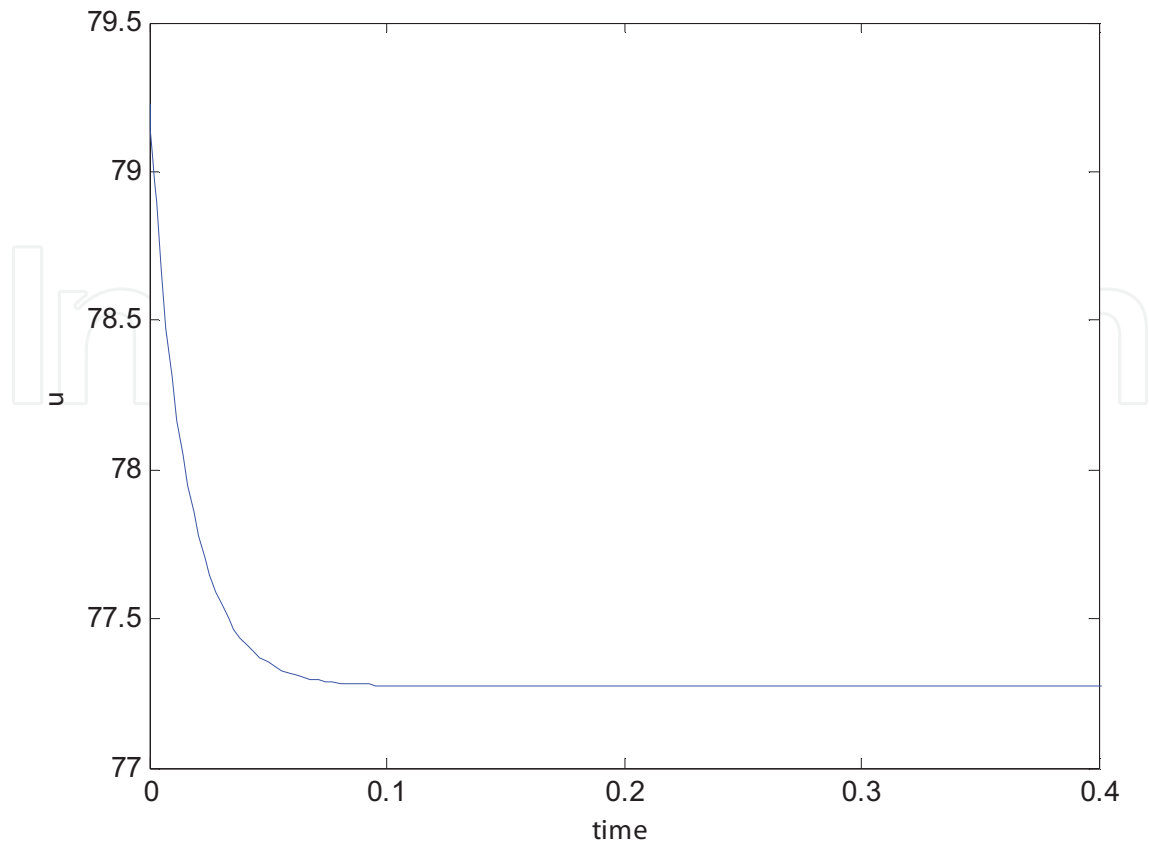


Figure 3. State responses of  $x_2(t)$ .



**Figure 4.** Control trajectory of system.

The membership functions of state  $x_1$  are shown in **Figure 1**.

Then, solving LMIs (23) and (24) for  $\rho = 0.45$ ,  $\lambda = 1.02$  and  $\delta = 0.11$ ,  $\tau_1 = \tau_2 = 2$ ,  $\sigma_1 = 0$ ,  $\sigma_2 = 0$  gives the following feasible solution:

$$\begin{aligned}
 P &= \begin{bmatrix} 4.2727 & -1.3007 \\ -1.3007 & 6.4906 \end{bmatrix}, Q_1 = \begin{bmatrix} 14.1872 & -1.9381 \\ -1.9381 & 13.0104 \end{bmatrix}, Q_2 = \begin{bmatrix} 3.1029 & 1.2838 \\ 1.2838 & 2.0181 \end{bmatrix}, \\
 R_1 &= \begin{bmatrix} 8.3691 & -1.3053 \\ -1.3053 & 7.0523 \end{bmatrix}, R_2 = \begin{bmatrix} 5.2020 & 2.2730 \\ 2.2730 & 1.0238 \end{bmatrix}, \varepsilon = 1.8043, \\
 K_1 &= [-0.4233 \quad -0.5031], K_2 = [-0.5961 \quad -0.7049], K_3 = [-0.4593 \quad -0.3874].
 \end{aligned}$$

**Figures 2–4** illustrate the simulation results of applying the non-fragile fuzzy controller to the system (25) with  $x'_e = [3.6626 \quad 2.5443]^T$  and  $u'_e = 77.7272$  under initial condition  $\varphi(t) = [1.2 \quad -1.8]^T$ ,  $t \in [-2 \quad 0]$ . It can be seen that with the fuzzy control law, the closed-loop system is asymptotically stable and an upper bound of the guaranteed cost is  $J_0 = 292.0399$ . The simulation results show that the fuzzy non-fragile guaranteed controller proposed in this paper is effective.

## 6. Conclusions

In this paper, the problem of non-fragile guaranteed cost control for a class of fuzzy time-varying delay systems with local bilinear models has been explored. By utilizing the Lyapunov

stability theory and LMI technique, sufficient conditions for the delay-dependent asymptotically stability of the closed-loop T-S fuzzy local bilinear system have been obtained. Moreover, the designed fuzzy controller has guaranteed the cost function-bound constraint. Finally, the effectiveness of the developed approach has been demonstrated by the simulation example. The robust non-fragile guaranteed cost control and robust non-fragile H-infinite control based on fuzzy bilinear model will be further investigated in the future work.

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