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The Post-Modern Transcendental of Language in Science and Philosophy

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Abstract

In this chapter I discuss the deep mutations occurring today in our society and in our culture, the natural and mathematical sciences included, from the standpoint of the "transcendental of language", and of the primacy of language over knowledge. That is, from the standpoint of the "completion of the linguistic turn" in the foundations of logic and mathematics using Peirce's algebra of relations. This evolved during the last century till the development of the Category Theory as universal language for mathematics, in many senses wider than set theory. Therefore, starting from the fundamental M. Stone's representation theorem for Boolean algebras, computer scientists developed a coalgebraic first-order semantics defined on Stone's spaces, for Boolean algebras, till arriving to the definition of a non-Turing paradigm of coalgebraic universality in computation. Independently, theoretical physicists developed a coalgebraic modelling of dissipative quantum systems in quantum field theory, interpreted as a thermo-field dynamics. The deep connection between these two coalgebraic constructions is the fact that the topologies of Stone spaces in computer science are the same of the C^*-algebras of quantum physics. This allows the development of a new class of quantum computers based on coalgebras. This suggests also an intriguing explanation of why one of the most successful experimental applications of this coalgebraic modelling of dissipative quantum systems is just in cognitive neuroscience.

Keywords: semiotics, Boolean algebras, algebra of logic, category theory, coalgebras, quantum field theory

1. Introduction: the semiotic interpretation of the transcendental of language

In this chapter, I suggest interpreting our present age, characterized by deep changes in every realm of the society and of the culture—science and philosophy included—as a Post-

Modern Age, not in the usual nihilist interpretation of Post-Modernity, but in a constructive way. This can be more easily understood if we see at the three ages, *Ancient*, including both the Greek and the Latin ages, *Modern* and *Post-Modern*, from the 'transcendental' standpoint. That is, from the standpoint of the *ontological foundation of truth*, so to characterize the Ancient Age with the 'Transcendental of Being', the Modern Age with the 'Transcendental of Knowing' and the Post-Modern Age with the 'Transcendental of Language'. In fact, during the Ancient Age—Middle Age included—the foundation of truth of the predicative sentences is the *being* of things existing independently of human thought and language, either in the *ideal realm* of the Platonic 'supercelestial world' ('hyperuranium') or in the *natural realm* of the Aristotelian physics. On the contrary, during the Modern age, because of the crisis of the Aristotelian cosmology, consequent to the birth of the Galilean science, the foundation of truth depends on Descartes' and Newton's principle of *evidence*,[1] and then it depends on human knowledge.

On the contrary, Otto Apel introduced the expression 'Transcendental of Language' in the contemporary philosophical debate [1, 2], for signifying the primacy of language over knowledge in our Post-Modern Age. Apel introduced this terminology originally together with Jürgen Habermas, who afterwards refused this connotation of his philosophy and social ontology, as well as the connotation of our age as 'post-modern' [3], because of the prevailing 'anti-modernist' and 'nihilist' meaning that this term acquired in the contemporary debate [4]. Anyway, apart from these terminology questions, what these authors and the others like me (e.g., the American philosopher John Deely, in his monumental textbook 'Four Ages of Understanding' [5]) want to signify as characterizing our age is the completion of the 'linguistic turn', only initiated by Gottlob Frege and Ludwig Wittengstein at the beginning of XX cent. This completion goes into the 'semiotic' direction depicted by Charles S. Peirce at the end of XIX cent., but effectively developed, both in science and philosophy, only during the last 50 years.

The illustration of this later completion in fundamental physics, as well as in logic and in computer science, all related with the 'algebra of relations' and then with the 'Category Theory' (CT), and the consequences for the anthropology and the epistemology, and more generally for the post-modern man that lost in this way his modern centrality, is the main object of this chapter.

Indeed, the scientific contribution of Peirce, often forgotten by philosophers, concerns precisely algebras since he practically 'invented' the algebra of relations [6], so that also his 'theory of

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[1]The "First Rule" of Descartes' *Discourse on Method* (1637), written as Introduction to his treatises on *Geometry, Optics*, and *Metreology* reads: '"never to accept anything as true if I didn't have *evident* knowledge of its truth"' [65]. In the Ancient English of his *Treatise of Optics* (1704), Newton stated: '"These Principles [the laws of Newtonian Mechanics] I consider, not as occult Qualities, supposed to result from the specifick Forms of Things, but as general Laws of Nature, by which the Things themselves are formed; their Truth appearing to us by Phaenomena, though their Causes are not yet discover'd. For these are manifest Qualities, and their Causes only are occult. (…) To derive two or three general Principles of Motion from Phenomena, and afterwards to tell us how the Properties and Actions of all corporeal Things follow from those manifest Principles, would be a very great step in Philosophy, though the Causes of those Principles not yet discover'd' [64, p. 376].

signs' or *semiotics* consists ultimately in the vindication that the fundamental irreducible relations in algebra, and then in logic and mathematics, are not the 'dyadic' ones, like in Boole's and Schröder's 'algebra of logic', but the 'triadic' relations. In this way, also his fundamental theory of 'semiosis', where the signifying character of 'signs', in the wider sense, and not only in the linguistic one, is based on its famous categorical distinction between 'firstness', 'secondness' and 'thirdness' as a triadic structure of relations underlying everything is signifying in language [7], in thought [8] and finally in nature itself [9], depends essentially on the irreducibility of triadic relations.

Therefore, if we approach the issue of the transcendental of language from the standpoint of *a social ontology of language*, without investigating over its 'semiotic' algebraic and pre-linguistic basis, we are almost completely missing the point in two senses.

1. Before all, we are continuing in the Modern prejudice of considering humans as the only actors of the communication interchanges, even though, because language is a social construction, such a 'transcendental subject' has in this approach a collective and not individual nature. This implies that, because the linguistic sign has necessarily a *conventional nature*, as De Saussure taught us [10], this incomplete approach to the post-modern linguistic turn leads us necessarily to an ontological conventionalism, and then back to the nihilist interpretation of our Post-Modern Age. However, De Saussure himself, who emphasized in his linguistics the conventional character of linguistic signs, suggests us an implicit reference to Peirce's semiotics, when he distinguishes in his famous Treatise between 'sign' and 'symbol'. The latter, indeed, is characterized for him as to the former by a 'natural bond' in the brain between 'signifier' and 'signified'. This makes effectively 'triadic' the semantic relationship in its 'physical' foundation, apart from the social construction of meanings by the linguistic interchanges. Even though De Saussure's 'semiology' never considered Peirce and his semiotics, nevertheless, such a triadic nature of symbol, and its intrinsic 'natural bond' in its pre-linguistic constitution, is copying with the 'semiotic naturalism' of Peirce's ontology [9]. This position led the American philosopher to define himself during all his career as a 'Scholastic realist' on a naturalistic basis that makes his position equidistant, both from a Platonic realism and from a linguistic conventionalism [11]. This suggestion introduces us immediately to the second main reason of the failure of the interpretation of our age as per a social ontology of language.

2. Indeed, such an ontology is completely missing the point of the deep changes we are facing in our culture, and in our society—in economy, and hence in politics, before all—as well as in fundamental sciences, namely, in physics, mathematics and logic. All these changes have, indeed, a common denominator in the *information revolution* we are facing today, and that will challenge much more our societies, our economies and finally our political and military systems, *on a worldwide dimension*, in the very next future. This revolution depends, on the one hand, on the amazing increasing of computing power of devices related to the actual and growing availability on the market of quantum computers that, in a quantum optics implementation in nano-technology, will not require like the actual quantum computers in electronic implementation to work at −273°C (thinks for instance at the famous D-Wave). In this way, they will be used practically like our electronic CPU's, but with a power and a velocity

of calculus some orders of magnitude higher, and with a power consumption of few watts. On the other hand, this revolution will depend on the development of the so-called *AI autonomous systems* (think at the self-driving cars, robots, drones, etc.), destined to change deeply every realm of our private and public, civil and military lives, on the other one. Despite these two branches of development seem independent as to each other, they are deeply connected from the theoretical standpoint, as we see. From the standpoint of social ontology, all this means that beside the '*conscious* communication agents'—that is humans, both individually and collectively intended—there exist also the '*unconscious* communication agents', i.e., the computational systems, in their networking with humans, to be considered as main actors of the actual social scene. They are acquiring an ever-growing role in the interchanges of information, of goods and of services within our societies, changing completely the financial and labour markets in economy, but also the access to cultural contents, the formation and the control of public opinions and, finally, the notion of democracy itself. This is making suddenly obsolete the classical and modern philosophical interpretations of our societies based on the centrality of individuals, and the related Gibbs' statistical approach to economic and social sciences. This approach is indeed based on the interchanges among individual actors like gas molecules of Boltzmann thermodynamics and the related notion of system *stability at equilibrium* of statistical mechanics, and therefore based on the related notion of *information asymmetry*—a notion that is at the basis also of Shannon's statistical notion of information in communication engineering [12]—as the core of market stability. Not casually the 1970 Nobel Prize in economics, P.A. Samuelson—the founder of the prestigious 'MIT School of Economics' counting among its members an impressive list of other Nobel Prizes—dedicated the first two chapters of his momentous textbook *Foundations of economic analysis* to Gibbs' thermodynamics, as the inspiring model of his approach [13]. And not casually the notion of information asymmetry obtained the Nobel Prize in 2001 to G. Akerlof, M. Spence and J.E. Stiglitz for their 'analyses of markets with asymmetric information'. However, an ever-growing usage of the Internet and of automatic computer-based financial exchanges acting worldwide on the markets in a second-fraction timing is evidently reducing this asymmetry, if not destroying at all [14]. The physical frame of reference is no longer 'a gas' of individual economical agents, but 'a fluid of interacting and ever changing 'condensates' of economical agents. Therefore, no longer the statistical mechanics, but the *condensed matter physics,* dealing with dissipative systems, persistently acting in *far-from-equilibrium conditions,* must be the frame of reference for modelling the actual situation of financial markets. In a word, the new situation makes obsolete the classical statistical techniques of financial analysis and prevision. This theoretical failure made the market recurrent crises as unpredictable as stronger earthquakes during a persistent earthquake swarm. In other terms, the lack of suitable means of control and prevision over the actual financial jungle of markets makes them so vulnerable to financial speculations, and then our economic systems, and our democracies too, so fragile, because effectively out from any possible control and political or ethical orientation towards the common good. Roughly speaking, the modern optimistic confidence into the 'invisible hand' acting on markets of Adam Smith risks to be today without any mathematical foundation. As Samuelson rightly emphasized at his time, this was Gibbs' statistics supposing a stability at equilibrium, by the

reciprocal compensation of the 'forces' in the markets. In our 'Communication Age', this hand is often completely paralyzed! Also in this sense, and overall in this sense from the social standpoint, we are living today in a Post-Modern Age.

Therefore, in the next section of this chapter, we illustrate 'the paradigm-shift' occurring in quantum field theory (QFT), and then in quantum computations, before all for dealing with the challenges of condensed matter physics in dissipative quantum systems. At the same time, this paradigm shift is strictly related in theoretical computer science with the research of innovative solutions to the 'big data' issue, and particularly with the challenge constituted by the *infinite data streams* modelling. They are characterized, indeed, by the continuous changes of the inner long-range correlations, and then by the necessity of readapting continuously the 'degrees of freedom', or the 'dimensions', of the computing system representation space. No classical statistical tool can perform such a task, but, on the contrary, it is what characterizes the dynamic principle of the 'doubling of the degrees of freedom' between a system and its thermal-bath in the *coalgebraic modelling* of quantum dissipative systems in the so-called *thermal QFT*. From the mathematical standpoint, the problem is indeed the same. In fact, there exists an evident convergence between the coalgebraic approach in thermal QFT, developed by theoretical physicists during the last 20 years, and the coalgebraic approach to computing systems (automata) interpreted as *labelled state transition systems.* This was developed simultaneously, but till recent years independently, by the theoretical computer scientists, precisely for dealing with the *dynamical* modelling of data stream, in the framework of 'Universal Coalgebra' as general theory of systems, and then outside the Turing paradigm of universality in computations [15].

On the other hand, since this coalgebraic approach to Boolean algebra semantics in computer science is developed in the framework of CT logic, this offers us in the final section of this chapter, for a systematic comparison between the phenomenological and the semiotic approach to the problem of meaning, so to describe more precisely the epistemological role of human consciousness, and its unicity, in our Post-Modern Age.

2. The topological interpretation of quantum field theory and of quantum computing in the framework of the category theory

2.1. The topological interpretation of quantum computing and quantum field theory

Also for solving the just remembered problems of representation and control on data streams, and more generally for solving the computational issues related with the famous 'big data' problems, emerging in any field of contemporary human and natural sciences—for which not only human minds, but also the formal apparatus of standard logic and mathematics are impotent—a new generation of quantum computing systems is object of the most advanced research.

This improvement is based on the so-called 'topological quantum computing', or 'topological QC' [16, 17], which, on its turn, is based on the operator algebras [18, 19], and then on a

'topological interpretation of quantum field theory', or 'topological QFT' [20], as fundamental theory of condensed matter physics, as well as of elementary particle physics 'beyond the Standard Model'.

The experimental proof of the insufficiency of the 'Standard Model' (SM) as theory about the ultimate constituents of matter has been awarded by the Nobel Prize in Physics in 2015. Furtherly, the promising results in the realm of topological QFT and topological QC, which led also to the discovery of 'exotic' phases of matter, have been awarded with the Nobel Prize in Physics for this year 2016. This emphasizes the absolute relevance of the research program of topological quantum computing, leading computer science beyond the classical, logistic, 'Turing Universality' principle in computation, in the sense that the topological QC paradigm is wider than Turing's one, because including it. It is based indeed on 'Algebraic Universality', and more significantly for our aims, on a 'Coalgebraic Universality' principle [15].

For understanding intuitively which is the deep paradigm shift in the ontology of physical reality related to QFT, let us start from the illustration of the difference between the mechanical vacuum of Newtonian and Laplacian mechanics, and the quantum vacuum (QV) of QFT (for such a reconstruction, see Refs. [21, 22]).

The QV has to be intended, indeed, as the dynamic substrate of all force fields connecting dynamically everything in the universe, and as the bounded energy reservoir of everything that exists in our universe and even of whichever possible universe in an (hypothetical) multiverse. In the QV ubiquitous present, everything is immersed from 'its inside' (the material constituents), and from 'its outside' (the environment). The same elementary particles constituting the material substrate of whichever physical thing are to be interpreted dynamically, given that their same mass has ultimately a dynamic justification, via the famous 'Higgs field'. Elementary particles, indeed, in the QFT framework are as many 'quanta' of the relative force field, given that, not only the gauge bosons (the massless photons, gluons, the massive W and Z bosons) of the interaction force fields of the SM (respectively, the electromagnetic, the strong and the weak forces), but also the Higgs boson and the relative field, as well as the massive fermions of SM (the quarks and the leptons (neutrinos and electrons)), i.e., the elementary 'building bricks' of the macroscopic bodies have to be conceived in QFT like as many massive or massless quanta of the relative material (fermions) or interaction (gauge bosons) force fields.

Finally, the same macroscopic bodies of our everyday experience, ourselves included, are constituted by 'condensations' of the elementary constituents (molecules, atoms, quarks and leptons, etc.), at different level of matter complexity (see the notion of QV 'foliation'), are depending on as many 'phase coherences' or coherent modes of oscillation of the relative force fields, determining the 'long-range quantum entanglements', and then the macroscopic unity of each body, as well as their reciprocal differences. Each of these 'phase coherence domains' or 'matter phases' depends, via the famous 'Goldstone theorem', which is the core of the Higgs mechanism in QFT, on a 'spontaneous symmetry breakdown' (SSB) of the QV.

Each SSB, in other terms, depends on the 'modes of (phase) coherent oscillations' of some force fields—either the material or the interaction ones. The 'quanta' of these coherent modes are a

third type of non-massive and non-energetic bosons (i.e., non-associated to any specific force field), beside the gauge bosons and the Higgs' one, the so-called 'Nambu-Goldstone bosons' (NGB's). They appear normally in the equations of QFT and are observed, measured and denoted as 'quasi-particles', for their strange properties. They, indeed, disappear without residuals—i.e., without violating the First Principle of Thermodynamics, because they are not 'quanta of energy' like the gauge bosons—with the field phase coherence that they 'order'. In this way, each phase coherence domain is characterized by a univocal 'fingerprint' corresponding to the value of the condensate of NGB's determining this phase. Therefore, they assume different denominations, according to the different phases of matter they determine. For example, in solid state physics, they assume the name of 'phonons', by determining the different phases—liquid, solid or crystalline of materials by breaking the rotational 'Galileian symmetry of molecule mechanical vibrations. Or, they assume the name of 'magnons', because determining in some metals their 'ferromagnetic phase', by breaking the rotational symmetry of the magnetization vectors, orienting them into one only direction.

Roughly speaking, this means that what microscopically links together the molecules of the plastic casing of my mouse, or the molecules of the wood desk on which the mouse is staying, are the different phase coherences of the oscillation modes of their respective material and electromagnetic force fields, ultimately depending on the long-range quantum correlations (entanglements) among these fields. Just as, what distinguishes the different liquid or solid phases of the same material are, respectively, the longitudinal, or the longitudinal and the transversal long-range correlations of the mechanical oscillations of the molecules of that material, 'breaking' the spherical 'Galilean symmetry' of the molecule mechanical vibrations that, on the contrary, holds 'unbroken' in the gas phase of the same material.

Ontologically, the distinction between material and interaction fields with their particle-like (bosonic and fermionic) quanta, on one side in the QV, and, on the other side, the NGB's as quanta of the coherent modes of oscillations of the material fields, emerging from the QV by SSB's like as many 'ordering principles' in the constitution of the complex structure of particles and then of macroscopic bodies, by the principle of QV foliation obviously recall the 'double consti-tution' matter-form of the Aristotelian ontology of nature. Not only because in QFT like in Aristotelian physics no mechanical vacuum exists, but overall because in this ontology, the 'natural forms' of bodies emerge as *ordering principles,* through the concourse of purely physical causes, from the universal substrate of the 'primary matter', according to the improper modern translation of the Aristotelian term of *próte dynamis*, 'primary dynamism'. The connection of the *próte dynamis* with QV as dynamic substrate of any physical entity is straightforward in this framework. This "education" of natural forms from the 'primary dynamism' of matter is, indeed, the core of the Aristotelian 'causal justification' of the physical forms (natural kinds) in nature, against the *dualism* of the Platonic ontology of nature where 'forms' are inserted from the outside of matter intended in a purely mechanistic way. Of this extrinsic character of 'forms' as to 'matter' the Galileian-Newtonian ontology is the 'representational counterpart', because banishing the forms in physics into the *mathematical formalisms* of a representational mind, modern conceptual-ist counterpart of the Platonic hyperuranium (see below Section 3 and [22–24]).

2.2. Some epistemological consequences

Anyway, QFT is evidently an ontological paradigm shift as to the mechanistic one of the modern Newtonian and Laplacian physics, conceiving the physical body as 'isolated' in the *mechanical vacuum*. This ontology has in the solipsism of the Kantian 'transcendental subject' the necessary counterpart for justifying a 'representational epistemology', in which truth is based on *evidence*, as Descartes first realized, and therefore it is based on the self-consciousness of some knowing subject.

In this way, the ontological truth is based on a consciousness state ('evidence'), according to the Modern Transcendental of Knowledge of Descartes, Newton and Kant. At that time, at the down of Modernity, John Poinsot's (1589–1644) opposed to this approach a 'proto-semiotic' interpretation of Aristotle's and Aquinas' logic and ontology, according to which truth depends on an identity of structure (effectively a 'homomorphism' or bijective mapping) on causal basis, from the formal structure of a thing—that, as such, is always an element of *a natural kind*[2]—onto the formal structure subject-predicate of a sentence referring to it. As far as such a correspondence becomes aware in humans, we have a 'conscious knowledge'. In this way, Poinsot anticipated the post-modern primacy of language over knowledge that, for using a famous Heidegger expression, was a *Holzweg* during the Modern Age, re-proposed independently at the end of Modernity by Ch. S. Peirce [5, 25].[3]

Poinsot, indeed, in his treatise *De Signis,* the first treatise of semiotics in the Modern Age [26], individuated a third ontological type of relations beside the classical 'real' (causal or physical) and 'rational' (mental) relations of the Scholastic tradition: the 'linguistic relations' (*relationes secundum dici*). In Poinsot's proto-semiotic framework, indeed, like in Aquinas' one, the truth of the sentence depends on its causal relation from the things, so that the *logical structure* subject-predicate of the sentence 'mirrors dually' the *causal structure* 'species-thing' (or 'genus-species'). The human knowledge, therefore, is the conscious self-representational after-effect of such an *onto-logical* foundation of truth, so to vindicate a primacy of *ontology over epistemology*. In this way, in human knowledge, the real 'thing' becomes an 'object-for-a-subject', in the sense of awareness of the outcome of a relational pre-conscious foundational process of truth in language. In this way, Poinsot gives us a semiotic relational interpretation of Aquinas' fundamental statement that is at the basis of his ontological theory of truth, per which: 'being is the first known by the intellect… however knowledge is a sort of effect of truth (*effectus quidem veritatis*)' (Aquinas, *Quaestione Disputatae De Veritate,* I,1). The following scheme exemplifies Poinsot's theory.

From this scheme, it is evident that this 'proto-semiotic' foundation of truth in language consists in the unconscious representation (mapping) of *real relations* from a thing, into the *logical ones* of a predicative sentence (= 'transcendental relations', in the figure), by a *reflexive*

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[2]For example, a horse is a member of the genus of mammalians on a causal basis, in the sense that the members of a given species/genus share the same causal web justifying their existences. Effectively, it is well-known that in genetics the members of a given species share common sequences of DNA, so that from the DNA is possible to reconstruct the phylogenetic history of a given species.

[3]Effectively, Peirce did not know Poinsot's works, even though both depend on the teaching of the works of the so-called *Cambricenses* teachers [5]. For this reason, along all his career, Peirce defined himself as a 'Scholastic realist'.

relation (= first 'rational relation', in the figure). This has the double role of *reversing* the arrow directions between the first two so to make them *dual* as to each other, and of *preceding* the further *self-reflexive* relation (= second 'rational relation', in the figure), by which the 'thing' is represented as an 'object' in a self-conscious mind.

The connection of this naturalistic ontology of knowledge with QFT as fundamental physics of the cognitive neuroscience and of topological QC will be discussed in the following. On the contrary, here it is important to emphasize the strict relationship between Poinsot and Peirce that historically makes of Poinsot the first philosopher who, in the history of Western thought, defined the 'triadic' structure of a *signifying* relation, i.e., of a sign. This idea is hidden in the definition by Poinsot of a 'relation in language' (*relatio secundum dici*) as a *transcendental relation* (see **Figure 1**), in two senses (see, Poinsot, *De Signis*, 574b14–575a19. In [26, p. 82]):

1. Because it does not belong to the category of relations that are all *dyadic*, that is, they are a 'being-to'(*esse ad*), whereas a 'sign' is a 'being-for' (*esse per*), and therefore it is a *triadic relation*;

2. Because a linguistic relation can apply to all the other categories and then *it transcends them*.

In this way, the 'proto-semiotic' ontology of Poinsot consists in enriching the Western ontology, not only with a new 'ontological category' of beings, the *signs*, beside the *rational* ones in mind, and the *real* ones in physics. They are a new type of *relations*, the *semiotic relations*, designed by Poinsot with the term of '*transcendental* relations', because of their fundamental role in language of connecting whichever 'category' of beings and relations. Specifically, they are able to connect 'rational relations', relating *mental* 'objects' (concepts) among themselves, with 'real relations' (causes), relating *natural* 'things' among themselves, via the power of signs of 'expressing' (being-for) both rational 'objects' and natural 'things'. Therefore, they are able to extend the function of 'signifying' also to the other two ontological categories, of 'objects', as signifying *things*, and to 'things' (effects) as signifying '*other things*', i.e., *causes*, e.g., a healthy blood as signifying a healthy body, so to effectively *bridging* 'objects' with 'things', and then 'rational relations' (mental relations) with 'real relations' (causal relations).

All the richness of this proto-semiotic ontology gets lost with the modern epistemological foundation of truth, so to confirm that, if the Middle-Age was the 'dark age' for physical and

Figure 1. Poinsot's scheme of the foundation of truth in language as preceding its knowledge.

mathematical sciences, the Modern Age was the 'dark age' for ontological and metaphysical sciences. In fact, the epistemological foundation of truth on 'evidence' of modern philosophy and science, from Galilei, to Descartes, to Newton and Kant, moves the real things, reduced to 'objects-for-a-subject', and their real and linguistic relations reduced to 'rational relations' in mind, in the double sense:

1. Of the rarefied, fascinating realm of the human 'abstract thought' of the pure mathematics evidences; and

2. Of their 'observer-related' application to empirical evidences, according to this modern epistemological interpretation of the Galileian method, in which mathematics precedes and guides the empirical observation.

Effectively, the self-representational 'evidence' as foundation of truth in Descartes and his followers 'cuts', because of its self-representational (self-conscious) nature, any relationship with the 'outer reality', just as the Newtonian calculus has 'to cut' any dynamic interaction of a mechanical system in the mathematical formalism of calculus, by supposing the abstract *mechanical vacuum*, for stopping the derivative order at the second one, by the 'inertia principle'. That is, for considering the acceleration as a constant, and so granting the abstract integrability of a function, and then the geometrical (kinematic) representation of the dynamics of a mechanical system.

S. de Laplace extended the Newtonian method to "many body physics", where, because of the many bodies simultaneously interacting, their isolation from any interaction can be abstractly granted only by supposing the so-called "asymptotic condition". It constitutes the core of the Laplacian "perturbative methods". Synthetically, in the (false) supposition that the properties of an isolated and of an interacting body are "always" the same, each of the components of a many body system is studied in an asymptotic condition, i.e., at an infinite spatio-temporal distance from each other, so to grant their isolation condition. Afterward, this characterization is applied in the interaction condition, interpreted as a "perturbation" of the asymptotic one. This formalism is the core of the modern statistical mechanics, extending the Newtonian mathematical analysis from geometry to the matrix algebra. Afterward, L. Boltzmann and J. W. Gibbs extended this method to the statistical thermodynamics of systems stable at equilibrium (gases). Finally, J. Von Neumann extended the matrix formalism of statistical mechanics to the formalism of Hilbert spaces in quantum mechanics (QM), so to make of the asymptotic condition the core of R. Feynmann's diagram formalism of QFT, interpreted as a "second quantization" with respect to QM.

Let us deepen, therefore, the core of the alternative paradigm of thermal QFT, we illustrated intuitively before [23]. The coalgebraic formalism underlying its modelling, both in physics and in computer science, is able indeed not only to recover all the richness of the semiotic ontology of Poinsot, but to give it the support of the formal rigour of the axiomatic method in logic and mathematics, as we see in the following of this chapter.

2.3. QFT as a thermal field theory and its topological modelling

In fact, historically and scientifically, which are the origins of the QV notion, and why they are so fundamental and unavoidable in QFT? QV is the only possible explanation at the

fundamental level, of the *Third Principle of Thermodynamics* ('The entropy of a system approaches a constant value as the temperature approaches zero'). Indeed, the Nobel Laureate Walter Nernst first discovered in 1912 that for a given mole of matter (namely an ensemble of an Avogadro number of atoms or molecules), for temperatures close to the absolute 0, T_0, the variation of entropy ΔS would become infinite (by dividing by 0).

Nernst demonstrated that, for avoiding this catastrophe, we must suppose the molar heat capacity C is not constant at all, but vanishes, in the limit $T \to 0$, so to make ΔS finite, as it must be. This means, however, that near the absolute 0°K, there is a mismatch between the variation of the body content of energy and the supply of energy from the outside. We can avoid such a paradox, only by supposing that such a mysterious inner supplier of energy is the vacuum. This implies that the absolute 0°K is unreachable. In other terms, there is an unavoidable fluctuation of the elementary constituents of matter. The ontological conclusion for fundamental physics is that we cannot any longer conceive physical bodies as isolated, because *they are always intrinsically 'open' to the thermal fluctuations of QV*. This means that the usage of the asymptotic condition of perturbative methods that by 'Feynman's diagrams' is at the core of the Standard Model (SM) formalism is, in some proper sense, 'falsifying' the intimate thermal nature of the physical reality. In the SM, indeed, the mechanistic prejudice still holds in interpreting *fermions* as 'particles', and *gauge bosons* (photons, gluons, and W and Z bosons), as quanta of the three fundamental force fields (electromagnetic, strong and weak forces, respectively), by which fermions interact among themselves. On the contrary, in thermal QFT, both fermions and gauge bosons must be interpreted as quanta of the relative force fields, of *material* force fields (fermions) and of *interaction* force fields (gauge bosons), respectively.

In this picture, the QV, differently from the mechanical vacuum separating the inert particles of the Newtonian mechanics, emerges as a dynamic continuum of force fields connecting everything in nature, so to justify a topological representation of QFT [20]. This explains also the progressive disaffection of physicists towards perturbative methods when dealing with the many unresolved problems of the 'physics beyond the SM'. Anyway, for this discovery, eliminating the notion of the 'inert isolated bodies' in the mechanical vacuum of the Newtonian mechanics, Walter Nernst is a chemist who is one of the founders of the modern quantum physics.

All this is, indeed, the starting point of S. Umezawa's 'thermo-field dynamics' (TFD), as an alternative interpretation of QFT in quantum thermodynamics, with respect to its statistical mechanics interpretation, because vindicating the primacy of dynamics over kinematics in physics [27, 28]. During the last 20 years, however, by the integration of TFD with the fundamental 'Goldstone Theorem' [29], and then with the infinitely many *spontaneous symmetry breakdowns* (SSB's) of the QV, all compatible with the QV ground state and that are at the basis of the Higgs mechanism, Umezawa's TFD formalism received an essential improvement. This goes into the direction of the topological QFT, because of the 'dynamic rearrangement of symmetries' that each SSB intrinsically implies. Let us see more deeply this essential point [21].

Generally, since when they were theoretically defined by Goldstone's theorem, SSBs have an essential role in the local gauge theory by Higgs field, because to each SSB corresponds the emergence of 'long range quantum correlations (entanglements)' we previously designed as

'phase coherence domains' of the force fields [30]. In fact, in QFT, the 'Stone-Von Neumann Theorem', at the basis of Von Neumann's classical formalism for QM [31], *per se* does not hold. This theorem states that, for systems with a finite number of degrees of freedom, which is always the case in QM, the representations of the canonical commutation relations (CCRs) are all 'unitarily equivalent as to each other'.[4] On the contrary, in QFT, because of the fundamental 'Haag Theorem' (1959), and of the related infinitely many SSB's of the QV [29], the number of the degrees of freedom is not finite, and infinitely many unitarily inequivalent representations of the canonical commutation (bosons) and anti-commutation (fermions) relations exist in the QV. This means that in QFT based on statistical mechanics, the choice of the finite 'orthonormal basis of the Hilbert space', *per se* infinite dimensional, depends on the observer. This led to the never ending epistemological discussions about the 'objectivity' of QM, and of QFT (quantum thermodynamics included) as far as both based on statistical mechanics and of its asymptotic equilibrium condition (the so-called 'KSM-condition') [32].

In TFD sense, on the contrary, 'QFT can be recognized as an intrinsically thermal quantum theory' [21, p. ix], because, for the Third Principle of Thermodynamics, all quantum systems are energetically open to QV fluctuations in the background. Of course, each open QFT system can recover its Hamiltonian character, because of the necessity of anyway satisfying the energy balance condition ($\Delta E = 0$) of each QFT system with its thermal-bath. This can be represented by 'doubling' each state of the Hilbert space with the correspondent 'entangled state' of the thermal-bath, each doubled state representing a given 'phase coherence' emerging from the QV by an SSB.

This is the core of the fundamental principle of the 'doubling of the degrees of freedom'(DDF) of thermal QFT, which is essential also for modelling quantum computing architectures based on DDF as a dynamic 'deep learning' strategy [33]. The essential improvement as to early Umezawa's formalism is that such an openness of a QFT system can be mathematically formalized by the 'algebra doubling', between a q-deformed Hopf coalgebra (thermal-bath) mapping its structure onto its 'dual' q-deformed Hopf algebra (system), where q is a thermal parameter [22], intrinsically related with the 'Bogoliubov transformation', appearing in any process of particle 'creation-annihilation' in quantum thermodynamics. This means that in this topological approach to thermal QFT, the dynamics itself, and not the observer, determines the finite 'orthonormal basis of the doubled Hilbert space' by the DDF principle [20, 33].

To conclude, in QFT, the Heisenberg uncertainty relation of QM between the *statistical* wave and particle representations of a quantum system must be rewritten as relating *dynamically* the uncertainty on the number of the field quanta with the uncertainty on the field phase, namely:

$$\Delta n \Delta \varphi \geq \varphi(\hbar) \tag{1}$$

where n is the number of quanta of the force field, and φ is the field phase. If $\Delta n = 0$, φ is undefined, so that it makes sense to neglect the waveform aspect in favour of the individual,

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[4]We recall that at the basis of QM formalism, there exists the fundamental D. Hilbert's discovery that the canonical variables of classical mechanics (the position q and the momentum p), which *per se* do not commute with each other because of the uncertainty principle, however commute each with the Fourier transform of the other. In this way, each pair of them constitutes a CCR.

particle-like behaviour. On the contrary, if $\Delta\varphi = 0$, n is undefined because an extremely high number of quanta are oscillating together according to a well-defined phase, i.e., within a given phase coherence domain. In this way, it would be nonsensical to describe the phenomenon in terms of individual particles behaviour, since the collective modes of the force field prevail.

In QFT, there is, therefore, a duality between two dynamic entities: the fundamental force field phase and the associated quantum particles that are simply the (fermionic/bosonic) quanta of the associated (material/interaction) field. In such a way, the long-range quantum entanglements associated with SSBs, and determining a 'phase coherence domain' into the QV, do not imply any odd relationship between particles like in QM. Entanglements are simply the expression of the continuous, topological character of force fields, of their phases and of their relations. To sum up, according to such a view, Schrödinger's wave function of QM appears to be only a statistical, observer-related, 'blanket' of a finest structure of the dynamic nature of reality.

Of course, the *probability measures* associated with this dynamic interpretation of QFT are related with the so-called 'Wigner function'. The main difference with Schrödinger wave function is that not only the former, differently from the latter, is defined on the phase space of the system. What is much more fundamental is that the Wigner function uses the notion of *quasi-probability* [34], and not the notion of probability of the classical Kolmogorov axiomatic theory of probability [35]. Indeed, the notion of quasi-probability allows regions integrated under given expectation values *do not represent mutually exclusive states*, because of the reinterpretation of the uncertainty principle, so to violate the first axiom of Kolmogorov's theory. That is, the separation of variables in such distributions is not fixed, but, as it is the rule in the case of phase transitions (think, intuitively, at the phase transition between the gas, the liquid and the solid states in condensed matter), can evolve dynamically, even though, via the DDF principle, our representation can match this evolution, by the associated measure of the *minimum of the free-energy* [20, 33].

2.4. From set theory to category theory as universal language for mathematics

This last evidence introduces us in the necessity of dealing with a change of perspective also in fundamental mathematics and logic. Topological QFT and topological QC are indeed generally based on the 'Algebraic Universality' given that, following an intriguing analogy, in quantum mathematics sets are represented by Hilbert spaces, subsets by Hilbert space sub-algebras, and instead of functions over sets, we have operators over Hilbert spaces [36]. This change of perspectives contributed to the affirmation of Category Theory (CT) as a universal language for mathematics and logic, in a proper sense 'more general' than set theory.

Using CT, indeed, it is possible to discover and to define relationships among different theories that it would be impossible discovering and defining otherwise. In a proper sense, CT is a sort of axiomatic outcome of Peirce's intuition of a purely relational categorical approach to theories, also because of CT native dependence on the algebra of relations, of which Peirce was the pioneer. Let us sketch briefly some notions of CT, using a sort of synthetic handbook of CT, inserted as the introductory paper, explicitly thought for physicists and philosophers [37], in a collection of papers devoted to apply several CT structures, particularly *coalgebras*, in topological QFT, string theories included [38].

The principal difference as to set theory is that in CT the *primitives* are: (1) *morphisms* or *arrows*, *f*, *g*—intended as a generalization of notions such as 'function', 'operator', 'map', etc.; (2) the *compositions of arrows*, $f \circ g$; and (3) two 'mappings', $dom(\bullet)$ and $codom(\bullet)$ assigning a domain and a codomain to each arrow. In this way, even set elements in CT are to be considered as domain-codomains of morphisms, and, more generally any 'object' x in CT corresponds to the domain of a reflexive morphism I_x, i.e., to an 'identity' relationship.

Therefore, any *structure-preserving collection* of 'objects', 'morphisms' and of the two 'mappings' $dom(f)$ and $cod(f)$, assigning to each morphism f its domain-codomain of objects, constitutes a *category* in CT. In this sense, fundamental examples of categories in mathematics are: **Set** (sets and functions), **Grp** (groups and homomorphisms), **Top** (topological spaces and continuous functions), **Pos** (partially ordered sets and monotone functions), **Vect** (vector spaces defined on numerical fields and linear functions), etc. Particularly, the category of **Pos** is fundamental in logic. Indeed, partial ordering is a structure of ordering relations, ≤, among sets satisfying simultaneously:

- $x \leq x$ (Reflexivity).

- $x \leq y \wedge y \leq x \Rightarrow x = y$ (Antisymmetry).

- $x \leq y \wedge y \leq z \Rightarrow x \leq z$ (Transitivity).

The structure of 'total ordering' of sets, and the relative category **Tos** of totally ordered sets, satisfies Antisymmetry and Transitivity, but instead of Reflexivity, it satisfies the ordering property:

- $x \leq y \vee y \leq x$ (Totality).

That is, for all sets, an ordering relation is defined. Nevertheless, the category **Tos** lacks in an 'object' as to **Pos,** because the ordering relation ≤ that **Tos** uses is no longer an object in it, and since does not satisfy any longer reflexivity like in **Pos**. Therefore, **Tos** is a subcategory of **Pos.** In fact, fundamental posets are the real number set ($\mathbb{R}$,≤), and the power set $\mathcal{P}$ of a given set X ($\mathcal{P}(X)$,⊆).

Another fundamental notion of CT is the notion of *functor F*, that is, a 'morphism between categories' sending all the objects, arrows and compositions from a category $\mathcal{C}$ into another $\mathcal{D}$, i.e., $F : \mathcal{C} \rightarrow \mathcal{D}$ so to justify a *homomorphism*—corresponding in set theory to a 'bijective mapping'[5]—between the categories $\mathcal{C}$ and $\mathcal{D}$. Of course, for each category $\mathcal{C}$, there exists an *endofunctor* mapping a category onto itself: $\mathcal{C} \rightarrow \mathcal{C}$. Moreover, the application of the functor can be 'covariant', if it preserves all the objects and the directions of morphisms and the orders of

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[5]For sake of completeness, to an 'injective mapping' in set theory corresponds the functorial mapping of *monomorphism* in CT, and to a 'surjective mapping' an *epimorphism*. What is nice in using CT as a metalanguage of mathematics is that the meta-notion of 'functor' as 'morphism of morphisms' is not *per se* a second order notion, as they are in logic the notion of 'function of functions' or 'class of classes'. That is, by CT, we can have a universal axiomatic metalanguage of logical and mathematical theories that does not imply *per se* any quantification over constants of the object-language, so to avoid all the problems related with the incompleteness of the higher order predicate calculus—even though it is possible to represent in CT higher-order logics. On the contrary, we can formalize in CT logic the logical quantifiers of the predicate calculus like as many *adjoints* of an algebraic structure over the power-set of a given set, so 'to extend to an algebraic form the usual Tarski model-theoretic semantics for first-order logic' [37, p. 45].

compositions between the two categories. On the contrary, the application of a functor G is 'contravariant' if it preserves all the objects, but reversing all the directions of the morphisms (i.e., from $A \to B$, to $GA \leftarrow GB$), and the orders of the compositions (i.e., from $f \circ g$ to $Gg \circ Gf$). In this case, the target category of the functor is the opposite as to the source category. That is, a functor G is contravariant, if $G : \mathcal{C} \to \mathcal{D}^{\mathrm{op}}$.

The notion of 'opposite category', per which a category $\mathcal{C}$ is dual as to its opposite $\mathcal{C}^{\mathrm{op}}$, leads immediately to the fundamental *principle of duality*[6] in logic, according to which a statement S is true in/about $\mathcal{C}$ if and only if its dual S^{op} obtained from S by reversing all the arrows is true in/about $\mathcal{C}^{\mathrm{op}}$. That is, S and S^{op} are 'dually equivalent', i.e., $S \leftrightarrows S^{\mathrm{op}}$, which is different from the ordinary equivalence between statements defined in/about the same category, i.e., $S \leftrightarrow S'$. What is important to emphasize for our aims is that in computational set theoretic semantics, the dual category of $\mathbf{Set^{op}}$ is more significant than $\mathbf{Set}$, given that a generic conditional in logic 'if…then', e.g., 'for all x', if x is a horse then it is a mammalian, is true if and only if the 'mammalian set' dually includes the 'horse set' with all its subsets. Therefore, the semantics of a given statement is set theoretically defined on the power set $\mathcal{P}(X)$ of a given set X. Categorically, indeed, the power set functor $\mathcal{P}$ is a covariant endofunctor $\mathbf{Set} \to \mathbf{Set}$, mapping each set X to its power set $\mathcal{P}(X)$ and sending each function $f : X \to Y$ to the map sending $U \subseteq X$ to its image $f(U) \subseteq Y$, that is:

$$X \mapsto \mathcal{P}(X),\ (f : X \to Y \mapsto \mathcal{P}(f) := S \mapsto \{f(x) \mid x \in S\} \tag{2}$$

vice versa, the contravariant set functor $\mathcal{P}^{\mathrm{op}}: \mathbf{Set^{op}} \to \mathbf{Set}$ sends each function $f : X \to Y$ to the map which sends $V \subseteq Y$ to its inverse image $f^{-1}(V) \subseteq X$, but, of course, preserving all the objects. Therefore,

$$\mathcal{P}^{\mathrm{op}}(X) := \mathcal{P}(X); \mathcal{P}^{\mathrm{op}}(f : X \to Y) \mapsto \mathcal{P}(Y) \to \mathcal{P}(X) := T \{x \in X \mid f(x) \in T\} \tag{3}$$

Moreover, other useful categorical dual constructions can be significantly formalized in CT that we cannot define here, but that have an immediate significance for us because both the topological formalism of quantum physics and of quantum computation are plenty of exemplifications of their usage. For instance, the notions of 'left' and 'right adjoints' of functions and operators, the notions of 'universality' (uniqueness) and 'couniversality', of 'products' and 'coproducts', of 'limits' and 'colimits' interpreted, respectively, as 'final' and 'initial' objects of two categories related by a third category of 'indexing functors', so to grant the mapping, via a 'diagonal functor', of all the objects and morphisms of one category into the other. Practically, all the objects and the operations that are usefully formalized in set theory, and then in calculus and logic—including the 'exponentiation' operation for forming function spaces, and the consequent 'evaluation function' over function domains—can be usefully formalized also in CT, with a significant difference, however. Instead of considering objects and operations for what they 'are' as it is in set theory, in CT we are considering them for what they 'do' [37, p. 53],

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[6]Effectively, the notion of *duality* is well-known in logic, mathematics and physics that are plenty of dual notions. For example, in mathematics, a function and its inverse are dual, just as 'and' and 'or' in logic, according to the De Morgan laws, or a function and its Fourier transpose in physics. See [66] for a survey.

so to fulfil in formal way the primacy of *pragmatics* over syntax and semantics that the semiotic interpretation of logic by Moore borrowed from his teacher Peirce. To conclude, CT significantly completes in an axiomatic way that is absolutely lacking in Peirce's approach—and this constitutes its fundamental weakness—Peirce's 'pragmatic' approach as a research programme for formal logic and mathematics, in terms of a 'formal semiotics'.

2.5. An a'pplication of CT logic to quantum physics and quantum computing

Coming back to topological QFT an QC in the light of CT notions just illustrated, two fundamental categories we met already in Section 2.3, and that can be made dual for the contravariant application of the same functor Ω are those of coalgebras, $\mathcal{A} \to \mathcal{A} \times \mathcal{A}$ and of algebras, $\mathcal{A} \times \mathcal{A} \to \mathcal{A}$, given that often they are not dual at all. This latter is the case, for instance, of fundamental structures in QM like *Hopf algebras*, characterizing all the calculations over any lattice of quantum numbers. A Hopf algebra H is indeed a 'bi-algebra', because it is characterized by two types of operations, i.e., *coproducts* (coalgebra: $H \to H \times H$) and *products* (algebra: $H \times H \to H$), which are both *commuting* and defined over a *field K* with a *K*-linear *map S*: $H \to H$, or *antipode*. This is evidently a *covariant* mapping, i.e., a vectorial covariant mapping sending commuting *coproducts* over commuting *products*, and *counits* ε over *units* η, so that the following diagram of **Figure 2** commutes:
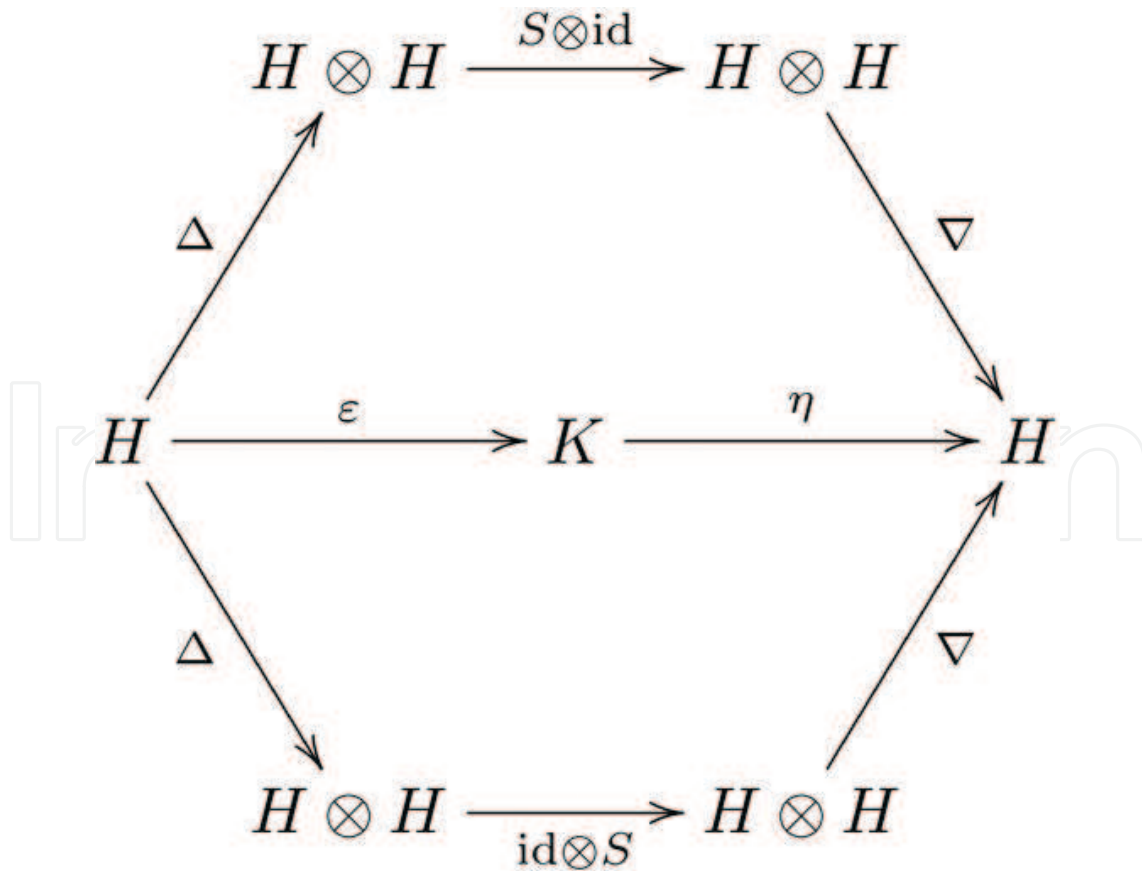
Figure 2. Commuting diagram of a Hopf algebra *H*.

In this way, any Hopf algebra is 'self-dual'—in the sense that the dual of a Hopf algebra is always a Hopf algebra, as expressed by the symmetry of the above diagram—just as any Hilbert space, but also like any Boolean algebra are. Now, the role of a Hopf bialgebra in QM calculations over a lattice of quantum numbers emerges immediately when we consider that the 'algebraic half' of the Hopf bialgebra, $H \times H \to H$, applies when we have to calculate, for instance, the energy of a single particle, whereas the 'coalgebraic half', $H \to H \times H$, applies when we have to calculate the total energy of two particles in the same quantum state (coproducts are effectively sums). In this case, the commutativity also of the coproducts makes perfectly sense, because the total energy does not change by interchanging between themselves the two particles that as such, for the Heisenberg uncertainty, are indistinguishable in a quantum state.

The situation changes completely when we deal with dissipative quantum systems of thermal QFT we discussed in Section 2.3, where the total energy concerns the system state and the thermal-bath state that *are not interchangeable at all*. In other terms, in the *q-deformed Hopf coalgebras*—where the 'deformation parameter' *q* breaking the symmetry of the Hopf bialgebra is a *thermal* parameter—the co-products are *non-commutative,* just as the associated 'doubled Hilbert space', where each state of the system is 'mirrored' by a thermal-bath state (the 'tilde state' in the symbolism below), according to the DDF principle introduced in Section 2.3. The *q*-deformed Hopf coalgebra of a dissipative QFT system, indeed, describes the doubling of the degrees of freedom $a \to \{a \times \tilde{a}\}$ and of the state space $\mathcal{F} \to \mathcal{F} \times \tilde{\mathcal{F}}$ with the operators a and $\tilde{a}$ acting on $\mathcal{F}$ and $\tilde{\mathcal{F}}$, respectively. In this case, in the associated doubled Hilbert space, what are commuting are the associated *operators $A(\theta), \tilde{A}(\theta)$* [33][7]:

$$A(\theta) = A\cosh\theta - \tilde{A}^{\dagger}\sinh\theta \tag{4}$$

$$\tilde{A}(\theta) = \tilde{A}\cosh\theta - A^{\dagger}\sinh\theta \tag{5}$$

where, θ is the 'angle' of a Bogoliubov transformation, strictly related to the deformation parameter q, and the canonical commutation relations are:

$$[A(\theta), A(\theta)^{\dagger}] = 1, [\tilde{A}(\theta), \tilde{A}(\theta)^{\dagger}] = 1 \tag{6}$$

All other commutators being equal to zero. Eqs. (4) and (5) are nothing but the *Bogoliubov transformations* for the $\{A, \tilde{A}\}$ couple, evidently applied in a *reversed* way, characterizing any *phase transition* of a QFT system, that is, any process of 'creation-annihilation of particles' from the QV, according to the relation (1) above. In other terms, the Bogoliubov transformations provide an explicit realization of the *contravariant mapping* between a q-deformed Hopf coalgebra and its dual q-deformed Hopf algebra, where the 'reversal of the arrows' has an immediate physical significance in the correspondent *reversal of the energy arrow* characterizing the energy balance in any dissipative system.

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[7]Effectively, we are working here in the hyperbolic function basis $\{e^{+\theta}, e^{-\theta}\}$, i.e., on bosons, and not on the circular function basis $\{e^{+i\theta}, e^{-i\theta}\}$ corresponding to fermions [20].

On the other hand, since each dissipative QFT system is characterized by a pair of a q-deformed Hopf coalgebra, $qHCoalg$ and a q-deformed Hopf algebra, $qHAlg$, each pair being *univocally characterized* by a different value of the q parameter, it is possible to demonstrate [33] the *dual equivalence* between the category of q-deformed coalgebras, and q-deformed Hopf algebras. In fact, by using in a contravariant way the vectorial mapping on Hilbert spaces related to the Bogoliubov transform T^*, i.e., by using in a contravariant way the endofunctor T characterizing the category **qHCoalg**, we can obtain:

$$\mathbf{qHCoalg}(\mathbf{T}) \leftrightharpoons \mathbf{qHAlg}(\mathbf{T}^*) \tag{7}$$

Now, for computer scientists, in general, the categorical duality coalgebra-algebra, for the contravariant application of the same functor Ω, i.e., **Coalg(Ω)** $\leftrightharpoons$ **Alg(Ω^{op})**, are important, just as the category of **Set^{op}** as to **Set**, discussed before for the logicians. In fact, the primacy of **Coalg** as to **Alg** depends on the fact that in **Coalg,** we are not constrained like in **Alg** by the necessity that all the endofunctors must be polynomials because of the 'Fundamental Theorem of Algebra'. In this way, coalgebras appear to be more suitable for modelling *non-linear systems*, as far as by a functorial contravariant mapping, they might 'induce' their structure onto dually homomorphic algebraic structures, and specifically on *Boolean Algebras*. Let us deepen shortly this point, also for the strict correlation with quantum computing.

Effectively, one of the pillars of topological QC is M. Stone's *representation theorem for Boolean algebras*. This theorem demonstrates the isomorphism between a Boolean algebra (BA) and a partially ordered set defined over Stone's topological spaces [39], i.e., over spaces sharing the same topology of C^*-algebras. That is, the algebras associated with Hilbert spaces in topological QFT, via the famous GNS construction [19]. Stone's representation theorem associates each Boolean algebra A with a Stone topological space $S(A)$, in the sense of the isomorphism existing between a Boolean lattice A and a partial ordering of *clopen sets*[8] over $S(A)$. It is important to emphasize that the category of Boolean algebras **BAlg** and of the Stone spaces, **Stone**, are dual, in the sense that a monotone function from the Boolean algebra A to the Boolean algebra B is dual to a continuous function in the opposite direction from the Stone space $S(B)$ to the Stone space $S(A)$. Afterwards, in 1988, S. Abramsky demonstrated the dual equivalence, for the contravariant application of the so-called 'Vietoris functor', V, of the category of coalgebras defined on Stone spaces, **SCoalg,** and the category **BAlg**, where the 'Vietoris space' is a vector space by which the mapping from one structure to the other can be formally justified, via the so-called 'Vietoris construction' [40]. In this way, we can say that each Stone coalgebra 'induces' its own structure over the corresponding Boolean algebra, i.e., **SCoalg(V)** $\leftrightharpoons$ **BAlg(V^{op})** so that Abramsky's fundamental result is the core of the 'Universal Coalgebra' as general theory of systems [15]. Finally, the clopen sets of **SCoalg** are 'Non-wellfounded' (NWF), that is, they satisfy P. Aczel's non-standard NWF set theory, based on the so-called 'anti-foundation axiom', which refuses the 'regularity axiom' of the standard ZF set theory, so that set *self-inclusion* is allowed, and then *unbounded chains* of set inclusions [41].

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[8]More precisely, the isomorphism is with an *ultrafilter* (or maximal partial ordering of subsets, with the exclusion of the empty-set, of the power-set of a given set, ordered by inclusion) of clopen sets, i.e., of open sets closed by other open sets because defined on intervals of real numbers. For the notion of clopen sets, think, for instance, at the sets associated with points inside a circle, where the only closed sets are the points constituting the circle border, i.e., the circumference.

This means that a 'total ordering' of all sets cannot be justified in this set-theoretic semantics, that is, not all sets are comparable according to the ordering relation (≤). On the other hand, we can always represent in NWF set theory the relationship superset-subset among subsets of a given set by *directed graphs*, where the nodes are subsets, the edges are inclusion relations and the root is the superset. On this basis, P. Aczel demonstrated that in NWF set theory, there exists an ultimate root of all the set directed graphs by his powerful 'Final Coalgebra Theorem' [42]. The final root of all NWF sets is, in fact, like the 'universal class' *V* of the standard set theories, but with a fundamental difference. All the set elements are not actually existing in it, because no total ordering of all sets is here allowed. On the contrary, they can be progressively 'unfolded' from the root as a sequence of sets by the principle of *coinduction* (↓) of set 'reversed orderings' (≥) for a 'final coalgebra', which is obviously *dual* as to the usual *induction* principle (↑) of set 'orderings' (≤) for an 'initial Boolean algebra', but equally effective as method of set definition and proof [15, 43].

All this led J. Rutten to define the principle of *Universal Coalgebra* as dual to 'Universal Algebra', and as general theory of dynamic and computation systems, both modelled as *labelled (indexed) state-transition systems* [15]. This principle allows, indeed, to define the semantics of functional programming on the physical states of the machine, as far as coalgebraically modelled. Moreover, by the construction of the so-called 'infinite state black-box machine', characterized by a 'final coalgebra' for the category of 'diagonal functors' ($\Omega \times \mathcal{I}$), where $\mathcal{I}$ is a set of 'indexes' mapping each coalgebra of a given category onto its final one for the endofunctor Ω [15, 44], it is possible to model computations on (infinite) data streams that, as we recalled at the beginning, is crucial for computer science. In fact, in CT, it is possible to formalize the notion of 'observational equivalence' as dual to the algebraic notion of 'equivalence by congruence' [44]. This notion is evidently important also for quantum physics, given that, because of Heisenberg uncertainty principle, the computations can be performed not on states, per se, but only on some 'state observables', effectively on the operators over Hilbert spaces.

To conclude, it is possible to demonstrate that the category of **q-HCoalg** satisfies the construction of the 'infinite state black-box machine' for modelling in computer science the notion of 'QV-foliation' characterizing the coalgebraic interpretation of thermal QFT illustrated above. Indeed, the set of *q* deformation parameters gives us the set $\mathcal{I}$ of indexes for the endofunctor T characterizing the category **q-HCoalg** [33]. This evidence not only explains why one of the most significant experimental confirmation of thermal QFT is the modelling of 'long-term memories' or 'deep-beliefs' in cognitive neuroscience that we discuss in the concluding section. It also opens the way to use the DDF principle for dealing with the issue of the so-called 'deep-learning' as to infinite streams in quantum computing. Effectively, we are prototyping in Italy, by the nanotechnology labs of the National Research Council (CNR) in Bologna, a specific architecture of optical quantum computer modelled on these principles [33].

3. Conclusions: consequences for the epistemology

What is highly significant in the coalgebraic modelling of dynamic systems, and of their logic I just illustrated, is that in conductive reversed (≥) partial orderings, as far as defined on NWF sets where no total ordering is allowed, instead of using the usual transitive rule in the ordering by inclusion relation: $((x \supseteq y \wedge y \supseteq z) \rightarrow x \supseteq z)$ that as such supposes a 'linear' ordering,

with 'jumps' between ascendants and descendants, and then a *total ordering* (see Section 2.4), we can use, in the 'set unfolding' process by coinduction from the root (superset), the 'weaker' Euclidean transitive rule: $((x \supseteq y \wedge x \supseteq z) \to y \supseteq z)$, or $((x \supseteq y \wedge x \supseteq z) \to z \supseteq y)$.

Moreover, because both $(y \supseteq z)$ and $(z \supseteq y)$ are allowed in inclusions following the Euclidean rule of transitivity, it is possible to generate, by coinduction from the same root, *equivalence classes of sets*, because of the antisymmetric rule characterizing any partial ordering. In fact, because no total ordering holds, it is possible that from one superset, different inclusion paths following the Euclidean rule derive, like in an evolutionary tree by successive non-linear 'bifurcations', so that, as it must be in each partial and not total ordering, some subsets are not comparable by an ordering relation $(\leq)$, even though, ultimately, sharing the same root. They belong indeed to different inclusion paths. In this way, properly, a superset here only 'admits' $(\ni)$ not 'includes' $(\subset)$ subsets. Just it happens in biology where properly the 'mammalian' root 'admits' not 'includes' as its own subsets 'horses', 'cats', 'dolphins', etc., given that they derive from the same root, but following different and reciprocally irreducible 'unfolding' (evolutionary) paths.

Finally, it is evident that in coalgebras defined on NWF-sets, it is possible to formalize also modal logics [45], as Abramsky first emphasized in his visionary contribution of 1988, and then S. Moss demonstrated (see [40, 46], and for updated syntheses [44, 47]). This means that in coalgebraic logic, we can develop a first-order *modal semantics* of Kripke models—as distinguished from second-order modal semantics for Kripke frames—according to the notion of 'local truth' [48], based on the notion of *dual equivalence* $(\leftrightarrows)$ by a *bounded morphism* $\left(\xleftarrow{\rightleftarrows} \right)$, i.e., by a contravariant functorial mapping between Kripke models. What, intuitively, all this means for our aims is that, because modal coalgebras admit only a *stratified (indexed) usage* of the necessity operator $\square$ and of the universal quantifier $\forall$, since a set *actually* exists as far as effectively *unfolded* by a co-inductive procedure, the semantic evaluations in the Boolean logic effectively consist in a *convergence* between an inductive 'constructive' $(\uparrow)$ procedure, and a co-inductive 'unfolding' procedure $(\downarrow)$. Namely, they effectively consist in the superposition *limit/colimit* between two concurrent inductive/coinductive computations. This is the core of Abramsky notion of *finitary objects* as 'limits of finite ones', definable only on NWF sets, finitary objects that according to him are the most proper objects of the mathematical modelling of computations [40]. If we come back to Poinsot's proto-semiotic scheme of **Figure 1** in Section 2.2, we realize that the coalgebraic logic is a formal fulfilment of this intuition at the down of Modern Age.

More generally, indeed, we can apply it not only to mathematical logic, but also to philosophical modal and intensional logics, according to the programme of 'formal philosophy' (formal ontology, formal epistemology, formal ethics, etc.) with evident applications to the computational simulation of intentional tasks, as far as they can be modelled only in intensional logics, and then in computational systems not based on the Turing paradigm, just like also human brains are [49, 50].

Effectively, as far as their fundamental physics obeys a thermal QFT like any biological system, they obey a 'Coalgebraic Universality' principle [15] in their computations [33], just as it holds in human brains during intentional tasks. It is indeed not casual that during the last ten years, the QV-foliation in QFT has been successfully applied to solve dynamically the capacity problem of the long-term memories—namely, the 'deep beliefs' in the computer science

jargon—in the living brain, interpreted as a 'dissipative brain', i.e., 'entangled' with its environment (thermal-bath) via the DDF formalism [24, 51–53].

In a formal ontology based on the semiotic naturalism—that is a coalgebraic modal logic based on thermal QFT—all this, roughly speaking, means that it is *logically* true that the *(sub-)class* of horses is a *member* of the *(super-)class* of mammalians *iff, dually,* it is *ontically* (dynamically) true that a *co-membership* of the *species* of horses to the *genus* of mammalians occurs, from some step n onward of the universe evolution (= 'natural unfolding' of a biological evolution tree). That is:

$$\Box_{\forall n > m} \left(\underbrace{horse \in mammmmalian}_{\text{Boolean Algebra}(\Omega*)} \quad \underset{\xleftarrow{\hspace{1cm}}}{\overset{\rightleftarrows}{}} \quad \underbrace{horse \ni mammalian}_{\text{Stone Coalgebra}(\Omega)} \right) \tag{8}$$

$$\text{Bounded Morphism}$$

In other terms, we are faced here with an example of a 'functorially induced' homomorphism, from a coalgebraic *natural structure* of *natural kinds* (genus/species) into a *logical structure of predicate domains* (class/sub-class), as an example of modal *local truth,* applied to a theory of the ontological natural realism, in the framework of an evolutionary cosmology [23, 54], where it is nonsensical to use not indexed (absolute) modal operators and quantifiers, given that physical laws emergence depends on the universe evolution. In parenthesis, this gives also a solution to the otherwise unsolved problem, in Kripke's relational semantics, of the denotation of *natural kinds* (the denoted objects of common names, such as 'horses' or 'mammalians' in our example) and of the connected *causal theory of reference,* but on a 'naturalistic' basis, and not 'social' one, like in S. Kripke's [55], and I. Putnam's [56] theories.

To conclude, such a formal ontology of the natural realism of which I illustrated here only some basic principles and that I present systematically in a book actually in preparation [57] is able to give Post-Modern Age an ontology including both conscious (humans) and unconscious (computers) communication agents as the main actors of our Information Age, as I stressed in Section 1 of this chapter.

Therefore, for a final illustration between the modern 'Transcendental of Knowledge' and the post-modern 'Transcendental of Language', according to the 'complete linguistic turn' of Peirce semiotics, I introduce as an exemplification the comparison between Husserl's criticism, and Peirce's criticism to Schröder's first volume of his book on the 'algebra of logic' [58], which was the first historical proposal of a 'mathematical logic'. Peirce's contribution consists, indeed, in the proposal of an 'algebra of relations', correcting in a semiotic/semantic way the early formalistic proposal of an 'algebra of logic' by Ernst Schröder, without any necessary reference to a knowing, conscious subject [6], and consistent with his ontology of a 'semiotic naturalism' [9]. Edmund Husserl also shared this same criticism against Schröder formalism, almost in the same years, but independently from Peirce's semiotics. In fact, Husserl criticized Schröder formalistic approach to algebraic logic from the standpoint of the Transcendental of Knowledge [59], i.e., according to the subject-object intentional relationship, proper of phenomenological foundation of formal logic and mathematics [60].

In other terms, following Poinsot's early suggestion, updated to the actual situation, we can say that the proper of humans as conscious communication agents in our Information Age is the

abstract, because self-conscious knowledge of logical and mathematical truths, requiring for its formalization a second order set-theoretical semantics. This applies *globally* to an *infinite* universe of logical and mathematical *objects,* apart from any 'morphism' we can define on them, constituting the universal class *V* of a given formal system. *V,* therefore, defines the 'universe' of the objects with which a given axiomatic system is dealing with, i.e., the abstract objects 'formally existing' in the system. Now, 'necessary and sufficient condition' for the membership to *V* is that all its members satisfy *a self-identity relationship,*[9] a condition stated for the first time, in the history of Western thought, in Plato's Dialogue *Parmenides,* the dialogue in which Plato's metaphysics reaches its most consistent development.

The core of the modern transcendentalism, consists therefore, from Descartes and Kant on, Husserl included, in identifying 'self-identity' with 'self-evidence', so to justify in the usual logical jargon, the denotation of the members of *V* as 'objects' (as-to-a-subject) constituting the 'universe' of a given axiomatic system. Therefore, if we compare the definition of the *standard* notion of sets, just at the beginning of Fraenkel's *Abstract Set Theory* book and Husserl's parallel passage about the formal ontology of *independent objects* as 'parts of wholes' in his *Logical Investigations,* the common dependence on a Platonic ontology is a notion shared by both authors. For Fraenkel, indeed—who always affirmed the necessity for pure mathematicians of embracing a Platonic ontology [61]—both the 'intuition of objects' and 'collecting objects into an aggregate' are 'intellectual acts' [62, p. 6]. Let us compare the following two passages, respectively, of Husserl and of Fraenkel, for realizing this key point of the whole question of the modern transcendentalism in the foundations of logic and mathematics. Husserl:

> Seen in their mutual interrelations, contents presented together on any occasion fall into two main classes: independent and non-independent contents. We have independent contents wherever the elements of a presentational complex (complex of contents) by their very nature permit their separated presentation; we have dependent contents [i.e., "wholes"] wherever this is not the case[63, p. 6].

Fraenkel:

> Definition of set. A set or aggregate is a collection of definite, distinct objects of our intuition or of our intellect, to be conceived as a whole (unity)[62].

Where the two approaches diverge, it is about the different logical value to be attributed to *evidence.* For Husserl, such 'objects' of a logical system, and the relative 'axioms', as far as self-

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[9]'*V* is, by definition, the class of all those elements which are self-identical; i.e., since everything is self-identical, *V* is simply the class of all elements' [67, p. 144].

evident, are *apodictic* or 'absolute'. On the contrary, contemporary mathematics does not trust in 'evidence' because dependent on historical factors. In this way, modern mathematics, by embracing from B. Riemann on, the *axiomatic method,* can attribute only a *hypothetical* value to logical and mathematical truths, because 'relative' to the limited universe of objects defined through the axioms of a given theory. The role of *evidence* then remains only as to the *logical primitives,* on which the *meaning* of all the well-formed formulas—axioms, definitions and theorems, via the formation and deduction rules—ultimately depends in any formal system, as K. Gödel himself emphasized many times in his writings.

Today, however, this is not the full story. Besides the abstract way of humans of dealing with logical and mathematical truths, there exists also the way of the *unconscious communication agents* of dealing with *local truths.* They are based on a coalgebraic first-order semantics for Boolean algebras, for defining the representation space of these agents, and its continuous re-adaptation on the hidden correlations of data streams. As we anticipated in the Introduction, and now it is (I hope) more evident to us, this new generation of computational systems is not engaged in any 'imitation game' with our conscious *minds*. In the limit, indeed, they are imitating at last the *computational dynamics* of our pre-conscious *brains.* For this reason, they do not suffer the computational limits of second-order logics, and then of the 'Universal Turing Machine'. Therefore, they might support, integrate and only in this sense, substitute our minds —and the classical Turing-like computational architectures—in all these tasks, where they— our minds and the classical computers, as far as simple extensions of our minds—are destined inevitably to fail. These tasks are, indeed, ultimately reducible to only one: *reckoning with the complexity of reality,* in whichever natural, social, economic realm, without any unsustainable waste of time, and of computational resources. This is the challenge, but also the hope of our present time, as far as we become aware of it. We are humans, at last!

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