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Lagrangian Model-Based Fault Diagnosis in a PVTOL

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Abstract

A Lagrangian formalism is used to model a PVTOL in order to obtain an aircraft model. The Euler-Lagrange model of the PVTOL is used to develop an algorithm for fault diagnosis. Diagnosis implies the detection, isolation and identification of a fault. The considered approach is based on the knowledge of a system model as well as the model of the possible faults. The idea is to use non-linear decoupling approach to derivate a set of subsystems, each related to a specific fault or a set of faults. An observer-based residual generation is designed for each subsystem, this structure allows the fault detection and isolation stage, for fault identification a kind of approximated inversion algorithm to meet the different diagnostic levels. The results are obtained taking advantage of the structure given by the Euler-Lagrange modelling of the PVTOL as well as from recent results related to observer design and fault identification.

Keywords: fault diagnosis, Lagrangian systems, non-linear systems, observers, fault isolation

1. Introduction

Nowadays, the unmanned aerial vehicles (UAVs) represent a big boom in the electronic industry, thanks to their versatility and largely due to the falling cost of the electronic parts and the UAV by itself. UAV is a kind of an aerial vehicle that is able to take off vertically, such as helicopters and some special airplanes, and it is represented by the planar vertical take-off landing (PVTOL) aircraft model. Note that PVTOL aircraft models represent more than only UAV systems. Reliability requirements in aerial vehicles bring the necessity of a fault detection and isolation schema. In general, they are non-linear systems, and so a non-linear inspired strategy for the detection and

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isolation of faults could also be used. An idea consists in taking advantage of the structure given by the Lagrangian model of a PVTOL in order to develop an algorithm for the detection, isolation and identification of faults.

Many research studies dealing with the fault detection and isolation (FDI) problem have been already published, most of them deals with linear systems, see for instance Refs. [1–3]. On the other hand, for non-linear systems, some solutions exist, based on the inherited characteristics, see Refs. [4, 5] for more details. The most common approach used for FDI is the hardware redundancy; however, this approach normally represents an increment in weight and economical cost of the aircraft. In order to avoid this problem, some mathematical relations could be used, the simplest way is to compare two or more internal signals, having as goal to create a residue, which, in fact will be zero if the system is working normally and different from zero if not. In order to create such relations it is common to exploit some intrinsic characteristics of the systems. See for instance Ref. [6]. Diagnosis for the PVTOL system has been considered previously using a Hamiltonian formalism [7].

A Lagrangian formalism is used to model a PVTOL in order to obtain an aircraft model. The Euler-Lagrange model of the PVTOL is used to develop an algorithm for fault diagnosis. Diagnosis implies the detection, isolation and identification of a fault. The considered approach is based on the knowledge of a system model as well as the model of the possible faults. The idea is to use non-linear decoupling approach to derivate a set of subsystems, each related to a specific fault or a set of faults. An observer-based residual generation is designed for each subsystem. Detection and isolation of faults can be reached at this stage, for fault identification a kind of approximated inversion algorithm to meet the different diagnostic levels. The results are obtained taking advantage of the structure given by the Euler-Lagrange modelling of the PVTOL as well as from recent results related to observer design and fault identification.

Fault diagnosis algorithms can be developed for a more or less general Euler-Lagrange model of a system, which, in fact, also include a PVTOL system. Fault diagnosis includes detection, isolation and identification of faults. In order to meet a diagnosis task, an observer-based residual generator is designed in order to determine whether a fault is present. A decoupling approach is used in order to guarantee also a fault isolation task. As discussed, both steps could be systematically developed for the considered system model. Further, fault isolation is approached using a kind of approximated system inversion to develop approximated fault estimation through dynamic inversion of the corresponding residual equation. The schema is shown using a specific example of a PVTOL. As presented in the results, the proposed approach can be used effectively for the diagnosis of a PVTOL system.

2. Lagrangian modelling

There is a huge amount of literature about Lagrange's equations of movement, however, see for example Ref. [8]. The structure of a *Planar-Vertical-Take-Off* and *Landing* (PVTOL) is represented in **Figure 1**.

The absolute linear position to the PVTOL is defined in the inertial frame $x - y - z$ axes with two generalized coordinates $\xi^T \overset{\Delta}{=} [y \quad z]$. One additional generalized coordinate, the angular position,

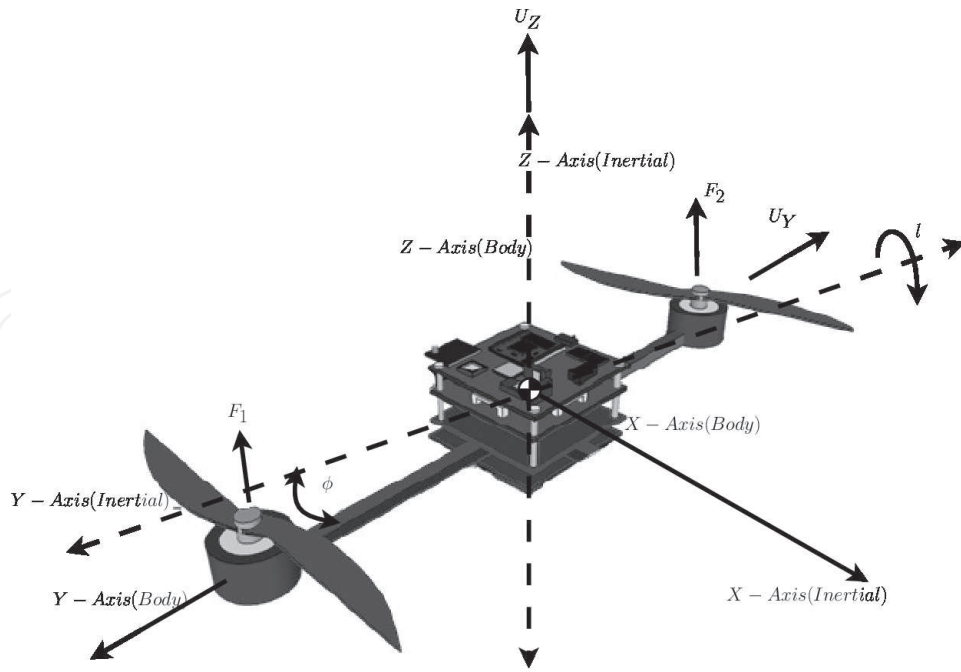**Figure 1.** Schema of the PVTOL system.

is defined in the inertial frame. Note that the pitch angle θ, i.e. the rotation angle around the y-axis, and yaw angle Ψ, i.e. the rotation of the PVTOL around the z-axis, are zero. The only angular movement is the roll angle ϕ, i.e. the rotation around the x-axis.

$$\xi = \begin{bmatrix} y \\ z \end{bmatrix}, \quad \eta = \phi, \quad q = \begin{bmatrix} y \\ z \\ \phi \end{bmatrix} \tag{1}$$

The origin of the body frame (also the origin of the inertial frame) is the centre of mass of the PVTOL system. The PVTOL is assumed to have a symmetric structure with the two arms aligned with the body x-axis. The inertia is represented by J_x.

The Lagrangian is defined as the sum of kinetic energy minus the potential energy (E_{pot}). In the case of the PVTOL, the kinetic energy consist of two parts, one related to the translational energy (E_{tran}) and the second related to the rotational energy (E_{rot}).

$$\mathcal{L}(q,\dot{q}) = E_{tran} + E_{rot} - E_{pot} \tag{2}$$

The movement equations of Lagrange are given by

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}}\right] - \frac{\partial \mathcal{L}(q,\dot{q})}{\partial q} = \begin{bmatrix} f_y \\ f_z \\ \ell \end{bmatrix} \tag{3}$$

where f_y represent the generalized forces on the y-axis, f_z represent the generalized forces on the z-axis and ℓ is the torque.

For the PVTOL results:

$$E_{\text{tran}} = \frac{1}{2}m[\dot{y} \quad \dot{z}]\begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} \tag{4}$$

$$E_{\text{rot}} = \frac{1}{2}J_x\omega^2 = \frac{1}{2}J_x\dot{\phi}^2 \tag{5}$$

$$E_{\text{pot}} = mgz \tag{6}$$

So that the Lagrangian results

$$\mathcal{L}(q,\dot{q}) = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\dot{z}^2 + \frac{1}{2}J_x\dot{\phi}^2 - mgz \tag{7}$$

and the terms

$$\frac{\partial \mathcal{L}}{\partial \dot{q}} = \begin{bmatrix} m\dot{y} \\ m\dot{z} \\ J_x\dot{\phi} \end{bmatrix} \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \begin{bmatrix} m\ddot{y} \\ m\ddot{z} \\ J_x\dot{\omega} \end{bmatrix} = \begin{bmatrix} m\ddot{y} \\ m\ddot{z} \\ J_x\ddot{\phi} \end{bmatrix} \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial q} = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix} \tag{10}$$

The generalised forces (in the inertial frame) are given by

$$f_y = \cos(\phi)U_y - \sin(\phi)U_z \tag{11}$$

$$f_z = \sin(\phi)U_y + \cos(\phi)U_z \tag{12}$$

where U_z represents the total thrust force (the sum of the forces of each rotor), acting on the z-axis of the body frame. U_y corresponds to the side forces on the y-axis of the body frame. The moment acting on the rolling angle is given by ℓ.

The movement equations are given by

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_x \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} U_y \\ U_z \\ \ell \end{bmatrix} \tag{13}$$

3. Diagnosis approach

Fault diagnosis aim to detect the fault occurrence in the functional units of the system, as well as to classify the different faults and to determine the type, magnitude and cause of faults, which leads to undesired behaviour of the whole system. The fault diagnosis can be achieved by

hardware redundancy or software redundancy also called analytical redundancy. One technique of fault diagnosis is the model-based fault diagnosis, which employs software redundancy.

In the model-based fault diagnosis technique, the system behaviour is online reconstructed by a mathematical model, which is implemented in the software form. In this scheme, the system model run in parallel to the system and both of them are driven by the same control inputs. Thus, in the fault-free case, reconstructed system variables by the system model follow the corresponding real system variables and show a derivation in the faulty case.

A comparison of the measured system variables with their estimates by the system model is called residual. Thus, a residual signal includes the fault effect, and ideally if the residual signal is different from zero, then a fault has occurred otherwise the system is fault free. The residual generation process is carried out in two stages, first, the system outputs have to be estimated, then, the difference between those signals and the signal coming from sensors is computed [9].

Figure 2 shows the general scheme for residual generation using a model-based fault diagnosis technique.

In this contribution, a fault diagnosis for systems with model Euler-Lagrange is presented. A model-based fault diagnosis technique with analytical redundancy is used to obtain a residual generation.

Consider a dynamic system without faults described by the following Euler-Lagrange equations

$$\frac{d}{dt}\left[\frac{\partial L(q,\dot{q})}{\partial \dot{q}}\right] - \frac{\partial L(q,\dot{q})}{\partial q} = \tau,$$
$$y_o = q, \tag{14}$$

where $\tau \in \mathbf{R}^n$ is the vector of generalized forces, $q \in \mathbf{R}^n$ is the vector of generalized coordinates, L is the Lagrangian and y is the vector output.

In this work, additive faults in control input and sensor are considered. The Euler-Lagrange model of the faulty system is defined as
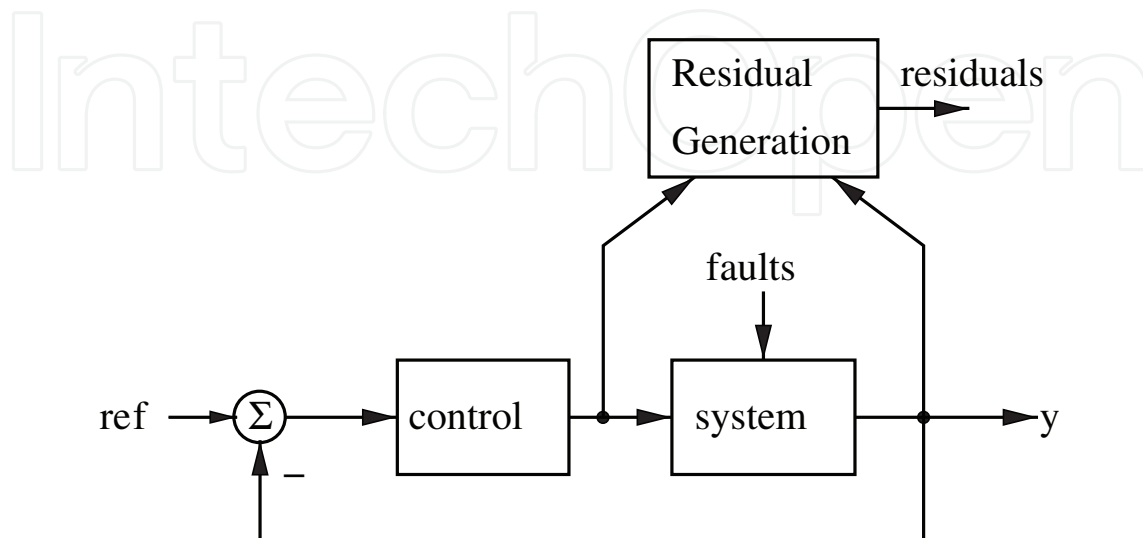

Figure 2. General scheme for residual generation.

$$\frac{d}{dt}\left[\frac{\partial L(q,\dot{q})}{\partial \dot{q}}\right] - \frac{\partial L(q,\dot{q})}{\partial q} = (\tau + QF_a),$$

$$y_f = (q + NF_s),$$

(15)

where $F_a \in \mathbf{R}^n$ is the vector of control input faults, $Q \in \mathbf{R}^{n \times n}$ is a constant matrix, $F_s \in \mathbf{R}^n$ is the vector of sensor faults and $N \in \mathbf{R}^{n \times n}$ is a constant matrix.

Assumption 1. *Consider an Euler-Lagrange system with faults described by Eq. (15) and the system behaviour is on line reconstructed by the Euler-Lagrange system without faults Eq. (14), then the faults presented in the system Eq. (15) can be detected by the residual generator*

$$r(t) = y_f(t) - y_o(t)$$

(16)

4. Application results

The method presented in the previous section is applied in a PVTOL. Only additive faults are taken into account, the faults could affect sensors (y, z and ϕ) and the control inputs (u_z, u_y and ℓ), the faulty case is restricted to one fault at a time, meaning that it is assured that if a fault appears, it is impossible that another fault occurs. Once the fault has occurred, it still presents until the end of the simulation.

The faulty PVTOL system is defined as

$$\begin{bmatrix} m\cos(\phi) & m\sin(\phi) & 0 \\ -m\sin(\phi) & m\cos(\phi) & 0 \\ 0 & 0 & J_x \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} mg\sin(\phi) \\ mg\cos(\phi) \\ 0 \end{bmatrix} = \begin{bmatrix} U_y \\ U_z \\ \ell \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{uy} \\ f_{uz} \\ f_{\ell} \end{bmatrix},$$

(17)

$$y_o = \begin{bmatrix} y \\ z \\ \phi \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{sy} \\ f_{sz} \\ f_{s\phi} \end{bmatrix},$$

(18)

where f_{uy} is the fault in the control input U_y, f_{uz} is the fault in the control input U_z, f_{ℓ} is the fault in the control input ℓ, f_{sz} is the fault in the sensor of position in the vertical movement, f_{sy} is the fault in the sensor of position in the horizontal movement and $f_{s\phi}$ is the fault in the sensor of angle ϕ.

Among the different detection methods available in the literature [2], the threshold-based approach is one of the most common thanks to its simplicity and accuracy. The principle of this approach is based on the idea that the parameters of the system (e.g. mass and dimensions) could vary because of the measurement or estimation errors. Those data are used to determine a threshold, which is computed by varying the internal parameters of the system in a certain ± percentage. This is carried out in order to avoid false alarms caused by the difference between the mathematical model and the real system.

The internal parameters of the PVTOL that may vary are the mass (m) and the inertia (J_x), in order to cover the worst case scenario, both parameters vary at the same time $+10$ and -10%. As a result, the thresholds are fixed as depicted in **Figure 3**.
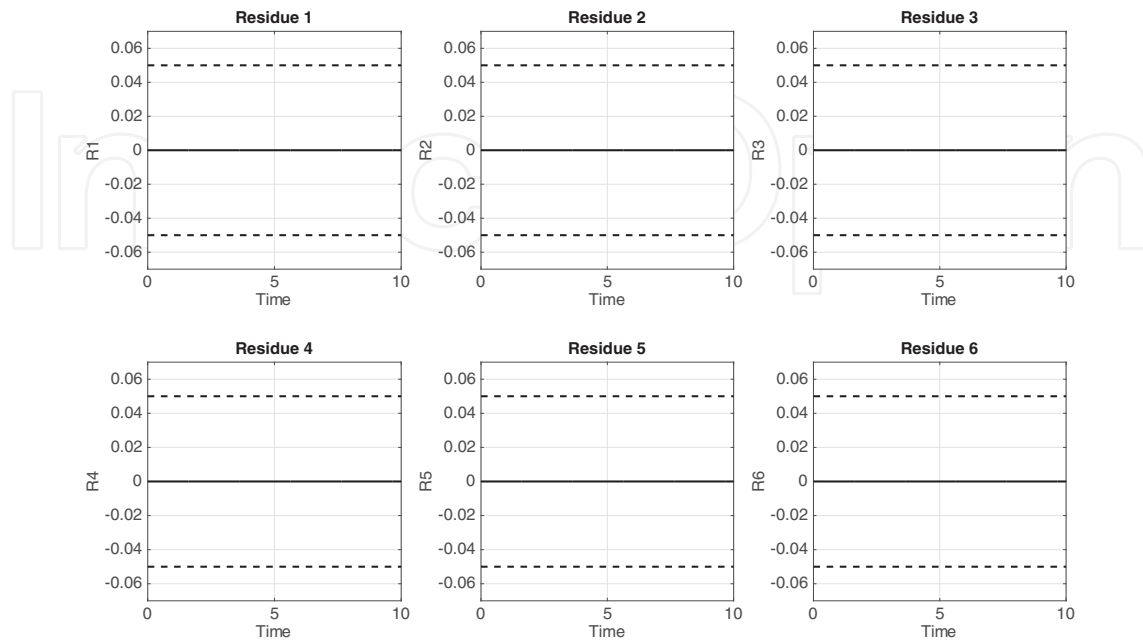

Figure 3. Amplitudes of the detection thresholds. - -, detection threshold; —, residues.

As explained in the beginning of this section only additive faults are taken into account, since the controller is designed to stabilize the system in hover flight, the fault amplitude is defined as a percentage of the initial value for sensors and a percentage of the maximum amplitude of the input control. This percentage is fixed $\pm 10\%$ for sensors and $\pm 5\%$ for control inputs. The faults are triggered 7 seconds after the beginning of the simulation and it is persistent until the end.

In order to detect the fault, six different residues are computed, for this, it is assumed that the entire state is available, according to the previous section as follows:

$$R_1 = y_f - y \tag{19}$$

$$R_2 = z_f - z \tag{20}$$

$$R_3 = \phi_f - \phi \tag{21}$$

$$R_4 = \dot{y}_f - \dot{y} \tag{22}$$

$$R_5 = \dot{z}_f - \dot{z} \tag{23}$$

$$R_6 = \dot{\phi}_f - \dot{\phi} \tag{24}$$

where the suffix $_f$ denotes the signal coming from the sensor.

4.1. Sensor faults

The sensor faults considered in this work affect the vertical measurement (z), the horizontal (y) and the inclination angle (ϕ), as explained before the amplitudes of the faults are 0.2 m, 0.1 m and 0.2°, respectively. The controller is designed to decouple the sensor faults, as a result each fault is independent of the others and by consequence all of them are detectable and isolable, thanks to their different fault signatures. **Figures 4–6** depict the sensor faults.
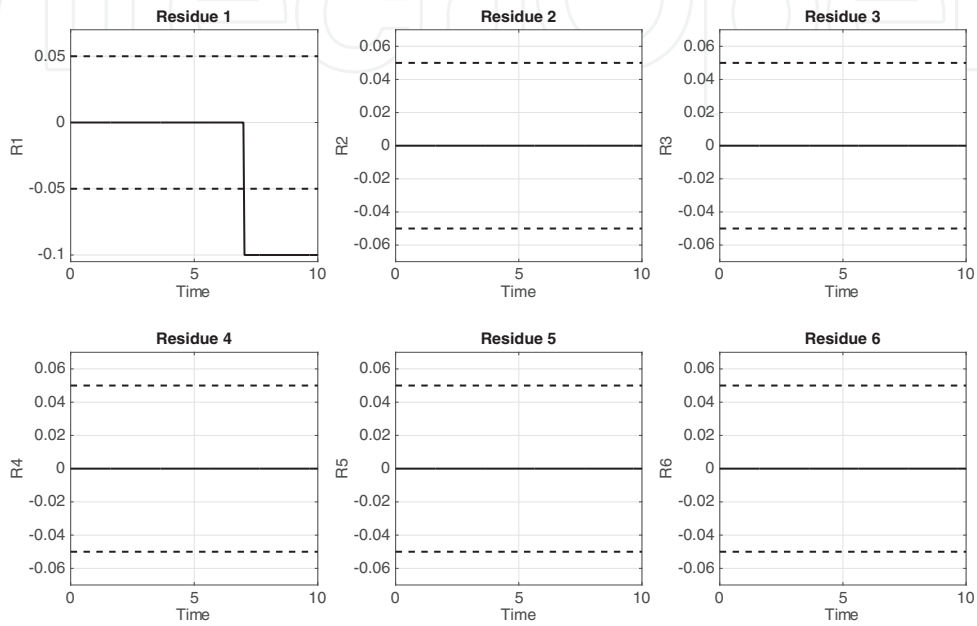

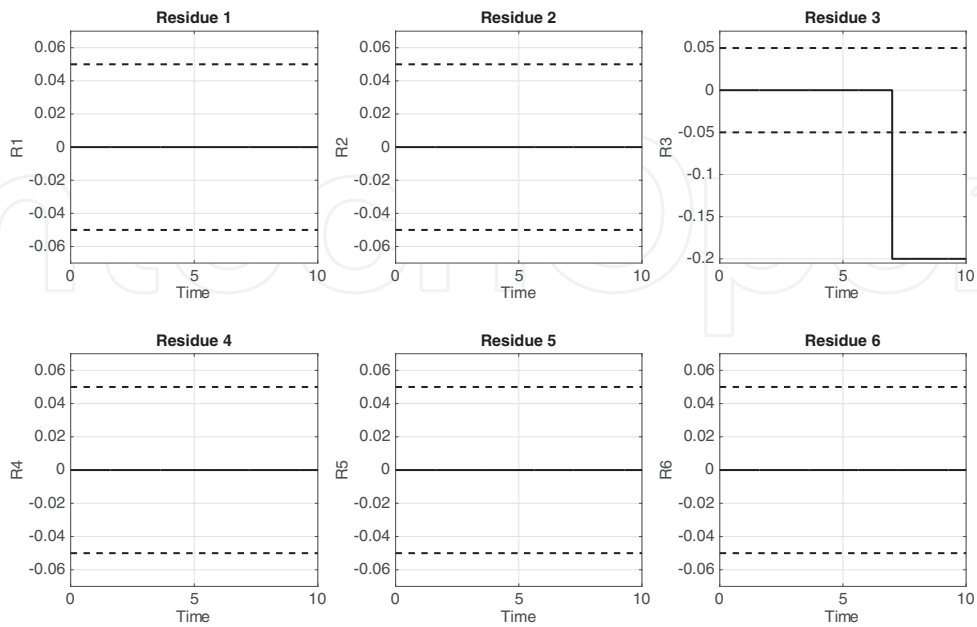**Figure 4.** Fault affecting y sensor. - -, detection threshold; —, residues.

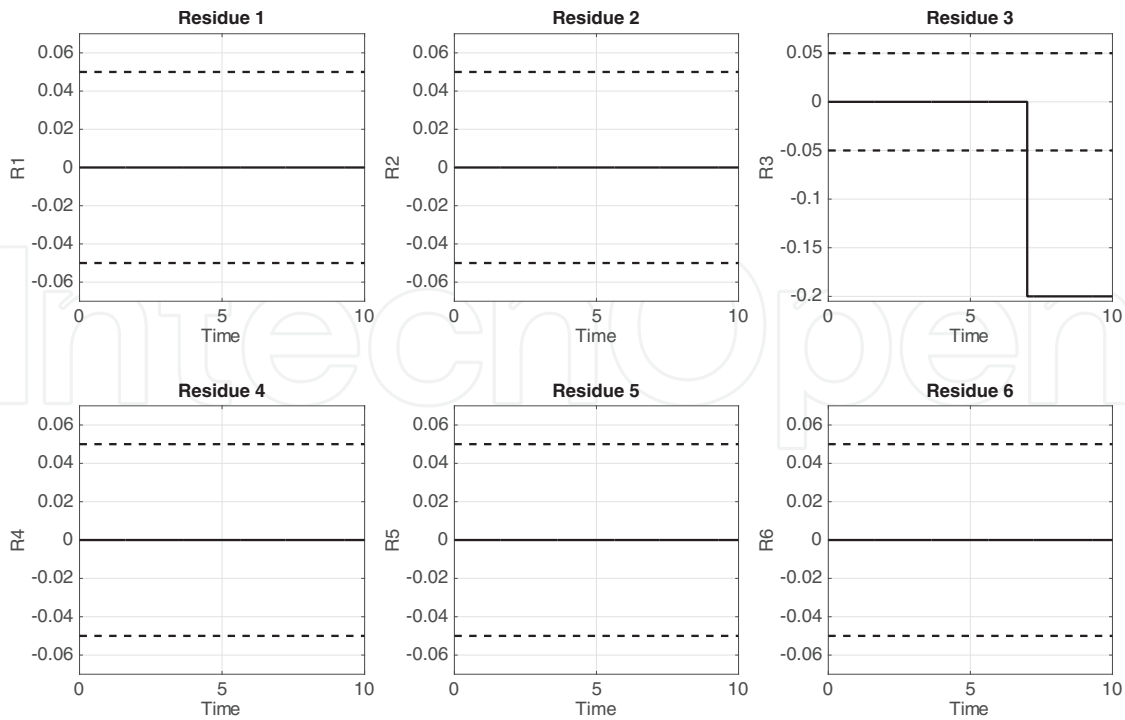**Figure 5.** Fault affecting z sensor. - -, detection threshold; —, residues.

Figure 6. Fault affecting ϕ sensor. - -, detection threshold; —, residues.

Once the residue exceeds the detection threshold, the fault is considered detected and it will be isolable if and only if the fault signature is different among the others. As expected, and thanks to the controller design every fault affecting the sensors is isolable. The fault signatures are presented in **Table 1**. *X* means that the residue exceeds the threshold; *O* means that even if the residue is affected, it does not surpass the threshold and by consequence this residue is not triggered.

Fault	R1	R2	R3	R4	R5	R6
Sensor y	X	O	O	O	O	O
Sensor z	O	X	O	O	O	O
Sensor ϕ	O	O	X	O	O	O

Table 1. Fault signatures of sensors.

4.2. Control inputs faults

The fault amplitudes of the control inputs are fixed by obtaining the 5% of the maximum size of them during an unfaulty simulation, after this processes the amplitudes are fixed to 0.5, 0.425 and 0.008 for u_y, u_z and ℓ, respectively. Thanks to the controller design, the faults affecting the control inputs are detectable and isolable. **Figures 7–9** shows that the detection threshold is exceeded once a fault occurs, by consequence, fault detection is accomplished. **Table 2** depicts the fault signatures, it is straightforward to see that they are all different, this behavior confirms that every single fault is detected and isolated.
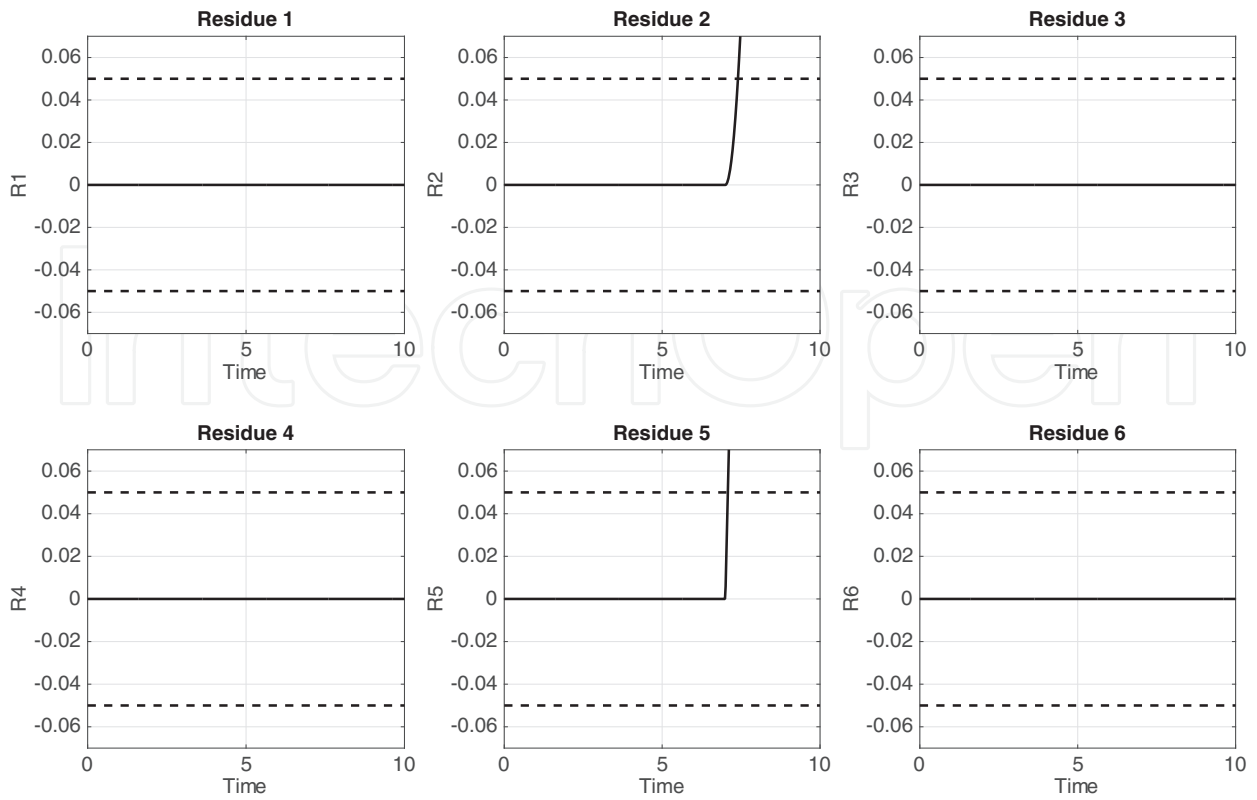
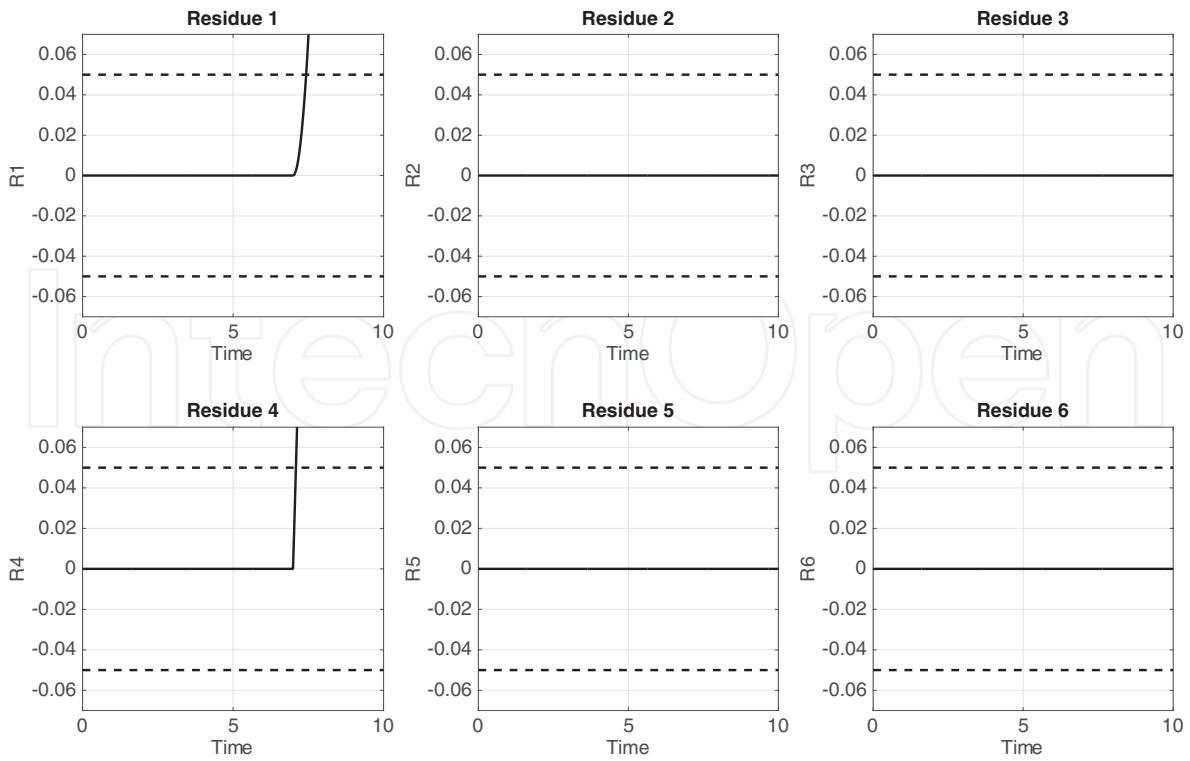**Figure 7.** Fault affecting u_z. - -, detection threshold; —, residues.

Figure 8. Fault affecting u_y. - -, detection threshold; —, residues.
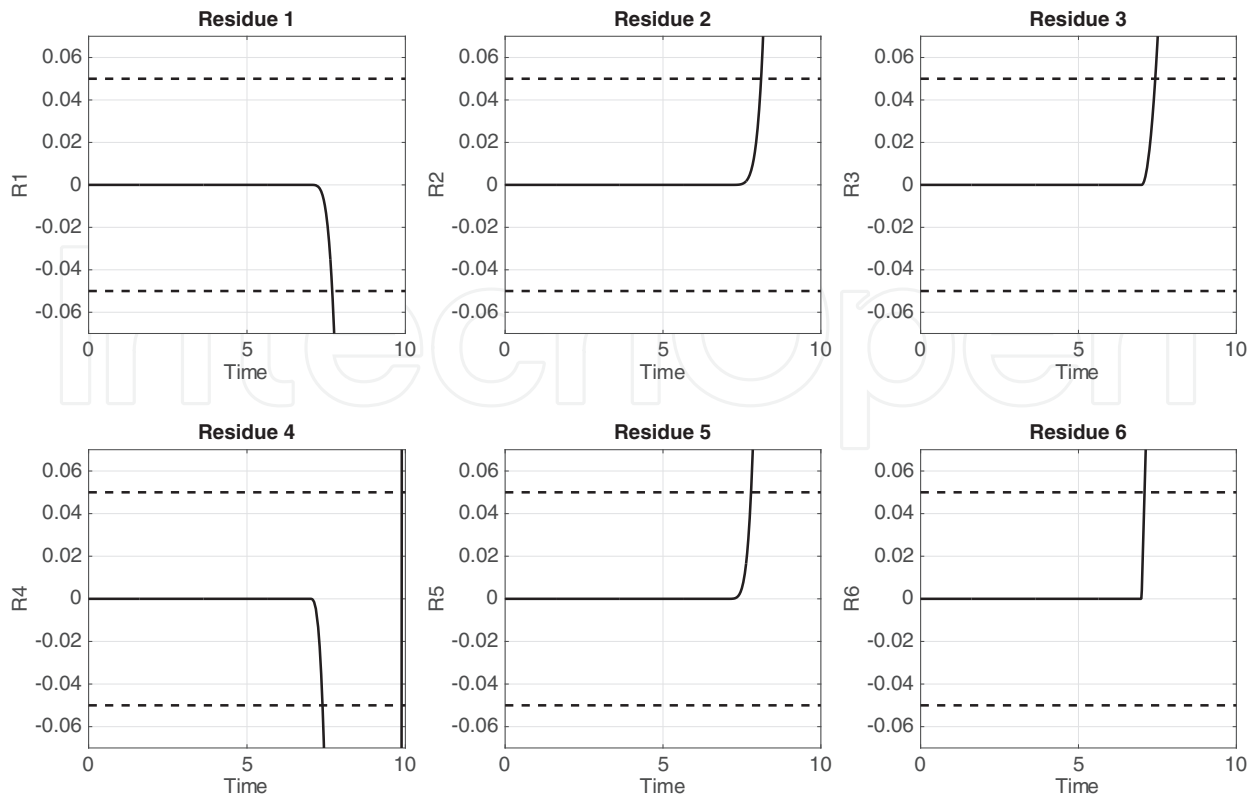
Figure 9. Fault affecting ℓ. - -, detection threshold; —, residues.

Fault	R1	R2	R3	R4	R5	R6
Control input u_z	O	X	O	O	X	O
Control input u_y	X	O	O	X	O	O
Control input ℓ	X	X	X	X	X	X

Table 2. Fault signatures of control inputs.

5. Conclusion

This work presents a fault detection and isolation approach applied to Lagrangian systems.

Every fault is detectable and isolable as it can be seen in **Tables 1** and **2**, this result is obtained thanks to the special design of the state feedback controller, by consequence the faults affecting the sensors are easily isolable, and the residues affected during a control input fault are those related to the measures affecting the states, for instance a fault affecting u_y triggers the residues related to the y and $\dot{y}$ measures; on the other hand, the faults affecting the control input ℓ trigger all the residues, this is because the direct relation between this control input and the ϕ measure, as could be seen in Eq. (13) besides the ϕ measure appears in the other states, as a result every residue is triggered; however, this signature is unique, by consequence this fault is considered detected and isolated.

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