Chapter 5

A Multi-Period Synthesis Approach to Designing Flexible Heat-Exchanger Networks

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Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/66694

Abstract

This chapter presents a new synthesis method for designing flexible heat-exchanger networks. The methodology used involves a two-step approach: In the first step, a multi-period network is designed for a large number of critical operating periods using a finite set of operating points which lie within the uncertain parameter range, while considering the impact of potential fluctuations in periodic durations of each of the chosen critical points on the network. In the second step, the flexibility of the resulting multi-period network of the first step is tested using very large, randomly generated set of finite potential operating points together with their periodic durations. The key criteria used in determining the finite set of operating points that would participate in the initial multi-period network synthesis of the first step are the nominal operating points, the extreme operating points in terms of heat-load requirements as well as their length of periods. This implies that the resulting flexible network can feasibly transfer heat irrespective of possible fluctuations in periodic durations for any of the potential process-operating points. The solutions obtained using the new approach compare favourably with those in the literature.

Keywords: heat-exchanger networks, multi-period, synthesis, flexible networks, mathematical programming

1. Introduction

An efficiently designed heat-exchanger network (HEN) can be used to achieve significant reductions in energy consumption and pollutant emission into the environment by chemical plants. However, most design methods that have been presented for the synthesis of HENs have assumed fixed process-operating parameters. However, in reality, process parameters may fluctuate around some nominal operating points due to various factors such as changes in environmental conditions, plant start-ups/shutdowns, changes in product quality demand, and so on. In some other cases, the process parameters may deliberately be moved away from their set point/nominal conditions due to reasons such as planned transition from one product quality to another. For these cases, even though the set point tracking of the process parameters is required in ensuring a smooth transition to the new set of operating points, the network of heat exchangers still have to be flexible to handle these new set of operating conditions. Despite the fact that these new set of operating points lie within the possible range of variability of the process parameters, their length of duration needs to be taken into consideration while designing a flexible heat-exchanger network.

The methods that have been used for the synthesis of flexible heat-exchanger networks have been both sequential [1–3] and simultaneous in nature [4–16]. Some of the sequential methods are an automated multi-period version of the mixed integer linear programme (MILP) transshipment model [17] and the non-linear programme (NLP) minimum investment network cost model [18]. The simultaneous methods have mostly been based on a multi-period version of the simplified stage-wise superstructure (SWS) model [19]. Some of the existing design methods for flexible HENs may only be feasible to transfer heat for a finite set of process-operating parameters for which the network is designed in what is known as multi-period networks [4–8]. In the multi-period networks [4–8], each period of operation is distinct in that the process parameters, as well as the length of periods for each of the periods of operations, are known upfront. Since multi-period networks are capable of transferring heat within the specified finite set of operating periods, they can be termed flexible networks. However, their degree of flexibility may only be limited to the set of finite operating points for which the network is designed. The degree of flexibility of the SWS-based multi-period networks has been improved through the use of the timesharing mechanism [9], where heat exchangers may be shared by different stream pairs in more than one period of operation. This is unlike other SWS-based multi-period models where either the average area [4] or the maximum area [5–7] of the same pair of streams exchanging heat in the same stage of the superstructure, and in different periods of operations, is used as the representative heat-exchanger area in the objective function.

According to Jiang and Chang [9], a major shortcoming of the average area or maximum area approach, as presented in the literature, is that an exchanger may be overdesigned for some periods of operations such that when unforeseen changes in period durations occur, the multi-period network may not be feasible to transfer heat any longer, or it may require a significantly higher utility flows. Even though the timesharing approach overcomes some of the aforementioned shortcomings of the other SWS-based methods [4–7], the complexities involved in having to thoroughly clean exchangers during the process of exchanger swapping can be enormous. Furthermore, additional costs and complex controllability issues will be incurred due to excessive piping and associated instrumentations. Hence, some other synthesis methods that result in networks that have a greater degree of flexibility have been presented in the literature [10–15]. One of the methods entails [10] a three-step approach where a network is designed based on a finite set of operating points in the first step. In the second step, the resulting network from the first step is tested for flexibility using the active set strategy [20]. The third step entails using integer cuts to exclude non-qualifying networks in the flexibility tests. Chen et al. [11] extended the aforementioned method by modifying the flexibility analysis step such that area restrictions are not considered during a first step of flexibility analysis but are considered in a later step.

Chen and Hung [12] extended the method of Chen and Hung [10] with some modifications to the synthesis of flexible heat and mass exchange networks. The flexibility test of this method is carried out on a large number of randomly generated parameters within the range of uncertainty. Another method [13] in the area of flexible HEN designs used a two-stage strategy which is based on the SWS model for the synthesis of flexible and controllable networks. Li et al. [14] developed a two-step approach for flexible HENS. The first step entails the synthesis of the network structure using the nominal set of operating conditions. The flexibility of the resulting network is further improved through a structural union with the topology of the critical operating points. In the second step, the areas of the heat exchangers obtained in the first step are further optimised considering flexibility and total annual cost (TAC). It should be known that in the method of Li et al. [14], the area optimisation is only done after obtaining structures that qualify from the flexibility step. This implies that a true simultaneous optimisation may somewhat be omitted. The method has the advantage that it can synthesise flexible networks with non-convex feasible regions. Li et al. [15] used a simulated annealing and decoupling strategy to determine the flexibility index of large-scale non-convex HEN optimisation problem.

It is worth stating at this point that, apart from the use of the multi-period version of the SWS model, a common feature of most of the flexible HENS methods is that they involve a first step where a candidate single period or a multi-period network is synthesised for a minimum total annual cost (TAC) scenario, followed by a flexibility analysis step. For the first step, the candidate multi-period network has mostly been made comprising few periods of operations which may include the nominal operating conditions and the critical operating points. The critical operating points are the periods of operations that require the maximum heat load. The authors of this chapter are of the view that if these sets of operating points that are used to generate the candidate multi-period network are carefully chosen, there may not be a need for complex and mathematically intensive flexibility analysis step, especially in small- to medium-scale HENS problems. This implies that as many as possible critical operating points that lie within the overall range of potential disturbance/fluctuation should be selected for participation in the candidate multi-period network synthesis of the first step. A further criterion that needs to be considered while designing the representative candidate multi-period network, which has erstwhile been ignored by existing flexible HENS methods, is the length of periods for each of the critical operating points used to generate the representative candidate multi-period network in the first step. The existing methods [5, 9–14] used the average costs of utility usage by each period of operation present in the first-step candidate multi-period network to determine its minimum TAC and associated network structure as shown in Eqs. (1) and (2)

$$
\begin{aligned}
\min TAC = AF \cdot \Bigg\{ & \sum_{i\epsilon H}\sum_{j\epsilon C}\sum_{k\epsilon K} CFz_{i,j,k} + \sum_{i\epsilon H}\sum_{CU} CFz_{i,cu} + \sum_{j\epsilon C}\sum_{HU} CFz_{j,hu} \\
& + \sum_{i\epsilon H}\sum_{j\epsilon C}\sum_{k\epsilon K} AC_{i,j,k} A_{i,j,k}^{AE_{i,j}} + \sum_{j\epsilon C} AC_{j,hu} A_{j,hu}^{AE_{j,hu}} + \sum_{i\epsilon H} AC_{i,cu} A_{i,cu}^{AE_{i,cu}} \Bigg\} \\
& + \left[\sum_{p\epsilon P} \frac{DOP_p}{NOP} \sum_{j\epsilon C} CUHq_{j,hu,p} + \sum_{p\epsilon P} \frac{DOP_p}{NOP} \sum_{i\epsilon H} CUCq_{i,cu,p} \right]
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\min TAC = \frac{1}{NOP+1} & \left[\sum_{p\epsilon P}\sum_{i\epsilon H} CUCq_{i,cu,p} + \sum_{p\epsilon P}\sum_{j\epsilon C} CUHq_{j,hu,p} \right] \\
& + \sum_{i\epsilon H}\sum_{j\epsilon C}\sum_{k\epsilon K} AC_{i,j,k} A_{i,j,k}^{AE_{i,j}} + \sum_{j\epsilon C} AC_{j,hu} A_{j,hu}^{AE_{j,hu}} + \sum_{i\epsilon H} AC_{i,cu} A_{i,cu}^{AE_{i,cu}}
\end{aligned}
\tag{2}
$$

where *AF* is the annualisation factor, *CF* is the fixed charge for heat exchangers, *AC* is the area costs for heat exchangers, *AE* is the area cost exponent for heat exchangers, *CUH* and *CUC* are the costs of hot and cold utilities, respectively, *DOP* is the duration of period *p*, *NOP* is the number of periods/operating conditions, $A_{i,j,k}$ is the area of heat exchanger for hot and cold-process stream pairs *i,j* in interval *k*. $A_{j,hu}$ and $A_{i,cu}$ are the area of heat exchangers exchanging heat between hot utility and cold-process streams and cold utility and hot-process streams, respectively. *H,C,HU,CU* are the set of hot streams, cold streams, hot utilities and cold utilities, respectively. It should be known that the area $A_{i,j,k}$ is the representative heat exchange area which, as explained previously, are used by the same pair of streams exchanging heat in the same interval of the superstructure at different periods of operations.

Eqs. (1) and (2) are the objective functions used in determining the TAC for the first-step initial candidate multi-period network that is later tested for flexibility using various kinds of approaches in some of the existing methods [5, 9–14]. It can be seen that the utility cost calculation component of these equations will result in allotting equal contributions, in terms of utility usage durations, for each of the periods of operations present in the first-step candidate multi-period network. This implies that the candidate multi-period network that is designed at the initial step, and later tested for flexibility, may be limited based on the fact that it is designed with the assumption that these initial candidate critical points have equal-period durations. Since TAC is being solved for at the first step, the objective functions in Eqs. (1) and (2) will aim to simultaneously minimise both utility consumption and investment costs. The investment cost is influenced by the size of heat-exchanger areas and the number of units. Allowing this limitation at the first step of the flexible network synthesis process means that the flexibility analysis step needs to be sophisticated so as to compensate for this limitation. This is because some candidate networks that lie in the uncertain process parameter range that are tested in the flexibility step may be disqualified from being included in the flexible network feasible space due to the fact that Eqs. (1) and (2) were used as the objective functions for generating the initial candidate multi-period network. Furthermore, even at the flexibility testing stage, the feasible solution space may further be limited or constrained based on the fact that equal-period duration scenario was assumed. The authors of this chapter are of the view that adequately incorporating period durations at the stage of generating the candidate multi-period network is vital so as to reduce the degree of complexity of the flexibility tests that would be carried out subsequently. Moreover, it can be said that a HEN is flexible not only when it is able to feasibly transfer heat for scenarios where each of the potential possible operating points that lie within the range of disturbance/fluctuation has equal lengths of periods, but also when their lengths of periods are significantly different from each other, as well as being uncertain. This implies that the total annual cost of a flexible HEN is not fixed but depends on which operating points (including period durations) within the uncertain range of variability are active.

2. Problem statement

Given a set of hot-process streams and a set of cold-process streams, which have to be cooled and heated, respectively. Given also are the supply and target temperatures and the flow rates of these streams. Hot and cold utilities are also available. The task is to synthesise a flexible heat-exchanger network which is optimally operable (i.e. featuring a minimum TAC network) for any unforeseen process-operating parameter point lying within an uncertain operating range.

3. Methodology

The methodology adopted entails the use of a multi-period version of the simplified stage-wise superstructure (SWS) of Yee and Grossmann [19], as presented in Refs. [5, 8]. The SWS is shown in **Figure 1**. In this superstructure, each hot-process stream and each cold-process stream has the option of splitting within each interval and each period of operation where it exists so as to exchange heat with streams of the opposite kind in intervals 2 and 3. The hot- and cold-process streams are then taken to their final temperatures in intervals 1 and 4 through heat exchange with utilities of the opposite kind in each period of operation where the process streams exist. The details of the superstructure can be found in Refs. [5, 7].

3.1. Model equations

The detailed multi-period HENS models used in this chapter are shown in the appendix. For detailed explanations of each of these equations, the reader is referred to the multi-period SWS HENS model of Verheyen and Zhang [5] and Isafiade et al. [7]. The maximum area approach as introduced by Verheyen and Zhang [5] for HENS is also used in this chapter for flexible HENS as shown in Eq. (3)

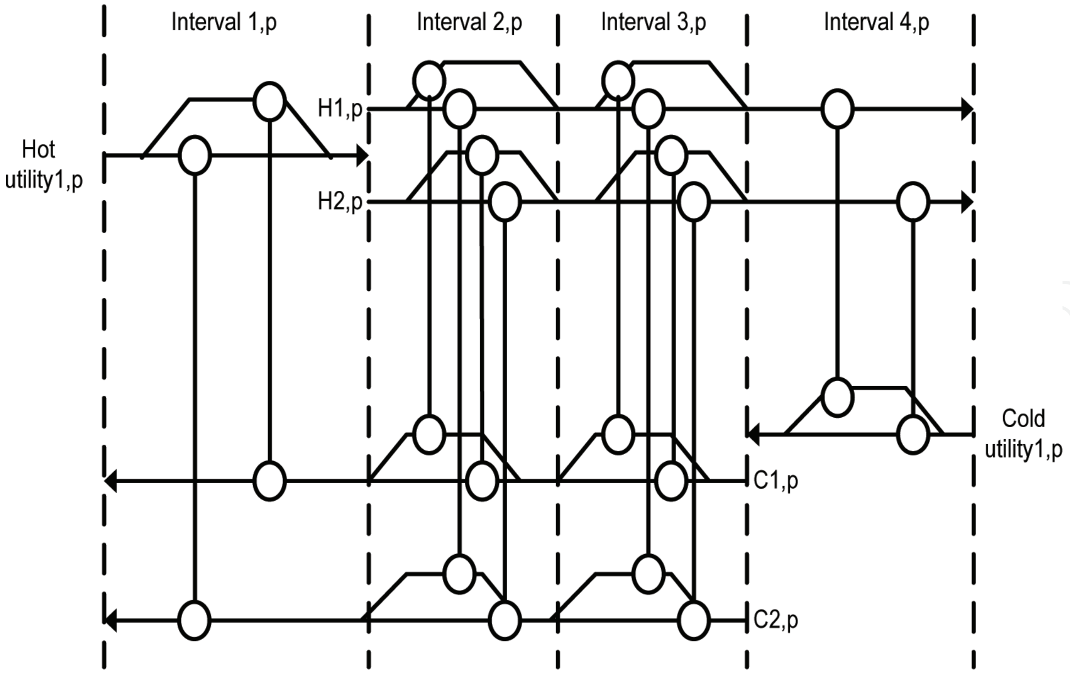

Figure 1. Multi-period version of SWS model.

$$A_{i,j,k} \geq \frac{q_{i,j,k,p}}{(LMTD_{i,j,k,p})(U_{i,j})} \tag{3}$$

The maximum area, $A_{i,j,k}$, is then included in the objective function shown in Eq. (4). However, it should be known that Eq. (4), which is the objective function used in this study, was introduced by Isafiade and Fraser [6] for multi-period networks having specified process parameter points

$$\begin{aligned} \min\Bigg\{ &\left[\frac{DOP_p}{\sum_{p=1}^{NOP} DOP_p}\sum_{i\in H} CUC \cdot q_{i,j,k,p} + \frac{DOP_p}{\sum_{p=1}^{NOP} DOP_p}\sum_{i\in H} CUH \cdot q_{i,j,k,p}\right] \\ &+AF\left[\sum_{i\in H}\sum_{j\in C}\sum_{k\in K} CF_{ij} \cdot z_{i,j,k} + \sum_{i\in H}\sum_{j\in C}\sum_{k\in K} AC_{i,j,k} \cdot A_{i,j,k}^{AE_{i,j}}\right]\Bigg\} \forall i \\ &\in H, j\in C, k\in K, p\in P \end{aligned} \tag{4}$$

It is worth mentioning that the terms in the first square bracket of Eq. (4) are the annual operating cost terms. The presentation of these terms by Isafiade and Fraser [6] adequately allocates the contribution of each hot/cold utility to the annual operating cost of the flexible network based on each operating periods of duration. This is unlike the objective functions shown in Eqs. (1) and (2), which are used in most existing methods, and which make an implicit assumption that each operating parameter point within the uncertain range would operate at an equal/average period duration. Whereas this may not always be true because any of the parameter points may dominate at any point in time, hence the network needs to be flexible enough to handle unforeseen period durations.

For the example solved in this chapter, the solver DICOPT, which uses CPLEX for the MILP and CONOPT for the NLP sub-problems, has been used. The solver environment used is GAMS [21]. The machine used operates on Microsoft® Windows 7 Enterprise™ 64 bit, Intel® Core™ i5-3210M processor running at 2.50 GHz with 4 GB of installed memory.

3.2. Solution approach

Based on the foregoing explanations, in this chapter, the following procedure is adopted in generating the candidate initial multi-period network and the subsequent flexibility tests:

1. Identify a large set of critical/extreme candidate-operating parameter points which lie within the full disturbance range. The candidate points are then translated into a multi-period problem having a large number of periods with specified period durations. The resulting set of periods of operations should represent critical points that include the nominal conditions, maximum heat-load-required conditions, minimum heat-load-required conditions and a combination of each of these scenarios.

2. Solve the identified critical set of periods in Step 1 as a multi-period mixed integer non-linear programming (MINLP) problem with equal-period durations for each of the periods of operations.

3. Use the selected matches for the equal-period duration scenario in Step 2, as well as the areas of these matches to initialise the multi-period problem created in Step 1. This implies that the areas of the matches in the multi-period model of this third step should be fixed to the areas obtained for the equal-period scenario in Step 2. Solve the resulting model as an MINLP a number of times. For each time that the model is solved, each period of operation is made to dominate the total period durations, for example, 99.1% of the time, while the remaining time length is shared equally among the other periods of operations. This is necessary so as to examine the flexibility of the network obtained in Step 2 to feasibly transfer heat in a cost-efficient manner irrespective of the period durations of the participating critical operating points. However, if the network obtained in Step 2, which now has fixed areas, is infeasible for any of the critical periods being tested, the fixed areas would be adjusted until all heat loads in all periods are satisfied in terms of heat exchange. The adjustment should be done using the exchanger sizes obtained in Step 2 for the period concerned. The adjustment should continue until a compromise minimum TAC is obtained. The purpose of this step is to identify the critical exchanger areas (i.e. the maximum) that would be able to transfer heat in the final flexible network not only based on operating parameters but also based on unforeseen fluctuations in period durations of any of the potential operating points. It should be known that in order to identify the contribution of each dominant period of operation to the total utility usage, as well as the structure of the final flexible network, the objective function for multi-period HENS as used by Isafiade and Fraser [6] needs to be employed.

4. This step, which is the flexibility analysis step, entails generating a large number of operating points, as large as 100 periods of operations for small- to medium-scale problems, which are then appended to the multi-period network having fixed areas obtained

in Step 3. The model at this stage is solved with further minor adjustments to exchanger areas, if needed, so as to accommodate as many operating points as possible.

4. Example

One example, which was adapted from Chen and Hung [10–12], has been used to illustrate how the newly presented methodology works. The example was first presented by Floudas and Grossmann [3] and has also been solved by other authors [14, 15]. It has two hot streams, two cold streams, one hot utility (steam) and one cold utility (cooling water). **Table 1** shows the stream parameters and other costing details.

In the example, for hot stream 1, the heat capacity flow rate fluctuates around a nominal value of 1.4 kW/K by a magnitude of ±0.4, while its supply temperature fluctuates around a nominal value of 583 K by a magnitude of ±10. For cold stream 2, the heat capacity flow rate fluctuates around a nominal value of 2.0 kW/K by a magnitude of ±0.4, while its supply temperature fluctuates around a nominal value of 388 K by a magnitude of ±5. The objective in this example is to develop a flexible HEN that can feasibly transfer heat for the specified disturbance range in a minimum TAC network. In solving this problem, Chen and Hung [10] identified four extreme operating points within the uncertain process parameter range which include those for nominal conditions, maximum area, maximum cooling load and maximum heating load. The second, third and fourth sets of operating points, that is, the maximum area, maximum cooling load and maximum heating load, respectively, were appended one after the other, to the candidate network generated using the nominal operating conditions.

In solving this problem using the new method developed in this chapter, the first step entails identifying 10 sets of operating points (termed periods) that would be used to generate the base candidate multi-period network. The parameters for the 10 periods are shown in **Table 2**. The parameters listed in **Table 2** are then solved in Step 2 as a multi-period problem having 10 periods of operations with equal-period durations and unequal-period durations using Eqs. (3)

Stream	**Heat capacity flow rate FCp (kW/K)**	**Supply temperature $T^S(K)$**	**Target temperature $T^t(K)$**	**Cost (\$/kWh)**
Hot-process stream 1, $H1$	1.4 ± 0.4	583±10	323	-
Hot-process stream 2, $H2$	2.0	723	553	-
Cold-process stream 1, $C1$	3.0	313	393	-
Cold-process stream 2, $C2$	2.0 ± 0.4	388 ± 5	553	-
Hot utility, $HU1$	-	573	573	171.428×10^{-4}
Cold utility, $CU1$	-	303	323	60.576×10^{-4}

Operating hours = 8600 (hours/year), heat exchanger capital cost function = $4333A^{0.6}$ (\$/year), capital annualisation factor (AF) = 0.2, A in m^2, overall heat transfer coefficient (U) for all matches = 0.08 kW/(m^2·K^{-1}), $\Delta T_{\min}$ = 10 K.

Table 1. Stream, cost and capital equipment data for the example.

Period	Stream	$T^S(K)$	$T^t(K)$	FCp (kW/K)	Enthalpy (kW)	Period	Stream	$T^S(K)$	$T^t(K)$	FCp (kW/K)	Enthalpy (kW)
1	$H1$	583	323	1.4	364	6	$H1$	593	323	1.8	486
	$C2$	388	553	2	330		$C2$	388	553	2.0	330
2	$H1$	593	323	1.8	486	7	$H1$	573	323	1.0	250
	$C2$	383	553	2.4	408		$C2$	388	553	2.0	330
3	$H1$	573	323	1.0	250	8	$H1$	583	323	1.4	364
	$C2$	393	553	1.6	256		$C2$	383	553	2.4	408
4	$H1$	593	323	1.8	486	9	$H1$	583	323	1.4	364
	$C2$	393	553	1.6	256		$C2$	393	553	1.6	256
5	$H1$	573	323	1.0	250	10	$H1$	593	323	1.0	270
	$C2$	383	553	2.4	408		$C2$	383	553	2.0	340

Table 2. Periods of operations used to generate candidate network.

Dominant period	Equal	1	2	3	4	5	6	7	8	9	10
Period duration	10%	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1
TAC ($)	38,133	38,340	34,458	38,715	46,573	41,852	41,219	31,025	32,425	38,964	30,689

Table 3. Total annual cost for each dominant period.

and (4) as well as Eq. (A1) in the appendix. For the unequal-period duration scenario, each of the selected periods of operations is made to dominate the total length of periods for all the 10 periods by a significant amount. The purpose of this is to ensure that the final flexible network is able to cater for all possible scenarios including the worst-case scenarios in terms of heat-exchanger area and utility requirement, irrespective of the duration of period for each of the parameter points lying within the full disturbance range. The resulting solution for each scenario is shown in **Table 3**. The average solution generation time for each of the solutions in **Table 3** is 5 S of CPU time. The TAC shown in the first column of **Table 3** is for a case where all periods have equal duration, that is, each period contributes 10% of the total period duration. The second column is for a case where the parameter points of period 1 dominates the total period durations by 99.1%. The same scenario applies for the rest of the columns, that is, each period concerned dominates the total period duration by 99.1%.

Step 3 requires that the selected matches, as well as their areas, for the equal-period duration solution network (TAC = 38,133 $ in **Table 3**) and the network obtained for the most dominant period (i.e. period 4, TAC = 46,573 $ in **Table 3**) be identified from **Table 3**. **Table 4** shows the selected matches, as well as their areas, for the two cases. It should be known that the solution network for the equal-period scenario has more units compared with that of a case where the dominant period (i.e. period 4) is considered. This implies that the solution for the dominant period will result in a simpler network with fewer units, but with higher operating cost. In this chapter, the solution of the equal-period scenario is used in subsequent steps so as to get a TAC

Equal-period scenario		**Period 4 dominating**	
Match	**Area (m^2)**	**Match**	**Area (m^2)**
HU1,C1,1	4.71	HU1,C2,1	43.04
H1,C1,5	32.74	H1,C1,5	14.08
H1,C2,4	6.04	H1,CU1,6	57.34
H1,CU1,6	57.34	H2,C1,5	4.77
H2,C1,5	4.77	H2,C2,3	17.26
H2,C2,2	27.36		
Total area	132.96		136.49

Table 4. Selected matches for equal-period duration and most dominant period.

Dominant period	**Equal**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Period duration	10%	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1
TAC (\$/yr)	38,134	39,396	35,064	37,127	47,235	41,479	43,419	31,247	33,669	40.943	31,763

Table 5. TAC for equal period and one dominating period at a time in the flexible network tested for 10 periods.

that competes with those presented in the literature. However, the resulting network of the equal-period scenario is still tested for flexibility to feasibly transfer heat in scenarios of unequal-period durations.

Step 3 further requires that the matches selected (including their areas) in the equal-period case be used to initialise the multi-period MINLP model of the 10-period problem data shown in **Table 2**, by fixing the areas of the matches to those of the equal-period scenario shown in **Table 4**. Note that the model is still solved as an MINLP at this stage, despite the fact that the matches and areas are fixed, because in getting compromise solutions for a flexible network, not all matches in **Table 4** (for the equal-period case) may be selected, in fact matches which do not exist on the table may even be added to the network. Solving the 10-period candidate multi-period network of **Table 2**, using the fixed areas of the matches shown in **Table 4** for the equal-period case, gives a range of TACs for each period dominating one at a time as shown in **Table 5**.

In Step 4, the flexibility of the network obtained in Step 3 was then further tested for more randomly generated parameter points lying within the full disturbance range by solving a large multi-period model with equal-period durations. The model at this stage is initialised using the matches, as well as their areas, shown for the equal-period case in **Table 4**. The network was not feasible for a case having 100 periods of equal durations, so the areas of the network were adjusted to obtain new set of areas as shown in **Table 6**.

After the adjustments, the total network area increased from 132.96 m^2 in **Table 4** to 135.4 m^2 in **Table 6**. The resulting network was then feasible to transfer heat in a cost-efficient manner

Match	Area (m^2)
HU1,C1,1	4.8
H1,C1,5	32.8
H1,C2,4	6.1
H1,CU1,6	59.7
H2,C1,5	4.8
H2,C2,2	27.4
Total area	135.4

Table 6. Heat-exchanger areas for final flexible network for the example.

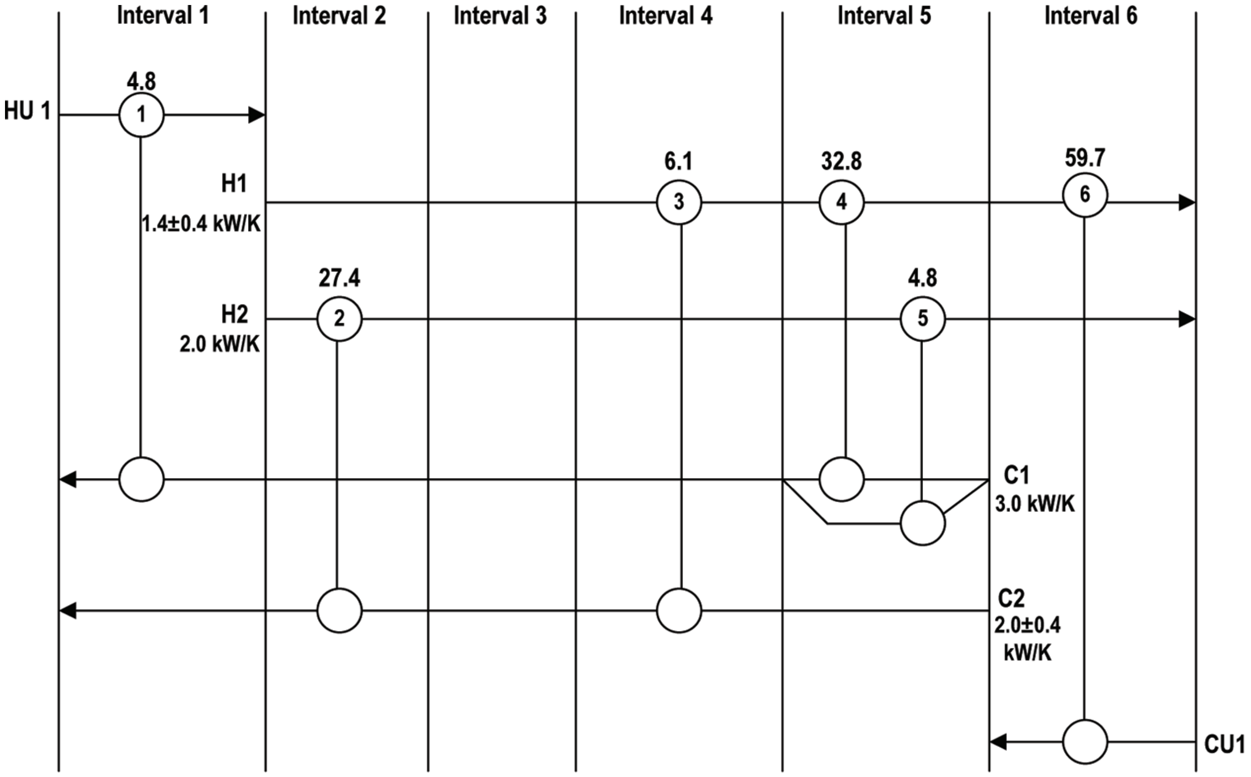

Figure 2. Final flexible structure of this study.

for all of the 100 possible periods of operations. The TAC of the 100-period scenario, which was obtained in 20.94 S of CPU time, is 38,992 $. The flexible network, which is shown in **Figure 2**, is deemed flexible at this stage; hence, it is selected as the final flexible network. In this figure, the areas of each of the heat exchangers are indicated on the units. It should be known that only the final flexible network obtained in Step 4 is shown because the network structure remains unchanged in each of the steps. **Table 7** shows a comparison of the final solution obtained in this example with those of other papers. It is worth mentioning that the solutions of other works, which are shown in **Table 7**, are presented for a case where the periods have equal duration. However, in

Costs	Floudas and Grossmann [3]	Chen and Hung [10]	Li et al. [14]	This work
Annual operating cost ($)	10,499	11,772	11,866	8554
Annual capital cost ($)	39,380	30,104	28,626	30,438
Total annual cost ($)	49,879	41,876	40,492	38,992

Table 7. Comparison of solutions for the example.

Dominant period	Equal	1	2	3	4	5	6	7	8	9	10
Period duration	10%	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.1
TAC ($/yr)	38,429	39,698	35,365	37,428	47,537	41,759	43,720	31,549	33,926	41,245	32,065

Table 8. Total annual cost for each dominant period in final flexible network for the example.

reality, the period durations may not be equal; hence, there is a need to also test or demonstrate the flexibility of the final network for unequal-period durations as done in this chapter. The final network obtained in this study has been tested for a 10-period scenario where period durations may be unequal and the TAC that should be anticipated for cases where each of the periods in the set of 10 periods dominates by 99.1% of the time is shown in **Table 8**. It is expected that the TAC that would be obtained when any of the 100 possible periods of operations dominates significantly by 99.1% of the time, or less, will not be too different from what is shown in **Table 8**.

Figures 3 and **4** show the final flexible networks obtained by other authors as found in Refs. [12, 14]. What is common to these two structures is that they both have six units, out of

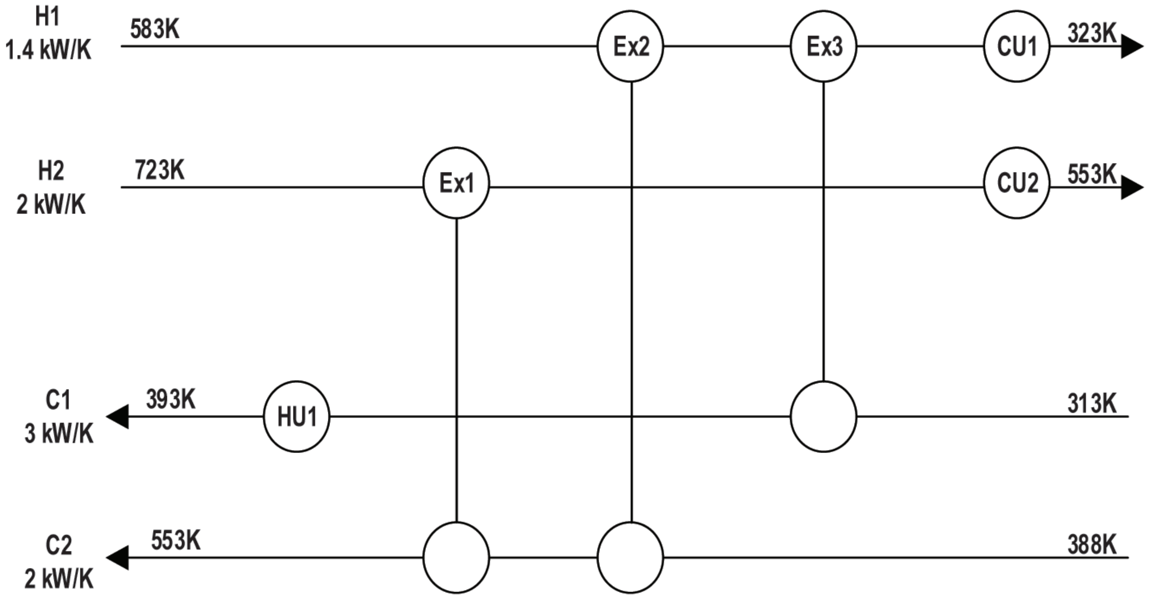

Figure 3. Final flexible HEN structure of Li et al. [14].

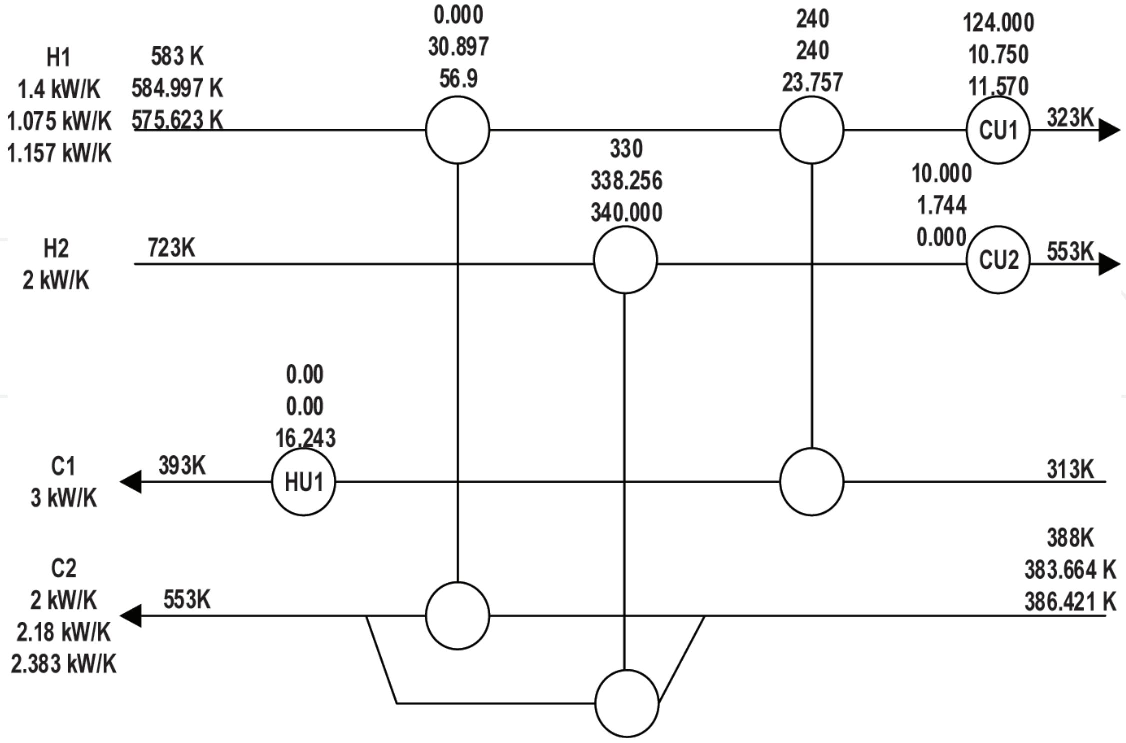

Figure 4. Final flexible HEN structure of Chen and Hung [12].

which two are coolers. This is unlike the structure obtained in this chapter which has only one cooler.

5. Conclusions

This chapter has presented a new simplified synthesis method for designing small- to medium-sized flexible heat-exchanger networks using a mixed integer non-linear programming multi-period synthesis approach. The new method improves on currently available methods in the literature based on the fact that the final flexible network is selected considering the possibility of one or more sets of process parameter points dominating the total period durations more than others. This is essential so as to effectively plan for utility management. Key limitations of the new method are its tediousness of application, especially in large-scale problems, due to the fact that the impact on the overall network TAC of the possibility of each set of selected critical points dominating the total period duration needs to be investigated in a sequential manner. Also, during the flexibility tests, heat-exchange areas need to be manually adjusted, and there is no specific criterion to consider for the adjustments.

Appendix

The set of equations shown in Eq. (A1) was used in the multi-period model of this study. The details of each of these equations can be found in Verheyen and Zhang [5] and Isafiade et al. [7].

$$\left\{\begin{array}{l}
\left.\begin{cases}(T^{s}_{i,p}-T^{t}_{i,p})F_{i,p}=\sum\limits_{k\in K}\sum\limits_{j\in C}q_{i,j,k,p} i\in H p\in P\\ (T^{t}_{j,p}-T^{s}_{j,p})F_{j,p}=\sum\limits_{k\in K}\sum\limits_{i\in H}q_{i,j,k,p} j\in C p\in P\end{cases}\right\}\text{overal energy balances}\\
\left.\begin{cases}(t_{i,k,p}-t_{i,k+1,p})F_{i,p}=\sum\limits_{j\in C}q_{i,j,k,p} k\in K i\in H p\in P\\ (t_{j,k,p}-t_{j,k+1,p})F_{j,p}=\sum\limits_{i\in H}q_{i,j,k,p} k\in K j\in C p\in P\end{cases}\right\}\text{stage energy balances}\\
\left.\begin{cases}t_{i,k=2,p}=T^{s}_{i,p} i\in H p\in P\\ t_{j,K-1,p}=T^{s}_{j,p} j\in C p\in P\end{cases}\right\}\begin{array}{c}\text{assignment of superstructure}\\ \text{inlet temperatures}\end{array}\\
\left.\begin{cases}t_{i,k,p}\geq t_{i,k+1,p} i\in H p\in P\\ t_{j,k,p}\geq t_{j,k+1,p} j\in C p\in P\end{cases}\right\}\text{temperature feasibility}\\
\left\{q_{i,j,k,p}-\Omega_{p}z_{i,j,k}\leq 0\right\}\text{logical constraint for heat load}\\
\left.\begin{cases}dt_{i,j,k,p}\leq t_{i,k,p}-tc_{i,j,k,p}+\phi(1-z_{i,j,k})k\in K i\in H j\in C p\in P\\ dt_{i,j,k+1,p}\leq th_{i,j,k+1,p}-t_{j,k+1,p}+\phi\ (1-z_{i,j,k})\ k\in K i\in H j\in C p\in P\end{cases}\right\}\begin{array}{c}\text{exchanger approach}\\ \text{temperatures}\end{array}\\
\left\{dt_{i,j,k,p}\geq\varepsilon\ k\in K i\in H j\in C p\in P\right\}\text{bound for approach temperature}\\
\left\{LMTD_{i,j,k,p}=\frac{2}{3}\left((dt_{i,j,k,p})(dt_{i,j,k+1,\mathrm{p}})\right)^{1/2}+\frac{1}{3}\left(\frac{(dt_{i,j,k,p})+(dt_{i,j,k+1,\mathrm{p}})}{2}\right)\right\}\begin{array}{c}\text{logarithmic mean}\\ \text{temperature difference}\end{array}\\
z_{i,j,k}\in\{0,1\}\\
t_{i,k,p},t_{j,k,p},q_{i,j,k,p},dt_{i,j,k,p}\geq 0
\end{array}\right\} \tag{A1}$$

Nomenclature

CU	Cold utility
HENS	Heat-exchanger network synthesis
HU	Hot utility
MILP	Mixed integer linear programme
NLP	Non-linear programme
MINLP	Mixed integer non-linear programme
SWS	Stage-wise superstructure

Indices

i	Hot-process streams and hot utilities
j	Cold-process streams and cold utilities
p	Index representing period of operation ($p = 1,\ldots NOP$)
k	Stage boundaries

Sets

H	Hot-process streams and hot utilities
C	Cold-process streams and cold utilities
P	Period of operation
K	Stage boundaries or temperature locations (where $K-1$ is the set defining the stages)

Parameters

$T^s_{i,p}$	Supply temperature of hot streams (process and utility streams) for period p, K
$T^t_{i,p}$	Target temperature of hot streams (process and utility streams) for period p, K
$T^s_{j,p}$	Supply temperature of cold streams (process and utility streams) for period p, K
$T^t_{j,p}$	Target temperature of cold streams (process and utility streams) for period p, K
$F_{i,p}$	Heat capacity flow rate of hot stream i in period p, W/K
$F_{i,p}$	Heat capacity flow rate of cold stream j in period p, W/K
$U_{i,j}$	Overall heat transfer coefficient, W/(m^2·K)
AF	Annualisation factor
$CF_{i,j}$	Fixed cost for heat exchangers, \$/yr
$AC_{i,j}$	Area cost coefficient
AE	Area cost index
DOP_p	Duration of each period p
NOP	Number of periods
CUH_i	Cost per unit of hot utility i, \$/(W·yr)
CUC_j	Cost per unit of cold utility j, \$/(W·yr)
Ω_p	Upper bound for heat exchanged between match i and j in period p, W
ϕ	Upper bound for driving force in match i,j in period p, K

Variables

$A_{i,j,k}$	Area of match between hot stream i and cold stream j in interval k, m^2
$y_{i,j,k}$	Binary variable representing a match between hot stream i and cold stream j in stage k
$q_{i,j,k,p}$	Heat load of a match between hot stream i and cold stream j in stage k and in period p, W/K
$t_{i,k,p}$	Temperature of hot stream i at stage boundary k in period p, K
$t_{j,k,p}$	Temperature of cold stream j at stage boundary k in period p, K
$th_{i,j,k,p}$	Exit temperature of hot stream i from match i,j,k in period p, K
$tc_{i,j,k,p}$	Exit temperature of cold stream j from match i,j,k in period p, K
$fh_{i,j,p,k}$	Heat capacity flow rate split hot stream i in match i,j,k in period p, W/K
$fc_{i,j,p,k}$	Heat capacity flow rate split cold stream j in match i,j,k in period p, W/K
$dt_{i,j,k,p}$	Driving force in match i,j at stage boundary k and period p, K

TAC	Total annual cost of the network, including annualised capital cost and the cost of hot and cold utilities, \$/y

Acknowledgements

This study is supported by the National Research Foundation of South Africa (Grant numbers: 85536 and 87744) and the Research Office of the University of Cape Town, South Africa, and Sasol. The funds are gratefully acknowledged.

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