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Thickness and Thermal Conductivities of the Walls and Fluid Layer Effects on the Onset of Thermal Convection in a Horizontal Fluid Layer Heated from Below

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Additional information is available at the end of the chapter

Abstract

The thermal boundary conditions have important effects on the hydrodynamics of a thermo-convective fluid layer. These effects are introduced through the Biot number under the Robin type boundary conditions. The thermal conductivity and thicknesses of the walls are key properties to bridge two known ideal situations widely studied: the fluid layer bounded by two insulating walls and the fluid layer bounded by two perfect thermal conducting walls. This chapter is devoted to the physical mechanisms involved in the thermal boundary conditions, its influence on the linear stability of the fluid layer and its implications with the pattern formation. A review of very important investigations on the subject is also given. The role of the thermal conductivities and thicknesses of the walls is explained with help of curves of criticality for the thermoconvection in a horizontal Newtonian fluid layer.

Keywords: thermal convection, boundary condition, hydrodynamic stability, Biot number, patterns

1. Introduction

The present work is devoted to the study of some important physical properties and geometrical configurations that may modify the pattern formation in Newtonian fluid layers. The theory presented here may be of interest for a number of applications such as for the control of convective motions [1, 2], for the study of movements in the mantle of the earth [3], in the

study of convective cell formation in the surface of the sun [3] and in biotechnological appliances involving the Rayleigh convection phenomena [4].

The formation of patterns is a very interesting subject in fluid mechanics. This topic involves complex physics and mathematics [5]. From the physical point of view, various parameters influence the onset of convection and later the evolution of the formed patterns. Some variables affecting the patterns are,

- type of fluid: Newtonian or non-Newtonian
- the properties of the fluid (like density and viscosity) and properties of the bounding surfaces (like thickness and thermal conductivity), among others

This chapter focuses on the points given in the above-mentioned list. The bounding surfaces have become an interesting topic of study since the boundary conditions are mathematically written according to their nature [6, 7]. The thickness and thermal conductivity of the walls and the fluid layer are strongly related to the familiar eigenvalue Rayleigh number and to the wavenumber. They are also related to two classical approximations commonly found in hydrodynamic stability. These two classical approximations are:

- insulating walls and
- perfect thermal conducting walls.

For short, the insulating wall approximation correspond to constant heat flux boundary conditions while the perfect thermal conducting walls approximation correspond to the constant temperature boundary conditions. The critical Rayleigh and wavenumber are (R_c = 720, k_c = 0) and (R_c = 1707.96, k_c = 3.12), respectively. Then the purpose of considering the thickness and thermal conductivity of the walls are to bridge ideal approximations to the problem of thermal convection and to provide critical conditions that better simulate the lab experiments.

The boundary conditions are of paramount importance for proper understanding of the physical phenomenon of thermal convection [1, 6, 7], for comparison between theoretical and experimental data and for its control [1]. As new technologies and appliances develop, more sophisticated mathematical models are needed. A good example for the previous statement is that of the manufacturing of corrugated surfaces [8, 9] in which the formed convective pattern is deposited on the lower boundary after evaporation of the solvent. This may occur for convection in polymer solutions which are composed of polymeric chains and solvents.

This chapter is organized as follows. In Section 2, a general formulation for the natural convection in a horizontal fluid layer heated from below is given along with some data on the basic state of the temperature. Section 3 presents a brief explanation on how the thermal boundary conditions are related to the linear hydrodynamic stability. In Section 4, a discussion on the basic state of the temperature is presented. Some points about the effect of the thermal conductivities and thicknesses of the walls on the pattern formation are discussed in Section 5. Section 6 is devoted to list some challenges in hydrodynamic stability that are

connected to the thermal boundary conditions. Finally, in Section 7 a general discussion on the subject is given.

2. The problem of convection in a fluid layer

The importance of the thermal boundary conditions can be seen from the point of view of the familiar problem of Rayleigh thermal convection in a horizontal infinite fluid layer vertically bounded by two solid and rigid walls [10, 11]. Consider the scheme presented in **Figure 1** which shows the thermal and geometrical properties of the bounding walls. This extension to the problem of convection has been presented in Ref. [2], and studied by Cerisier et al. [7] and Howle [1], among others.

The two problems of thermal convection that have been widely studied are that of bounding insulating walls (see Refs. [11, 12] for more details) and that of bounding perfect thermal conducting walls (see Refs. [10, 13] form more details). These two cases can be mathematically expressed as,

- $dT/dz = 0$ at the boundaries. For insulating walls (according to Chapman et al. [11]).

- $T = 0$ at the boundaries. For perfect thermal conducting walls (according to Chandrasekhar [10]).

On the other hand, it is well known that lab experiments and technological developments are not restricted to these ideal cases. In other words, more general boundary conditions are needed to satisfy the requirements of intermediate cases, as represented in **Figure 2**. Mathematically speaking, the proper thermal boundary conditions for non-ideal situations are those of the Robin type. This is a boundary condition encompassing both cases mentioned above.
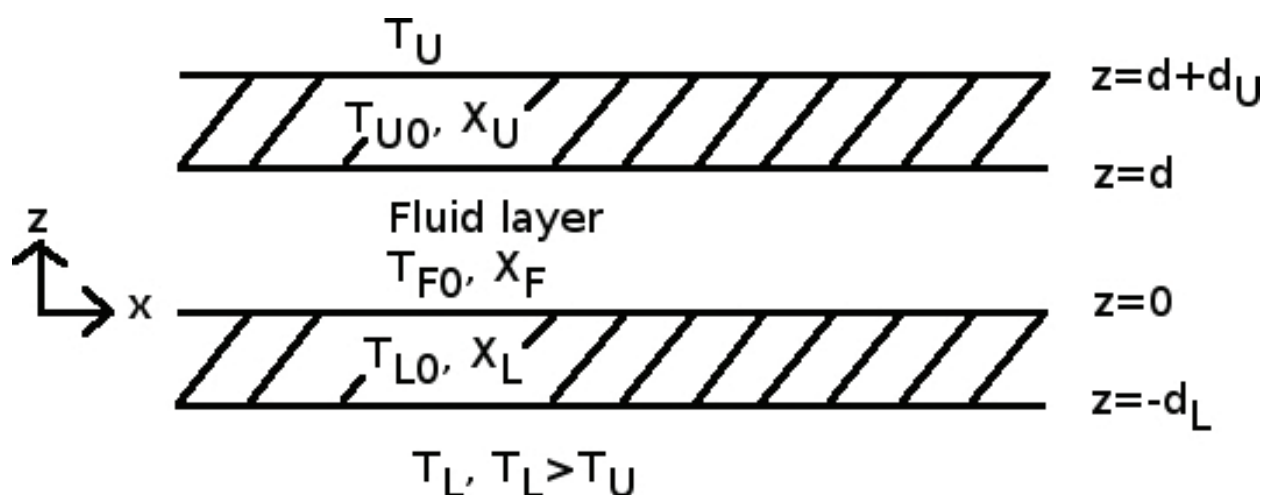

Figure 1. Scheme for the problem of Rayleigh convection including the thickness and thermal conductivities of the walls. $T_{L,U}$ stand for the constant temperature at the lower and upper walls, $T_{L0,U0,F\ 0}$ stand for the basic state temperature profile at the lower wall, upper wall and fluid layer; while $X_{L,U,F}$ represent the thermal conductivities of the lower wall, upper wall and fluid layer, respectively. Dimensional variables are used.
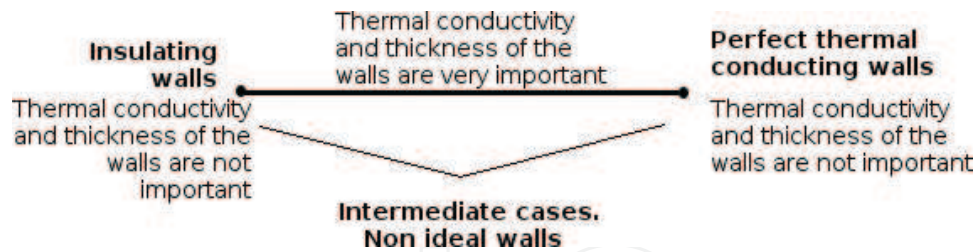
Figure 2. A heuristic explanation about the connection of the thermal and geometrical properties of the walls with the boundary conditions from one ideal case to the other.

When considering bounding walls of non-ideal properties the Biot number appears in the boundary conditions. The Biot number actually is derived from the geometrical and thermal properties of the walls and the working fluid allowing mapping the critical conditions (mainly, critical Rayleigh number, R_c and critical wavenumber, k_c) for the onset of convection from the insulating to perfect conducting walls, as shown in **Figure 2**. Some of investigations have been carried out to fill this gap in the theory of hydrodynamic stability in Newtonian [7, 14, 15] and non-Newtonian fluids [6], as well. Another implication of the thermal conductivities and thicknesses of the bounding walls and that of the fluid layer is the more general temperature profile in the basic state. Even, two more temperature profiles in the basic state appear, one for the lower wall and another one for the upper wall (see **Figure 1**).

The temperature profiles are then defined as:

$$T_{F0} = -z + T_U + 1 + \overline{X_U}\,\overline{d_U} \tag{1}$$

$$T_{L0} = -\overline{X_L}z + T_L + 1 - \overline{X_L}\,\overline{d_L} \tag{2}$$

$$T_{U0} = \overline{X_U}(1 + \overline{d_U} - z) + T_U \tag{3}$$

where the variables in Eqs. (1)–(3) are in non-dimensional form (the reader may see Refs. [6, 7] for more details). Here, $X_U = X_F/X_U$, $X_L = X_F/X_L$, $d_U = d_U/d_F$ and $d_L = d_L/d_F$. Notice that Eqs. (2) and (3) are not considered in studies related to the limiting cases of insulating and perfect conducting walls.

Eq. (1) represents the temperature profile of the fluid layer, Eq. (2) represents the temperature profile of the lower bounding wall and Eq. (3) represents the temperature profile of the upper wall. These temperature profiles may be easily obtained by considering the set of boundary conditions for the temperature below. These conditions assure the continuity and smoothness of the temperature across the whole system including the two walls and the fluid layer (as seen in **Figure 1**):

$$T_{L0} = T_L \text{ at } z = -d_L \tag{4}$$

$$T_{U0} = T_U \text{ at } z = d + d_U \tag{5}$$

$$T_{F0} = T_{L0} \text{ at } z = 0 \tag{6}$$

$$\overline{X_L} \frac{dT_{F0}}{dz} = \frac{dT_{L0}}{dz} \text{ at } z = 0 \tag{7}$$

$$T_{F0} = T_{U0} \text{ at } z = d \tag{8}$$

$$\overline{X_U} \frac{dT_{F0}}{dz} = \frac{dT_{U0}}{dz} \text{ at } z = d \tag{9}$$

where Eqs. (4)–(9) are given in non-dimensional form. As the differential equations to be solved to calculate T_{F0}, T_{L0} and T_{U0} are homogeneous of a single second order term, the solutions are linear polynomials. This means that the three temperature profiles in the basic state lie over a straight line going from $z = -d_L$ to $z = d + d_U$, if continuity and smoothness are expected.

3. Importance in the linear stability

In the linear stability of a fluid layer, its basic state is subjected to small perturbations. This is made to determine whether the fluid layer is stable or not. The linear stability is featured by two parameters, for steady situations: the critical Rayleigh and wavenumbers. **Figures 3** and **4** show the critical points for the two ideal cases mentioned above.

The basic state for the fluid temperature as given in Eq. (1) conveys information not only of the fluid properties but also of the walls through the parameters X_U, d_U and T_U. Unfortunately, the information of the thermal and geometrical properties pass only to the boundary conditions and leaving the governing differential unchanged. This is valid for cases in which the structure of the equations allows only the derivative of the basic state temperature profile in the equation for the perturbation of the temperature.

An example case is that of the convection of Rayleigh. The differential equations for this problem are:

$$Pr^{-1}\sigma\left(\frac{d}{dz^2} - k^2\right)W(z) - \left(\frac{d}{dz^2} - k^2\right)^2 W(z) = Rk^2\theta(z) \tag{10}$$

$$\left[\sigma - \left(\frac{d}{dz^2} - k^2\right)\right]\theta(z) = \frac{dT_{F0}}{dz}W(z) \tag{11}$$

where W and θ are the perturbations for the velocity and the temperature of the fluid, and Pr is the Prandtl number. $\sigma = \sigma_{R+i}\sigma_i$, with σ_R being the growth rate of the perturbations and σ_i, the frequency of oscillation. It is well known that there is no frequency of oscillation in the case of Rayleigh convection, so that if $\sigma_R = 0$ is set, then R_c = 720, 1707.96, for the insulating and perfect thermal conducting walls are obtained.

At this point, no information of the basic state is given to Eqs. (4) and (5) since only dT_{F0}/dz is required. This may represent a limitation to the model since the linear stability of the system
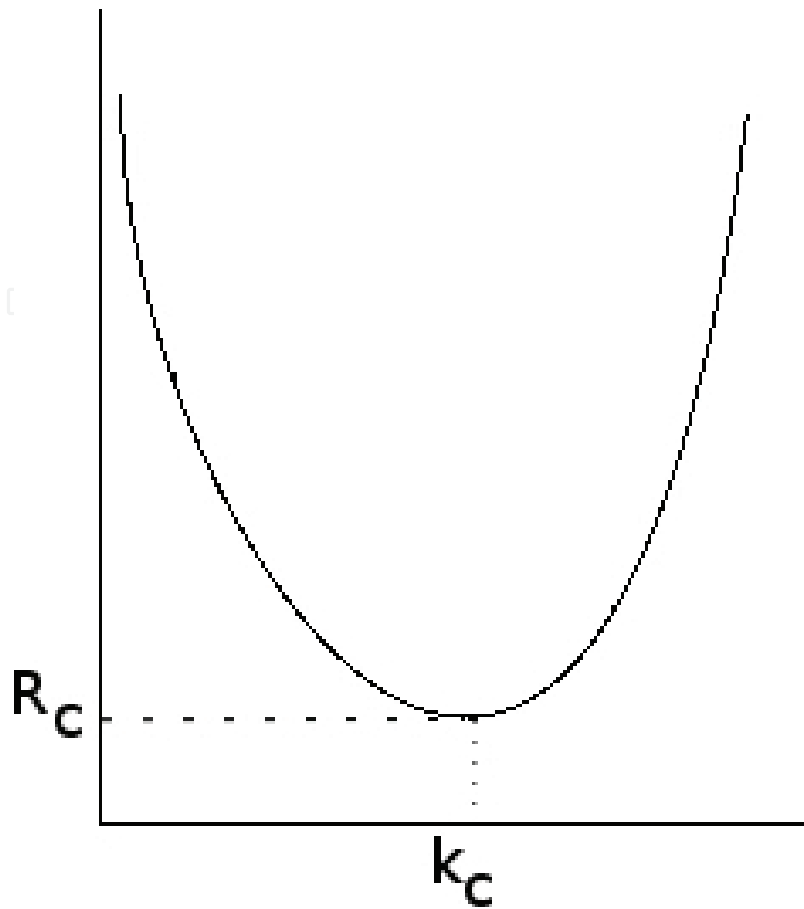
Figure 3. A curve showing the critical point (k_c, R_c) for steady convection of a fluid layer bounded by two insulating walls.

comes from the basic state. The only way to introduce the effect of the thermal conductivities and thickness of the walls and the fluid layer is through the Biot number in the Robin type boundary conditions. These can be expressed as:

$$\left(\left[\frac{d\theta}{dz} - \frac{\sqrt{k^2 + \overline{\kappa_L}\sigma}\,\theta}{\overline{X_L}\tanh[\sqrt{k^2 + \overline{\kappa_L}\sigma}\,\overline{d_L}]}\right]\right)_{z=0} = 0 \tag{12}$$

$$\left(\left[\frac{d\theta}{dz} - \frac{\sqrt{k^2 + \overline{\kappa_U}\sigma}\,\theta}{\overline{X_U}\tanh[\sqrt{k^2 + \overline{\kappa_U}\sigma}\,\overline{d_U}]}\right]\right)_{z=1} = 0 \tag{13}$$

where $\overline{\kappa_L} = \kappa_L/\kappa_F$ and $\overline{\kappa_U} = \kappa_U/\kappa_F$ are ratios of the thermal diffusivities of the walls to that of the fluid layer. The Biot number is a key component of the Robin type thermal boundary conditions and according to Eqs. (12) and (13), the Biot number for the lower wall is:

$$B_L = \frac{\sqrt{k^2 + \overline{\kappa_U}\sigma}}{\overline{X_U}\tanh[\sqrt{k^2 + \overline{\kappa_U}\sigma}\,\overline{d_U}]} \tag{14}$$

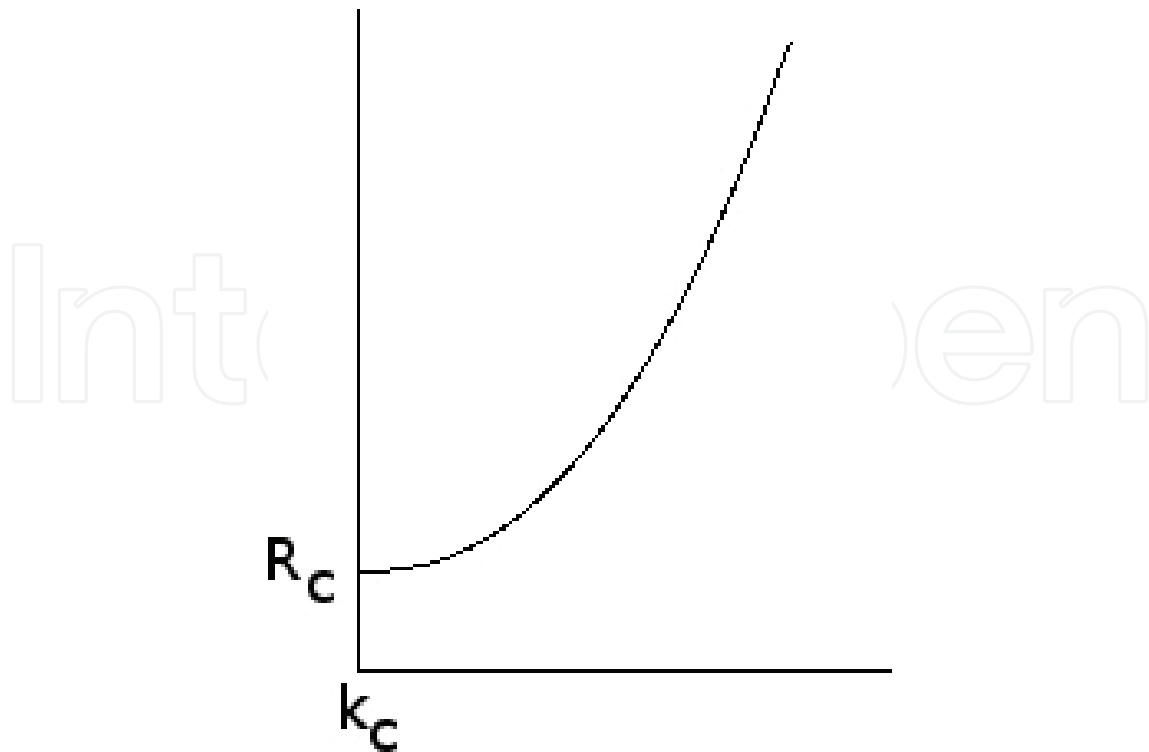while for the upper bounding wall, its corresponding Biot number is

Figure 4. A curve showing the critical point (k_c, R_c) for steady convection of a fluid layer bounded by two perfect thermal conducting walls.

$$B_U = \frac{\sqrt{k^2 + \overline{\kappa_U}\sigma}}{\overline{X_U}\tanh[\sqrt{k^2 + \overline{\kappa_U}\sigma}\,\overline{d_U}]} \tag{15}$$

The set of Eqs. (4)–(7) represent the complete eigenvalue problem for the Rayleigh number R. Then, by mapping with the thermal conductivity ratios $\overline{X_L}$ and $\overline{X_U}$ from limiting values of zero to infinity it is possible to bridge the two ideal case mentioned above. The importance of the thermal and geometrical properties are shown in the following set of curves of criticality, given in **Figures 5** and **6**.

The middle region of the curves in **Figures 5** and **6** show two graphs that collapse in the extremes. This middle region clearly shows the effect of the walls thicknesses which disappears as the thermal conductivities ratio approaches zero or a very large magnitude. This behaviour can be easily explained by recalling the two occurring ideal situations when the thermal conductivity ratios $\overline{X_L}$ and $\overline{X_U}$ are zero or infinity, for an insulator or a perfect thermal conductor. No matter how large the thicknesses of the perfect thermal conducting or perfect insulating walls are, the heat shall be transferred instantaneously. This can be also mathematically seen from Eqs. (12) and (13) since in the limit of insulating walls, the boundary conditions reduce to $\frac{d\theta}{dz} = 0$ and in the limit of perfect thermal conducting walls, these conditions give $\theta = 0$. In this last sentence, it should be remarked that the thicknesses of the bounding walls, and that of the fluid, vanish.
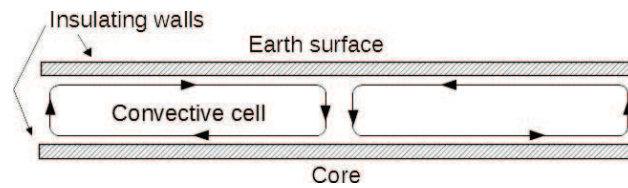
Figure 5. Curves of criticality for the Rayleigh number showing the effect of the thermal conductivities and thickness of the walls on the Rayleigh convection in a Newtonian fluid. For these curves of criticality $\overline{X_L} = \overline{X_U}$ and $\overline{d_L} = \overline{d_U}$ was set.
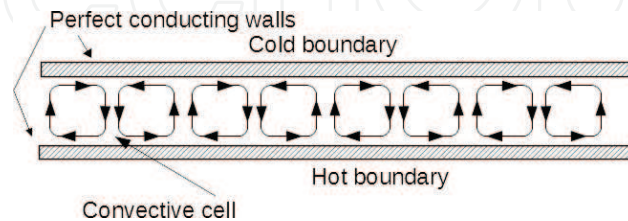

Figure 6. Curves of criticality for the wavenumber number showing the effect of the thermal conductivities and thickness of the walls on the Rayleigh convection in a Newtonian fluid. For these curves of criticality $\overline{X_L} = \overline{X_U}$ and $\overline{d_L} = \overline{d_U}$ was set.

As it can be seen, from **Figure 5**, in the extremes of the horizontal axis, R_c = 720 and 1707.96. Correspondingly, from **Figure 6**, in the extremes of the horizontal axis k_c = 0 and R_c = 3.12. The data presented in the set of curves of criticality were first reported by Riahi [14, 15] and later by Cerisier et al. [7].

Pérez-Reyes and Dávalos [6] presented a study of the influence of the thermal and geometrical properties on the convection of viscoelastic Maxwell fluids. They found a behaviour similar to that shown in **Figures 5** and **6** and reported the appearance of a codimensional-two point. This is due to the competition between stationary and oscillatory convection to destabilize the system. Besides, if the linear stability is changed, then the nonlinear stability results are to be changed too.

4. About the basic state for the temperature

At this point, some qualitative information may be given about the basic state for the temperature given in Eqs. (1)–(3). The obvious question is: are the parameters $\overline{X_L}$ and $\overline{X_U}$, and the Eqs. (1)–(3) useless? It should be mentioned that these are not used in the computation of the data shown in **Figures 5** and **6**. This is a direct consequence of the symmetry of the equations and of the adimensionalization of the problem. In fact, the Rayleigh number in **Figures 5** and **6** is modified by a factor $1/(1 + \overline{X_L d_L} + \overline{X_U d_U})$. This is a shortcut in the solution to the problem. Furthermore, the proper basic state should be one including the three basic states, Eqs. (1)–(3) which would have the same form as that of Eq. (3).

This last equation may become important for problems represented by differential equations with additional terms to the base model as shown in Eqs. (10) and (11). For example, a

comparison of the basic state for the temperature in any of the ideal cases would show that at the boundaries $T_{F0} = T_L$ and $T_{F0} = T_U$, respectively. Eq. (3) does not match these results at the boundaries due to the effect of the thickness of the walls. However T_{F0} may satisfy the same requirements through T_{L0} and T_{U0}.

The additional terms to the base, shown in Eqs. (10) and (11), model equations for the hydrodynamics in the fluid layer could come from variations in the viscosity of the fluid, for example. In situations where the viscosity varies with temperature, its effect appears in the viscous term of the momentum balance equation. This type of problems has been studied by Palm et al. [16], by Wall and Nagata [17] and by Wall and Wilson [18], among others. The working equations of these studies show that a temperature dependent viscosity may introduce terms requiring T_{F0} and dT_{F0}/dz. This can be seen in the following set of equations corresponding to the problem of thermal convection in a fluid layer with temperature dependent viscosity being heated from below:

$$C_1 \frac{d^4 W}{dz^4} + \overline{\gamma} \frac{d^3 W}{dz^3} + C_2 \frac{d^2 W}{dz^2} - k^2 \overline{\gamma} \frac{dW}{dz} + C_3 W + Rk^2 \theta = 0 \tag{16}$$

$$W + \frac{d^2 \theta}{dz^2} + C_4 \theta = 0 \tag{17}$$

where the coefficients are defined as:

$$C_1 = \overline{\gamma}(T_{F0} - T_L) - 1 \tag{18}$$

$$C_2 = Pr^{-1}\sigma + k^4[\overline{\gamma}(T_{F0} - T_L) - 1] \tag{19}$$

$$C_3 = -Pr^{-1}\sigma k^2 - 2k^2[\overline{\gamma}(T_{F0} - T_L) - 1] \tag{20}$$

$$C_4 = -(\sigma + k^2) \tag{21}$$

So that the second and third terms in right-hand side of Eq. (1) should appear in the final eigenvalue problem explicitly. These additional terms may be of interest for a proper understanding of the convective phenomena.

5. Influence on the pattern formation

The pattern formation in convective systems is a subject widely studied. One common approach to the study of convective patterns is the problem of pattern selection in a given geometry [5]. The formation of convective cells is highly dependent on the boundary conditions. This is true not only because of the mathematical structure of the boundary conditions but also because of the nature of the bounding surfaces. In the limiting case of insulating walls are very large convection cells of slow motion. A good example of this system is the mantle convection occurring between the earth core and its surface. **Figure 7** shows a simple scheme of a convection cell driven by the difference of temperature between the core and the
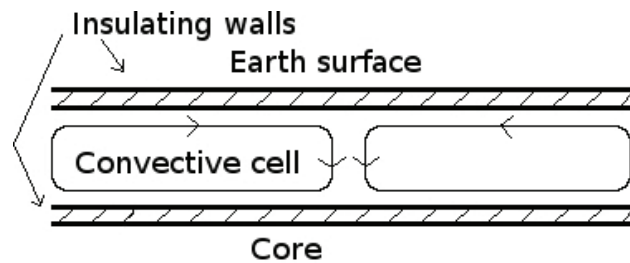
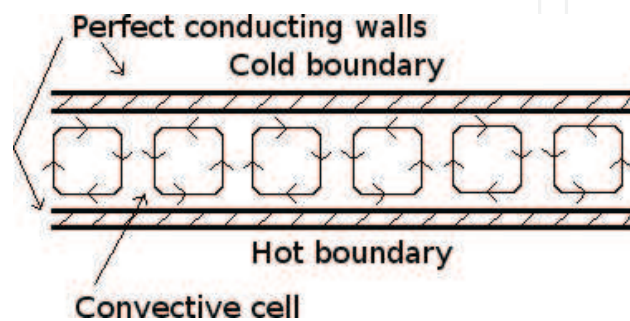**Figure 7.** A simple representation of the mantle convection.

Figure 8. A simple representation of the thermal convection in a fluid layer bounded by two perfect conducting walls.

surface. It has been demonstrated that these convection cells are very large and that the liquid rock material moves slowly [3, 12, 19]. Thus, a feature of this type configuration is the presence of large convection cells of slow moving fluid, since the critical wave number is zero ($k_c = 0$).

The case of perfect conducting walls is quite different. In this configuration, more than one convection cell appear since the critical wavenumber is finite ($k_c = 3.12$). A representation of this case is given in **Figure 8**. Another consequence of the idealized perfect thermal conducting walls is a faster re-circulating fluid motion in comparison with the previous described situation. The study of these ideal cases includes a variation in which one of the walls is a perfect thermal conductor while the other is considered an insulating wall.

The study of the effect of the boundaries on the pattern formation has called the attention of a number of researchers and diverse cases have been studied. Chapman and Proctor [20] were interested in the behaviour of the system when non-ideal walls were considered so that they applied an analytical approximation for poorly thermal conducting boundaries. Chapman and Proctor [20] were able to calculate critical wavenumbers different from zero which made more sense for experimentalists. Additionally, Proctor [21] studied the selection of patterns in finite domains for rolls, square, rectangles and hexagonal patterns. The approximation of poorly thermal conducting walls becomes so interesting and tractable that these ideas were extended to problems of double diffusion by Proctor [22] and by Cox [23] for example. Other areas of the fluid mechanics have used similar ideas like in magnetohydrodynamics by Dávalos-Orozco [24] and in convection of second order fluids by Dávalos-Orozco [25], for example.

The pattern formation is studied as a nonlinear problem which is linked to the results of the linear hydrodynamic stability of the system. Therefore, introducing more general thermal boundary conditions shall convey more information to the study of the behaviour of the moving fluid. Some efforts have been made including the thermal and geometrical properties of the walls by Riahi [14], for example. The thermal conductivities and thicknesses of the walls shall clearly modify the convective cells. One clear effect would be on the size of the patterns. It is possible that the selected structure remains at least not for set domains like squares, rectangles, etc.

6. Some problems to engage

Most of the problems with analytical solutions and interesting physical mechanisms in it have already been studied. In fact, fluid mechanics is called a mature area of physical sciences. The remaining problems are complex, mostly without analytical solutions and with many variables involved.

Some interesting problems to study in hydrodynamic stability, linked to the thermal boundary conditions, are mentioned here.

- There is lack of information about the role of the thermal diffusivities of the walls and the fluid layer for intermediate values of the ratio of thermal conductivities. In the literature, it has been assumed that the fluid layer and the bounding walls have the same thermal diffusivities.

- The Robin type thermal boundary conditions are assumed to convey pore physical information to the eigenvalue problem. To the best knowledge of the author, there are no reports about experiments to corroborate this.

- Nonlinear problems about pattern selection shall become more difficult to handle. Perhaps, more efficient numerical computations would be needed.

7. Discussion

A number of problems have been discussed here. The physical implications of the thermal boundary conditions were highlighted in terms of the thermal conductivities and thicknesses of the bounding walls. Although only the classical horizontal infinite two-plate configuration was considered, the main ideas can be used to understand more complex geometries.

The ratio of thermal conductivities allow the mapping of the critical conditions for the onset of convection from insulating walls ($\overline{X} \to \infty$) to perfect thermal conducting walls ($\overline{X} \to 0$). On the other hand, small thicknesses ratio destabilize the system while large thicknesses ratio help to stabilize the fluid layer. The physical mechanisms behind this behaviour is explained through thermal diffusion times across the bounding walls and the fluid layer. These observations are valid for fixed values of the thermal diffusivities.

It is shown from the qualitative point of view, that additional terms in the base model equations of the problem of Rayleigh convection may carry more information from the temperature basic states of the bounding walls and fluid layer to the eigenvalue problem. The temperature basic states of the walls may help to understand the physical mechanisms involved in diverse thermal convection problems.

Acknowledgements

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