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Joining in Nonrigid Variation Simulation

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Abstract

Geometrical variation is closely related to fulfillment of both functional and esthetical requirements on the final product. To investigate the fulfillment of those requirements, Monte Carlo (MC)-based variation simulations can be executed in order to predict the levels of geometrical variation on subassembly and/or product level. If the variation simulations are accurate enough, physical tests and try-outs can be replaced, which reduce cost and lead-time. To ensure high accuracy, the joining process is important to include in the variation simulation. In this chapter, an overview of nonrigid variation simulation is given and aspects such as the type and number of joining points, the joining sequence and joining forces are discussed.

Keywords: variation simulation, joining, spot welding, clip fastener, welding, sequence, holding forces, assembly forces

1. Introduction

Computer-aided tolerancing (CAT) is a research area within computer-aided technologies. Tolerances are a way to describe the allowed variation in a specific dimension or feature. Variation on part level will, together with a number of other influencing factors, control the variation on product level of an assembled product. To fulfill assembly requirements as well as functional and esthetical requirements of the final product, the allowed tolerances play an important role. **Figure 1** illustrates a product not fulfilling the esthetical requirement of parallelism of a split line. These kinds of problems can be caused by geometrical variation. However, tight tolerances are costly and tolerancing is therefore a balancing between cost and quality [1]. There are also other methods, such as stability analysis [2], to decrease the sensitivity to geometrical variation on part level. In that way, the geometrical quality of the assembly can be improved without tightening tolerances with increased cost as a consequence.



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Generally, methods to minimize the effects of geometrical variation are gathered under the term "geometry assurance" [3].

Within CAT, variation simulation is used to support the choice of design concepts and tolerances. Variation simulation is used to predict the geometrical result of a final assembly, given part variation as inspection data or tolerances and information about the joining and assembly process. By the use of variation simulation, different design concepts can be compared early on in the product development cycle and the predicted variation on assembly level can be compared to geometrical assembly requirements. In this way, the product development time and cost can be reduced and the geometrical quality of the final assembly or product can be increased. There are several different software tools for variation simulation, such as 3DCS [4], VSA [5] and RD&T [6].

In this chapter, both the terms variation and deviation are used. On individual basis, a part or an assembly can deviate from their nominal value in a critical dimension. The reason for this



Part variation from stamping

Variation due to joining process

Variation due to manual assembly

Variation in final product

Figure 1. A nonparallel split line, caused by variation due to the stamping, joining and assembly processes.



Figure 2. Factors affecting geometrical deviations of a subassembly.

is usually a large number of underlying factors. The effects from these factors can lead to a mean shift, where all individual parts in a population deviate from nominal values in a similar way, or to a variation affecting the deviations of the individual parts in a population independently. Usually, a deviation is caused by a combination of the two phenomena. If a sample of parts or subassemblies is considered, there will be a mean shift and a variation within the sample.

The accuracy of the simulations is a critical issue—if the simulations shall be trustworthy and simulation results shall replace physical tests and prototypes, the difference between simulated result and real result must be minimized. To reach a good agreement, it is important to include in the simulation as many as possible factors that influence the result in reality. In **Figure 2**, different factors affecting the geometrical result of an automated joining process in the body shop are outlined.

The main factors affecting the geometrical variation and mean shift of a subassembly are:

- **Part variation**, i.e., how a sample of parts differs from nominal geometry. This variation is due to variation in material properties and variation in previous manufacturing processes. A good estimate of part variation is necessary to include in the variation simulation. This can be done by using part tolerances or inspection data. How to include inspection data in the simulation is treated in [7, 8].
- **Fixturing**, i.e., how the parts are positioned in the assembly fixtures during joining. This is usually referred to as the locating scheme [9, 10]. The positions of locating points (also referred to as locators) and additional clamps are important since they control the robustness (i.e., sensitivity to part variation). A rigid part has six degrees of freedom which are locked by the master locating points, while a nonrigid part can be overconstrained and additional clamps are added to the master locating points.

Also deviations in the fixture caused by lack of repeatability [11, 12] and the clamping sequence [13] affect the result. The forces applied in the clamps constitute another important factor. A too low clamping force can result in a locator not be able to force the part to nominal position and a too high clamping force may lead to unwanted steering due to high friction between the part and the clamp. Friction and steering directions are also related to the size and shape of the clamp [14]. Fixtures are compared to the parts to be joined very stiff and are therefore modeled as rigid bodies in this chapter.

• The joining process, including the number, position and sequence of joining points. For some joining processes, such as spot welding, the result might also be influenced by variations in the welding gun, leading to that the welding gun forces the assembly to a non-nominal position. Furthermore, the heat generated by the spot welding process can lead to local deformations, usually considered to be of minor importance [15]. If there is a large gap between the parts to be joined, due to part or fixture variation, and the force applied by the joining tool is too low to close the gap, this will lead to failing joints, which of course affect the geometrical outcome and also the strength of the assembly. Spot welding is a frequently used joining method in automotive industry, but also methods such as riveting, clinching, clip fastening and adhesive joining occur.

- Gravity can affect part positions and lead to parts being assembled in erroneous positions.
- **Material properties** such as stiffness and density. The part deviations are usually assumed to be small compared to the part dimensions in order to speed up the calculations in nonrigid variation simulation. The material models are linear, and it is assumed that the changes in the stiffness matrix due to part and positioning variation can be neglected. Only elastic behavior is taken into consideration.
- **Collisions** between parts lead to forces making the parts deform. In the simulation model this can be taken care of using contact modeling [16, 17].

In this chapter, the main focus is on the joining process and on how to include that in variation simulation. The joining methods considered are mainly spot welding, riveting, bolts, clip fasteners and continuous welding. Some attention is also paid to how to minimize the geometrical variation caused by the joining process, especially by finding good joining sequences.

2. Nonrigid variation simulation

In nonrigid variation simulation, the parts are allowed to bend, i.e., there is no assumption that the parts are rigid. To perform a nonrigid simulation, finite element analysis (FEA) is integrated in the simulation. The benefits of nonrigid variation simulation, compared to the rigid one, are in many cases an increased accuracy and also the ability to overconstrain parts and assemblies, calculate forces and how the parts are bent, to see the effects from gravity, include effects from heat, etc.

FEA in nonrigid modeling is used to discretize the geometry using a set of finite elements gathered in a mesh. All the elements in the mesh are joined by shared nodes. In nonrigid analysis, the displacements in the nodes are studied. The magnitudes of the displacements depend on what forces that are applied to the geometry. The forces can be caused by, for example, variation in single parts, assembly fixtures and the joining process.

To calculate the forces, the stiffness of the material must be known. The stiffness information is gathered in the so-called stiffness matrix and describes how resistant each node is to deflection when exposed to an applied force.

The sum of the forces must be in balance and FEA is used to solve the equilibrium equations describing this. Assembly deviations can be predicted through the use of this method.

The direct Monte Carlo (MC) simulation, combined with FEA, is a standard technique for the variation simulation of nonrigid parts. However, since a large number of runs are required to achieve satisfactory accuracy in a direct Monte Carlo simulation, the simulation time will be very long since a new FEA calculation must be executed in each run. Liu and Hu [18] presented a technique called method of influence coefficients (MICs) to solve this problem. The main idea of the proposed method is to find linear relationships between part deviations and assembly spring-back deviations. A set of sensitivity matrices, constructed using FEA, describes that linear relationship. These sensitivity matrices can then be used in the

simulations, and in this way the number of FEA calculations can be reduced. Camelio et al. [19] applied the method to a multistation system. Dahlström and Lindkvist [20] combined MIC with contact modeling. Contact modeling is a method to avoid that parts virtually penetrate each other during assembly. Contact modeling was also treated in [16, 17].

2.1. Simulation methodology

To include joining sequence in a nonrigid variation simulation methodology, the steps described below are generally necessary to include in the calculations to be able to predict variation and mean shift in critical dimensions of a subassembly:

Step 1: The parts are positioned in their assembly fixtures, and overconstrained locating systems (i.e., clamps) are applied. To clamp non-nominal parts, forces are applied.

Step 2: The parts are joined together according to a defined joining sequence. The gaps in the joining points are closed one by one using the defined sequence.

Step 3: When the last joint is set, the assembly is released from the clamps and is allowed to springback.

These steps can be included in a Monte Carlo (MC)-based variation simulation [17]. The assembly forces and the geometrical variation of the final assembly are calculated in each MC replication. For a node-node based balanced joining method, such as spot welding using a balanced gun, an overview of the modeling of Steps 1–3 is given below.

Modeling of step 1: Positioning/clamping the parts in the assembly fixture.

The parts *a* and *b* are positioned in their assembly fixtures. Overconstrained locating systems, i.e., clamps are applied. The magnitude of the gaps to be closed in the clamping points are collected in the vectors $\{\mathbf{u}_p^a\}$ and $\{\mathbf{u}_p^b\}$, respectively. The gaps are closed by applying the forces $\{\mathbf{F}_p^a\}$ and $\{\mathbf{F}_p^b\}$, respectively. The part stiffness matrices are represented by $[K_p^a]$ and $[K_p^b]$. The relations between the required forces and the gaps are described in Eqs. (1) and (2):

$$\left\{ F_{p}^{a} \right\} = \left[K_{p}^{a} \right] \left\{ u_{p}^{a} \right\}$$

$$\left\{ F_{p}^{b} \right\} = \left[K_{p}^{b} \right] \left\{ u_{p}^{b} \right\}$$

$$(1)$$

$$(2)$$

Modeling of step 2: Welding the parts, weld point (wp) i = 1, ..., N

To close the gap in weld point (wp) *i*, a force $\{F_A^i\}$, where the index *A* stands for assembly, is applied and the relation becomes:

$$\left\{\mathbf{F}_{\mathbf{A}}^{\mathbf{i}}\right\} = \left[\mathbf{K}_{\mathbf{A}}^{\mathbf{i}-1}\right] \left\{\mathbf{u}_{\mathbf{A}}^{\mathbf{i}-1}\right\} \tag{3}$$

After setting wp *i*, the assembly is released from its fixture and will then springback (the release step is executed since it gives clearer and easier calculations compared to accumulating

the resulting forces for each wp). To calculate the springback, the stiffness matrix $[K_A^i]$, is used. This matrix describes the stiffness of the assembly after wp *i* is set and is determined using Eq. (4).

After the springback calculation, the assembly is brought back to its clamped position again, and the forces $\{F_A^i\}$ required to do this are registered. Both clamping forces and forces due to contact modeling are taken into consideration.

The stiffness matrix K_A^i is updated for every new wp by adding a matrix $\left[K_{wp(ia,ib)}^i\right]$ to the stiffness matrix from the previous step. The added matrix describes how the new wp locks three translations and three rotations. This is described in Eq. (4):

$$\left[\mathbf{K}_{\mathbf{A}}^{\mathbf{i}}\right] = \left[\mathbf{K}_{\mathbf{A}}^{\mathbf{i}-1}\right] + \left[\mathbf{K}_{\mathbf{wp}(\mathbf{ia},\mathbf{ib})}\right] \tag{4}$$

For the very first wp, the matrix $[\mathbf{K}_{\mathbf{A}}^0]$ refers to the original part stiffness matrices, i.e., one for each part. The deviation used for the first welding point, $\{\mathbf{u}_{\mathbf{A}}^0\}$, corresponds to the part deviations.

Modeling of step 3: Final springback

When all welding points are set and the assembly is unclamped it will springback due to previously applied forces. To simulate this springback, a force $\{F_a\}$ corresponding to the force $\{F_A^N\}$, but in the opposite direction, is applied. Using the relation

$$\{F_a\} = \begin{bmatrix} K_a^N \end{bmatrix} \{u_a\}$$
(5)

the final assembly deviation $\{u_a\}$ can be calculated:

$$\{\mathbf{u}_{\mathbf{a}}\} = \left[\mathbf{K}_{\mathbf{a}}^{\mathrm{N}}\right]^{-1}\{\mathbf{F}_{\mathbf{a}}\} \tag{6}$$

The above description was intentionally kept simple in order to provide a first rough picture of the modeling. It is in fact much more complicated. Still much simplified, the following summarizes what information that needs to be calculated in order to predict the final springback of an assembly.

Step 1: The parts are positioned/clamped in their fixtures. Forces are applied to clamp the nonnominal parts. Overconstrained locating systems are often applied, locking the part in more than six degrees of freedom. Simplified, the following are established in this first step:

- a) Position and orientation of the parts (or subassemblies) to be assembled, by locking each part in six degrees of freedom.
- **b)** Deformation of the parts due to possible overconstrained clamping of the parts to be assembled.
- c) The resulting penetrations or gaps arising between the mating surfaces. In the following this is referred to as the penetration state.

Step 2: The parts are welded together in a predefined welding sequence. The gaps in the weld points are closed one by one. The force needed to close a gap interplay with the forces arising in the mating surfaces, which in turn affects the penetration state. Solving the resulting force pairs can be done using contact modeling, using iterative [17] or more advanced optimization methods, e.g., quadratic programming [16]. The following is calculated for each welding point in this step:

- a) Force pair needed to close the gap in the welding point and the resulting force pairs necessary to hinder penetration of the parts.
- **b**) The resulting deformation of the parts due to introduced weld and contact forces.
- c) The resulting changes of clamping forces.

Step 3: After the spot welds are set, the assembly is unclamped and is allowed to springback. The following is then calculated, constraining the assembly in only six degrees of freedom:

- a) Changes in stiffness of the parts due to the added welds.
- **b)** Changes of the part geometries and the springback of the assembly, due to the release of contact and clamp forces.

Step 1b, Step 1c, Step 2 and Step 3 are iterated until all the spot welds are set.

Step 4: After the last spot weld is set, the assembly is unclamped and is allowed to springback. The resulting springback is established positioning the assembly in a targeting fixture. This fixture can be either constrained to evaluate the total springback or overconstrained to match the locating system used in the tolerancing of the assembly. A bit simplified, the following steps are repeated taking into account the new locating system:

- a) Step 1, to calculate the final penetration state.
- **b)** Steps 2a and 2b, to calculate the final shape of the assembly.

As previously mentioned, the above steps can be included in a Monte Carlo (MC)-based variation simulation [21]. The joining forces and the geometrical deviation of the final assembly are calculated in each MC replication. The method described in this section is implemented in the variation simulation tool RD&T [6], which is used for the case studies presented in this chapter.

For large assemblies with dense meshes and many weld points, the model size and the simulation time has been an issue, although the MIC is used. However, there has recently been some work done in this area to reduce the simulation models by Lindau et al. [22].

2.2. Case study

In **Figure 3**, a case study of an A-pillar can be seen. This assembly consists of two parts which are joined with nine spot welds. The joining sequence used in industrial production is seen in the figure. For this case study, there exist inspection data for both part A and part B. This data can, together with information about the locating schemes, be used as input to a variation simulation. The result from the simulation, based on the method described above, is then



Figure 3. An A-pillar assembly. The two parts are joined by nine spot welds.

compared to inspection data on assembly level in a number of inspection points, see **Figure 4**. The data is originating from industrial settings, but it should be pointed out that this subassembly was chosen for analysis since there have been a lot of difficulties associated with it. The level of variation in the subassembly has been unacceptably high and the behavior of particularly part A has been hard to predict.

The case study is used to illustrate the difference between rigid and nonrigid modeling and also how the inclusion of contact modeling and joining sequence can affect the simulation result.

The inspection data on assembly level is blue in **Figure 4**. As can be seen, the rigid prediction (black line) is quite far from the inspection data. Nonrigid modeling but without contact modeling (pink line) gives approximately the same result as the rigid modeling. When adding contact modeling (red line) a much better agreement with inspection data is achieved and when also welding sequence is added to the simulation model (green line), additional improvements can be reached.

The inspection points 51–59 are all positioned on part A. This part is 0.97 mm thick compared to 1.6 mm for part B. During inspection, the locating scheme from part B is used. These



Figure 4. A comparison between simulated data and inspection data.

Model	Rigid	No contact	Contact	Contact and wp seq.
Correlation	0.26	0.02	0.87	0.87

Table 1. Correlation between simulated data and inspection data for different modeling alternatives.

factors simply that it is more difficult to predict the springback for part B than for part A, and from **Figure 4** it can be seen that there is a bigger difference between the different simulation results for the points on part A. Correlation values, which measure the linear relationship between simulated data and inspection data, for inspection points on part A can be seen in **Table 1**. As can be seen, the correlation increases when more influencing factors are taken into account in the simulation model. However, for this case study there is no difference between the correlation values with and without welding sequence. The deviations between simulated and inspected values are though slightly less for the model where welding sequence is included.

For the full simulation model the correlation is 0.87 which is a very high value. This indicates that the trends are captured very well by the simulation model. However, there is still a gap between simulated and real values for some of the inspection points. This needs to be closed by refinement of the simulation methodology and/or better input data. The input data for this case study consist of inspection data on part level. The parts should ideally be measured unclamped, i.e., not forced to a specific shape. In this case, additional clamps were applied, leading to an overconstrained condition and information loss on the actual springback shapes of the parts [23].

3. The type and number of joining points

In this section, different joining methods will be discussed as well as how to choose geometry points. Geometry points are the set of joining points that are assumed to lock the geometry of a subassembly. These points are set first in a joining sequence.

There are many different joining methods and as mentioned in the Introduction, the ones considered here are joining by:

- Bolts, rivets, clinches and spot welds: These are treated together, since they all are assumed to lock two parts together in all degrees of freedom. Therefore, they can be modeled in the same way. The heat induced by the spot welding, and possible local deformation caused by this, is not considered here. Neither are local deformations caused by clinching.
- Clip fasteners: clip fasteners are often used to join dissimilar material, such as plastic parts to more rigid, structural parts. The clip fasteners can lock the parts in varying degrees of freedom depending on their configuration. Clip fasteners are discussed in Section 6.
- Continuous welding: continuous welding is used to join two structural parts. The main challenge in modeling continuous welding with respect to geometrical deviation is that the heat generated by the welding process leads to deformations of the parts. Continuous welding is discussed in Section 7.

In FEA-based variation simulation the joining is modeled by constraining the allowed displacement of one or several nodes from one part to corresponding nodes on the other part. These corresponding nodes are in previous work [16, 17] referred to as node-pairs. The constraints set on the nodes in a node-pair depend on the type of joint to be modeled. Depending on the type, the relative displacement of the corresponding nodes are locked in one to six degrees of freedom (dof), where three dofs prevent possible translation in orthogonal directions and three dofs prevent the rotations.

3.1. Geometry points

The geometrical outcome of the assembly is of course affected by the number and position of the joining points. Generally, the choice of joining points is based on requirements of strength of the assembly and also the accessibility for the joining tools.

The geometry of an assembly is mostly affected by the first joining points when the joining is done in a sequence. To set all joining points might be a time-consuming task, and for spot welding in automotive industry, the set of spot welds is often divided into two. The first set of points, the geometry points, is supposed to lock the geometry of the assembly. After the geometry points are welded, the assembly can be released from its fixtures and the remaining points, the respot points, or simply the respots, can be welded in any of the following stations with minimal effects on the geometry. Which respot that should be set in which station is a question about balancing the capacity between stations and robots to minimize cycle time [24]. By this procedure the cycle time can be kept down and the welding sequence for the respots is chosen mainly with respect to cycle time. To maintain a high geometrical quality and a low



Figure 5. Factors affecting the choice of geometry points. The results are based on interviews with engineers in automotive industry.

cycle time, the division of a set of welding points into geometry points and respots is an important task. Generally, the three criteria below should be taken into consideration when choosing the set of geometry points.

Criterion 1: assure geometry robustness and product functions. The assembly should be geometrically robust, i.e., insensitive to variation, and the key features of the product should be assured. This criterion is evaluated after all joining points are set, but since the joining sequence affects the outcome, this is necessary to keep in mind when choosing the geometry points.

Criterion 2: assure rigidity of parts to withstand forces. The assembly should, after setting the geometry points, be stiff enough to withstand the applied forces from the subsequent processes, such as forces from the robot's gripper, welding guns and handling operations between stations.

Criterion 3: minimizing the influence of respots. A minimal influence from the respots on the geometry of the assembly is sought.

In industry, the choice of geometry points is often based on experience. A small interview study, based on 14 engineers from two different companies in the automotive industry, has been performed in a master thesis [25]. The study focused on spot welding and shows that choosing geometry points is a complex task and a lot of different considerations need to be taken. In **Figure 5**, an overview of factors affecting the choice is given. The factors are categorized using the criteria 1–3. As can be seen, factors such as design concept, locating schemes and material properties affect the choice.

Based on the interviews, some best practice guidelines to select a set of geometry points were established:

• Choose geometry points close to the locators and clamps.

- Spread the geometry points over the surfaces to be joined.
- Make sure that there are geometry points to lock the main axis of the assembly, i.e., choose geometry points on surfaces with different normal directions.

This complex process of choosing geometry points can be supported by the use of variation simulation. Different sets of geometry points can be compared and for each set, the criteria listed above can be evaluated in software for variation simulation.

A case study, consisting of five parts, is used to illustrate the idea, see **Figure 6**. Here, a color coding of the difference between geometrical deviations from nominal values before and after the respots are set is shown for four different sets of geometry points. Blue colors correspond to a very small difference and red values indicate bigger values. Therefore, from this aspect, the second set of geometry points is preferable.



Figure 6. The differences between geometrical deviation before and after the respots are set for four different sets of geometry points.

4. Joining sequences in variation simulation

The joining sequence is well known to affect the geometrical outcome of an assembly. In **Figure 7**, a subassembly of a reinforcement and a torsional stiffness bracket is shown. The upper part and the lower part are joined using seven spot welds, seen in the left part of the figure. The result is evaluated using the locating scheme for the lower part, why most of the variation in the assembly is seen on the upper part. Red color indicates high level of variation and blue colors indicate no, or low, variation.



Figure 7. The importance of joining sequence for geometrical variation in an assembly. Left: the different spot welds. Middle: geometrical variation in the assembly for a bad joining sequence. Right: geometrical variation in the assembly for a good joining sequence.

The color-coded pictures in the middle and to the right show the simulated result for two different spot welding sequences. The major difference between the two different sequences is that the good sequence (picture to the right) starts with the spot weld set in *z*-direction (wp 6 in the figure), while that spot weld is set last in the picture in the middle.

This example shows the importance of choosing a good joining sequence. The joining sequence is usually set early on in the product development phase and by finding a good sequence the geometrical variation can be reduced without additional costs.

The sequence should be chosen in such a way that requirements on both geometrical quality and cycle time can be fulfilled. As mentioned in the previous section, the sequence of the geometry points is often chosen with respect to geometrical quality and the sequence of the respots with respect to cycle time.

The joining sequence is often discussed in terms of spot welding sequence, but the problem of finding a suitable joining sequence is of course not less important for joining with rivets, bolts or clip fasteners.

There are two main problems with finding a suitable joining sequence. The first is that this is a fast growing problem. The number of possible sequences for *n* joining points is *n*! If there are, for example, 10 different joining points the number of possible joining sequences is 3,628,800. Even with a fast simulation tool where each sequence can be evaluated in minutes it will take many years to test all sequences. The second problem is that in most cases there is no analytical function relating the sequence to the geometrical variation of the assembly. The relation between input and output is also highly nonlinear since for some input, the parts collide with each other and the assembly is deformed.

There is some research done on optimization of spot welding sequences. Shiu et al. [26] investigate the relationship between stress build-up due to different spot welding sequences and the resulting dimensional variation. General guidelines for welding sequences are also

established. A method for quality and throughput optimization based on a systematic search algorithm was suggested in [27].

Liu and Hu [28] present two principles for minimizing the dimensional variance, namely to weld from weak to strong and to weld simultaneously if possible. This work was used as a starting point in [29] where different general strategies for finding optimal spot welding sequences were evaluated on eight different industrial case studies. The most promising strategy was to investigate how a disturbance in each weld point w_i affects the gap in the unwelded joining points and in each step choose the welding points w_i that affected the other points the most. The strategy worked out quite well for the case studies, but the results were not completely convincing.

The major part of the work done in joining sequence optimization has however been based on genetic algorithms. Liao [30] is using genetic algorithms to find the optimal number and position of spot welds. The objective function to be minimized is the weighted sum of the deviations and/or variations in a number of predefined inspection points. Xie and Hsieh [31] minimize the deformation in a number of user-defined points by using genetic algorithms. They also take cycle time into consideration. The developed algorithm is implemented in the software EAVS (elastic assembly variation simulation). Only deformation, no variation, is considered in their work. Genetic algorithms are also used by Segeborn et al. [32, 33] to minimize geometrical variation and cycle time.

A drawback of GA is that even though not all possible sequences need to be tested, there is still a need to run a non-negligible portion of all sequences, which might be time consuming. Furthermore, it is difficult to interpret the result and understand the reasons for a certain sequence being the best one.

An interview study, presented below, has been made to clarify how experienced engineers in industry find good spot welding sequences. Even though the choice of sequence is highly case-dependent and based on craftsmanship, some guidelines have been reached.

4.1. Geometry-dependent guidelines for choosing spot welding sequence

This study is a continuation of the study described in Section 3 about how to choose geometry points. According to the engineers participating in the study, the task of choosing geometry points and finding a suitable sequence can be done independently and the set of geometry points should of course be established before the sequence is set. This reduces the problem of finding good sequences significantly.

Two special cases where the welding sequence has a significant influence on the geometrical quality of the subassembly were pointed out. These are illustrated using basic sketches in **Figure 8**.

The first case is described in **Figure 8(a)**, where one thinner and one thicker part is joined with weld points in different directions.

By looking at the requirements and shape of the assembly, it can be realized that an important requirement is that the bottom surfaces meet. Thus, the best option is to start with wp1 and close the biggest gap to secure this requirement. Since the inner flanges are thinner than the



Figure 8. Two typical cases when a spot welding sequence should be chosen. The green circles with numbers indicate different welding points.

outer ones, they will have the ability to bend after welding the middle point. If wp2 and wp3 should have been set first, this would result in a stiffer assembly and forcing the parts together in wp1 would then probably have caused a distortion of the assembly. By closing the gap in wp1 first, no new gaps in wp2 or wp3 will be introduced and thereby no distortion will be added when welding these points.

In **Figure 8(b)**, part variation leads to a gap between the parts to be joined. Here, the welding sequence should be chosen in such a way that the material flow is controlled. The adjustable locator is assumed to only steer in the normal direction of the surface, i.e., the material is allowed to glide under the locator when a force is applied by the welding gun. The preferred sequence here is to first set wp2, since it has the biggest effect on material flow. To start with the weld point that affects the material flow the most might seem as an easy task when looking at the sketch in **Figure 8**. However, **Figure 8** illustrates only two of many possible situations and only two dimensions. For three dimensions, this is more complex and simulation software can be a good support.

5. Simulation of distribution of joining forces

When joining two parts, a force must be applied to press the parts to be joined against each other before the actual joining takes place. In resistance spot welding, the parts must be

pressed against each other in order to allow the current to pass through both parts and melt them together. If a clip joint is used, a force is needed to apply the clip fastener, and so on. The quality and strength of the achieved joint are usually affected by the magnitude of the applied force.

Part variation and variation in assembly fixtures may lead to gaps between the parts in the locations of the joining points. These gaps need to be closed, and some of the predefined joining forces are then consumed for closing the gaps. Therefore, the variation in initial gaps needs to be taken into consideration when defining suitable joining forces. This variation can be predicted using the methods described in the earlier sections. The term joining force refers to the total force applied to join the parts and the term assembly force is used for the force needed to close the initial gap. The assembly force is by that a subset of the total welding force. The total joining force is usually constant, and therefore it is important to minimize the variation in assembly forces in order to achieve joints of good quality.

A too low force will obviously lead to a joint of low quality, and also a too high force might lead to a joint that will decrease in strength. For spot welding, a too high electrode force will increase the welding contact zone and thereby decrease the electric resistance, leading to a too small nugget size. In this work the variation in assembly forces is considered. This should generally be more interesting than looking at the mean value of the forces, since the total force applied from the welding gun can be adjusted to fit a certain level of assembly forces. A large variation in the assembly forces is more difficult to compensate.

Murakawa and Ueda [34] investigated the force needed to close an initial gap by applying parametric curves and approximation curves to two simple structures. Due to the variation in magnitude of the initial gap, the force required to close it will of course also vary.

Wärmefjord et al. [35] described a method for predicting variation in assembly forces due to part variation. In [36] the effect of variation in spot weld position on assembly forces was investigated.

To calculate the distribution of assembly forces, Eq. (3) is used. This is done for a case study shown in the left part of **Figure 9**. This is a D-pillar consisting of two parts joined by four spot welds. The four spot welds lead to 4! = 24 different possible spot welding sequences. The predicted values of the level of variation for the different spot welding sequences are illustrated together with the predicted maximum assembly force in any of the four spot welds. The maximum force equals the range in this case, since the minimum force is 0 N for every sequence.

The bars represent a value of the geometrical variation given by $6s_{RMS}$. The total root mean square (RMS) value for standard deviation is a general measure of the variation in all nodes of the assembly. For each node j, j = 1, ..., k, the magnitude of the variation in x-, y- and z-direction is calculated as the sum of the variations in the three directions:

$$s_j^2 = s_{jx}^2 + s_{jy}^2 + s_{jz}^2 \tag{7}$$

The RMS value is then calculated:

$$s_{RMS} = \sqrt{\frac{s_1^2 + \dots + s_k^2}{k}}$$
 (8)

The worst value (sequence 21) is 35% larger than the best value (sequence 3).

The maximum assembly force needed to close the gap, i.e.,

$$F_{A}^{\max} = \max_{i} \{F_{A}^{i}\}, \quad i = 1, ..., 4$$
 (9)

for the four different spot welds in every sequence is illustrated by the black line in **Figure 9**. The maximum assembly force for welding sequence 23 is 0.46 kN. This is a value similar to the sequence demanding the least assembly force, sequence number 9. It can be noted that these two sequences have similar levels of geometrical variation. Therefore, by choosing sequence 9 instead of sequence 23, the range of the assembly forces can be reduced without changing any tolerances or affecting the level of geometrical variation.

The maximum assembly force for a certain sequence does not, of course, occur in every Monte Carlo replication. The joining process should, however, be designed with those extreme values taken into consideration.



Figure 9. Simulation results for a D-pillar assembly.

6. Clip fasteners

Another kind of joining method is the use of clip fasteners. Clip fasteners are often used to join parts of dissimilar materials, such as plastic parts and more rigid, structural parts. From a modeling perspective, the major differences between clip fasteners and spot welds are that:

- **1)** Spot welds are usually assumed to lock all degrees of freedom of a node. A clip fastener is often allowed to slide in one or two directions.
- **2)** Spot welds are modeled using a node-node contact. Clip fastener is usually covering an area, see **Figure 10**. Therefore, for non-nominal parts, levering effects affecting the whole subassembly will rise and this must of course be included in the model.

For clip fasteners, an area for the clip should be defined and also what degrees of freedom that are locked by the clip. Also play should be possible to model. Both those issues are, in the approach presented here, solved using contact modeling. This is included in the theory described in Section 2.

Normally, the purpose of a contact pair is to draw two parts apart to avoid penetration. The same method can however be used to push two nodes together in a certain direction. By defining double contact pairs with opposite directions, the parts are joined together, but still no penetration is allowed. Assuming a rectangular clip fastener, as the gray rectangle in **Figure 11**, a double contact pair is defined in each corner (red circles in **Figure 11**). In this way the area of a clip fastener can be taken into account. Circular contact areas between the fastener and the part can be modeled by using additional contact points. Using the contact modeling approach, contact pairs can be defined for several directions, so the clip fastener can be modeled to lock movement in *X*, *Y* and/or *Z* direction in the coordinate system.

It is however quite common that a clip fastener is locking a part completely in one or two direction and that a play is allowed in one direction. In this section, a case study consisting of a wheel arch extension joined to a rear fender with five clip fasteners, see **Figure 12**. For the case study, circular clip fasteners in a slot hole, with 5 mm play in one direction are used, see **Figure 13**. The play can easily be modeled using the contact modeling approach. Instead of pushing two contact points together, a nominal allowed distance is defined where the points can move. If a point is outside this distance, the contact modeling is used to push the point back to an allowed position.

In the case study, the fender is made of sheet metal and the extension of the polymer acrylonitrile butadiene styrene (ABS). The clip fastener encircled in **Figure 12** is investigated more in detail.

Both the level of geometrical variation and the joining forces are of interest to predict. The joining sequence will affect the result, but in the case study here, a simplified model is used where all clip fasteners are assumed to be set simultaneously. Then the joining force equals the



Figure 10. Illustration of clip fasteners.



Figure 11. The clip fastener modeling approach.



Figure 12. A wheel arch extension joined to a rear fender using five clip fasteners.

holding force, which is the force the clip fastener must withstand after the parts are joined. There exist a number of different clip fastener alternatives and the expected holding forces affect the choice of fastener. Since the two parts are of different material, they will respond differently to changes in temperature. The effect from this on the holding forces will be investigated in future work.

The bars in **Figure 13** show how the result for holding forces and the RMS value of geometrical variation (see Eq. (8)) differs depending on modeling strategy. Two different modeling alternatives are compared:

- A node/node based approach where two nodes, one from each part, are locked together. This is the traditional modeling approach for clip fasteners.
- A new modeling approach, as suggested in this section, where the area of the clip fastener is taken into account. This modeling alternative is more realistic compared to the node/node based alternative.

For each model, a scenario with and without play is tested. 3000 Monte Carlo iterations are run and the results are illustrated in **Figure 14**. The results presented in the upper part of **Figure 14** are the maximum value of the holding forces (see Eq. (9)), taken over all iterations, in the clip fastener encircled in **Figure 12**. As seen from **Figure 12**, the node-node based joint with no area is differing from the other values. This is due to the lack of levering effects and this shows the



Figure 13. A clip fastener where play is allowed in one direction (indicated by red lines).



Maximal holding force [N]

Figure 14. Forces and geometrical variation for the different modeling alternatives.

importance of inclusion of the joint area. For both modeling alternatives, the versions without play result in higher forces. This is expected due to the extra constraints.

In the lower part of **Figure 14**, the RMS value, measuring geometrical variation on assembly level, is shown. The variation is reduced with more constraints, i.e., without play. Variation is not as sensitive as holding forces to changes in the area of the clip fastener and the difference between the node-node based model and the clip fastener model is not as large as when the holding forces were predicted. However, there is still a difference and to improve the accuracy of the variation simulation it can be concluded that the area of clip fasteners should be included in the simulation model, and this is extra important when simulating holding forces. It is also necessary to be able to include plays in the modeling.

7. Continuous welding and the effect from heat

To include the effects from continuous welding in variation simulation is a difficult challenge. The heat induced by the welding gives rise to deformations of the parts and also changes the material properties. Welding simulations are today done on nominal parts and each simulation is very time consuming. Simulation times of many hours are not uncommon. To include this in a Monte Carlo based variation simulation where thousands of iterations are run is not possible. However, it has been shown that the distortion introduced by welding is dependent on part variation and assembly fixture variation [37, 38], so in order to obtain an accurate result, a new welding simulation need to be run for each Monte Carlo iteration, i.e., there is a need for methods for predicting the effects of non-nominal welding.

Also for welding simulation based on nominal conditions the long simulation time and the large memory consumption has been a problem. Therefore methods have been proposed aiming at reducing computation time and memory consumption. These methods are usually based on the assumption that the main driving force causing distortion in the welding process is the shrinkage of the material that has been melted by the weld gun, and then cooled down to solidify and recover its stiffness. These methods are based on applying a strain to the structure, see for example [39] for inherent strain, [40] for models based on different strains in the longitudinal and the transverse direction and [41, 42] for volumetric shrinkage.

Test has been done to use the methods based on volumetric shrinkage for approximating nonnominal welding, see [43]. This method comprises three steps: the first step involves calculating the temperature during welding to approximate the section of the material that is melted. This can be done by simulating the heat transfer as the weld gun traverses the weld joint. However, although the heat transfer simulation is generally faster than the mechanical part of welding simulation, this is generally too slow. Instead, since the temperature surrounding the weld gun during welding in many cases can be considered to be constant it is possible to use a steady state heat calculation using an Eulerian frame. This requires the simulation of only one time step. The second step involves the approximation of the melted section of the structure. Using the result from the steady state heat simulation from step one, where the weld gun is positioned somewhere along the weld joint with the temperature distribution applied to the mesh, a surface is traveling through the weld path. This surface has a local coordinate system with the origin at the weld joint for a discrete set of succeeding points; it has further one axis pointing at the direction of the tangent of the weld path which is also the normal to this surface, and another axis pointing in the surface of the welded pieces. The third axis is pointing in the outward normal direction to this surface. As this surface is traveling through the weld path, it records the position, in this plane, for all nodes above the melting temperature. Hence, a 2-dimensional projection of the melted volume surrounding the weld gun is achieved. The smallest 2-dimensional convex hull is calculated from these projections.

In the third and final step, we are again traveling through the weld path with this surface. This time all nodes that are encapsulated in the tube that is the result of the 2-dimensional convex hull moving through the weld path are collected.





These nodes are assigned a temperature that corresponds to the temperature difference between melting temperature and room temperature. An elastic structural calculation gives us the final distortion after welding. An illustration of the different steps is given in **Figure 15**.

The method explained above only involved one thermal FEA and one elastic structural FEA. Therefore, it is much less time consuming compared to transient welding simulation. Therefore, it can be used for iterative use in, for example, variation simulation or optimization. However, since it is an approximate method it is suggested that transient welding simulation is used in later development stages to ensure that right design decisions have been made.

The method described above is called the steady-state, convex hull, volumetric-shrinkage method, or the SCV-method. The method was first proposed in [43] where it was compared to transient simulation, and developed to account for variation of the convex hull along the weld joint in [44] and the use of a temperature distribution in the melted zone for better approximation of the shrinkage in [45].

8. Summary and outlook

In this chapter nonrigid variation simulation has been described. This method is used to predict geometrical variation and mean shifts in critical dimensions of a subassembly or a final assembled product. For nonrigid parts, the inclusion of their nonrigid behavior is necessary in order to achieve good accuracy in the simulations. Moreover, a number of other factors affecting the geometrical outcome of an assembly need to be taken into account. In this chapter, how to model different joining processes in nonrigid variation simulation has been in focus. Among the joining processes that can be modeled are point-based methods locking two parts together in all degrees of freedom in a node-pair. In this class of joining, spot welding is

one of the most commonly used, but there are also clinching, riveting and bolting. How to model clips fastener has also been discussed. The clip fasteners are often used to join dissimilar materials, such as plastic and more structural parts. They often lock the part in less than six degrees of freedom and they also often cover an area, which can lead to levering effects and this must of course be included in the variation simulation. Furthermore, continuous welding has been discussed. The major difference compared to the other joining methods is that the geometrical distortion induced by the heat source must be included.

Topics such as the joining sequence, which strongly affect the geometrical variation on assembly level, and the number of geometry points have also been discussed. It is shown that how inclusion of the joining sequence leads to increased correlation between simulated and real result for an industrial case study.

Nonrigid variation simulation is for most cases where the parts are not completely rigid more accurate than the rigid simulation. Drawbacks can be lack of input data, support from variation simulation tools and more computer expensive calculations. The method described here is implemented in a commercial tool for variation simulation and there is ongoing work to reduce simulation time and model sizes. However, this is a critical issue for nonrigid variation simulation and there is still work need to be done in this area. Furthermore, the simulation tools need to be refined even more to be able to handle, for example, different materials and their material models and large deformations (outside the linear regime).

Moreover, with an increased access to inspection data in the footsteps of Industry 4.0, fast algorithms for quick adjustments of the assembly process with respect to, for example, joining sequences and position of locators, based on individual part deviations need to be developed.

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