Chapter 6

Modeling, Simulation, and Results of Their Use in Railway Vehicle Dynamics Studies

Krzysztof Zboinski

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/62105

Abstract

This chapter focuses on problems related to building mathematical and numerical models of railway vehicle dynamics and then using these models in the process of vehicle dynamics simulation. Finally, the results of such simulations devoted to selected dynamical problems are presented, highlighting the importance of powerful tools such as both the modeling and the simulation. The dynamical problems selected for the presentation concern railway vehicle stability and importance of kinematics accuracy for the description of the dynamics. These selected problems focus on the vehicle dynamics in a curved track, both in the circular and transition sections. Type of the chapter should be defined as the review paper, however, based on the authors' own results in the main.

Keywords: Railway vehicle, vehicle dynamics, curved track, transition curve, numerical simulation

1. Introduction

The present review chapter is based on the authors' results gathered and published in subsequent parts for many years of his work in the field of railway vehicle dynamics with focus on a curved track motion. The idea of the chapter is to combine all the results on the one hand and to select them suitably on the other hand. Both these demands are fulfilled in order to give a picture of comprehensiveness of the combined issues and not overload the reader with excessive details that could spoil the presentation of the chapter's main aim. The aim is to present the outstanding role of modeling, simulation, and results of their use, in a shortened way, in the contemporary research questions of the railway vehicle dynamics. However, this is going to be done through exploitation of the author's own results. The reference could be done here to [1], the comprehensive monograph in Polish (371 pages). This chapter is profiled differently, however, and results are presented in the concise way and internationally accessible form, i.e. in English.

1.1. The fundamentals in rail vehicle dynamics

It is rather a well-known fact (e.g. Refs. [1–3]), however sometimes being forgotten, that in rail vehicle dynamics, perturbations of vehicle motion relative to vehicle general motion are of the primary interest. The general motion of vehicle is of lesser interest as it is predefined by the track alignment (shape). Hence, in the general view, the vehicle reproduces the motion imposed by the track centerline shape, as the vehicle is guided by the track (rails). Such a guided motion is already known. Instead, the perturbed motion relative to the track centerline appears to be of real importance in the rail vehicle dynamics problems.

Therefore, the description of such a perturbed motion makes the bases in the rail vehicle dynamics. As recently shown by the author in Ref. [1, 2], there are three options to perform such description.

Option 1. In this option, one adopts coordinates relative to inertial reference system of type A in Figure 1 and any formalism of equations building valid for motion relative to A:

$$fd\mathrm{B} = fd\mathrm{Z} \quad \Rightarrow \quad \{x = x(t), \dot{x} = \dot{x}(t)\} \quad \Rightarrow \quad \{x' = x - u(t), \dot{x}' = \dot{x} - v(t)\} \tag{1}$$

where *fd* is the operator representing any chosen formalism; B is the inertia forces in relation to A; Z is the external (active) forces; $x,\ \dot{x}$ are the coordinates and velocities, respectively; and $u(t),\ v(t),\ w(t)$ are the known functions of time representing displacements, velocities, and accelerations of transportation, that is, of system A' in relation to A.

Option 2. In this option, one adopts coordinates relative to noninertial reference system of type A' in Figure 1 and any formalism adapted to motion description relative to A' (the direct methods of relative motion dynamics):

$$fd\,\mathrm{B}' = fd\,\mathrm{Z} + fp\,\mathrm{P}(\dot{u}, \ddot{u}) \quad \Rightarrow \quad \{x' = x'(t), \dot{x}' = \dot{x}'(t)\} \tag{2}$$

where B' is the inertia forces in relation to A'; P is the imaginary forces (inertia forces arising from and dependent on the transportation); and *fp* is the operator of the imaginary forces, appropriate for the formalism adopted.

Option 3. In this option, one takes coordinates relative to A' and any formalism valid in A. In practice, this is any of the variational principles of mechanics or the formalism arising directly from it:

$$\left.\begin{aligned} fz\mathrm{B} &= fz\mathrm{Z} \\ x &= x' + u(t) \\ \dot{x} &= \dot{x}' + v(t) \\ \ddot{x} &= \ddot{x}' + w(t) \end{aligned}\right\} \quad \Rightarrow \quad \begin{matrix} \{x = x(t), \dot{x} = \dot{x}(t)\} \\ \{x' = x'(t), \dot{x}' = \dot{x}'(t)\} \end{matrix} \tag{3}$$

http://dx.doi.org/10.5772/62105

where *fz* is the operator representing the variational principle.

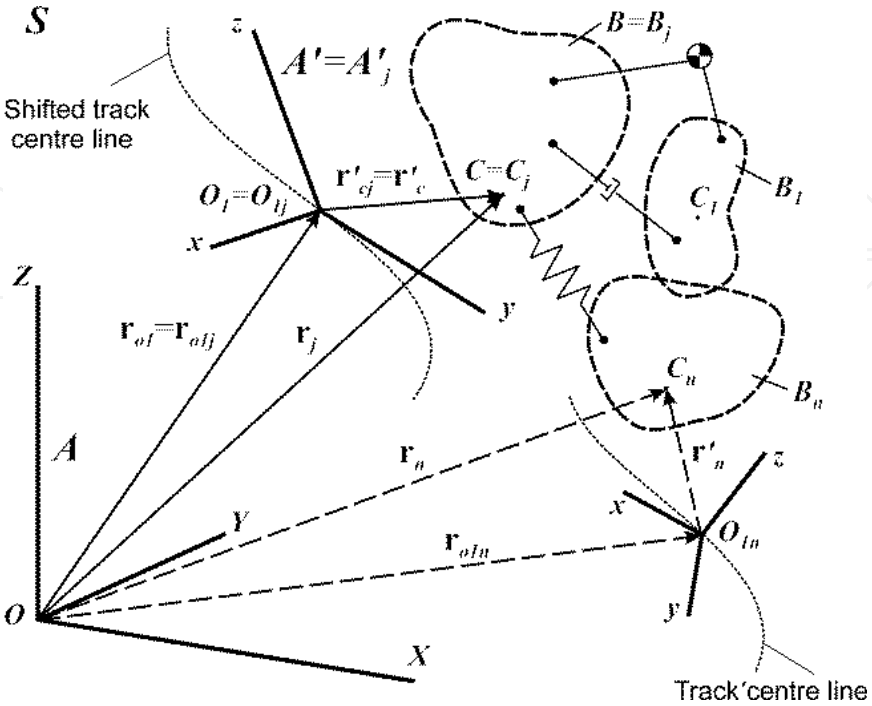

Figure 1. Multibody system *S* and useful coordinate systems

Commenting on Eqns. (1–3), one should note that the left-hand side equalities represent dynamical equations of motion. In case of Eq. (3), the equations of motion are supplemented with kinematical relations. They express absolute variables with the relative ones and need to be introduced into Eq. (31) when equations of motion are being solved. The general form of Eqns. (1–3) serves any form of the vectorial and scalar equations as well as the matrix form in certain cases. Meaning of the dynamical equations of motion is typical. It becomes obvious as B represents inertia forces relative to absolute (inertial) system, Z represents external forces, B′ represents inertia forces relative to moving (noninertial) system, and P represents imaginary forces (inertia forces depending on the transportation, i.e., motion of noninertial system relative to the inertial one). Moreover, the operators *fd*, *fp*, and *fz* do not change the meaning of the forces. The operators are introduced as a reminder that particular formalisms may need application of some specific coordinates, velocities, recording, and operations to make use of the formalism.

The curly brackets represent solutions of the equations of motion. As the equations are second-order ordinary differential equations, the solutions are displacements (coordinates) and velocities, both the linear and angular ones in general.

It is worth emphasizing that Eq. (2) is the only one that needs explicit form of the imaginary forces to be recorded. In contrast, in case of Eqns. (1) and (3), the inertia forces are also taken into account, however, in an inexplicit way. This is because $B = B' + P$. In addition, if one uses the same formalism of building the equations to build Eq. (2) or to build either Eqns. (1) or (3), then form of the forces B and B′ is identical. The only difference is with their meanings that refer to absolute and relative motions. In order to highlight this, the superscript " ′ " is used. It is used, in fact, to distinguish all the relative variables in Eqns. (1–3) and also in the subsequent text.

Based on the denotation explained above, one can note in the curly brackets that Eq. (2) is the only one that leads directly to the solution representing the relative coordinates and velocities. Eq. (2) is advantageous as the relative variables are those of interest in most of rail-vehicle dynamics problems. In case of Eq. (1), the relative solution can be obtained indirectly, whereas in the case of Eq. (3) a part of the solution represents absolute variables and another the relative ones.

Discussing briefly the practical use of particular options, it has to be stated at first that numbers and types of application differ. The approach defined with Eq. (1) is definitely used most rarely. The approach defined with Eq. (3) is the one among all three that is used most often in the commercial software packages for automatic generation of equations of motion (AGEM). This includes the software suitable for the systems in a rail vehicle type. Currently, such software (e.g., MEDYNA, VAMPIRE, SIMPACK, or VI-RAIL codes) are used quite often mainly in solving many contemporary engineering problems. However, their use in scientific research is questionable. It is contested because of the so-called black box problem. Thus, following the formal scientific methodology, the researcher himself and the others should precisely know how the problem was resolved. Therefore, the assumptions and the way of modeling have to be elucidated, which allows others to repeat the study or to compare it with results for other similar but different approaches (for different assumptions and methods of modeling). Examples of use of this approach to build the models and the simulation software for the scientific purposes can also be found. However, for such purposes, the option defined with Eq. (2) seems to be used more frequently, where different levels of accuracy are used in practice. Thus, some authors neglect selected terms of the imaginary forces. The importance of these terms in the rail vehicle dynamics is comprehensively discussed by the author of the present chapter in Ref. [4].

2. Equations of motion based on dynamics of relative motion approach

In his studies, the author of the present chapter himself practices and is a supporter of option 2 described earlier. This option defined with Eq. (2) exploits direct results of the dynamics of relative motion. Both, the variables used and the equations of motion, are defined directly in relation to moving coordinate systems in type A' in Figure 1. In his studies, the author applied a few formalisms of equation building. He applied them in both the traditional and numerical approaches. In the first approach, equations of motion were derived on paper first and then implemented in the simulation software in order to solve them numerically. In the second approach, the AGEM was applied by the author to build and solve equations numerically. Lagrange type II equations (e.g., see Ref. [5]) were in use in the traditional approach. In the AGEM approach, the author exploited Kane's equations as presented in Refs. [1, 2, 6]. Here, five examples of the equations valid in noninertial systems will be presented. These are matrix equations for a single rigid body based on Newton and Euler equations; state–space form of Newton–Euler equations for a single rigid body; state–space form of Newton–Euler equations for the multibody system (MBS) with constraints; Kane's equations in its general form; and Huston's form of Kane's equation suitable in AGEM. Besides, the method valid for Kane's equations will be presented, which enables building the equations for the holonomic and nonholonomic systems based on the equations for the free (unconstrained) system. Derivations for all these equations can be found in Refs. [1, 2].

2.1. The Newton–Euler equations

Based on the fundamental kinematical relations for relative motion, relating absolute and relative velocities and accelerations,

$$\boldsymbol{p}_C = \boldsymbol{a}'_C + \boldsymbol{a}_{o1} + \boldsymbol{\varepsilon} \times \boldsymbol{r}'_C + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}'_C) + 2\boldsymbol{\omega} \times \boldsymbol{v}'_C \tag{4}$$

$$\boldsymbol{\theta} = \boldsymbol{\omega} + \boldsymbol{\omega}' \tag{5}$$

$$\boldsymbol{\alpha} = \frac{d\boldsymbol{\theta}}{dt} = \frac{d\boldsymbol{\omega}}{dt} + \frac{d\boldsymbol{\omega}'}{dt} = \boldsymbol{\varepsilon} + \frac{d'\boldsymbol{\omega}'}{dt} + \boldsymbol{\omega} \times \boldsymbol{\omega}' = \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}' + \boldsymbol{\omega} \times \boldsymbol{\omega}' \tag{6}$$

and vectorial forms of Newton and Euler equations, the vectorial equations of relative motion for translation and rotation of a single free rigid body can be obtained as those presented in Refs. [7, 1, 2]. Based on these vectorial equations, their matrix forms can be recorded in several ways as shown, e.g., in Refs. [1, 2, 8–10]. The forms as in Refs. [1, 2] are as follows:

$$m\mathbf{a}'_C = -m(\mathbf{a}_{o1} + \hat{\mathbf{r}}'_C\boldsymbol{\varepsilon} + \hat{\boldsymbol{\omega}}\hat{\boldsymbol{\omega}}\mathbf{r}'_C + 2\hat{\boldsymbol{\omega}}\mathbf{v}'_C) + \mathbf{R}_C = \mathbf{Q}_1 + \mathbf{R}_C \tag{7}$$

$$\mathbf{J}\boldsymbol{\varepsilon}' = -\hat{\boldsymbol{\theta}}\mathbf{J}\boldsymbol{\theta} - \mathbf{J}\boldsymbol{\varepsilon} - \mathbf{J}(\hat{\boldsymbol{\omega}}\boldsymbol{\omega}') + \mathbf{T}_C = \mathbf{Q}_2 + \mathbf{T}_C \tag{8}$$

The meanings of the denotations present in the above Eqns. (4–6) are as follows: $\boldsymbol{p}_C$, $\boldsymbol{\theta}$, $\boldsymbol{\alpha}$ are the absolute acceleration of body mass centre C, absolute angular velocity of body B, and absolute angular acceleration of body B; $\boldsymbol{r}'_C$ is the radius vector of C in O_1xyz system; $\boldsymbol{v}'_C$, $\boldsymbol{a}'_C$ are the relative velocity and relative acceleration of C in relation to O_1xyz; $\boldsymbol{\omega}'$, $\boldsymbol{\varepsilon}'$ are the relative angular velocity and relative angular acceleration of B; $\boldsymbol{a}_{o1}$, $\boldsymbol{\omega}$, $\boldsymbol{\varepsilon}$ are the absolute acceleration of O_1 origin (i.e., in relation to $OXYZ$), transportation angular velocity, and transportation angular acceleration (i.e., of O_1xyz in $OXYZ$). In Eqs. (7, 8), m, $\mathbf{J}$ are the mass and inertia tensor (its matrix representation) of body B in relation to C; $\mathbf{R}_C$, $\mathbf{T}_C$ are the matrix of resultant external forces acting on B and matrix of resultant external torques acting on B in relation to C; and $\mathbf{Q}_1$,

$\mathbf{Q}_2$ are the matrices representing sums of the inertia terms. The remaining denotations are matrices that have their counterparts in the vectors defined in Eqns. (4–6).

One should realize here that in order to express all matrices in Eqns. (7, 8) through their elements selection of the vector bases $\boldsymbol{i}_m$ (m=1, 2, 3) for Eqns. (7) and (8) is necessary.

Equations (7, 8) can easily be recorded in one matrix equation:

$$\mathbf{I}\dot{\mathbf{x}}'_{\mathrm{II}} = \mathbf{Q} + \Lambda \tag{9}$$

where

$$\dot{\mathbf{x}}'_{\mathrm{II}} = [\dot{\mathbf{x}}'_{\mathrm{IIi}}]^{\mathrm{T}} = [\mathbf{a}'_{Cj}, \varepsilon'_j]^{\mathrm{T}} = [\dot{\mathbf{v}}'_{Cj}, \dot{\omega}'_j]^{\mathrm{T}} \quad (i = 1,...,6; j = 1,\ 2,\ 3); \tag{10}$$

$$\mathbf{I} = \begin{bmatrix} m\mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix}; \qquad \mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix}; \qquad \Lambda = \begin{bmatrix} \mathbf{R}_C \\ \mathbf{T}_C \end{bmatrix} \tag{11}$$

and $\mathbf{E}$ is the unit matrix. Besides, for any vector $\boldsymbol{c}$ the denotation $\frown{\mathbf{c}}$ represents the vector's skew-symmetric matrix, while generally the matrix $\mathbf{I}$ is not the symmetrical one.

On analyzing Eq. (10), the variables $\mathbf{x}'_{\mathrm{II}}$ and $\mathbf{x}'_{\mathrm{I}}$ (velocities and coordinates) are the same as those in Newton and Euler equations. They define translation of the centre C and rotation of B around C. In their case, the linear kinematical relation holds $\dot{\mathbf{x}}_{\mathrm{I}} = \mathbf{K}(\mathbf{x}_{\mathrm{I}})\mathbf{x}_{\mathrm{II}}$.

It can be shown that the equation similar to Eq. (9) can be obtained when arbitrarily chosen set of variables $\mathbf{w}'_{\mathrm{II}}$ and $\mathbf{w}'_{\mathrm{I}}$ are adopted. If matrix $\mathbf{K}$ is not singular then for variables

$$\mathbf{x}'_{\mathrm{I}} = \mathbf{X}_{\mathrm{WI}}(\mathbf{w}'_{\mathrm{I}},\mathbf{t}); \dot{\mathbf{x}}'_{\mathrm{I}} = \mathbf{H}(\mathbf{w}'_{\mathrm{I}},\mathbf{t})\dot{\mathbf{w}}'_{\mathrm{I}} + \mathbf{h}(\mathbf{w}'_{\mathrm{I}},\mathbf{t})$$

and linear kinematical relations

$$\dot{\mathbf{w}}'_{\mathrm{I}} = \mathbf{G}(\mathbf{w}'_{\mathrm{I}})\mathbf{w}'_{\mathrm{II}}$$

the equation can be written as follows:

$$\mathbf{x}'_{\mathrm{II}} = \mathbf{K}^{-1}\dot{\mathbf{x}}'_{\mathrm{I}} = \mathbf{K}^{-1}(\mathbf{H}\dot{\mathbf{w}}'_{\mathrm{I}} + \mathbf{h}) = (\mathbf{K}^{-1}\mathbf{H}\mathbf{G})\mathbf{w}'_{\mathrm{II}} + \mathbf{K}^{-1}\mathbf{h} = \Omega\mathbf{w}'_{\mathrm{II}} + \varsigma \tag{12}$$

while the derivative equals

$$\dot{\mathbf{x}}'_{\mathrm{II}} = \mathbf{\Omega}\dot{\mathbf{w}}'_{\mathrm{II}} + \dot{\mathbf{\Omega}}\mathbf{w}'_{\mathrm{II}} + \dot{\varsigma} \tag{13}$$

After the introduction of Eq. (13) in (9), one gets

$$\hat{\mathbf{I}}\dot{\mathbf{w}}'_{\mathrm{II}} = \hat{\mathbf{Q}} + \hat{\Lambda} \tag{14}$$

where

$$\hat{\mathbf{I}} = \mathbf{I}\Omega; \quad \hat{\mathbf{Q}} = [\mathbf{Q} - \mathbf{I}(\dot{\Omega}\mathbf{w}'_{\mathrm{II}} + \dot{\varsigma})]; \quad \hat{\Lambda} = \Lambda \tag{15}$$

As shown explicitly in Refs. [1, 2], the result represented by Eq. (14) can be generalized to the case of constraint system with holonomic and nonholonomic constraints (e.g., see Ref. [7]). Then it can be extended so that the inertia matrix becomes symmetrical one what enables to get the corresponding inverse matrix and finally to solve the equations. This can be done through the left-hand side multiplication of the equation by the transpose matrix $\mathbf{\Omega}^{\mathrm{T}}$. As a result of the described operations, one arrives at

$$\breve{\mathbf{I}}\dot{\mathbf{w}}'_{\mathrm{II}} = \breve{\mathbf{Q}} + \breve{\Lambda} + \breve{\Lambda}_{\mathbf{z}} \tag{16}$$

where

$$\breve{\mathbf{I}} = \Omega^{\mathrm{T}}\hat{\mathbf{I}} = \Omega^{\mathrm{T}}\hat{\mathbf{I}}\Omega \quad ; \quad \breve{\mathbf{Q}} = \Omega^{\mathrm{T}}\hat{\mathbf{Q}}; \tag{17}$$

$$\breve{\Lambda} = \mathbf{\Omega}^{\mathrm{T}}\hat{\Lambda}; \qquad \breve{\Lambda}_{\mathbf{z}} = \mathbf{\Omega}^{\mathrm{T}}\hat{\Lambda}_{\mathbf{z}} = \mathbf{\Omega}^{\mathrm{T}}\mathbf{\Phi}^{\mathrm{T}}(\mathbf{w}'_{\mathrm{I}},\mathbf{t}) \tag{18}$$

In the equations above, **t** is the indicator of the dependence on time *t*; $\mathbf{\Phi}$, $\boldsymbol{\lambda}$ are the so-called constraint matrix and column matrix of Lagrange's multipliers (e.g., see Refs. [8, 1, 2]). If one is going to solve Eq. (16) then all equations of constraints have to be added to the system of equations to make it possible.

When one is not interested in values of the constraint (internal) forces $\breve{\mathbf{\Lambda}}_{\mathbf{z}}$ then it is reasonable to express equations of motion for the reduced set of independent variables $\mathbf{y}'_{\mathrm{I}}$ and y'_{II}. This reduces the number of equations by the number of constraints. The relation at the velocity level between maximum and reduced (minimum) set of the variables is represented through matrix representation of explicit constraint equations at the velocity level:

$$\mathbf{w}'_{\mathrm{II}} = \varphi\mathbf{y}'_{\mathrm{II}} + \xi \tag{19}$$

After Eq. (19) is exploited in Eq. (14), the state–space equation in the independent generalized velocities takes the following form:

$$\tilde{\mathbf{I}}\dot{\mathbf{y}}'_{\mathrm{II}} = \tilde{\mathbf{Q}} + \tilde{\Lambda} \tag{20}$$

where

$$\tilde{\mathbf{I}} = \varphi^{\mathrm{T}}\hat{\mathbf{I}}\varphi \quad ; \qquad \tilde{\mathbf{Q}} = \varphi^{\mathrm{T}}[\hat{\mathbf{Q}} - \hat{\mathbf{I}}(\dot{\varphi}\mathbf{y}'_{\mathrm{II}} + \dot{\xi})] \quad ; \qquad \tilde{\Lambda} = \varphi^{\mathrm{T}}\hat{\Lambda} \tag{21}$$

Note that inertia matrix $\tilde{\mathbf{I}}$ is symmetrical due to the left-hand side multiplication by $\boldsymbol{\varphi}^{\mathrm{T}}$. The kinematical relations corresponding to Eq. (20) in the linear matrix form can be expressed as $\dot{\mathbf{y}}'_{\mathrm{I}} = \hat{\mathbf{Y}}_{\mathrm{I}}(\mathbf{y}'_{\mathrm{I}})\mathbf{y}'_{\mathrm{II}}$. In case nonholonomic constraints exist in the system, they have to be provided to make the solution of Eq. (20) possible.

The final Eqns. (16, 20) are valid for a single rigid body. It was shown in Ref. [8] for inertial systems that forms valid for a single body can directly be generalized to any number of rigid bodies. As in principle the structure of equations for noninertial systems differs in additional inertia terms of correction character only (see, e.g., Eqns. (2, 3)), this result can be extended to noninertial systems. In terms of the notation, it is trivial and means that Eqns. (16) and (20) valid in noninertial systems remain unchanged for any number of rigid bodies.

2.2. Kane's equations

Any of the formalisms of analytical mechanics valid in inertial systems can be adapted to describe the relative motion in the noninertial system. The author of the chapter performed such an adaptation for Kane's equation. It was done in a formal manner, that is, corresponding equations of relative motion were derived as shown in Refs. [6, 2, 1].

The partial velocities are fundamental to original Kane's approach [11]. The corresponding relative linear and angular partial velocities were introduced by the author as shown in Refs. [6, 1, 2]. Let us introduce them for the simple nonholonomic system [11] of l degrees of freedom composed of j (j=1,...,n) rigid bodies by defining linear v'_j and angular ω'_j relative velocities of the mechanical system:

$$\boldsymbol{v}'_j = \sum_{\rho=1}^{l} \frac{\partial \boldsymbol{v}'_j}{\partial u_{j\rho}} u_\rho + \frac{\partial \boldsymbol{r}'_j}{\partial t} = \sum_{\rho=1}^{l} \boldsymbol{v}'_{j\rho} u_\rho + \boldsymbol{v}'_{jt} = \sum_{m=1}^{3} \left(\sum_{\rho=1}^{l} v'_{j\rho m} u_\rho + v'_{jtm} \right) \boldsymbol{i}_m^{(j)} \tag{22}$$

$$\boldsymbol{\omega}'_j = \sum_{\rho=1}^{l} \frac{\partial \boldsymbol{\omega}'_j}{\partial u_{j\rho}} u_\rho + \frac{\partial \boldsymbol{\varphi}'_j}{\partial t} = \sum_{\rho=1}^{l} \boldsymbol{\omega}'_{j\rho} u_\rho + \boldsymbol{\omega}'_{jt} = \sum_{m=1}^{3} \left(\sum_{\rho=1}^{l} \omega'_{j\rho m} u_\rho + \omega'_{jtm} \right) \boldsymbol{i}_m^{(j)} \tag{23}$$

where $\boldsymbol{v}'_{j\rho}$, $\boldsymbol{\omega}'_{j\rho}$, $\boldsymbol{v}'_{jt}$, $\boldsymbol{\omega}'_{jt}$ are the functions of generalized coordinates $q_1,...,q_k$ and time t. At the same time, $\boldsymbol{v}'_{j\rho}$, $\boldsymbol{\omega}'_{j\rho}$ are the ρth relative partial velocity of body B_j mass centre C_j (or any point P_j) and ρth relative angular partial velocity of B_j in relation to A', respectively; $v'_{j\rho m}$, $\omega'_{j\rho m}$ are the scalar components of $\boldsymbol{v}'_{j\rho}$, $\boldsymbol{\omega}'_{j\rho}$, called coefficients of partial velocities; u_ρ is the quasi-velocity $(\rho=1,...,l)$; m is the indicator of unit vectors (base vectors) $\boldsymbol{i}_m^{(j)}$ (m=1, 2, 3) defining axes directions of reference systems, where $\boldsymbol{v}'_j$ and $\boldsymbol{\omega}'_j$ are expressed (there is no need for $\boldsymbol{i}_m^{(j)}$ to have the same directions for translations and rotations and for all bodies B_j).

Form of the adapted Kane's equations is as follows:

$$\begin{aligned}
&\sum_{j=1}^{n}\boldsymbol{v}'_{j\rho}\cdot\left(-m_j\boldsymbol{a}'_j\right)+\sum_{j=1}^{n}\boldsymbol{v}'_{j\rho}\cdot\boldsymbol{R}_{Cj}\\
&\quad+\sum_{j=1}^{n}\boldsymbol{v}'_{j\rho}\cdot\left[-m_j\boldsymbol{a}_{o1j}-m_j\boldsymbol{\varepsilon}_j\times\boldsymbol{r}'_j-m_j\boldsymbol{\omega}_j\times\left(\boldsymbol{\omega}_j\times\boldsymbol{r}'_j\right)-2m_j\boldsymbol{\omega}_j\times\boldsymbol{v}'_j\right]\\
&\quad\quad+\sum_{j=1}^{n}\boldsymbol{\omega}'_{j\rho}\cdot\left(-\mathbf{J}_j\cdot\boldsymbol{\varepsilon}'_j-\boldsymbol{\omega}'_j\times\mathbf{J}_j\cdot\boldsymbol{\omega}'_j\right)+\sum_{j=1}^{n}\cdot\boldsymbol{\omega}'_{j\rho}\cdot\boldsymbol{T}_{Cj}\\
&\quad\quad\quad+\sum_{j=1}^{n}\boldsymbol{\omega}'_{j\rho}\cdot\left[-\mathbf{J}_j\cdot\boldsymbol{\varepsilon}_j-\boldsymbol{\omega}_j\times\mathbf{J}_j\cdot\boldsymbol{\omega}_j-2\boldsymbol{\omega}'_j\times\left(\mathbf{J}_j-0{,}5\vartheta_j\mathbf{E}\right)\cdot\boldsymbol{\omega}_j\right]\\
&\quad\quad\quad\quad=\sum_{j=1}^{n}\boldsymbol{v}'_{j\rho}\cdot\boldsymbol{R}^{*}_j+\sum_{j=1}^{n}\boldsymbol{v}'_{j\rho}\cdot\boldsymbol{R}_j+\sum_{j=1}^{n}\boldsymbol{v}'_{j\rho}\cdot\boldsymbol{R}^{**}_j\\
&\quad\quad\quad\quad\quad+\sum_{j=1}^{n}\boldsymbol{\omega}'_{j\rho}\cdot\boldsymbol{T}^{*}_j+\sum_{j=1}^{n}\boldsymbol{\omega}'_{j\rho}\cdot\boldsymbol{T}_j+\sum_{j=1}^{n}\boldsymbol{\omega}'_{j\rho}\cdot\boldsymbol{T}^{**}_j=0\qquad(\rho=1,...,l)
\end{aligned}\tag{24}$$

Most of the denotations in Eq. (24) are as already defined. Here, the meanings in Eqns. (4–8) are helpful. Note, however, that in Eq. (24) the j rigid bodies are described, while in Eqns. (4–8) just one. Besides, subscript C was omitted in $\boldsymbol{r}'_C$ resulting in $\mathbf{r}'_{Cj}\equiv\mathbf{r}'_j$. The undefined is ϑ – trace of $\mathbf{J}$, that is, $\vartheta=J_{11}+J_{22}+J_{33}$. In addition, $\boldsymbol{R}^*_j$, $\boldsymbol{T}^*_j$ are the inertia forces and torques, respectively, of motion in relation to noninertial systems of A' type; $\boldsymbol{R}^{**}_j$, $\boldsymbol{T}^{**}_j$ are the resultant imaginary forces and torques, respectively; while $\boldsymbol{R}_{Cj}\equiv\boldsymbol{R}_j$ and $\boldsymbol{T}_{Cj}\equiv\boldsymbol{T}_j$.

Equation (24) can be expressed in the shortened matrix form as follows [6, 2, 1]:

$$F_\rho^{\ *}+F_\rho+F_\rho^{\ **}=0\qquad(\rho=1,...,l)\tag{25}$$

where scalar components are defined with

$$F^*_\rho=\sum_{j=1}^{n}(\boldsymbol{R}^*_j\cdot\boldsymbol{v}'_{j\rho}+\boldsymbol{T}^*_j\cdot\boldsymbol{\omega}'_{j\rho})\ =R^*_{jm}v'_{j\rho m}+T^*_{jm}\omega'_{j\rho m}\tag{26}$$

$$F_\rho = \sum_{j=1}^{n} (\boldsymbol{R}_j \cdot \boldsymbol{v}'_{j\rho} + \boldsymbol{T}_j \cdot \boldsymbol{\omega}'_{j\rho}) \quad = R_{jm} v'_{j\rho m} + T_{jm} \omega'_{j\rho m} \tag{27}$$

$$F^{**}_\rho = \sum_{j=1}^{n} (\boldsymbol{R}^{**}_j \cdot \boldsymbol{v}'_{j\rho} + \boldsymbol{T}^{**}_j \cdot \boldsymbol{\omega}'_{j\rho}) = R^{**}_{jm} v'_{j\rho m} + T^{**}_{jm} \omega'_{j\rho m} \tag{28}$$

In the above Eqns. (26–28), F^*_ρ, F_ρ i F^{**}_ρ are the generalized inertia forces (relative to non-inertial system(s)), generalized external forces (identical with those in inertial system(s)), and generalized imaginary forces; R^*_{jm}, R_{jm}, R^{**}_{jm}, T^*_{jm}, T_{jm} and T^{**}_{jm} are the scalar components of resultant forces and torques $\boldsymbol{R}^*_j$, $\boldsymbol{R}_j$, $\boldsymbol{R}^{**}_j$, $\boldsymbol{T}^*_j$, $\boldsymbol{T}_j$ and $\boldsymbol{T}^{**}_j$. The final forms of Eqns. (26–28) exploit results for inertial systems by Huston (e.g., see Refs. [12–14]). Also note that signs of sums over j and m are omitted in these final forms. The original Huston's convention is used here, in which summations over repeated indices have to be performed instead.

Equations (24–28) are not useful in AGEM approach as they are too close to the vectorial origin of Kane's equations. Nevertheless, final forms of Eqns. (26–28) can be treated as the initial ones for the process of deriving the useful form. The process is based on the Huston results [12–14] for the inertial scleronomic systems and present author's extensions [6, 2, 1] to non-inertial and rheonomic systems. Consequently, invoking results from Refs. [6, 2, 1] one can write

$$\Theta_{\rho\pi} \dot{u}_\pi = f_\rho \qquad (\rho = 1,\ldots,l) \tag{29}$$

where

$$\Theta_{\rho\pi} = m_j v'_{j\rho m} v'_{j\pi m} + J_{jsm} \omega'_{j\rho s} \omega'_{j\pi m} \tag{30}$$

$$\begin{aligned} f_\rho = F_\rho + F^{**}_\rho &- (m_j v'_{j\rho m} \dot{v}'_{j\pi m} u_\pi + J_{jsm} \omega'_{j\rho s} \dot{\omega}'_{j\pi m} u_\pi + e_{rsm} J_{jsw} \omega'_{j\pi w} \omega'_{jqr} \omega'_{j\rho m} u_\pi u_q) \\ &- [m_j v'_{j\rho m} \dot{v}'_{jtm} + J_{jsm} \omega'_{j\rho s} \dot{\omega}'_{jtm} \\ &+ e_{rsm} J_{jsw} \omega'_{j\rho m} (\omega'_{j\pi w} \omega'_{jtr} u_\pi + \omega'_{jqr} \omega'_{jtw} u_q + \omega'_{jtr} \omega'_{jtw})] \end{aligned} \tag{31}$$

Besides, in Refs. [6, 2, 1], the form of imaginary forces corresponding to Eqns. (30) and (31) was presented as follows:

$$\begin{aligned} F^{**}_\rho = &-m_j a_{o1jm} v'_{j\rho m} - e_{swm} m_j \omega_{js} (e_{wrq} \omega_{jr} r'_{jq}) v'_{j\rho m} \\ &- e_{rsm} J_{jsw} \omega_{jw} \omega_{jr} \omega'_{j\rho m} - e_{swm} m_j \alpha_{js} r'_{jw} v'_{j\rho m} - J_{jms} \alpha_{js} \omega'_{j\rho m} \\ &- 2e_{swm} m_j \omega_{js} v'_{j\rho m} v'_{j\pi w} u_\pi - 2e_{rsm} (J_{jsw} - 0{,}5\vartheta_j E_{sw}) \omega_{jw} \omega'_{j\rho m} \omega'_{j\pi r} u_\pi \\ &- 2e_{swm} m_j \omega_{js} v'_{j\rho m} v'_{jtw} - 2e_{rsm} (J_{jsw} - 0{,}5\vartheta_j E_{sw}) \omega_{jw} \omega'_{j\rho m} \omega'_{jtr} \quad (\rho = 1,\ldots,l) \end{aligned} \tag{32}$$

http://dx.doi.org/10.5772/62105

The supplements to Eq. (29) are kinematical relations:

$$\dot{q}_\sigma = P_{\sigma\rho}(q,t)u_\rho + p_\sigma(q,t) \ ; \qquad (\sigma = 1,...,k), \ (\rho = 1,...,l) \tag{33}$$

where in the above equations e_{rsm}, e_{swm} are the permutation symbols; k is the number of generalized coordinates of the nonholonomic system; $P_{\sigma\rho}$, p_σ are the functional matrix and vector (column matrix). Besides, the square bracket in Eq. (31) (being the extension to rheonomic systems) and p_σ vanish for scleronomic systems.

Equations (24, 25, 29, 32, 33) serve nonholonomic, holonomic, and free systems. In Eqns. (25, 29, 32, 33), the difference between particular systems is taken into account by suitable values of partial velocities' coefficients $v'_{j\rho m}$, $\omega'_{j\rho m}$. For holonomic systems l=k, while for free ones l=k=6n. In addition, equations for free and holonomic systems and kinematical relations are sufficient to be solved. For nonholonomic systems, equations of nonholonomic constraints have to be provided additionally to get the solution.

It was shown by the author in Refs. [6, 2, 1] for the rail vehicle systems, taking account of their moderate dimension, nonoccurrence of nonholonomic constraints, limited number of holonomic constraints, identical coordinates and transportation for each rigid body within vehicle model, that it can be reasonable to make use of equations (or particular type of forces) for free systems. Assuming holonomic and nonholonomic constraint equations as

$$u_\sigma = \sum_{\rho=1}^{l} B_{\sigma\rho} u_\rho + D_\sigma \qquad (\sigma = l+1,...,k) \tag{34}$$

$$u_s = \sum_{\rho=1}^{k} A_{s\rho} u_\rho + C_s \qquad (s = k+1,...,6n) \tag{35}$$

the following formula enables to build equations (or selected forces type) for constraint nonholonomic system based on the equations (or selected forces type) for free system [6, 2, 1]:

$$F_\rho = \tilde{F}_\rho + \sum_{s=k+1}^{6n} \tilde{F}_s A_{s\rho} + \sum_{\sigma=l+1}^{k} \tilde{F}_\rho B_{\sigma\rho} + \sum_{\sigma=l+1}^{k} \sum_{s=k+1}^{6n} \tilde{F}_s A_{s\rho} B_{\sigma\rho} \qquad (\rho = 1,...,l) \tag{36}$$

If the system is holonomic then Eq. (34) vanishes, while Eq. (36) is reduced to just two first addends. In Eqns. (34–36), the following so far undefined notations appear: u_ρ, u_σ, u_s are the independent, dependent through nonholonomic constraints, and dependent through holonomic constraints quasi-velocities, respectively; $A_{s\rho}$, $B_{\sigma\rho}$, C_s, and D_σ are the functional coefficients depending on generalized coordinates $q_1,...,q_k$ and time t; and F, $\tilde{F}$ are the representations of any type of generalized forces F^*, F, F^{**} as defined in Eqns. (25–28) or of their sum for constraint and free systems, respectively.

2.3. Discussion of the equations

Taking account of Refs. [15, 16], both the Newton–Euler and Kane's equations are those most often used in AGEM. This is an interesting fact because major differences between both approaches exist. Therefore, the equations are worthy of discussion in this context.

It can be seen in Section 2.1 that building Eq. (16) and (20) that represent receipt useful in AGEM are operations on matrices. To perform these operations, the matrices have to be defined with their components. To do this, one needs to adopt base vectors $\boldsymbol{i}_m^{(j)}$ for the chosen reference systems. Unfortunately, operation must be performed at the beginning of the equations building, that is, jet in Eqns. (7, 8). Despite simple forms of Eqns. (16, 20), the method of obtaining equations for a given mechanical system is not short. Every time one builds equations, he has to choose vector bases, define matrices, and perform matrix calculations as described in Eqns. (12, 13, 15, 17, 18, 19, 21). These calculations are multiple, often in each of the mentioned equations. In the case of nonholonomic systems and interest in constraint forces, Eq. (16) is used, which together with constraint equations form differential-algebraic equation (DAEs) set. Such a set is difficult to solve in general case. On the other hand, when constraint forces are of no interest then a reduced number of ordinary differential equation (ODE) set is used (Eq. (20)).

Equations (29–33) form the receipt useful in AGEM of Kane's equations. Their forms might seem discouraging as they are complex, especially while comparing with Eqns. (16, 20). This impression is misleading as equations possess serious advantages as well. The advantage is the same form of either scalar (Eq. (24)) or matrix equations (Eqns. (29–33)) for free, holonomic, and nonholonomic systems. The most distinguishing advantage of Kane's equations, appearing just for this formalism, is the moment the components of vectors and tensors are defined through the vector bases selection. This is done at the very end of the equations building. This is possible since Eqns. (24, 29–33) originate directly from Eq. (24) based on vectors. Therefore, Eqns. (24, 29–33) are valid for any vector basis. An additional advantage is the type of Kane's equations that are always ODEs.

3. Example objects, nominal models, and numerical models

3.1. The objects and their nominal models

Equations of motion of mechanical system are referred to as its mathematical model. In order to make use of the general methods of building equations of motion as shown in Section 2, the nominal model of particular vehicle (object) has to be first determined. In case of railway vehicle, its nominal model projects its structure and selected physical features of the structure elements. The nominal models of two example objects will be presented below. Both are of British origin and possess relatively simple structure that is suitable for the basic character of the author's research. The first model [1, 5, 17] corresponds to 2-axle hsfv1 freight car and is shown in Figure 2. The second one [1, 18] corresponds to 4-axle MKIII passenger car and is shown in Figure 3.

http://dx.doi.org/10.5772/62105

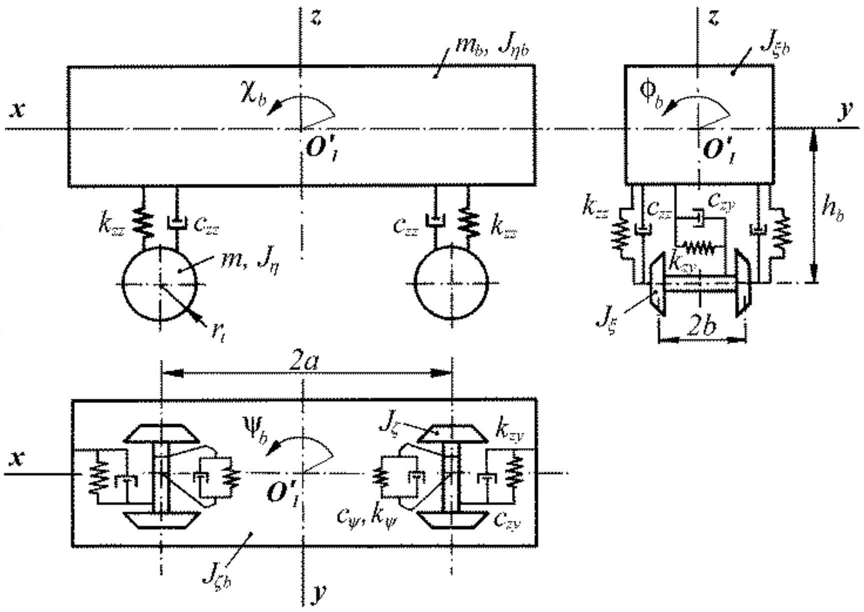

Figure 2. Nominal model of 2-axle freight car hsfv1 [1, 5, 17]

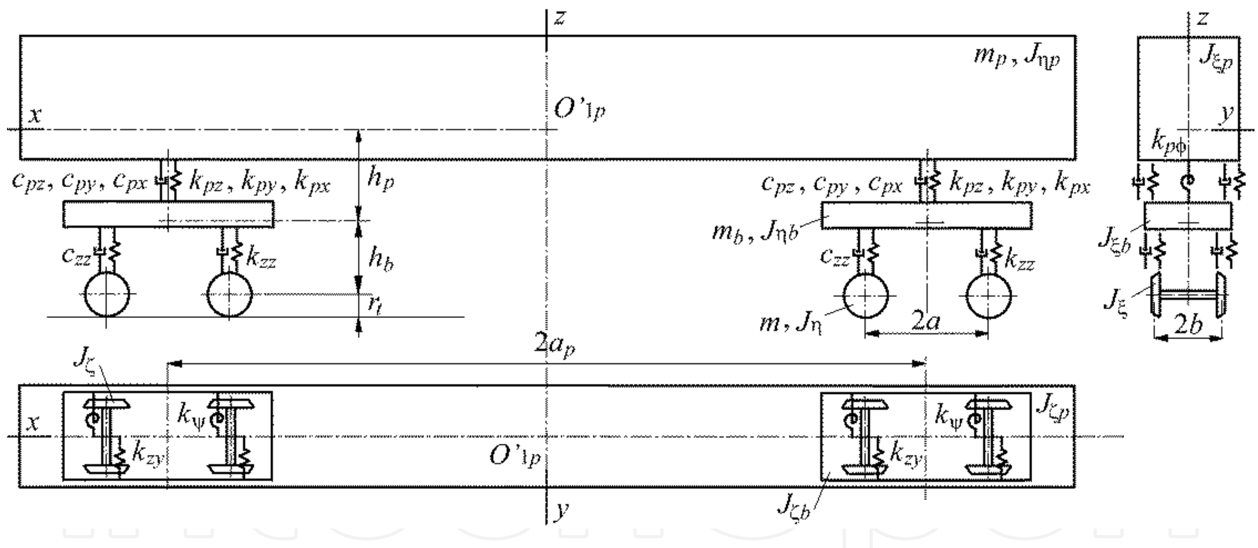

Figure 3. Nominal model of 4-axle passenger car MKIII [1, 18]

If the flexibility of track is going to be taken into consideration in the study, then some track nominal models should be adopted. In case one considers low-frequency dynamics of the vehicle motion (not more than 50 Hz) but not the higher-frequency phenomena in the track itself, then it is reasonable to build the track model that is composed of rigid bodies. Then the whole vehicle-track system model is a multibody model. Separate models for lateral (Figure 4) and vertical (Figure 5) directions as used by the author in his studies are presented below [1, 5, 17, 18].

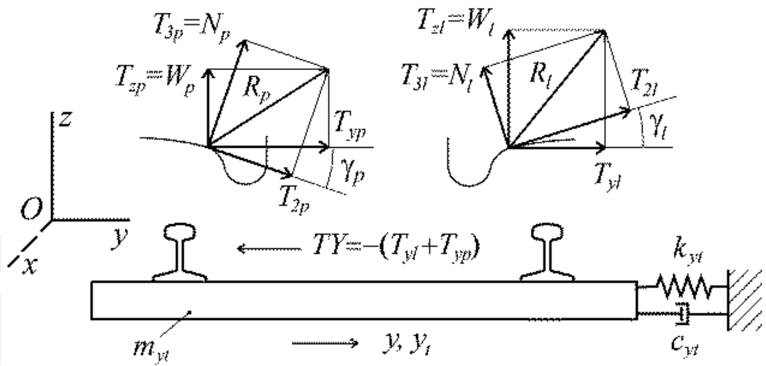

Figure 4. Nominal model of track flexible laterally [1, 17, 18]

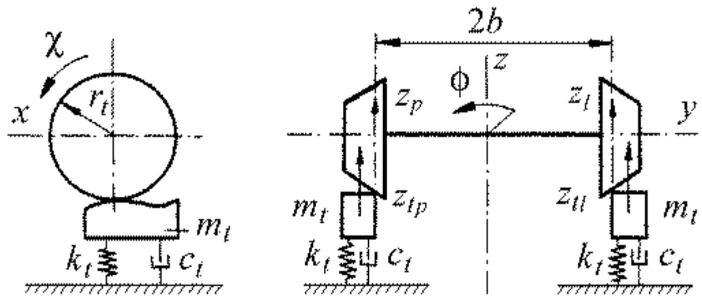

Figure 5. Nominal model of track flexible vertically [1, 17, 18]

The denotations used in Figures 2–5 are as follows: m is the mass; J_ξ, J_η, J_ζ are the inertia moments in longitudinal, lateral, and vertical directions, respectively; k, c are the stiffness and damping in the flexible elements in general; k_z , c_z , k_p , c_p are the stiffness and damping in the flexible elements of primary and secondary suspension, respectively; x, y, z are the linear displacements (coordinates) in longitudinal, lateral, and vertical directions, respectively (if used as indices, they indicate the corresponding directions); ϕ, χ, ψ are the rotations (angular coordinates) around axes of longitudinal, lateral, and vertical directions, respectively (if used as indices, they indicate the corresponding rotations); 2a, 2b are the vehicle or bogie base; h, r_t are the vertical dimensions and wheel rolling radius; N, T, W are the normal, tangential, and load forces in wheel/rail contact, respectively; γ is the contact angle. The indices used indicate p as the passenger car body or right-hand side, l the left-hand side; b the bogie or freight car body; and t the track. Values of the abovementioned parameters of models can be found in Refs. [1, 5, 17].

The whole hsfv1 car-track system model has 18 degrees of freedom (DOFs). The whole MKIII car-track model has as many as 38 degrees of freedom.

3.2. Numerical models

When mathematical models derived traditionally are available, based on general methods of the equations building as described in Section 2 and adopted nominal models as, for example,

http://dx.doi.org/10.5772/62105

those shown in Section 3.1, then they have to be converted to numerical models. When AGEM approach is used to build the equations, based again on the results of Section 2 and Section 3.1, then equations automatically form the numerical model. The numerical models are indispensable, since rail vehicle systems are multidimensional ones. In addition, their nonlinear versions are mainly used at present. The only way to solve dynamical differential equations of motion of such systems is numerical integration. In order to do this, the numerical model representing the mathematical model in a form understandable to computers must be built, which means that the equations have to be coded in some of the programming languages. In the case of this author, it is Fortran. Then, the equations are solved with use of one of the numerical methods of equation integration. The author uses Gear's method [19]. The software containing traditionally derived equations of motion or generating the equations automatically combined with the integration procedure is called the simulation software in railway vehicle dynamics.

In order to make use of the procedure, shortly described above, three additional major elements have to be provided. The first one is to adopt the model and then to build its numerical module to take account of relative kinematics arising from description in moving coordinate systems. Strictly speaking, the linear and angular velocities and accelerations of transportation, represented with functions $u(t), v(t), w(t)$ in Section 1.1 and with vectors $\boldsymbol{a}_{o1}$, $\boldsymbol{\omega}$, $\boldsymbol{\varepsilon}$ in Section 2, have to be determined. The author of this paper worked out a general method of their determination, which is a numerical calculation-oriented method [2, 1, 20]. Its generality arises from any three-dimensional track shape acceptable in the method. The most important requirement in the method is the description of the three-dimensional curve by parametric equations, with its length as the parameter. In fact, the three-dimensional case corresponds to transition curves (TCs) with the superelevation ramps, while circular (regular) curves (CC) and straight track (ST) are two- and one-dimensional special cases, respectively. The components, expressed analytically, of the velocities and accelerations of transportation for several types of TCs, CC, and ST can be found in Refs. [1, 20]. The sources for the algorithm to numerically determine the components for polynomial TCs are Refs. [21, 1].

The next element is tangential contact forces calculation. These forces arise from wheel/rail relative slip (creepage). Consequently, the longitudinal, lateral, and spin creepages are the inputs in the existing methods of the tangential forces calculation. Generally, one can talk about the exact and simplified numerical methods of nonlinear tangential contact forces calculation. The problem with the exact methods, as, for example, Kalker's CONTACT program [22], lies in the very slow calculations for the simulation purposes of rail vehicle dynamics. Thus, simplified methods are in use in the simulation programs (numerical models). One of them is Kalker's FASTSIM program [23]. This is just the software used by the present author in his models and so in the studies. Usually, the adopted value of the friction coefficient, necessary for the program to run, in the author's studies is μ=0.3. Other possibilities of tangential contact forces calculation are discussed in Refs. [24–27].

The last among three elements to be discussed is the problem of nonlinear contact geometry. This author uses ArgeCare RSGEO program [28] to address this. The program resolves normal contact problems and purely geometrical ones. As a result, contact areas become known and geometrical variables in the contact. The most important geometrical variables are instantaneous contact angles, rolling radii, and contact point positions on the wheel and rail for any lateral relative shift between wheel and rail. In addition, the program is capable of taking account of the influence of wheelset yaw angle on the parameters. Both the left- and right-hand side wheel and rail parameters are calculated in the same step. The results of ArgeCare RSGEO program (the author uses) are tabulated before simulations for a given wheel and rail pair of profiles as a function of the discrete lateral wheel/rail displacements and yaw angles. To get the contact parameters for any relative displacement and yaw rotation between wheel and rail, the tabulated data are linearly interpolated.

At the end, let us return to the issue already discussed at the beginning of Section 2. It is the way in which the equations of motion in the simulation software (numerical model) are introduced. Here, this refers to the traditionally derived and automatically generated equations of motion (AGEM approach). Now, we will provide some references corresponding to both approaches. If one is interested in samples of the equations traditionally derived by this author, then he could find them in Ref. [29]. If one is interested in details of the package ULYSSES and its core program TITAN co-built by this author, then he can find them in Refs. [1, 2, 6]. The software packages based on AGEM built by other authors are discussed in Refs. [15, 16].

4. Example results of the selected simulation studies

The author of the present chapter has exploited simulation in his studies for more than 20 years. There is no possibility to refer to all of them. Readers interested in earlier applications of simulation by the author can find reviews of the corresponding references in Ref. [20]. Similar review, however, also including current applications, is done in Ref. [1]. Total number of all author's applications runs to tens. Due to the limit of space, just two applications of simulation used in currently studied problems will briefly be discussed in the following sections. Their content will be focused more on the achievements, contributions, and final results than on the details of the methods or procedures used in those studies. Interested readers will be informed about the essential publications where the details can be found. These publications can also be treated as a main base source for the results discussed further. The secondary source is Ref. [1]. The applications in view concern the stability and kinematics issues.

Two other applications extensively studied at present are connected with TC shape optimization with the use of simulation and optimization methods as well as with dynamics of railway vehicles in TCs at velocities around the critical v_n. The example publications on these problems are Refs. [21, 30] and [31, 32], respectively.

4.1. Results of simulation use in rail vehicle nonlinear lateral stability studies

The serious interest of the author on the stability problems started with reference [17]. The most important results were presented many years later in Refs. [33, 34]. The fundamental reference to the methods used by this author, based on bifurcation approach and on usage of the simulation, is [35]. Recently, the remarkable contributions have been by H. True and his coauthors (e.g., see Refs. [36, 37]). Works by O. Polach (e.g., see Ref. [38]) are also interesting. Much more comprehensive and detailed literature review can be found in Refs. [1, 33, 34].

The meaning of stability is fundamental in the discussed problem. This author could describe its meaning in most of his works as follows. The used method of nonlinear lateral stability analysis regards formal theory of stability. By contrast, it is not so formal as compared with the methods based strictly on the theory. Instead, one finds the tool to be more practical. The simplification is based on adoption of certain assumptions as well as on behavioral expectations for the studied system. They arise from commonly existing knowledge about the systems in the type of rail vehicle. Therefore, one can skip formal adoption of some solution as the reference in the simplified approach. Moreover, he can skip introducing some perturbations into the system to examine whether the new solution stays close enough (in narrow vicinity) to the reference solution. Such a stay is required by the formal definition of the stability theory. Taking account of that, bifurcation plot building for the system is the main task in the method. However, formal verification whether the solutions enabling to build this plot are stable in completely formal sense is not such a task. The assumption exists in the method that any typical solution of rail vehicle system (either stationary or periodic) is stable. It might be accepted with care basing on understanding that periodic solutions in rail vehicle dynamics are self-exciting vibrations being governed through the wheel–rail tangential contact forces. Following that, the theory of self-exciting vibrations can be useful in order to predict/expect typical periodic behavior of the system. The adopted assumption enables to limit the number of simulations for different initial conditions but the same given velocity. Such simplified approach is described in Ref. [29]. In case of any doubts if obtained solution is stable or about the possibility of multiple solutions existence, more formal check for the stability is a must. Then, reasonably a denser sweeping over the initial conditions, to introduce the perturbation, is performed. Such a more accurate approach is called the extended analysis and is described in Ref. [30].

The author has been especially interested in the stability of rail vehicles in a curved track from the very beginning of his interest in stability problems. The studies were initiated as a result of the observed periodic solutions in a circular curve of track. They appear for velocities v greater than the nonlinear critical velocity v_n, exactly as in a straight track. The only difference relative to ST case is asymmetry of the results in relation to track centerline. Results of such kind are shown in Figure 6. Based on that fundamental observation, the studies were carried out focused on the influence of chosen factors on the stability in CC.

They involved curve radii R from the small to large ones and ST as well, where R=∞. The principles of these nonlinear stability analyses were adopted from the analysis in a ST. However, the important supplements to ST analysis were necessary. These supplements are shown in Figure 7. They finally resulted in the formally presented method of the nonlinear stability analysis in a curved track (CC).

It was successively improved for years and recently published in Refs. [33, 34]. The essence of this method is included in Figure 8.

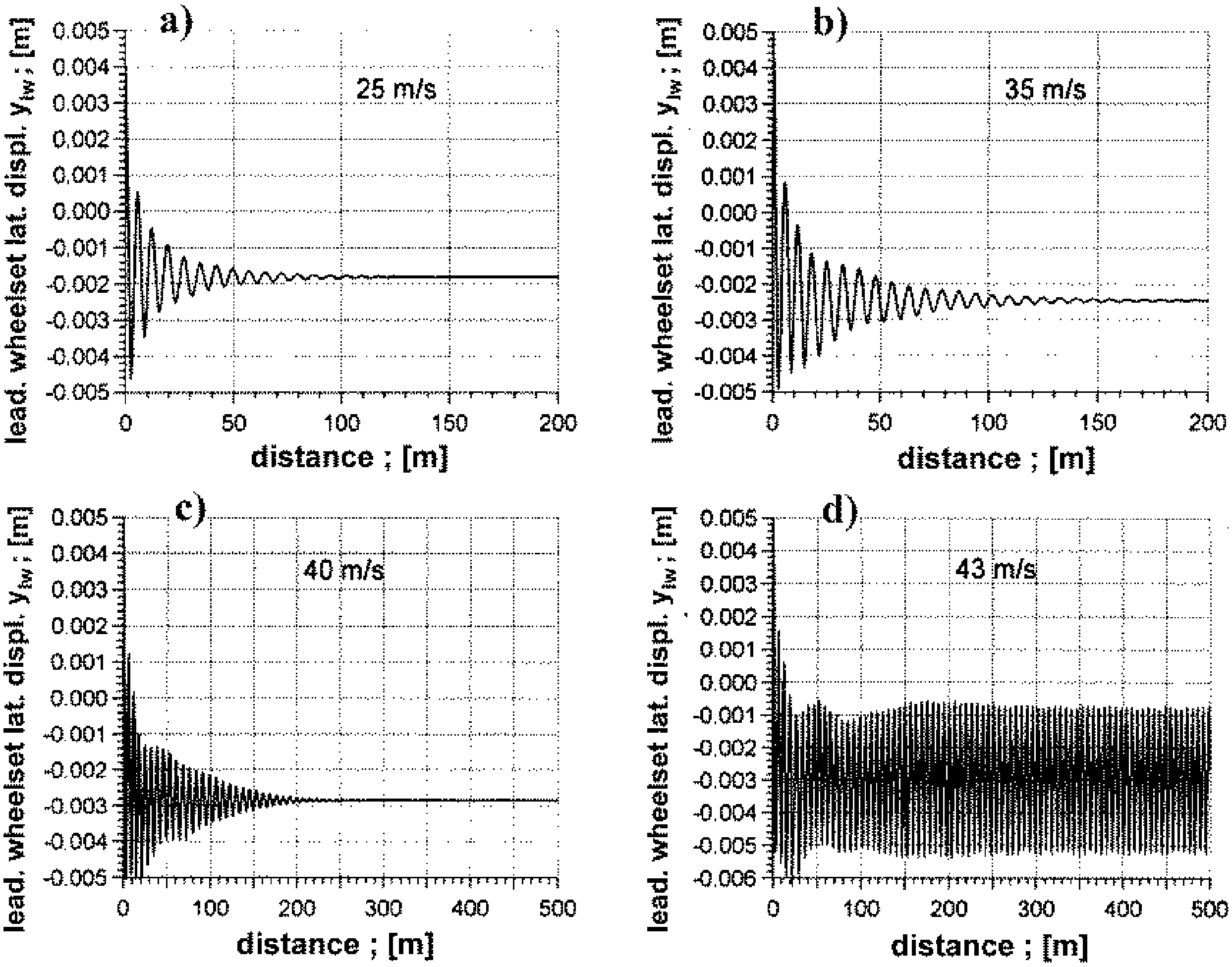

Figure 6. Stable stationary and periodic solutions for R=600 m and velocities v below and above v_n [1]

The meaning of the symbols and acronyms present in Figure 8 is as follows: v_c, v_n, v_s is the linear critical velocity, nonlinear critical velocity, velocity of calculation stop due to unbounded growth of the solution, respectively; sss, sps are the stable stationary and periodic solutions, respectively; uss, ups are the unstable stationary and periodic solutions, respectively; SNB, HSB are the saddle-node and Hopf's bifurcation point, respectively. All these notions are well known in the nonlinear stability studies in ST. The method consists in building a pair of bifurcation plots as in Figures 8a and 8b based on the results of simulation as shown in Figures 8c and 8d and obtained subsequently for whole range of velocity of motion v and given curve radius R. If one decides to include in both bifurcation plots results for whole range of R, then one obtains complex bifurcation plots. Such a pair of the complex bifurcation plots was named stability map in Ref. [39]. The example stability maps, as in Refs. [33, 1], are shown in Figures 9 and 10. Note that the unstable solutions are omitted in these figures as those are less important because they are not observable in the real systems.

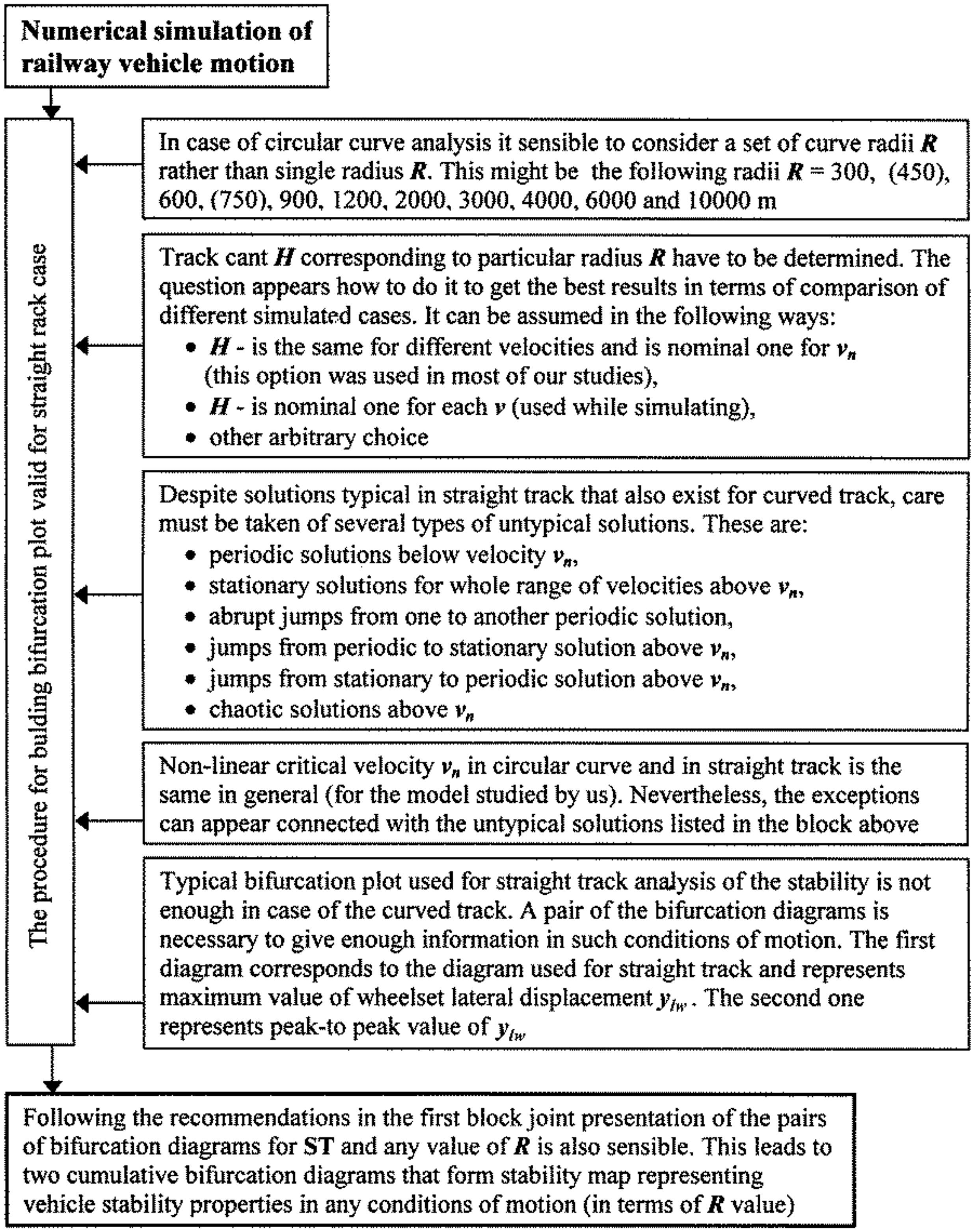

Figure 7. Supplements to the nonlinear lateral stability analysis in ST, necessary in CC analysis [33, 1]

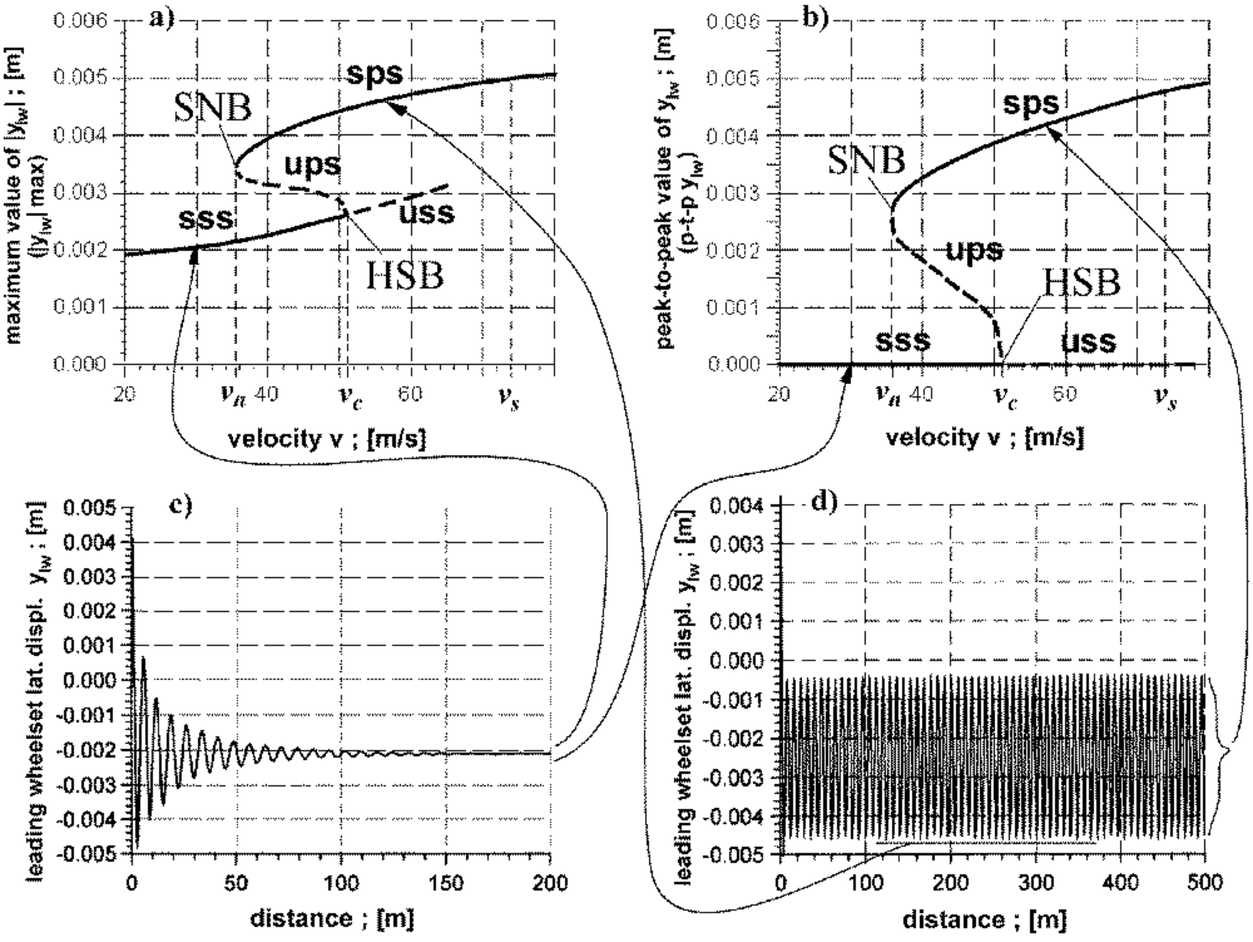

Figure 8. The essence of method of stability analysis in a CC, i.e., building the bifurcation plot [34, 1]

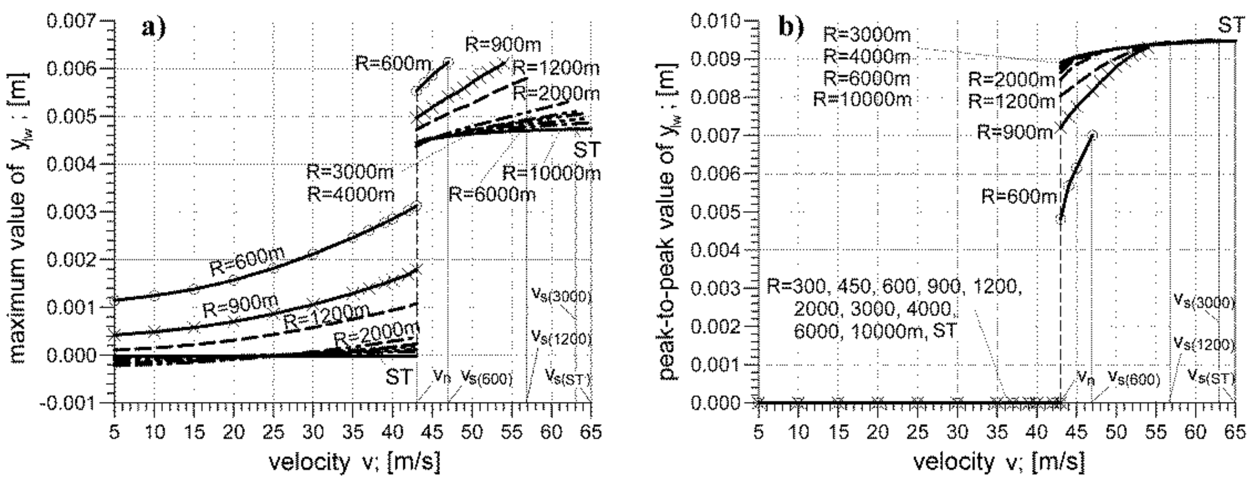

Figure 9. Stability map for hsfv1 car for S1002/UIC60 wheel and rail profiles: (a) leading wheelset lateral displacements y_{lw} and (b) peak to peak values of displacements y_{lw}

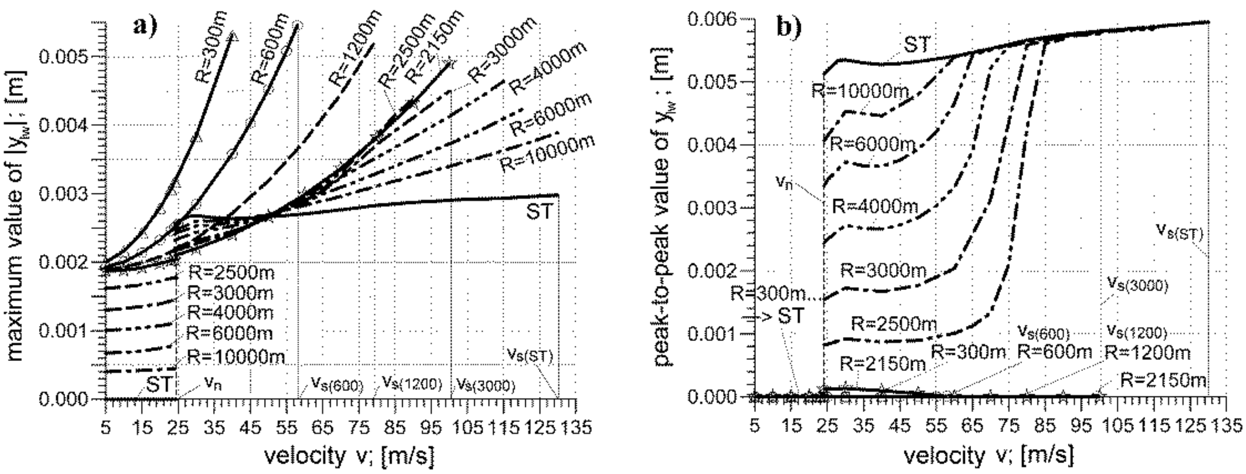

Figure 10. Stability map for hsfv1 car for SZDwheel/R65 wheel and rail profiles: (a) leading wheelset lateral displacement absolute values y_{lw} and (b) peak to peak values of displacements y_{lw}

The influence of several factors on the stability was studied based on the stability map technique. Among the factors studied are accuracy of wheelset's angle of attack determination, track superelevation, types of nominal wheel and rail profiles, type and value of the wheel and rail wear, stiffness and damping values in suspension, type of vehicle (car and bogie), rail inclination, way of mean rolling radius modeling, track gauge, and value of coefficient of friction. Figures 9 and 10 can represent part of the results for one of the factors, namely the wheel profile shape. Both maps were obtained for the same object, however, with different nominal wheel profiles. One can observe significant differences between both maps. One among the most important differences are nonlinear critical velocities v_n of 43 and 24.1 m/s, respectively. Another is the existence of quasi-static solutions for R<2150 m in Figure 10. This can be easily recognized as no vibrations appear, resulting in p-t-p y_{lw}=0, for such values of R in Figure 10b. In Figure 9, vibrations appear for all R values represented there. More differences are discussed in Refs. [33, 1].

The results in Figures 9 and 10 refer to 2-axle freight car hsfv1 of the model shown in Figure 2. Example results of stability studies for the model of MKIII passenger car shown in Figure 3 can be found in Ref. [18].

4.2. Use of the simulation in studying the influence of kinematics accuracy on vehicle dynamics in a curved track at variable velocity

Use of Option 2 from Section 1.1 to build mathematical and numerical models of vehicle-track systems by the present author, made it natural that he was always interested in the importance of accurate modeling relative kinematics connected with description in moving reference systems. This interest was amplified since an additional work is necessary as compared with description in absolute reference systems. Besides, some of the authors neglect additional terms (imaginary forces) in their equations of motion, without proper justification. Good examples of such works might be provided in Refs. [3, 40–42]. Author of this book chapter undertook two stage attempt to finally resolve the problem of particular inertia terms importance. The first one concerned vehicle motion with constant velocity. It finished with publication [5]. The second attempt concerned the motion with variable velocity and finished with publication [4]. Publication [4] is also verification of the results from [5], as motion with constant velocity was just a special case in Ref. [4]. Therefore, Ref. [4] is the main source for results forthcoming in the current section. These results were also published in Ref. [1]. Both Refs. [4] and [1] present the issue in a comprehensive form. Here, only samples of the results and most general conclusions will be represented.

The idea of the study was to compare the results for the vehicle model with all imaginary forces included with those for the model in which the imaginary forces were omitted. In order to precisely determine importance of particular terms types, the forces (and torques) were selectively, rather than totally, omitted. Let us now recognize the generalized imaginary forces according to Ref. [7]. Here, equation (24) will be useful. The forces' terms are present in the second line of this equation and the torques' terms are present in the fourth line of it. Note that all the terms are multiplied by the corresponding linear and angular partial velocities. Therefore, we can refer directly to content of the square brackets in lines 2 and 4. Making use of that idea, the 1st term in the square bracket in line 2 represents inertia forces of translation. The 2nd term in line 2 and 1st one in line 4 form inertia forces of rotation. The 3rd term in line 2 and 2nd one in line 4 correspond to centrifugal forces of inertia. The 4th term in line 2 and the 3rd in line 4 make gyroscopic forces. The abovementioned four categories of imaginary forces' terms appear in the equations as the terms' components corresponding to the direction a particular equation of motion describes. Finally, the omissions of all components for given directions were performed. In fact, these were longitudinal, lateral, and vertical directions. At the end, for the given direction, the analyses were performed to find the components of the terms of particular (practical) importance among all.

The variable velocity was realized in the studies with use of the uniform variable motion. The change in velocity is indirectly represented by the corresponding acceleration value a. Consequently, negative values of acceleration a mean deceleration, corresponding to vehicle braking, the positive values of a mean vehicle acceleration, corresponding to its speeding up, while zero value of a means motion with constant velocity, that is, v=const. Different intensity of braking or speeding up were realized with different acceleration values.

4.2.1. Scope of the studies and example results of simulation

Generally, the tests described in Ref. [4] represent seven different routes and two different vehicles. The routes included radii R=300, 600, 1200, and 2000 m and were composed in continuation either as ST, TC, and CC or as ST, TC, CC, TC, and ST. Thus, the entrance TCen and exit TCex transition curves were considered. The TCs were of the 3rd degree parabola type. Characterizing all the routes briefly, it has to be stated that their two general types existed. The first type corresponded to real railway conditions. The second one represented the case with unnaturally shortened TC in order to get stronger effects for research purposes. As concerns the two vehicles used in Ref. [4], the first one was hsfv1 car described in Section 3.1. Second was vehicle referred to as the 2-axle freight car in Ref. [5]. Both models have the same structure as shown in Figure 2, and they are supplemented with the same track models from Figures 3 and 4. This also results in the same number of DOFs for both of them. The major difference is that first model represents empty car, while the second one the laden car. The other minor differences in the parameters (see [5]) are negligible, here.

Two types of simulation results are presented below. The first type is imaginary forces being omitted in the study. They are a cause for the solution differences being of interest. The imaginary forces are shown in Figures 11, 14, and 16. The second type of simulation results is those representing the differences between solutions for the whole model and the model with particular imaginary force omitted. They are represented with Figures 12, 13, 15, and 17. Results for complete model are drawn with the solid line, while for the model with imaginary torques omitted with the dashed line.

Just two routes appear in the mentioned figures. The route I is as follows: ST (l=10 m, H=0 m); TCen (l=7 m, R_{min}=300 m, H_{max}=0.15 m); CC (l=30 m, R=300 m, H=0.15 m). Initial velocities for route I were between v_0=10.25 and 15.0 m/s, where the smallest value corresponded to a=0 m/s^2 and the biggest one to a=6 m/s^2. Route I includes TC with length unnaturally shortened for research purposes. Route II is as follows: ST (l=10 m, H=0 m); TCen (l=102, 4 m, R_{min}=600 m, H_{max}=0.128 m); CC (l=60 m, R=600 m, H=0.128 m); TCex (l=102.4 m, R_{min}=600 m, H_{max}=0.128 m); ST (l=40 m, H=0 m). Initial velocities for the simulations were v_0=19.91, 22.0, and 22.09 m/s. They corresponded to a=2, 0, and −2 m/s^2, respectively. Route II entirely corresponds to real railway conditions. In the above given brackets, the denotations are l is denoted as the length of the track section, R as the circular curve radius, and H as the circular curve superelevation. Values of the corresponding accelerations are given directly in the figures in round brackets.

It is worth noting in Figure 11 that it includes the biggest values of the imaginary torque obtained in the author's studies. As is seen, it is the roll imaginary torque $P_{\phi b}$ of hsfv1 car body related to longitudinal direction. Character of the torque changes is rectangular, that is, nonzero values of the torque exist for TC only. Values of this torque depend strongly on value of the acceleration a. The value is biggest for the biggest acceleration a=6 m/s^2 and smallest as well as equal to zero for a=0 m/s^2. The last fact means no influence of this torque on motion with constant velocity v=const. In the case of Figures 12 and 13, strong influence of omission of torque $P_{\phi b}$ from Figure 11 can be observed. The two mostly influenced coordinates are shown. They are roll ϕ_b and yaw ψ_b angles of hsfv1 car body, respectively. Note that the influence on angle ψ_b (Figure 13) is approximately one order of magnitude smaller than on angle ϕ_b (Figure 12).

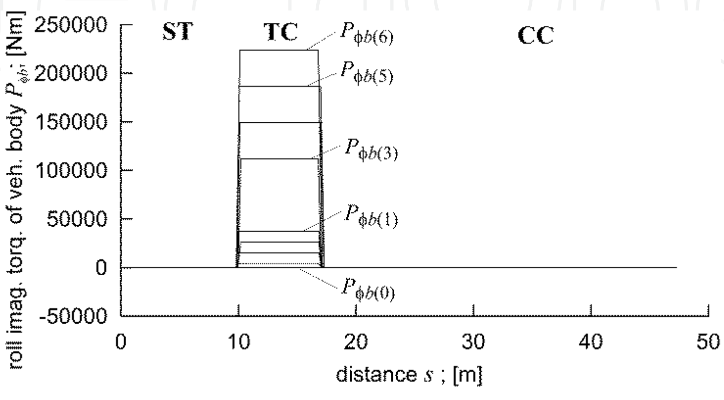

Figure 11. Roll imaginary torque of hsfv1 car body on route I for different accelerations

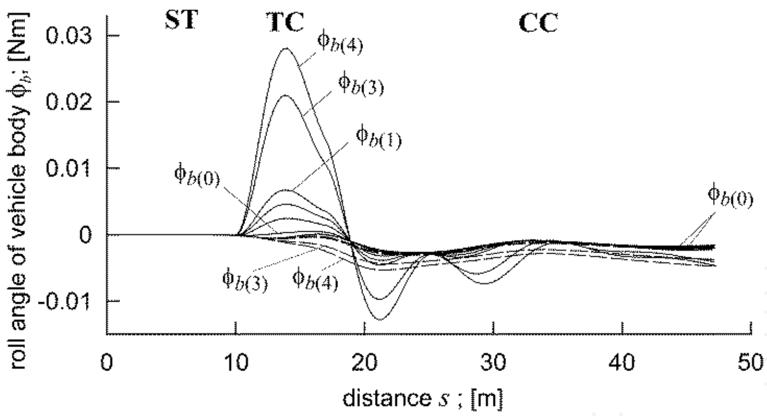

Figure 12. Roll angle of hsfv1car body on route I for different accelerations and roll imaginary torque of hsfv1car body omitted

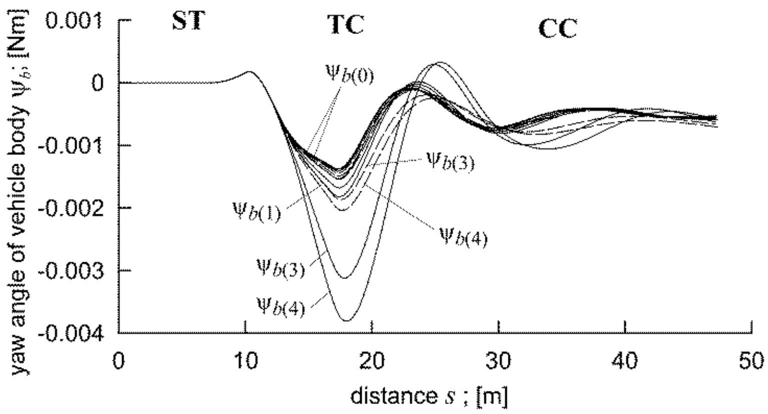

Figure 13. Yaw angle of hsfv1car body on route I for different accelerations and roll imaginary torque of hsfv1car body omitted

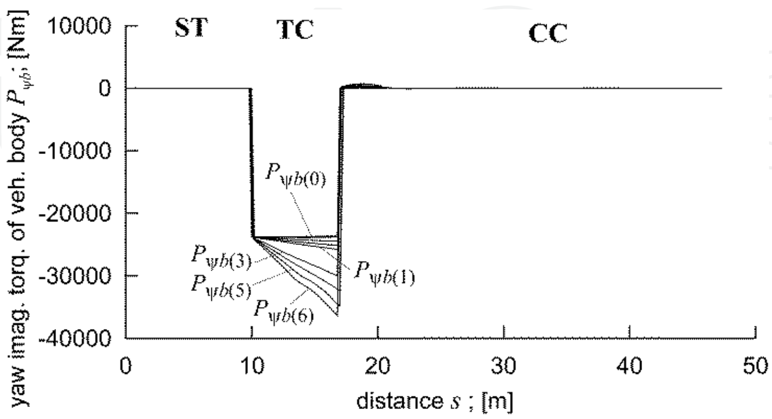

Figure 14. Yaw imaginary torque of hsfv1 car body on route I for different accelerations

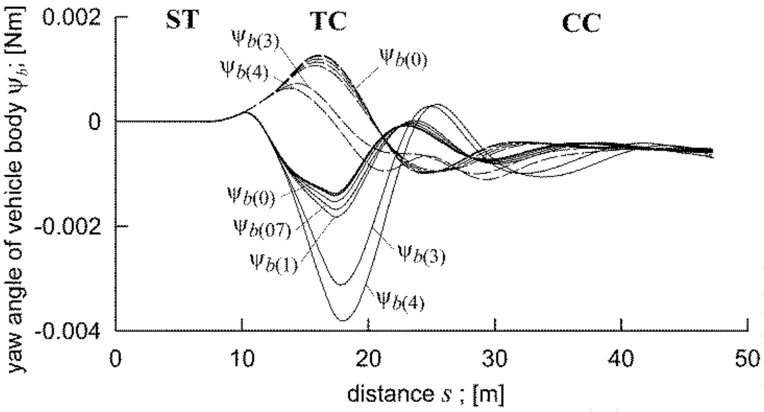

Figure 15. Yaw angle of hsfv1car body on route I for different accelerations and yaw imaginary torque of hsfv1car body omitted

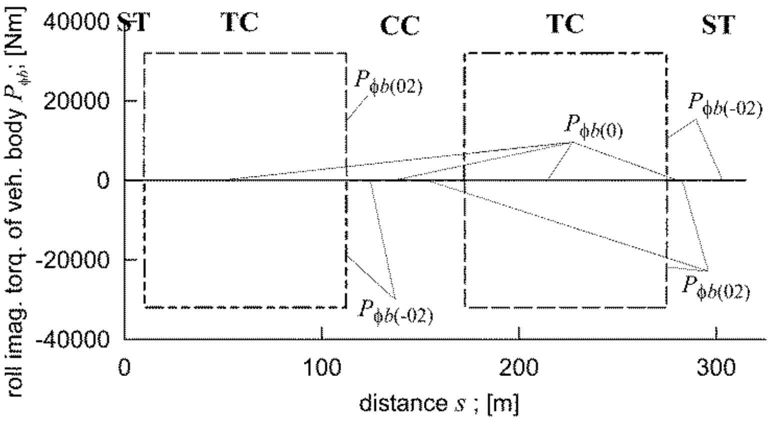

Figure 16. Roll imaginary torque of 2-axle freight car body on route II for different accelerations

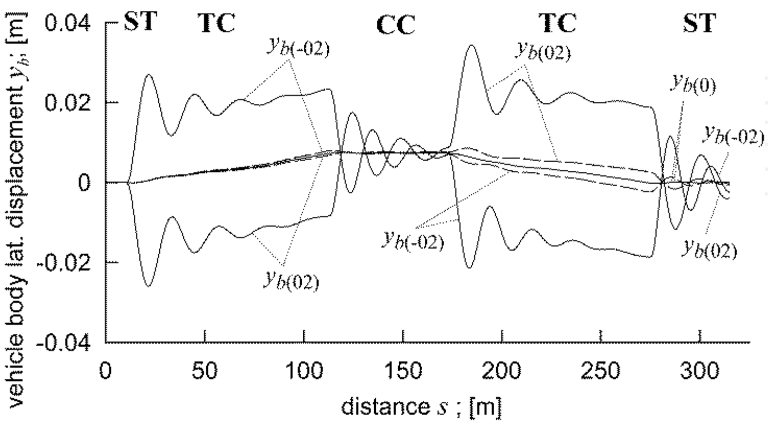

Figure 17. Lateral displacement of 2-axle freight car body on route II for different accelerations and roll imaginary torque of 2-axle freight car body omitted

In Figure 14, the value of the imaginary torque $P_{\psi b}$ of hsfv1 car body, related to vertical direction, is represented. Character of the torque changes corresponds to rectangle supplemented with triangle. The rectangle part is independent of the acceleration a. Therefore, it exists also for a=0 m/s^2. Increase of the supplementary triangle part grows with increase of a. Therefore, the torque value is biggest for the biggest acceleration a=6 m/s^2 and smallest for a=0 m/s^2. Overall, this means that in case of v=const. and a=0 m/s^2 influence of $P_{\psi b}$ omission exists; however, for a>0, it is bigger. This is also the case in Figure 15 for ψ_b, where very strong influence of the omission can be seen. The biggest influence for a=6 m/s^2 is of the same order of magnitude as in Figure 13 related to $P_{\phi b}$ omission. Therefore, it can be concluded that influences of $P_{\phi b}$ and $P_{\psi b}$ omissions on ψ_b values accumulate.

In Figure 16, the character of torque $P_{\phi b}$ for 2-axle freight car is in accordance with that in Figure 11. The difference exists, however, that results from route II configuration. One can observe opposite signs of $P_{\phi b}$ for the entrance TC (TCen) and exit TC (TCex). The sign becomes also opposite when opposite acceleration a sign is adopted. In Figure 16, this case corresponds to a=2 m/s^2 and a=−2 m/s^2. In Figure 17, for 2-axle freight car very strong influence of $P_{\phi b}$ omission on vehicle body lateral displacements y_b can be observed. The changes in signs for y_b correspond to those for the $P_{\phi b}$ described above provided one remembers that if $P_{\phi b}$>0 then y_b<0 and vice versa.

4.2.2. General conclusions from the study

Except the important influences described in Section 4.2.1, some other important influences were found and indicated in Ref. [4]. Therefore, except influences of $P_{\phi b}$ on ϕ_b, y_b, and ψ_b, the important influence on the wheelsets' lateral displacements y_{lw} and y_{tw} exists. Except the influence of $P_{\psi b}$ on ψ_b, very serious influence on the wheelsets yaw angles ψ_{lw} and ψ_{tw} exists. Assuming that lateral component (parallel to track plane in curved track) is known for its importance, the vertical component (perpendicular to the track plane) was studied. It appeared that the influence of this component omission on the results, especially vehicle body vertical displacements z_b, is also significant.

Conclusions from the studies, including track section, character of velocity, vehicle elements, direction defining equation, and purpose of calculations, were summarized in table form in Ref. [4]. Here, we present them in Table 1. This table is analogous to that in Ref. [4] in terms of merits. Nevertheless, Table 1 is differently arranged.

Generally, the most important are longitudinal and vertical direction terms related to car body. The influence of eventual term omission is particularly important in TCs. The centrifugal force is important in both the TCs and the CC sections. The influence in TCs increases as compared with CC when the vehicle moves with variable velocity. The stronger the change in velocity (both accelerating and braking), the greater the influence. The last column represents a direct recommendation of terms that should not be neglected.

Group of the imaginary forces	Term/line no. in Equation (24)	Force/Torque	Importance generally	Important sections of track	Important directions	Velocity constant/variable	Important elements of vehicle	Particularly important conditions	Practice/Research	General recommendations for omission
Inertia forces of translation	1/2	Force	Yes	ST, CC, TC	Long. ($O_1 x$)	Var.	All	Always, big $\|a\|$	p/r	**No**
Inertia forces of rotation	2/2	Force	No	---	---	---	---	---	---	Yes
	1/4	Torque	Yes	TC	Long. ($O_1 x$) vert. ($O_1 z$)	Var. const. (vert.)	Body	Short TC (long.& vert.), big $\|a\|$ (long.)	p/r	**No**
Centrifugal forces	3/2	Force	Yes	CC, TC	Lat. ($O_1 y$) vert. ($O_1 z$)	Const. var.	All (lat.), body (vert.)	Always, big v, small R	p/r	**No**
	2/4	Torque	No	---	---	---	---	---	---	Yes
Gyroscopic forces	4/2	Force	No	---	---	---	---	---	---	Yes
	3/4	Torque	Yes	CC	Long. $O_1 x$	Const.	Wheelset	Small R (R<600m), big v (v "/>v_c)	r	No

Table 1. Importance and recommendations for imaginary forces omission in rail vehicle dynamics

5. Concluding remarks

The author of this chapter has been involved in technological problems of railways for about 30 years. More precisely, his involvement concerns part of the railway system that are railway vehicles. Within the railway vehicles, many areas of studies can be mentioned as, for example, material issues, production technical issues, braking systems issues, traction systems issues, construction issues, exploitation issues, control issues, safety systems issues, etc. The author is interested in dynamics of vehicle motion with a special regard to the motion in curved sections of the track. It is obvious that the chapter was devoted to these last aspects. The author focused his efforts on giving the idea to the readers how important and powerful tools are methods of modeling and simulation when used for the purposes of the rail vehicle dynamics.

In Sections 1 and 2, the general methods of modeling dynamics in moving coordinate systems, useful for multibody systems of rail vehicle type, were represented. The Newton–Euler and Kane's equations in the form suitable in AGEM approach were discussed. At the end of Section 2.2, the method profiled for rail vehicle systems was also presented. These methods help build mathematical models of rail vehicle dynamics. In Section 3, example nominal models of rail vehicles were introduced, as used by the author. In that section, the issue of building numerical models corresponding to the mathematical ones was also discussed.

In Section 4, two examples of the author's simulation studies, with use of the numerical models, were discussed. The first example represented in Section 4.1 concerned the problem of rail vehicle stability in a curved track. Due to the author's systematic and consequent simulation studies, the method was formulated suitable for rail vehicle nonlinear lateral stability studies in a curved section of the track. The examples of the so-called stability map, being the most important result in the studies, were shown and briefly commented on. The stability maps approach was used by this author to study the influence of different factors on the results of stability analysis in a curved track.

The second example is represented in Section 4.2. It was devoted to the use of the simulation in studying the influence of kinematics accuracy on vehicle dynamics in a curved track at variable velocities. In fact, both the constant and variable velocity cases were of interest. Due to the consequent author's interest in this issue, it appeared possible, based on the direct results of simulation and equation terms analysis, to resolve the problem in full. The author presents tabular conclusion of the results in Section 4.2.2. The table states precisely which imaginary forces terms, in what conditions, and for which vehicle major elements (bodies) may have an important influence on the simulation results. The last column states that imaginary force of translation, imaginary torque of rotation, and centrifugal force should not be neglected in both theoretical and practical analyses (calculation). The gyroscopic forces should not be neglected in the theoretical issues as well.

Both described results of the simulation studies are original and important contributions to the knowledge on rail vehicle dynamics. These contributions would not be possible without the mathematical, nominal, and numerical models of vehicles build based on the modeling methods discussed in the chapter. Therefore, it is hoped that the readers have obtained an idea of the efficiency and importance of both the modeling and the simulation for the development of contemporary rail vehicle dynamics, and thus the railways in general.

Author details

Krzysztof Zboinski*

Address all correspondence to: kzb@wt.pw.edu.pl

Faculty of Transport, Warsaw University of Technology, Poland

References

[1] Zboinski K. Nieliniowa dynamika pojazdow szynowych w luku. Warsaw-Radom: Wydawnictwo Naukowe ITE PIB; 2012. 371 p. (monograph in Polish – translated title: Non-linear dynamics of rail vehicles in a curve)

[2] Zboinski K. Modelling dynamics of certain class of discrete multi-body systems based on direct method of the dynamics of relative motion. Meccanica. 2012;47(6): 1527–1551. DOI: 10.1007/s11012-011-9530-1

[3] Popp K, Schiehlen W. Fahrzeugdynamik. Stuttgart: B.G. Teubner; 1993. 308 p.

[4] Zboinski K. The importance of kinematics accuracy in modelling the dynamics of rail vehicle moving in a curved track with variable velocity. International Journal of Heavy Vehicle Systems. 2011;18(4):411–446.

[5] Zboinski K. Importance of imaginary forces and kinematic type non-linearities for description of railway vehicle dynamics. Proceedings of IMechE Part F Journal of Rail and Rapid Transit. 1999;213(3):199–210.

[6] Zboinski K. Relative kinematics exploited in Kane's approach to describe multibody-body system in relative motion. Acta Mechanica. 2001;147(1–4):19–34.

[7] Lurie AI. Analytical mechanics. 1st ed. Berlin, Heidelberg, New York: Springer-Verlag; 2002. 864 p.

[8] Roberson RE, Schwertassek R. Dynamics of multibody systems. Berlin, Heidelberg: Springer-Verlag; 1988. 460 p.

[9] Schiehlen W, editor. Dynamical analysis of vehicle systems. Vien, New York: Springer; 2007. 304 p.

[10] Schiehlen W, Eberhard P. Technische dynamik: modelle fur regulung und simulation. Stuttgart, Leipzig, Wiesbaden: B.G. Teubner; 2004. 252 p.

[11] Kane TR, Levinson DA. Dynamics: theory and applications. New York: McGraw-Hill College; 1985. 523 p.

[12] Huston R. Useful procedures in multibody dynamics. In: Bianchi G, Schiehlen W, editors. Proceedings of Dynamics of Multibody Systems IUTAM/IFT MM Symposium. Berlin, Heidelberg: Springer; 1986. p. 69–77.

[13] Huston RL. Multibody dynamics. New York: Butterworth-Heinemann; 1990. 416 p.

[14] Josephs H, Huston RL. Dynamics of mechanical systems. Boca Raton: CRC Press LLC; 2002. 776 p.

[15] Kortum W, Sharp R, dePater AD. Application of multibody computer codes to vehicle system dynamics - progress report to 12th IAVSD Symposium on a workshop

and resulting activities. Oberpfaffenhofen: Society for Engineering and Scientific Education; 1991.

[16] Schiehlen W, editor. Multibody systems handbook. Berlin, Heidelberg, New York, London, Paris, Tokyo, Hong Kong: Springer-Verlag; 1990.

[17] Zboinski K. Dynamical investigation of railway vehicles on a curved track. European Journal of Mechanics, part A Solids. 1998;213(3):1001–1020.

[18] Dusza M, Zboinski K. Simulation study of 4-axle railway vehicle stability analysis in a curved track. In: Materials of 23rd IAVSD Symposium on Vehicles on Roads and Tracks (electronic form); August 2013; Qingdao, China. 2013. Paper 2013-20.3-ID429.

[19] Gear CW. Numerical initial value problems in ordinary differential equations. Englewood Cliffs, New Jersey: Prentice -Hall Inc.; 1971. p. 253.

[20] Zboinski K. Numerical and traditional modelling of dynamics of multi-body system in type of a railway vehicle. Archives of Transport. 2004;16(3):81–106.

[21] Zboinski K, Woznica P. Optimisation of the railway transition curves' shape with use of vehicle-track dynamical model. Archives of Transport. 2010;22(3):387–407.

[22] Kalker JJ. User's manual for program CONTACT vers. 1 and 3. Delft: Delft University of Technology; 1986.

[23] Kalker JJ. A fast algorithm for the simplified theory of rolling contact. Vehicle System Dynamics. 1982;11:1–13.

[24] Iwnicki S, editor. Handbook of rail vehicle dynamics. Boca Raton, London, New York: CRC Press, Taylor and Francis Group; 2006. 526 p.

[25] Shabana AA, Zaazaa KE, Sugiyama H. Railroad vehicle dynamics: a computational approach. Boca Raton, London, New York: CRC Press, Taylor and Francis Group; 2008. 331 p.

[26] Lee SY, Cheng YC. Hunting stability analysis of higher-speed railway vehicle trucks on tangent track. Journal of Sound and Vibration. 2005;282(3–5):881–898.

[27] Kik W, Piotrowski J. A fast, approximate method to calculate normal load between wheel and rail and creep forces during rolling. In: Zobory I, editor. Proceedings of 2nd Mini-Conference Contact Mechanics and Wear of Rail/Wheel Systems ; Budapest. TU Budapest; 1996.

[28] Kik W. Comparison of behaviour of different wheelset-track models. Vehicle System Dynamics. 1992;20(suppl.):325–339.

[29] Kisilowski J, Knothe K, editors. Advanced railway vehicle system dynamics. Warsaw: WNT; 1991. 442 p.

[30] Zboinski K, Woznica P. Optimisation of railway transition curves with regard to wheel/rail wear. In: Pombo J, editor. Proceedings of the Second International Confer-

ence on Railway Technology: Research, Development and Maintenance; 2014; Corsica. Stirlingshire, UK: Civil-Comp Press; 2014. Paper 206. DOI: 10.4203/ccp.104.206

[31] Zboinski K, Golofit-Stawinska M. Simulation study into non-linear properties of railway vehicles in transition curve at velocities around the critical one. In: Materials of 23rd IAVSD Symposium on Vehicles on Roads and Tracks (electronic form); August 2013; Qingdao, China. 2013. Paper 2013-20.3-ID452.

[32] Zboinski K, Golofit-Stawinska M. Dynamics of 2 and 4-axle railway vehicles in transition curves above critical velocity. In: Pombo J, editor. Proceedings of the Second International Conference on Railway Technology: Research, Development and Maintenance; 2014; Corsica. Stirlingshire, UK: Civil-Comp Press; 2014. Paper 265. DOI: 10.4203/ccp.104.265

[33] Zboinski K, Dusza M. Self-exciting vibrations and Hopf's bifurcation in non-linear stability analysis of rail vehicles in a curved track. European Journal of Mechanics, part A Solids. 2010;29(2):190–203.

[34] Zboinski K, Dusza M. Extended study of railway vehicle lateral stability in a curved track. Vehicle System Dynamics. 2011;49(5):789–810.

[35] Huilgol RR. Hop-Friedrichs bifurcation and the hunting of a railway axle. Quarterly Journal of Applied Mathematics. 1978;36:85–94.

[36] True H. On multiple attractors and critical parameters and how to find them numerically: the right, the wrong and the gambling way. Vehicle system dynamics. 2013;51(3):443–459. DOI: 10.1080/00423114.2012.738919

[37] True H. On the theory of non-linear dynamics and its applications in vehicle system dynamics. Vehicle System Dynamics. 1999;31(5–6):393–421.

[38] Polach O. Characteristic parameters of nonlinear wheel/rail contact geometry. Vehicle System Dynamics. 2010;48(suppl.):19–36.

[39] Zboinski K, Dusza M. Analysis and method of the analysis of non-linear lateral stability of railway vehicles in curved track. Vehicle System Dynamics. 2004;41(suppl.): 222–231.

[40] Andersson E, Nilstam NG, Ohlsson L. Lateral track forcesat high speed curving. Comparison of practical and theoretical results of Swedish high speed train X2000. Vehicle System Dynamics. 1995;25(suppl.):37–52.

[41] Berghuvnd A, Stensson A. Dynamic behaviour of ore wagons in curves at Malmbanan. Vehicle System Dynamics. 1998;30(3–4):271–283.

[42] Scott GA. VAMPIRE user's manual. Report MR VDY 001. British Rail Research, Derby; 1989.

IntechOpen