Chapter 3

The Electromagnetic Force between Two Parallel Current Conductors Explained Using Coulomb's Law

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Additional information is available at the end of the chapter

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Abstract

In this book chapter the electromagnetic force between two parallel electric conductors has been derived, applying thereby the effects of propagation delay and the Special Relativity theory, taking thereby also into count the thus far neglected effects introduced by the voltage sources of both circuits. This has been done for a specific case consisting of two rectangular circuits, aligned to each other along one of the long sides, at a distance that is short compared to the long sides. The intention in doing so is to make a meaningful application of the concept of "two parallel conductors of infinite length", so that it is possible to make a complete calculation of the force between the two circuits, avoiding thus making a vague claim as for example Maxwell, saying that the other parts of the conductors do not contribute to the force. What is radically new in this interpretation is that it is Coulomb's law that is responsible for the force.

Keywords: Ampère's Bridge, Ampère's Law, Coulomb's Law, electromagnetic force, Lorentz force, Lorentz transformation, parallel conductors, propagation delay, retarded action, Special Relativity theory, Sagnac effect, time dilatation

1. Introduction

In several papers evidence has been presented that is able to refute the widely recognized electromagnetic theory of today [1-5]. One such fundamental law is Lorentz' force law. Already 1997 a paper presented mathematical proofs showing that this law is unable to explain the repulsive force between collinear currents, demonstrated in the case of Ampère's bridge [1]. Even Graneau's exploding wires and Hering's pump cause difficulties, when trying to use Lorentz' force law in order to explain the effects that have been registered [6-11]. Therefore it is most exciting to explain one of the most frequent applications of Lorentz force law, the attractive force exerted between two parallel conductors carrying a DC current. Confessedly, others have made efforts in this respect already near 200 years ago, most famous of them Ampère [12]. Successors, like Grassmann, have made serious efforts to make the Lorentz force (in his early pre-Lorentz formulation) appear to be in accordance with Ampère's results [13]. In a more recent paper, this claim has been discarded through a mathematical analysis of Grassmann's derivation [14]. Additionally, in order to introduce a new theory, it must be able not only to explain experiments that a recognized theory cannot, but also to explain the experiments that it apparently is successful in explaining. One crucial phenomenon is that of light, or electromagnetic radiation. In fact, it has been possible to explain this, too, using basically Coulomb's law [15-18]. One may mention also electromagnetic induction [3-5].

The traditional methods have the benefit of being able to predict certain experiments, but not all. A new method must therefore in order to be better both done the first thing, but also be able to explain more evidence. By going back to the most basic well-corroborated law, Coulomb's law, one would expect a possible solution, provided one is very careful and applies mathematical method in a very strict fashion.

2. Method

2.1. Description of a physical circuit describing two parallel conductors

A geometry is defined, with two parallel conductors aligned to the x axis in a Cartesian coordinate system. They are assumed to be of a length that is very long compared to the rectilinear distance. Already Maxwell complained that this kind of analysis is incomplete, if not taking into account the track along which the respective currents returns to its origin and that the apparent conflict between the theory of Ampère and that of Grassmann is related to this [19]. This is done here, too.

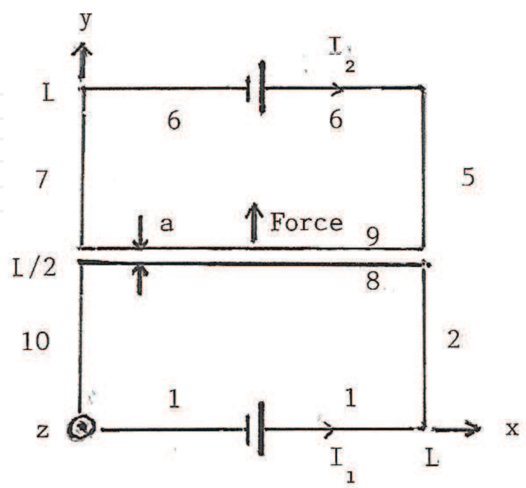

Figure 1. The configuration according to Ampère's bridge used in order to realize a "two-infinite-parallel-conductor" circuit

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2.2. Mathematical treatment of 'infinite length' and other approximations

A geometry is defined, with two parallel conductors aligned to the x axis in a Cartesian coordinate system. They are assumed to be of a length that is very long compared to the rectilinear distance between themselves. Already Maxwell complained that this kind of analysis is incomplete, if not taking into account the track along which the respective currents returns to its origin and that the apparent conflict between the theory of Ampère and that of Grassmann is related to this [20].This has been done here, too. What Maxwell did not specify closer, was the mathematical treatment of the concept 'infinite length'. It must of course be infinite with respect to a smaller entity in the circuit. Choosing quadrangle circuits, with side L and the distance between the two sides of respective quadrangle circuit that are aligned along each other are situated at a distance a. Treating the length of respective side as infinite will be mathematically expressed through

$$\mathbf{L} \gg \mathbf{a} \tag{1}$$

where L [m] denotes the length of the long side of respective circuit and a [m] the distance between the two parallel branches that are close to each other. Generally, what concerns the calculations, the integration results do not display higher order terms that are negligible with respect to the dominating terms. Therefore, a $\cong$ sign will often be used when accounting for the integration result.

2.3. The electromagnetic force between two currents

The two respective currents are being analyzed, using Coulomb's law, taking into account the effects of propagation delay and the Special Relativity Theory. The effects of the propagation delay were derived by this author in a paper 1997 [1], using thereby a different interpretation, than Feynman [21] and Jackson [22]. This author has been successful in showing what the fallacies are [2]. In the 1997 paper [1] it was crucial to the success in using Coulomb's law that the propagation delay was correctly being derived, both due to the "sending charges" of the "first conductor" and to the "receiving charges" of the "second conductor". Having done that analysis, it remains to take into account to the effects of the Special Relativity theory, especially the Lorentz transformation of lengths. Since that effect is related only to the relative movements of the two coordinate axes, and has nothing to do with the propagation delay an observer faces, it may be multiplied straightforwardly to the effect of propagation delay.

An electric current carried by a conductor implies that both the immobile lattice ions and the moving electrons contribute to the force exerted on other charges. In the case those are embedded in a neighboring electric conductor that is also carrying an electric current, they interact with both positive lattice ions and moving conductor electrons. This implies that altogether four kinds of interaction will take place, each demanding their own mathematical treatment respectively: from the positive ions of the first conductor to both kinds of charges of the second conductor and from the electrons of the first conductor to both kinds of charges of the second conductor. In the case of quadrangle circuits, the two respective currents appear both parallel and perpendicular to each other.

2.4. Coulomb's law, basic formulation

In order to integrate the contribution to the total force between two currents, carried by conductors, it is most suitable to use the differential force that an incremental segment gives rise to. If the immobile positive ions of both conductors are taken into account, one may write

$$\Delta\vec{\mathbf{F}} = \frac{\rho_1\rho_2\Delta\mathbf{x}_1\Delta\mathbf{x}_2\vec{\mathbf{u}}_\mathbf{r}}{4\pi\varepsilon_0\mathbf{r}^2} \tag{2}$$

where $\Delta\vec{F}$ [N] denotes the incremental the electric force vector between two points of the two circuits, ρ_1 denotes the line charge density of the first circuit [C/m], ρ_2 the line charge density of the second circuit [C/m] Δx_1 [m] an infinitesimal length element along the x direction of the first conductor, Δx_2 [m) an infinitesimal length element along the x direction of the second conductor, $\vec{r}$ [m] the distant vector from a point of the first conductor to a point of the second conductor $\vec{u}_r$ [m] a unit vector along the distant vector from a point of the first conductor to a point of the second conductor and ε_0 [F/m] denotes the permittivity of vacuum, assuming thus the currents being aligned along the x axis, but this will change throughout the chapter, dependent on which sections of the circuits are being treated, where the distant vector $\vec{r}$ has been dissolved into its three Cartesian components.

$$\vec{\mathbf{r}} = \left(\mathbf{x}_2 - \mathbf{x}_1, \mathbf{y}_2 - \mathbf{y}_1, 0\right) \tag{3}$$

For simplicity the z-coordinate has been chosen to zero, based on the model with the circuits situated in the x-y-plane as defined in Fig. 1.

The force between the two currents will appear as the y component of the total force, according to the following expression:

$$\Delta\vec{\mathbf{F}} \bullet \vec{\mathbf{u}}_\mathbf{y} = \frac{\rho_1\rho_2\Delta\mathbf{x}_1\Delta\mathbf{x}_2 \cdot \vec{\mathbf{u}}_\mathbf{r} \bullet \vec{\mathbf{u}}_\mathbf{y}}{4\pi\varepsilon_0\mathbf{r}^2} \tag{4}$$

A new variable has here been introduced, $\vec{u}_y$ [m] a unit vector along the positive y axis

The case when both conductors are parallel to each other, especially along the x axis.as for example between line 8 and 9, in the figure above, the attractive or repulsive forces between them may be described as the y-component of the force in Eq. (2), as expressed in Eq. (4) above.

In this case, where all charges are stationary, there will be neither a propagation delay effect nor a relativistic effect due to the Lorentz contraction of one or both coordinates.

2.5. The method deriving propagation delay

As was mentioned previously, the effects of propagation delay becomes relevant, when the charges are moving, so that the electric field due to a sending charge must be evaluated at an earlier time event than at the time, when the field was activated at a distant point. Going farther away, the travel time becomes longer. This is described in the following figure, which is based on the analysis first being done in another paper [1]:

The expression for the charge density that is being felt, or observed, at a point at a distance, when the charge density ρ_1 is due to individual charges moving with the velocity $\vec{v}_1$, was derived in the cited paper and is

$$\rho_{1,\mathbf{ret}} = \rho_1\left(1 - \frac{\vec{\mathbf{v}}_1 \bullet \vec{\mathbf{r}}}{\mathbf{rc}}\right) \tag{5}$$

where ρ_{1ret} [C/m] denotes the retarded charge density of the first circuit, $\vec{v}_1$ [m/s] the velocity of a charge element (electrons) of the first conductor and c [m/s] the speed of light, which may also be written:

$$\rho_{1,\mathbf{ret}} = \rho_1\left(1 - \frac{\mathbf{v}_1}{\mathbf{c}} \cdot \cos\theta\right) \tag{6}$$

where θ denotes the angle between the direction of the first current and the distant vector $\vec{r}$ [m].

This expression for the charge density will be used when the electrons are being studied at the first conductor.

In this connection it has to be mentioned that the traditional interpretation of propagation delay, as by Feynman [21] in his derivation of the Liénard-Wiechert potentials is fallacious [2]. However, additionally, there will appear a propagation delay effect also with respect to the charges receiving the action, since the farther away these charges are situated from the sending charges, the longer the way to travel, and hence, the charge density will appear to be smaller to the sender than what is the simultaneous charge density. Correspondingly, the expression for the charge density that is being felt by the sending charges, is also derived in the cited paper and is

$$\rho_{2',\mathbf{ret}} = \rho_2\left(1 - \frac{\vec{\mathbf{v}}_2 \bullet \vec{\mathbf{r}}}{\mathbf{rc}}\right) \tag{7}$$

where ρ_{2ret} [C/m] denotes the retarded charge density of the second circuit, $\vec{v}_2$ [m/s] the velocity of a line charge element (electrons) of the second conductor, which may also be written

$$\rho_{2,\mathbf{ret}} = \rho_2 (1 - \frac{\mathbf{v}_2}{\mathbf{c}} \cdot \cos\psi) \tag{8}$$

where ψ denotes the angle between the distant vector $\vec{r}$ [m] and the direction of the second current. This expression for the charge density will be used when the electrons are being studied at the second conductor.

2.6. Coulomb's law, taking into account the effects of propagation delay

The total force between two parts of respective circuit consists of the sum of the forces due to the four combinations of positive lattice ions and conduction electrons. Using the results of the preceding section, they are:

The first case is when the electric force due to positive charges of both conductors is being studied. The expression for the force will in this case be

$$\Delta\bar{\mathbf{F}}_{+\to+} \bullet \bar{\mathbf{u}}_\mathbf{y} = \frac{\rho_1 \rho_2 \cdot \bar{\mathbf{u}}_\mathbf{r} \bullet \bar{\mathbf{u}}_\mathbf{y} \cdot \Delta\mathbf{x}_1 \Delta\mathbf{x}_2}{4\pi\varepsilon_0 \mathbf{r}^2} \tag{9}$$

where $\Delta\vec{F}_{+\to+}$ [N] denotes the incremental force from the positive charges of the first conductor to the positive charges of the second conductor, still sticking to the case with both sections aligned along the x axis.

The second case, applying to conduction electrons of the first conductor affecting the positive immobile ions of the second conductor will be

$$\Delta\bar{\mathbf{F}}_{-\to+} \bullet \bar{\mathbf{u}}_\mathbf{y} = \frac{-\rho_{1,\mathbf{ret}} \cdot \rho_2 \cdot \bar{\mathbf{u}}_\mathbf{r} \bullet \bar{\mathbf{u}}_\mathbf{y} \cdot \Delta\mathbf{x}_1 \Delta\mathbf{x}_2}{4\pi\varepsilon_0 \mathbf{r}^2} \tag{10}$$

where $\Delta\vec{F}_{-\to+}$ [N] denotes the incremental force from the negative charges of the first conductor to the positive charges of the second conductor, thereby using Eq. (5) for the electrons, applying then the minus sign.

The third case, applying to the positive immobile ions of the first conductor exerting a force on the conduction electrons of the second conductor, will correspondingly be

$$\Delta\bar{\mathbf{F}}_{+\to-} \bullet \bar{\mathbf{u}}_\mathbf{y} = \frac{\rho_1 (-\rho_{2,\mathbf{ret}}) \cdot \bar{\mathbf{u}}_\mathbf{r} \bullet \bar{\mathbf{u}}_\mathbf{y} \cdot \Delta\mathbf{x}_1 \Delta\mathbf{x}_2}{4\pi\varepsilon_0 \mathbf{r}^2} \tag{11}$$

where $\Delta\vec{F}_{+\to-}$ [N] denotes the incremental force from the positive charges of the first conductor to the negative charges of the second conductor.

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Finally, the fourth case, applying to conduction electrons of the first conductor exerting a force on the conduction electrons of the second conductor will be:

$$\Delta\vec{\mathbf{F}}_{-\to-} \bullet \vec{\mathbf{u}}_{\mathbf{y}} = \frac{\rho_{1,\mathbf{ret}} \cdot \rho_{2,\mathbf{ret}} \cdot \vec{\mathbf{u}}_{\mathbf{r}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cdot \Delta\mathbf{x}_1 \Delta\mathbf{x}_2}{4\pi\varepsilon_0 \mathbf{r}^2} \tag{12}$$

where $\Delta\vec{F}_{-\to-}$ [N] denotes the incremental force from the negative charges of the first conductor to the negative charges of the second conductor.

Adding these four contributions, keeping in mind also that

$$\mathbf{I}_1 = \rho_1 \mathbf{v}_1 \tag{13}$$

where I_1 [A] denotes the current of the first conductor and

$$\mathbf{I}_2 = \rho_2 \mathbf{v}_2 \tag{14}$$

where I_2 [A} denotes the current of the second conductor and

$$\frac{1}{\varepsilon_0 \mu_0} = \mathbf{c}^2 \tag{15}$$

where μ_0 [N / A2] denotes the permeability of vacuum, gives rise to the following expression for the total electric force between two electric currents, carried by conductors, and has been earlier derived [1], :

$$\Delta\vec{\mathbf{F}} = \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \cos\theta \cos\psi \cdot \Delta\mathbf{x}_1 \Delta\mathbf{x}_2}{4\pi \cdot \mathbf{r}^2} \cdot \vec{\mathbf{u}}_{\mathbf{r}} \tag{16}$$

valid for the more general case, when the angles between two conductors may be chosen arbitrarily. In the case of two parallel conductors

$$\theta = \psi \tag{17}$$

This expression was also successful in predicting the repulsive force between the two parts of Ampère's bridge, whereas the Lorentz force wasn't [23].

2.7. Coulomb's law, taking into account also the effects of the Special relativity theory

The Special relativity theory implies that relative movement makes the extension of the moving things become smaller, as viewed from point of view of the laboratory system, thereby using the so-called standard configuration [24]. Hence, the vectors between moving and not-moving charges must necessarily be adjusted accordingly.

Hence, in order to derive more exact expressions for the electric force due to moving charges, all terms containing the distance vector between charges in the expressions above, even implicitly, must be modified by using the Lorentz contraction of space, more precisely the point of the vector that connects to a moving charge element [25]:

$$\mathbf{x}' = \frac{\mathbf{x} - \mathbf{vt}}{\sqrt{1 - \mathbf{v}^2 / \mathbf{c}^2}} \tag{18}$$

where x' [m] denotes the Lorentz transformation of the x variable of the first circuit, movement assun´med to take place along the x axis, i.e. Standard Configuration.where some authors prefer to use the term Lorentz factor [26] :

$$\gamma(\mathbf{v}) = \frac{1}{\sqrt{1 - \mathbf{v}^2 / \mathbf{c}^2}} \tag{19}$$

where the Lorentz factor $\gamma(v)$ is dimensionless. One thing that must be taken into account, when the Lorentz transformation is concerned, is that every single incremental charge element that is moving must be denoted its own specific Lorentz transformation, since the Lorentz transformation is in fact dealing with single points moving with a velocity v. This becomes evident, when realizing that it is one event that is observed from two different coordinate systems, i.e. inertial systems [27]. This way of using the Lorentz transformation was furthermore successful in explaining the Sagnac effect [28].

When performing the calculations, a simplification will be introduced that the electrons carrying the both currents are propagating with the same velocity, i.e.

$$\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v} \tag{20}$$

For convenience it is here repeated that $\vec{v}_1$ [m/s] denotes the velocity of a charge element (electrons) of the first conductor and $\vec{v}_2$ [m/s] the velocity of a charge element (electrons) of the second conductor.This assumption in turn leads to

$$\gamma(\mathbf{v}_1) = \gamma(\mathbf{v}_2) = \gamma(\mathbf{v}) \tag{21}$$

Some necessary preparations will also be needed before it is possible to perform the integrations, since the denominators of the terms that have to be integrated are on a form that makes integration on a closed form unfeasible, except for Eq. (9). Serial expansion of the denominators in the shape of binomial series [29] will make it possible to move terms of higher order of $(\frac{v}{c})$ embedded in the $\gamma(v)$ terms of the denominator up to the numerator.

The Lorentz transformation according to the Special Relativity theory will be applied. By practical reasons the calculations have been separated into two categories: the parts of the conductors being interacting with each other being parallel respectively perpendicular to each other. It may be remarked that the Lorentz transformation will affect the infinitesimal incremental length element dx in the denominator, so we will have

$$\rho^{\mathbf{R}} = \gamma(\mathbf{v}) \cdot \frac{\mathbf{dQ}}{\mathbf{dx}} \tag{22}$$

where ρ^{R} [C/m] denotes the Lorentz transformed line charge density.

Along a specified distance (according to the reference system K) along the positive direction of movement, there will apparently be more charges and if this would be the case for all directions of movement, as when the charges are turning back to their origin, hence in the opposite direction, charges would seem to have been "created". However, a second effect, the 'time dilation', will make the opposite thing with the incremental length elements in that direction, and, hence, the sum of charges will remain unchanged, independently of from which coordinate system one prefers to observe the events.This is described in the following.

Additionally, there is also an effect, time dilatation that has to be taken into account. This effect causes the observer of K to register different time proceeding dependent of the direction of movement. The basic formula describing time dilatation is [28], [30]:

$$\frac{\mathbf{dt}'}{\mathbf{dt}} = \gamma(\mathbf{v})(1 - \frac{\mathbf{v}}{\mathbf{c}^2}\frac{\mathbf{dx}}{\mathbf{dt}}) \tag{23}$$

Assuming the velocity of the moving electrons being v, makes Eq.(23) transform to:

$$\frac{\mathbf{dt}'}{\mathbf{dt}} = \gamma(\mathbf{v})(1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}) \tag{24}$$

where t [s] denotes the time according to the K inertial system, t' [s] the Lorentz transformed time being observed in K to take place in the K' system.

Eq. (24) may in turn be re-written to

$$\frac{\mathbf{dt}'}{\mathbf{dt}} = \frac{1}{\gamma(\mathbf{v})} \tag{25}$$

which implies that $\gamma(v)$ times more charge will flow through a cross section given a certain time.

but logically, if assuming that the movement takes place in the opposite direction, applying this on Eq. (23) leads to approximately

$$\frac{\mathbf{dt}'}{\mathbf{dt}} = \gamma(\mathbf{v}) \tag{26}$$

in that case,

since

$$(1 + \frac{\mathbf{v}^2}{\mathbf{c}^2}) \cong 1 - \frac{\mathbf{v}^2}{\mathbf{c}^2} \tag{27}$$

if neglecting higher order terms of $\frac{v}{c}$.

Eq. (24) may accordingly be re-written to

$$\frac{\mathbf{dt}'}{\mathbf{dt}} = (\gamma(\mathbf{v}))^3 \tag{28}$$

This implies that $(\gamma(v))^3$ times less charge will flow through a cross section given a certain time, i.e. one will have to divide the force by $(\gamma(v))^3$.

In the case of a rotating disc, the Sagnac effect describes, how light being propagated along the direction of rotation will travel a longer distance than a light beam sent counterclockwise along the same disc [28]. If for example two electron currents flow along the same positive axis, there will not be any time dilatation when comparing them, but if the currents are directed in opposite direction to each other, the effect will be doubled. However, in the case of perpendicular currents, there will be no such difference, since all movement is perpendicular to each other and, hence, there will not be any difference in propagation time in either direction. This all will become more evident, when defining the incremental force contributions due to every incremental displacement.

2.8. The parts of the two conductors being parallel, aligned along the x axis

The expressions for the electric force due to the four combinations of charges (9), (10), (11) and (12) will be modified, using the Lorentz transformation. In the first case, described by Eq. (9), there will be no change, since all the charges are being at rest and, accordingly, no relativistic effects will be observed:

$$\Delta\vec{\mathbf{F}}^{\mathbf{R}}_{+\to+} \bullet \vec{\mathbf{u}}_{\mathbf{y}} = \Delta\vec{\mathbf{F}}_{+\to+} \bullet \vec{\mathbf{u}}_{\mathbf{y}} = \frac{\rho_1\rho_2 \cdot \vec{\mathbf{u}}_{\mathbf{r}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cdot \Delta\mathbf{x}_1\Delta\mathbf{x}_2}{4\pi\varepsilon_0\mathbf{r}^2} \tag{29}$$

where

$$\vec{\mathbf{r}} = (\mathbf{x}_2 - \mathbf{x}_1, \mathbf{y}_2 - \mathbf{y}_1, 0) \tag{30}$$

where $\vec{r}$ [m] denotes the distant vector from a point of the first conductor to a point of the second conductor.

The case when conduction electrons of the first conductor are affecting the immobile, positive ions of the second conductor, as they are moving along the positive x axis, will be more complicated. Applying Eq. (22) and (25) leads implies that

$$\Delta\vec{\mathbf{F}}^{\mathbf{R}}_{-\to'+} \bullet \vec{\mathbf{u}}_{\mathbf{y}} = (\gamma(\mathbf{v}))^2 \frac{-\rho_1(1-\frac{\mathbf{v}_1}{\mathbf{c}}\cos\theta')\rho_2 \cdot \vec{\mathbf{u}}_{\mathbf{r}'} \bullet \vec{\mathbf{u}}_{\mathbf{y}}\Delta\mathbf{x}_1\Delta\mathbf{x}_2}{4\pi\varepsilon_0(\mathbf{r}')^2} \tag{31}$$

where θ' denotes the Lorentz transformed angle for this case, r' [m] the Lorentz transformed absolute value of the Lorentz transformed distance vector $\vec{r}'$ [m] and $\vec{u}_{r'}$ [m] a unit vector along the Lorentz transformed distant vector $\vec{r}'$ from a point of the first conductor to a point of the second conductor, where

$$\vec{\mathbf{r}}' = (\mathbf{x}_2 - \frac{\mathbf{x}_1}{\gamma(\mathbf{v}_1)}, \mathbf{y}_2 - \mathbf{y}_1, 0) \tag{32}$$

and

$$\cos\theta' = \frac{\mathbf{x}_2 - \frac{\mathbf{x}_1}{\gamma(\mathbf{v}_1)}}{\mathbf{r}'} \tag{33}$$

The case when the positive, immobile ions of the first conductors are is exerting a force on the conduction electrons of the second conductors, as they are moving along the positive x axis, will be also be more complicated. Applying Eq. (22) and (25) leads to:

$$\Delta^2\bar{\mathbf{F}}^{\mathbf{R}}_{+\rightarrow'-}\bullet\bar{\mathbf{u}}_{\mathbf{y}}=(\gamma(\mathbf{v})^2)\cdot\frac{-\rho_1\rho_2(1-\frac{\mathbf{v}_2}{\mathbf{c}}\cos\theta'')\cdot\bar{\mathbf{u}}_{\mathbf{r}''}\bullet\bar{\mathbf{u}}_{\mathbf{y}}\Delta\mathbf{x}_1\Delta\mathbf{x}_2}{4\pi\varepsilon_0(\mathbf{r}'')^2} \tag{34}$$

where θ'' denotes the Lorentz transformed angle for this case, r'' [m] the Lorentz transformed absolute value of the Lorentz transformed distance vector $\vec{r}''$ [m] and $\vec{u}_{r''}$ [m] a unit vector along the Lorentz transformed distant vector $\vec{r}''$ from a point of the first conductor to a point of the second conductor, where

$$\bar{\mathbf{r}}''=(\frac{\mathbf{x}_2}{\gamma(\mathbf{v}_2)}-\mathbf{x}_1,\mathbf{y}_2-\mathbf{y}_1,0) \tag{35}$$

$$\cos\theta''=\frac{\frac{\mathbf{x}_2}{\gamma(\mathbf{v}_2)}-\mathbf{x}_1}{\mathbf{r}''} \tag{36}$$

In the case either of the currents flows in the opposite direction, instead of Eq. (25) should be applied Eq. (28) together with Eq. (22) on the equation dealing with that current (i.e. either Eq. (31) or Eq. (34).

The last case, the conduction electrons of the first conductor affecting the conduction electrons of the second conductor, there will be no time dilation effect, but still a Lorentz contraction., This will give rise to the following expression:

$$\Delta\bar{\mathbf{F}}^{\mathbf{R}}_{-\rightarrow'-}\bullet\bar{\mathbf{u}}_{\mathbf{y}}=(\gamma(\mathbf{v})^2)\cdot\frac{\rho_1(1-\frac{\mathbf{v}_1}{\mathbf{c}}\cdot\cos\theta''')\rho_2(1-\frac{\mathbf{v}_2}{\mathbf{c}}\cdot\cos\theta''')\cdot\bar{\mathbf{u}}_{\mathbf{r}'''}\bullet\bar{\mathbf{u}}_{\mathbf{y}}\cdot\Delta\mathbf{x}_1\Delta\mathbf{x}_2}{4\pi\varepsilon_0(\mathbf{r}''')^2} \tag{37}$$

where

$$\bar{\mathbf{r}}'''=(\frac{\mathbf{x}_2}{\gamma(\mathbf{v}_2)}-\frac{\mathbf{x}_1}{\gamma(\mathbf{v}_1)},\mathbf{y}_2-\mathbf{y}_1,0) \tag{38}$$

and

$$\cos\theta'''=\frac{\frac{\mathbf{x}_2}{\gamma(\mathbf{v}_2)}-\frac{\mathbf{x}_1}{\gamma(\mathbf{v}_1)}}{\mathbf{r}'''} \tag{39}$$

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where θ''' denotes the Lorentz transformed angle for this case, r''' [m] the Lorentz transformed absolute value of the Lorentz transformed distance vector $\vec{r}'''$ [m] and $\vec{u}_{r'''}$ [m] a unit vector along the Lorentz transformed distant vector $\vec{r}'''$ from a point of the first conductor to a point of the second conductor. Both the conduction electrons of the first circuit and the conduction electrons of the second circuit are moving, thereby both implying the need for a Lorentz transformation. Hence,

$$\Delta\vec{\mathbf{F}}^{\mathbf{R}}_{-\rightarrow-} \bullet \vec{\mathbf{u}}_{\mathbf{y}} = \frac{\rho_1(1-\frac{\mathbf{v}_1}{\mathbf{c}}\cdot\cos\theta''')\cdot\rho_2(1-\frac{\mathbf{v}_2}{\mathbf{c}}\cdot\cos\theta''')\cdot(\mathbf{y}_2-\mathbf{y}_1)\cdot\Delta\mathbf{x}_1\Delta\mathbf{x}_2}{4\pi\varepsilon_0\cdot(\frac{1}{((\gamma(\mathbf{v}))^2)}\cdot(\mathbf{x}_2-\mathbf{x}_1)^2+(\mathbf{y}_2-\mathbf{y}_1)^2)^{\frac{3}{2}}}. \tag{40}$$

Adding the four contributions to the force, assuming the velocities being equal, expressed through Eq. (29), Eq. (31), Eq. (34) and Eq. (37) gives, thereby using Eq. (34) (39), the result for the case the currents are being directed along the same direction:

$$\Delta\mathbf{F}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cong \frac{\rho_1\Delta\mathbf{x}_1\cdot\rho_2\Delta\mathbf{x}_2\cdot(\mathbf{y}_2-\mathbf{y}_1)}{4\pi\varepsilon_0\mathbf{r}^5}\cdot((\frac{\mathbf{v}}{\mathbf{c}})^2(-1+\cos^2\theta)) \tag{41}$$

where $\vec{F}^{R}_{total}$ [N] denotes the total force due to Lorentz transformed entities, due to the sum of the contributions from all participating charges, for two specified sections of respective conductor, or, using Eq. (13), Eq. (14) and Eq. (15) one may as well write

$$\Delta\vec{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} = \frac{\mu_0\mathbf{I}_1\mathbf{I}_2\Delta\mathbf{x}_1\Delta\mathbf{x}_2\cdot(\mathbf{y}_2-\mathbf{y}_1)}{4\pi\cdot\mathbf{r}^3}\cdot(-1+\frac{(\mathbf{x}_2-\mathbf{x}_1)^2}{\mathbf{r}^2}) \tag{42}$$

One may already now observe the leading term −1 in Eq. (42), pointing to the attractive force between parallel electric currents.

When the direction is the opposite of either current, in the case either or both of the currents flows along the negative x axis, instead of Eq. (25) Eq. (28) would have to be applied together with Eq. (22) on the equation dealing with that current (i.e. either Eq. (31), Eq. (34) or Eq. (37).

Eq. (29) will remain unchanged, since in that case both charges are at rest, implying thus no relativistic effects.

The result is the following modified versions of these equations:

$$\Delta\vec{\mathbf{F}}^{\mathbf{R}}_{-\rightarrow'+} \bullet \vec{\mathbf{u}}_{\mathbf{y}} = (\gamma(\mathbf{v}))^4\,\frac{-\rho_1(1-\frac{\mathbf{v}_1}{\mathbf{c}}\cos\theta')\rho_2\cdot\vec{\mathbf{u}}_{\mathbf{r}'}\bullet\vec{\mathbf{u}}_{\mathbf{y}}\Delta\mathbf{x}_1\Delta\mathbf{x}_2}{4\pi\varepsilon_0(\mathbf{r}')^2} \tag{43}$$

$$\Delta^2 \bar{\mathbf{F}}^{\mathbf{R}}_{+\rightarrow'-} \bullet \bar{\mathbf{u}}_{\mathbf{y}} = \cdot \frac{-\rho_1 \rho_2 (1 - \frac{\mathbf{v}_2}{\mathbf{c}} \cos\theta'') \cdot \bar{\mathbf{u}}_{\mathbf{r}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \Delta \mathbf{x}_1 \Delta \mathbf{x}_2}{4\pi\varepsilon_0 (\mathbf{r}'')^2} \tag{44}$$

$$\Delta \bar{\mathbf{F}}^{\mathbf{R}}_{-\rightarrow'-} \bullet \bar{\mathbf{u}}_{\mathbf{y}} = (\gamma(\mathbf{v})^2) \cdot (1 + 2 \cdot (\frac{\mathbf{v}}{\mathbf{c}})^2) \cdot \frac{\rho_1 (1 - \frac{\mathbf{v}_1}{\mathbf{c}} \cdot \cos\theta''') \rho_2 (1 - \frac{\mathbf{v}_2}{\mathbf{c}} \cdot \cos\theta''') \cdot \bar{\mathbf{u}}_{\mathbf{r}'''} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cdot \Delta \mathbf{x}_1 \Delta \mathbf{x}_2}{4\pi\varepsilon_0 (\mathbf{r}''')^2} \tag{45}$$

The multiplicative term is the sum of the effect of both the forwards moving electrons of the first conductor and the backwards moving electrons of the second conductor.

Adding the four contributions to the force, setting the velocities equal, expressed through Eq. (29), Eq. (43), Eq. (44) and Eq. (45) gives, thereby using Eq. (39), the result for the case the currents are flowing opposite to each other, along the x axis:

$$\Delta \mathbf{F}^{\mathbf{R}}_{\mathbf{total}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cong \frac{\rho_1 \Delta \mathbf{x}_1 \cdot \rho_2 \Delta \mathbf{x}_2 \cdot (\mathbf{y}_2 - \mathbf{y}_1)}{4\pi\varepsilon_0 \mathbf{r}^5} \cdot ((\frac{\mathbf{v}}{\mathbf{c}})^2 (1 - \cos^2\theta)) \tag{46}$$

which apparently implies that the force will be of equal strength, but with opposite sign. In that case. Using Eq. (13), Eq. (14) and Eq. (15) one may as well write

$$\Delta \bar{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} = \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \Delta \mathbf{x}_1 \Delta \mathbf{x}_2 \cdot (\mathbf{y}_2 - \mathbf{y}_1)}{4\pi \cdot \mathbf{r}^3} \cdot (1 - \frac{(\mathbf{x}_2 - \mathbf{x}_1)^2}{\mathbf{r}^2}) \tag{47}$$

2.8.1. Branch 8 to 9

In order to calculate the force between the two branches 8 and 9, one has to insert

$$\mathbf{y}_2 - \mathbf{y}_1 = \mathbf{a} \tag{48}$$

where *a* [m] indicates the distance between the two parallel branches that are close to each other, in Eq. (42), or

$$\int \mathbf{d}\bar{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_2=0}^{\mathbf{L}} \int_{\mathbf{x}_1=0}^{\mathbf{L}} \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \mathbf{d}\mathbf{x}_1 \mathbf{d}\mathbf{x}_2 \cdot \mathbf{a}}{4\pi \cdot \mathbf{r}^3} \cdot (-1 + \frac{(\mathbf{x}_2 - \mathbf{x}_1)^2}{\mathbf{r}^2}) \tag{49}$$

Solving the integral gives the result

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$$\mathbf{F^R}_{\mathbf{total}}(8 \to 9) = \int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} \cong -\frac{4}{3}\frac{\mu_0 \mathbf{I_1 I_2 L}}{4\pi \cdot \mathbf{a}} \tag{50}$$

where $F^{R}{}_{total}(n \to n)$ [N] indicates the total force due to Lorentz transformed entities, due to the sum of the contributions from all participating charges, for two specified sections of respective conductor,

The negative sign implies that it is a question of an attractive force.

2.8.2. Branch 8 to 6

In this case the currents are of opposite direction and hence, instead of applying Eq. (25) on the electrons of branch 6, one will have to apply Eq. (28).

Here

$$\mathbf{y}_2 - \mathbf{y}_1 = \frac{\mathbf{L}}{2} \tag{51}$$

The summation of the contributions from all the charges will in this case be

$$\int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \int_{\mathbf{x}_2=0}^{\mathbf{L}} \int_{\mathbf{x}_1=0}^{\mathbf{L}} \frac{\mu_0 \mathbf{I_1 I_2 dx_1 dx_2} \cdot (\frac{\mathbf{L}}{2})}{4\pi \cdot \mathbf{r}^3} \cdot (1 - \frac{(\mathbf{x}_2 - \mathbf{x}_1)^2}{\mathbf{r}^2}) \tag{52}$$

Solving the integral gives the result:

$$\mathbf{F^R}_{\mathbf{total}}(8 \to 6) = \int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot \frac{2}{3} \cdot \left(\frac{9}{\sqrt{5}} - 1\right) \tag{53}$$

2.8.3. Branch 8 to the voltage source of branch 6

The claim that the voltage source is playing a role in the balance of forces between electric currents was shown already in an earlier paper [31] and accordingly, Eq. (49) would have to be replaced by an integral applying an impulse current instead of I_2, namely $.I_2 \cdot 3L \cdot \delta(x_2 - L/2,)$

It has to be observed that the direction of that current is opposite to I_2, implying thereby the need for using Eq. (42), i.e. the current I_1 has the same direction as $.I_2 \cdot L \cdot \delta(x_2 - L/2,)$.

In this case $y_2 - y_1 = \frac{L}{2}$

The total contribution to the force here will accordingly be:

$$\int d\vec{F}^{R}_{total} \bullet \vec{u}_y \cong \int_{x_2=0}^{L}\int_{x_1=0}^{L} \frac{\mu_0 I_1 \cdot (3L \cdot I_2 \delta(x_2 - \frac{L}{2}) \cdot dx_1 dx_2 \cdot (\frac{L}{2} + a)}{4\pi \cdot r^3} \cdot (-1 + \frac{(x_2 - x_1)^2}{r^2}) \tag{54}$$

Solving the integral gives the result:

$$F^{R}_{total}(8 \rightarrow VS6) = \int d\vec{F}^{R}_{total} \bullet \vec{u}_y \cong \frac{\mu_0 I_1 I_2}{4\pi} \cdot (-5\sqrt{2}) \tag{55}$$

where *VSn* indicates a voltage source with applied branch number.

2.8.4. Branch 1 to 9

Since the geometry is exactly the same as in the case, when branch 8 is affecting branch 6, the integral equation will be almost the same, even though here

$$y_2 - y_1 = \frac{L}{2} \tag{56}$$

The summation of the contributions from all the charges will in this case be

$$\int d\vec{F}^{R}_{total} \bullet \vec{u}_y \cong \int_{x_2=0}^{L}\int_{x_1=0}^{L} \frac{\mu_0 I_1 I_2 dx_1 dx_2 \cdot (\frac{L}{2})}{4\pi \cdot r^3} \cdot (1 - \frac{(x_2 - x_1)^2}{r^2}) \tag{57}$$

The integration result according to Eq. (53) can be used straightforwardly, since the small difference in $y_2 - y_1$ compared to Eq (51) will be negligible.

$$F^{R}{}_{tottal}(1 \rightarrow 9) = \int_{x_2=0}^{L} dx_2 \int_{x_1=0}^{L} dx_1 F^{R}_{total} \bullet \vec{u}_y \cong \frac{I_1 I_2}{4\pi\varepsilon_0} \cdot \frac{2}{3} \cdot \left(\frac{9}{\sqrt{5}} - 1\right) \tag{58}$$

2.8.5. Branch 1 to 6

In order to calculate the force between the two branches 1 and 6, one will have to set

$$y_2 - y_1 = L \tag{59}$$

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in Eq. (42), or

$$\int \mathbf{d\bar{F}^{R}_{total}} \bullet \mathbf{\bar{u}_y} \cong \int_{\mathbf{x_2}=0}^{\mathbf{L}} \int_{\mathbf{x_1}=0}^{\mathbf{L}} \frac{\mu_0 \mathbf{I_1 I_2 dx_1 dx_2} \cdot \mathbf{L}}{4\pi \cdot \mathbf{r}^3} \cdot (-1 + \frac{(\mathbf{x_2} - \mathbf{x_1})^2}{\mathbf{r}^2}) \tag{60}$$

Solving the integral gives the result

$$\mathbf{F^{R}}_{\mathbf{total}}(1 \to 6) = \int_{\mathbf{x_2}=0}^{\mathbf{L}} \mathbf{dx_2} \int_{\mathbf{x_1}=0}^{\mathbf{L}} \mathbf{dx_1 F^{R}_{total}} \bullet \mathbf{\bar{u}_y} \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot \frac{2}{3} \cdot \left(1 - \frac{3}{\sqrt{2}}\right) \tag{61}$$

2.8.6. Branch 1 to the voltage source of branch 6

In this case involving a voltage source, one will have to apply an impulse current instead of I_2, namely $.3I_2 \cdot L \ \cdot \delta(x_2 - L \ /2,)$

It has to be observed that the direction of that current is opposite to I_2, implying thus the need for using Eq. (42), i.e. the current I_1 has the same direction as $.I_2 \cdot L \ \cdot \delta(x_2 - L \ /2)$.

In this case $y_2 - y_1 = L$

The total contribution to the force here will be:

$$\int \mathbf{d\bar{F}^{R}_{total}} \bullet \mathbf{\bar{u}_y} \cong \int_{\mathbf{x_2}=0}^{\mathbf{L}} \int_{\mathbf{x_1}=0}^{\mathbf{L}} \frac{\mu_0 \cdot \mathbf{I_1} \cdot 3\mathbf{L} \cdot \mathbf{I_2} \cdot \delta(\mathbf{x_2} - \frac{\mathbf{L}}{2}) \cdot \mathbf{dx_1 dx_2} \cdot \mathbf{L}}{4\pi \cdot \mathbf{r}^3} \cdot (-1 + \frac{(\mathbf{x_2} - \mathbf{x_1})^2}{\mathbf{r}^2}) \tag{62}$$

Solving the integral gives the result:

$$\mathbf{F^{R}}_{\mathbf{total}}(\mathbf{VS1} \to 6) = \int \mathbf{d\bar{F}^{R}_{total}} \bullet \mathbf{\bar{u}_y} \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot (\frac{28}{5\sqrt{5}}) \tag{63}$$

2.8.7. Voltage source of branch 1 to branch 9

In this case involving a voltage source, one will have to apply an impulse current instead of I_1, namely $.I_1 \cdot 3L \ \cdot \delta(x_1 - L \ /2,)$

It has to be observed that the direction of that current is opposite to I_2, implying thus the need for using Eq. (42), i.e. the current I_2 has the same direction as $.I_1 \cdot 3L \ \cdot \delta(x_1 - L \ /2,)$.

In this case $y_2 - y_1 = \frac{L}{2}$

The total contribution to the force here will be:

$$\int \mathbf{d}\vec{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_2=0}^{\mathbf{L}} \int_{\mathbf{x}_1=0}^{\mathbf{L}} \frac{\mu_0 \cdot 3\mathbf{L} \cdot \mathbf{I}_1 \delta(\mathbf{x}_1 - \dfrac{\mathbf{L}}{2}) \cdot \mathbf{I}_2 \cdot \mathbf{dx}_1 \mathbf{dx}_2 \cdot (\dfrac{\mathbf{L}}{2})}{4\pi \cdot \mathbf{r}^3} \cdot (-1 + \frac{(\mathbf{x}_2 - \mathbf{x}_1)^2}{\mathbf{r}^2}) \tag{64}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{total}}(\mathbf{VS}1 \to 9) = \int \mathbf{d}\vec{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cong \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{4\pi} \cdot (-5\sqrt{2}) \tag{65}$$

2.8.8. Voltage source of branch 1 to branch 6

In this case involving a voltage source, one will necessarily have to apply an impulse current instead of I_1, namely $.I_1 \cdot 3L \ \cdot \delta(x_1 - L \ /2, \ )$

It has to be observed that the direction of that current is opposite to I_2, implying thus the need for using Eq. (42), i.e. the current I_2 has the same direction as $.I_1 \cdot 3L \ \cdot \delta(x_1 - L \ /2, \ )$.

In this case $y_2 - y_1 = L$

The total contribution to the force here will be:

$$\int \mathbf{d}\vec{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_2=0}^{\mathbf{L}} \int_{\mathbf{x}_1=0}^{\mathbf{L}} \frac{\mu_0 \cdot 3\mathbf{L} \cdot \mathbf{I}_1 \delta(\mathbf{x}_1 - \dfrac{\mathbf{L}}{2}) \cdot \mathbf{I}_2 \cdot \mathbf{dx}_1 \mathbf{dx}_2 \cdot \mathbf{L}}{4\pi \cdot \mathbf{r}^3} \cdot (-1 + \frac{(\mathbf{x}_2 - \mathbf{x}_1)^2}{\mathbf{r}^2}) \tag{66}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{total}}(\mathbf{VS}1 \to 6) = \int \mathbf{d}\vec{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cong \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{4\pi} \cdot (\frac{28}{5\sqrt{5}}) \tag{67}$$

2.8.9. Voltage source of branch 1 to the voltage source of branch 6

In this case Eq. (37) will again be used, but in this case replacing both the currents I_1 and I_2 in Eq. (42) with impulse currents, namely $I_1 \cdot 3L \ \cdot \delta(x_2 - L \ /2, \ )$ and $I_2 \cdot 3L \ \cdot \delta(x_2 - L \ /2, \ )$.

In this case $y_2 - y_1 = L$

The total contribution to the force will in this case be:

$$\int d\vec{F}^{R}_{total} \bullet \vec{u}_y \cong \int_{x_2=0}^{L} \int_{x_1=0}^{L} \frac{\mu_0 \cdot 3L \cdot I_1 \delta(x_1 - \frac{L}{2}) \cdot 3L \cdot I_2 \cdot \delta(x_2 - \frac{L}{2}) \cdot dx_1 dx_2 \cdot L}{4\pi \cdot r^3} \cdot (-1 + \frac{(x_2 - x_1)^2}{r^2}) \qquad (68)$$

Solving the integral gives the result:

$$F^{R}{}_{total}(VS1 \to VS6) = \int d\vec{F}^{R}_{total} \bullet \vec{u}_y \cong \frac{\mu_0 I_1 I_2}{4\pi} \cdot (-9) \qquad (69)$$

2.9. The parts of the two conductors being parallel, aligned along the y axis

In this case both branches are situated along the y axis, necessary changes in the equation for the force between parallel currents will be needed. The earlier results concerning currents aligned to the x axis, expressed in Eq. (42) and Eq. (47) must necessarily be modified in order to fit with these facts. This means that the expression dealing with currents moving both in the same y direction will obey the following equation

$$\Delta\vec{F}^{R}_{total} \bullet \vec{u}_y = \frac{\mu_0 I_1 I_2 \Delta y_1 \Delta y_2 \cdot (y_2 - y_1)}{4\pi \cdot r^3} \cdot (-1 + \frac{(y_2 - y_1)^2}{r^2}) \qquad (70)$$

Further, in the case the currents flow opposite to each other, the following equation will have to be chosen:

$$\Delta\vec{F}^{R}_{total} \bullet \vec{u}_y = \frac{\mu_0 I_1 I_2 \Delta y_1 \Delta y_2 \cdot (y_2 - y_1)}{4\pi \cdot r^3} \cdot (1 - \frac{(y_2 - y_1)^2}{r^2}) \qquad (71)$$

2.9.1. Branch 10 to 7

In this case the currents flow opposite to each other and one should therefore be tempted to use Eq. (71). However, since the two currents come close to each other at one point, thereby giving rise to a singularity that is impossible to treat straightforwardly, it is necessary to extend the definition of the 'thin conductors' carrying the respective currents, to let them have extension in the x direction, though small, so that $x_1 : 0 \to w$ and $x_2 : 0 \to w$. In that case it is furthermore more suitable to apply Eq. (71). This all will be done the following way:

$$\int d\vec{F}^{R}_{total} \bullet \vec{u}_y = \frac{1}{w^2} \cdot \int_{x_1=0}^{w} dx_1 \int_{x_2=0}^{w} dx_2 \int_{y_1=0}^{\frac{L}{2}-a} dy_1 \int_{\frac{L}{2}}^{L} dy_2 \frac{\mu_0 I_1 I_2 \cdot (y_2 - y_1)}{4\pi \cdot r^3} \cdot (1 - \frac{(y_2 - y_1)^2}{r^2}) \qquad (72)$$

The approximation that has to be done, when both the conductors are to be regarded as 'thin', i.e. $w \to 0$ and the distance between their meeting points are small, $a << L$, is to decide, which one is the very smallest. If choosing $w << a << L$

when solving the integral the following result will arise:

$$\mathbf{F^R}_{\mathbf{total}}(10 \to 7) = \int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot (-\ln\frac{\mathbf{L}}{\mathbf{a}} + \ln 4) \tag{73}$$

2.9.2. Branch 10 to5

In order to calculate the force between branch 10 and 5, one will have to insert $x_2 - x_1 = L$ in

Eq. (70). Since the currents of the two branches flow in the same direction, Eq (70) will be applied.

The summation of the contributions from all the charges will in this case be

$$\int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} = \int_{\mathbf{y=0_1}}^{\frac{\mathbf{L}}{2}-\mathbf{a}} \mathbf{dy_1} \int_{\mathbf{y_2=\frac{L}{2}}}^{\mathbf{L}} \mathbf{dy_2} \frac{\mu_0 \mathbf{I_1 I_2} \cdot (\mathbf{y_2 - y_1})}{4\pi \cdot \mathbf{r}^3} \cdot (-1 + \frac{(\mathbf{y_2 - y_1})^2}{\mathbf{r}^2}) \tag{74}$$

Solving the integral gives the result:

$$\mathbf{F^R}_{\mathbf{total}}(10 \to 5) = \int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot \frac{1}{3}(\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{5}} + \frac{\mathbf{a}}{\mathbf{L}}) \tag{75}$$

2.9.3. Branch 2 to 7

In order to calculate the force between branch 2 and 7, one will have to insert $x_2 - x_1 = -L$ in Eq (70). Since the currents of the two branches flow in the same direction, Eq. (70) will be applied. Here the summation of the contributions from all the charges will in this case be

$$\int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} = \int_{\mathbf{y=0_1}}^{\frac{\mathbf{L}}{2}-\mathbf{a}} \mathbf{dy_1} \int_{\mathbf{y_2=\frac{L}{2}}}^{\mathbf{L}} \mathbf{dy_2} \frac{\mu_0 \mathbf{I_1 I_2} \cdot (\mathbf{y_2 - y_1})}{4\pi \cdot \mathbf{r}^3} \cdot (-1 + \frac{(\mathbf{y_2 - y_1})^2}{\mathbf{r}^2}) \tag{76}$$

Solving the integral gives the result:

$$\mathbf{F^R}_{\mathbf{total}}(2 \to 7) = \int \mathbf{d\bar{F}^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot \frac{1}{3}(\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{5}} + \frac{\mathbf{a}}{\mathbf{L}}) \tag{77}$$

2.9.4. Branch 2 to 5

In this case the currents flow opposite to each other and one should therefore be tempted to use Eq. (71). However, since the two currents come close to each other at one point, thereby giving rise to a singularity that is impossible to treat straightforwardly, it will be necessary to extend the definition of the 'thin conductors' carrying the respective currents, to let them have extension in the x direction, though small, so that $x_1 : L - w \to L$ and $x_2 : L - w \to L$. In this case it is furthermore more suitable to apply Eq. (71). This all will be done the following way:

$$\int \mathbf{d\vec{F}^R_{total}} \bullet \mathbf{\vec{u}_y} = \frac{1}{\mathbf{w}^2} \cdot \int_{x_1=L-w}^{L} \mathbf{dx}_1 \int_{x_2=L-w}^{L} \mathbf{dx}_2 \int_{y_1=0}^{\frac{L}{2}-a} \mathbf{dy}_1 \int_{y_2=\frac{L}{2}}^{L} \mathbf{dy}_2 \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \cdot (\mathbf{y}_2 - \mathbf{y}_1)}{4\pi \cdot \mathbf{r}^3} \cdot (1 - \frac{(\mathbf{y}_2 - \mathbf{y}_1)^2}{\mathbf{r}^2}) \tag{78}$$

where w [m] indicates the width of the conductor. The approximation that has to be done, when both the conductors are to be regarded as 'thin', i.e. $w \to 0$, and the distance between their meeting points are small, so that $a << L$, is to decide, which one is the very smallest. If choosing $w << a << L$ when solving the integral will give rise to the following result:

$$\mathbf{F^R}_{\mathbf{total}}(2 \to 5) = \int \mathbf{d\vec{F}^R_{total}} \bullet \mathbf{\vec{u}_y} \cong \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{4\pi} \cdot (-\ln \frac{\mathbf{L}}{\mathbf{a}} + \ln 4) \tag{79}$$

2.10. The parts of the two conductors being perpendicular to each other, from y axis to x axis

In the case one branch is situated along the y axis and the other along they x axis, necessary changes in the equation for the force between parallel currents have to be undertaken. The equations describing the four different contributions to the force, due to the four combinations of charges, Eq. (29), Eq. (31), Eq. (34) and Eq. (37), will be used, but modified with respect to the new directions. As mentioned earlier, in Ch. 2.6, there will be no time dilation effect. This leads to the following equation for the total incremental force between the two branches, in the case both currents flow along the respective positive axis:

$$\Delta \mathbf{\vec{F}^R_{total}} \bullet \mathbf{\vec{u}_y} = \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \Delta \mathbf{y}_1 \Delta \mathbf{x}_2 \cdot (\mathbf{y}_2 - \mathbf{y}_1)^2 \cdot (\mathbf{x}_2 - \mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{80}$$

If one of the currents flows along a negative axis, the equation will change sign, so that:

$$\Delta \mathbf{\vec{F}^R_{total}} \bullet \mathbf{\vec{u}_y} = -\frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \Delta \mathbf{y}_1 \Delta \mathbf{x}_2 \cdot (\mathbf{y}_2 - \mathbf{y}_1)^2 \cdot (\mathbf{x}_2 - \mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{81}$$

2.10.1. Branch 10 to 9

In order to calculate the force between the two branches 10 and 9, one will have to insert $x_1=0$ and $y_2=\frac{L}{2}$ in Eq. (80), or

$$\int \mathbf{dF^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \int_{x_2=0}^{L} \int_{y_2=0}^{\frac{L}{2}-a} \frac{\mu_0 \mathbf{I_1 I_2 dx_2 dy_1} \cdot (\frac{\mathbf{L}}{2} - \mathbf{y_1})^2 \cdot \mathbf{x_2}}{4\pi \cdot \mathbf{r}^5} \tag{82}$$

Solving the integral gives the result

$$\mathbf{F^R}_{\mathbf{total}}(10 \to 9) \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot (\frac{1}{3}) \cdot (-\frac{\mathbf{a}}{\mathbf{L}} + \ln\frac{\mathbf{L}}{\mathbf{a}} + \frac{1}{\sqrt{5}} - 2 \cdot \ln 2 + \ln(\sqrt{5}+1)) \tag{83}$$

2.10.2. Branch 2 to 9

In order to calculate the force between the two branches 2 and 9, one will have to assume $x_1=L$ and $y_2=\frac{L}{2}$ in Eq. (80), or

$$\int \mathbf{dF^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \int_{x_2=0}^{L} \int_{y_2=0}^{\frac{L}{2}-a} \frac{\mu_0 \mathbf{I_1 I_2 dx_2 dy_1} \cdot (\frac{\mathbf{L}}{2} - \mathbf{y_1})^2 \cdot (\mathbf{x_2} - \mathbf{L})}{4\pi \cdot \mathbf{r}^5} \tag{84}$$

Solving the integral gives the result

$$\mathbf{F^R}_{\mathbf{total}}(2 \to 9) \cong \frac{\mu_0 \mathbf{I_1 I_2}}{4\pi} \cdot (\frac{1}{3}) \cdot (-\frac{\mathbf{a}}{\mathbf{L}} + \ln\frac{\mathbf{L}}{\mathbf{a}} + \frac{1}{\sqrt{5}} - 2 \cdot \ln 2 + \ln(\sqrt{5}+1)) \tag{85}$$

2.10.3. Branch 10 to 6

In order to calculate the force between the two branches 10 and 6, one has to insert $x_1=0$ and $y_2=L$ in Eq. (81), thus keeping in mind the change of direction of the second current with respect to the two preceding sections, in this case:

$$\int \mathbf{dF^R_{total}} \bullet \mathbf{\bar{u}_y} \cong \int_{x_2=0}^{L} \int_{y_1=0}^{\frac{L}{2}-a} \frac{\mu_0 \mathbf{I_1 I_2 dx_2 dy_1} \cdot (-(\mathbf{L} - \mathbf{y_1})^2) \cdot \mathbf{x_2}}{4\pi \cdot \mathbf{r}^5} \tag{86}$$

Solving the integral gives the result

$$\mathbf{F^R}_{\mathbf{total}}(10 \to 6) \cong \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{4\pi} \cdot (\frac{1}{3}) \cdot (\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}} - \ln 2 - \ln \frac{-1+\sqrt{5}}{-1+\sqrt{2}}) \tag{87}$$

2.10.4. Branch 2 to 6

In order to calculate the force between the two branches 10 and 6, one has to set $x_1 = L$ and $y_2 = L$ in Eq. (81), or:

$$\int \mathbf{dF^R_{total}} \bullet \bar{\mathbf{u}}_\mathbf{y} \cong \int_{\mathbf{x}_2=0}^{\mathbf{L}} \int_{\mathbf{y}_1=0}^{\frac{\mathbf{L}}{2}-\mathbf{a}} \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \mathbf{dx}_2 \mathbf{dy}_1 \cdot (-(\mathbf{L}-\mathbf{y}_1)^2) \cdot (\mathbf{x}_2 - \mathbf{L})}{4\pi \cdot \mathbf{r}^5} \tag{88}$$

Solving the integral gives the result

$$\mathbf{F^R}_{\mathbf{total}}(2 \to 6) \cong \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{4\pi} \cdot (\frac{1}{3}) \cdot (\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}} - \ln 2 - \ln \frac{-1+\sqrt{5}}{-1+\sqrt{2}}) \tag{89}$$

2.10.5. Branch 10 to Voltage source 6

In this case involving a voltage source, one will have to apply an impulse current instead of I_2, namely $.I_2 \cdot 3L \ \cdot \delta(x_2 - L \ /2, \ )$

In this case $x_1 = 0$ and $y_2 = L$

It has to be observed that the direction of that current is opposite to I_2, implying thus the need for changing sign compared to the integral equation for section 2.9.3 dealing with the forces between branch 10 and branch 6, so that instead Eq. (80) should be used, thereby expressing the total: contribution to the force here:

$$\int \mathbf{dF^R_{total}} \bullet \bar{\mathbf{u}}_\mathbf{y} \cong \int_{\mathbf{x}_2=0}^{\mathbf{L}} \int_{\mathbf{y}_1=0}^{\frac{\mathbf{L}}{2}-\mathbf{a}} \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \cdot 3\mathbf{L} \cdot \delta(\mathbf{x}_2 - \frac{\mathbf{L}}{2}) \cdot \mathbf{dx}_2 \mathbf{dy}_1 \cdot ((\mathbf{L}-\mathbf{y}_1)^2) \cdot \mathbf{x}_2}{4\pi \cdot \mathbf{r}^5} \tag{90}$$

Solving the integral gives the result:

$$\mathbf{F^R}_{\mathbf{total}}(10 \to \mathbf{VS}6) = \int \mathbf{d\bar{F}^R_{total}} \bullet \bar{\mathbf{u}}_\mathbf{y} \cong \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{4\pi} \cdot (-\frac{5}{\sqrt{2}} + \frac{56}{5\sqrt{5}}) \tag{91}$$

2.10.6. Branch 2 to Voltage source 6

In this case involving a voltage source, one will have to apply an impulse current instead of I_2, namely .$I_2 \cdot 3L \cdot \delta(x_2 - L / 2,)$

In this case $x_1 = L$ and $y_2 = L$

It has to be observed that the direction of that current is opposite to I_2, implying thus the need for changing sign compared to the integral equation for section 2.9.3 dealing with the forces between branch 10 and branch 6, so that instead Eq. (80) should be used, thereby expressing the total: contribution to the force here:

$$\int \mathbf{dF}^{\mathbf{R}}_{\mathbf{total}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_2=0}^{\mathbf{L}} \int_{\mathbf{y}_1=0}^{\frac{\mathbf{L}}{2}-\mathbf{a}} \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \cdot 3\mathbf{L} \cdot \delta(\mathbf{x}_2 - \frac{\mathbf{L}}{2}) \cdot \mathbf{dx}_2 \mathbf{dy}_1 \cdot ((\mathbf{L} - \mathbf{y}_1)^2) \cdot (\mathbf{x}_2 - \mathbf{L})}{4\pi \cdot \mathbf{r}^5} \tag{92}$$

Solving the integral gives the result:

$$\mathbf{F^R}_{\mathbf{total}}(2 \to \mathbf{VS}6) = \int \mathbf{d}\bar{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cong \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{4\pi} \cdot (-\frac{5}{\sqrt{2}} + \frac{56}{5\sqrt{5}}) \tag{93}$$

2.11. The parts of the two conductors being perpendicular to each other, from the x axis to the y axis

In the case one branch is situated along the x axis and the other along they y axis, necessary changes in the equation for the force between parallel currents have to be undertaken. The earlier results concerning currents aligned to the x axis, expressed in Eq. (42) and Eq. (47) must necessarily be modified in order to fit with these facts. This will lead to the following equation:

$$\Delta\bar{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} = \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \Delta\mathbf{x}_1 \Delta\mathbf{y}_2 \cdot (\mathbf{y}_2 - \mathbf{y}_1)^2 \cdot (\mathbf{x}_2 - \mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{94}$$

If one of the currents flows along a negative axis, the equation will change sign, so that:

$$\Delta\bar{\mathbf{F}}^{\mathbf{R}}_{\mathbf{total}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} = -\frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \Delta\mathbf{x}_1 \Delta\mathbf{y}_2 \cdot (\mathbf{y}_2 - \mathbf{y}_1)^2 \cdot (\mathbf{x}_2 - \mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{95}$$

2.11.1. Branch 1 to 7

In order to calculate the force between the two branches 1 and 7, one has to insert y_1=0 and x_2=0 in Eq. (94), and integrating, or

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$$\int \mathbf{dF}_{\mathbf{total}}^{\mathbf{R}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_1=0}^{\mathbf{L}} \int_{\mathbf{y}_2=\frac{\mathbf{L}}{2}+\mathbf{a}}^{\mathbf{L}} \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \mathbf{dx}_1 \mathbf{dy}_2 \cdot \mathbf{y}_2{}^2 \cdot (-\mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{96}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{tottal}}(1 \to 7) \cong \frac{\mathbf{I}_1 \mathbf{I}_2}{4\pi\varepsilon_0} \cdot (\frac{1}{3}) \cdot (-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{5}} - \ln\frac{1+\sqrt{5}}{1+\sqrt{2}}) \tag{97}$$

2.11.2. Branch 1 to 5

In order to calculate the force between the two branches 1 and 7, one has to insert y_1=0 and x_2=0 in Eq. (95), and integrating, or

$$\int \mathbf{dF}_{\mathbf{total}}^{\mathbf{R}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_1=0}^{\mathbf{L}} \int_{\mathbf{y}_2=\frac{\mathbf{L}}{2}}^{\mathbf{L}} \frac{-\mu_0 \mathbf{I}_1 \mathbf{I}_2 \mathbf{dx}_1 \mathbf{dy}_2 \cdot \mathbf{y}_2{}^2 \cdot (\mathbf{L}-\mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{98}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{tottal}}(1 \to 5) \cong \frac{\mathbf{I}_1 \mathbf{I}_2}{4\pi\varepsilon_0} \cdot (\frac{1}{3}) \cdot (-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{5}} - \ln\frac{1+\sqrt{5}}{1+\sqrt{2}}) \tag{99}$$

2.11.3. Branch 8 to 7

In order to calculate the force between the two branches 8 and 7, one has to insert $y_1 = \frac{L}{2} - a$ and x_2=0 in Eq. (52d'), or

$$\int \mathbf{dF}_{\mathbf{total}}^{\mathbf{R}} \bullet \bar{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_1=0}^{\mathbf{L}} \int_{\mathbf{y}_2=\frac{\mathbf{L}}{2}}^{\mathbf{L}} \frac{-\mu_0 \mathbf{I}_1 \mathbf{I}_2 \mathbf{dx}_1 \mathbf{dy}_2 \cdot (\mathbf{y}_2 - \frac{\mathbf{L}}{2} + \mathbf{a})^2 \mathbf{x}_1}{4\pi \cdot \mathbf{r}^5} \tag{100}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{tottal}}(8 \to 7) \cong \frac{\mathbf{I}_1 \mathbf{I}_2}{4\pi\varepsilon_0} \cdot (\frac{1}{3}) \cdot (-\frac{\mathbf{a}}{\mathbf{L}} + \ln\frac{\mathbf{L}}{\mathbf{a}} - \frac{1}{\sqrt{5}} + \ln(1+\sqrt{5})) \tag{101}$$

2.11.4. Branch 8 to 5

In order to calculate the force between the two branches 8 and 7, one has to insert $y_1 = \frac{L}{2} - a$ and $x_2 = L$ in Eq. (52c′), or

$$\int \mathbf{dF}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_1=0}^{\mathbf{L}} \int_{\mathbf{y}_2=\frac{\mathbf{L}}{2}}^{\mathbf{L}} \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2 \mathbf{dx}_1 \mathbf{dy}_2 \cdot (\mathbf{y}_2 - \frac{\mathbf{L}}{2} + \mathbf{a})^2 \cdot (\mathbf{L} - \mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{102}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{tottal}}(8 \to 5) \cong \frac{\mathbf{I}_1 \mathbf{I}_2}{4\pi\varepsilon_0} \cdot (\frac{1}{3}) \cdot (-\frac{\mathbf{a}}{\mathbf{L}} + \ln\frac{\mathbf{L}}{\mathbf{a}} - \frac{1}{\sqrt{5}} + \ln(1+\sqrt{5})) \tag{103}$$

2.11.5. The Voltage source of branch 1 to branch 7

In this case involving a voltage source, one will have to apply an impulse current instead of I_1, namely $.I_1 \cdot 3L \cdot \delta(x_1 - L/2,)$

In this case $x_2 = 0$ and $y_1 = 0$

It has to be observed that the direction of that current is opposite to I_1, implying thus the need for changing sign compared to the integral equation in Sec. 2.10.1 dealing with the forces between branch 1 and branch 7. Hence, instead Eq. (95) would have to be used, thereby expressing the total: contribution to the force here:

$$\int \mathbf{dF}^{\mathbf{R}}_{\mathbf{total}} \bullet \vec{\mathbf{u}}_{\mathbf{y}} \cong \int_{\mathbf{x}_1=0}^{\mathbf{L}} \int_{\mathbf{y}_2=\frac{\mathbf{L}}{2}+\mathbf{a}}^{\mathbf{L}} \frac{-\mu_0 \cdot \mathbf{I}_1 \cdot 3\mathbf{L} \cdot \delta(\mathbf{x}_1 - \frac{\mathbf{L}}{2}) \cdot \mathbf{I}_2 \mathbf{dx}_1 \mathbf{dy}_2 \cdot \mathbf{y}_2{}^2 \cdot (-\mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{104}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{tottal}}(\mathbf{VS}1 \to 7) \cong \frac{\mathbf{I}_1 \mathbf{I}_2}{4\pi\varepsilon_0} \cdot (\frac{1}{3}) \cdot (-\frac{1}{\sqrt{2}} + \frac{16}{5\sqrt{5}}) \tag{105}$$

2.11.6. The Voltage source of branch 1 to branch 5

In this case involving a voltage source, one will have to apply an impulse current instead of I_1, namely $.I_1 \cdot 3L \cdot \delta(x_1 - L/2,)$

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In this case $x_2 = L$ and $y_1 = 0$

It has to be observed that the direction of that current is opposite to I_1, implying thus the need for changing sign compared to the integral equation in Sec. 2.10.2 dealing with the forces between branch 1 and branch 5. Instead, Eq. (94) should be used, thereby expressing the total: contribution to the force here:

$$\int \mathbf{dF}^{\mathbf{R}}_{\mathbf{total}} \bullet \mathbf{\bar{u}_y} \cong \int_{\mathbf{x}_1=0}^{\mathbf{L}} \int_{\mathbf{y}_2=\frac{\mathbf{L}}{2}}^{\mathbf{L}} \frac{\mu_0 \cdot \mathbf{I}_1 \cdot 3\mathbf{L} \cdot \delta(\mathbf{x}_1 - \frac{\mathbf{L}}{2}) \cdot \mathbf{I}_2 \mathbf{dx}_1 \mathbf{dy}_2 \cdot \mathbf{y}_2{}^2 \cdot (\mathbf{L} - \mathbf{x}_1)}{4\pi \cdot \mathbf{r}^5} \tag{106}$$

Solving the integral gives the result:

$$\mathbf{F}^{\mathbf{R}}{}_{\mathbf{tottal}}(\mathbf{VS}1 \to 5) \cong \frac{\mathbf{I}_1 \mathbf{I}_2}{4\pi\varepsilon_0} \cdot (\frac{1}{3}) \cdot (-\frac{1}{\sqrt{2}} + \frac{16}{5\sqrt{5}}) \tag{107}$$

3. Judgment (assessment) of the calculations

If assessing all the calculations for the different parts of the circuits, it becomes evident that it is only one that dominates over all the others, namely the force between the parts of the two circuits aligned along each other at a distance a a that is small compared to all the other dimensions of the circuits that are of order L , or $\frac{a}{L} \to 0$. The result is given through Eq. (50), implying an attractive force between these parts of the conductors. The force is furthermore proportional to the length of the conductors and inverse proportional to their mutual distance.

4. Conclusion

The two respective currents have been thoroughly analyzed, using Coulomb's law, taking into account the effects of propagation delay and the Special Relativity Theory. The way the effects of the propagation delay have been derived is that of this author in a paper 1997 [1], which differs fundamentally from the traditional interpretation, as that of Feynman [21] and Jackson [22]. This author has been successful in showing what the fallacies are [2]. Basically, Feynman committed a mathematical fault with respect to the calculation of the propagation delay, when deriving the Liénard-Wiechert potentials [21]. In the 1997 paper by this author [1] it was crucial to the success of Coulomb's law that the effects of propagation delay had been correctly derived, both with respect to the "sending charges" of the "first conductor" and to the "receiving charges" of the "second conductor". The first effect gives account for the dependence of the first current in the expression for the electromagnetic force, the second one for the second current in that same expression, but of course, it is arbitrary, which one is treated as sending or receiving current. This treatment makes it possible to see a product between two currents in an application of Coulomb's law, and, hence, there will be no need for the Lorentz force. When that analysis has been done, it remains to take into account to the effects of the Special relativity theory, especially the Lorentz transformation of lengths. Since this effect is related only to the relative movements of the two coordinate axes, and has nothing to do with the propagation delay an observer faces, it may be multiplied straightforwardly to the effect of propagation delay. The expression for the force between the two currents are compared to the expression that Ampère arrive at and to Lorentz' force law. Thereafter follows a discussion of the pro et contra of respective model. The result in this article is based on two well-corroborated natural laws: Coulomb's law [23] and the Special relativity theory. Ampére in turn, derives his law in a strictly empirical sense [32], searching for similarities with Coulomb's law. However, since in his time, the individual electron had not yet been discovered and, secondly, the Special relativity theory had not been defined. Hence, Ampère had no other choice than to establish a fairly good empirical law. Lorentz (or first: Grassmann) faced the same problem, but his formula was derived through evident mathematical faults [15]. Nb. This term ′Ampère's force law′ is not the same law as that Jackson denotes Ampère's law. Please cf. the original paper by Ampère [13] and Jackson [24]. This would make it possible to create a continuous, logical chain, from the findings by Ampère to the established Maxwell electrodynamics. Assis has made an effort to prove that both Ampère's law and Grassmann's law produce the same result, when the forces within Ampère's bridge are being derived [15]. Admittedly, he concedes that they are not equal at every point, but in the integral sense, when a complete, closed electric circuit is taken into account. From a strictly mathematical pint of view, however, if two functions are not equal at every point, they don't express equal functions. This is taught in the most basic undergraduate courses. Anyhow, stating that all electric circuits are necessarily closed, he arrives at the conclusion that both laws are equally applicable on electric circuits.

To conclude, all three of them: Coulomb's law, Lorentz' force law and Ampère's force law can account for the attractive force exerted between two parallel electric conductors, carrying a current in the same direction. On the mere basis of the shape of the functions, it is not possible to decide, which one is best expressing physical reality, since the very measurements of currents involves a theory for the force between currents in the context of traditional measurement instruments. Hence, for every choice of model, there will necessarily appear a coupling constant that makes the measurements fit with the theory. Therefore, it remains to make a qualitative analysis of the three models. Above it has already been explored that Coulomb's has been used in a very strict manner, applying only the effects of propagation delay and the Special Relativity theory, whereas Ampère's force law is only expressing an empirical estimation of the force and the Lorentz force has been fallaciously derived, using Ampère's force law.

Hence, the conclusion to be drawn is that Coulomb's law gives the most comprehensive explanation to the force.

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