Chapter 6

# Electromagnetic Waves Excitation by Thin Impedance Vibrators and Narrow Slots in Electrodynamic Volumes

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Additional information is available at the end of the chapter

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**Abstract**

Linear vibrator and slot radiators, i.e., radiators of electric and magnetic type, respectively, are widely used as separate receiver and transmitter structures, elements of antenna systems, and antenna-feeder devices, including combined vibrator-slot structures. Widespread occurrence of such radiators is an objective prerequisite for theoretical analysis of their electrodynamic characteristics. During the last decades, researchers have published results which make it possible to create a modern theory of thin vibrator and narrow slot radiators. This theory combines the fundamental asymptotic methods for determining the single radiator characteristics, the hybrid analytic-numerical approaches, and the direct numerical techniques for electrodynamic analysis of such radiators. However, the electrodynamics of single linear electric and magnetic radiators is far from been completed. It may be explained by further development of modern antenna techniques and antenna-feeder devices, which can be characterized by such features as multielement structures, integration, and modification of structural units to minimize their mass and dimensions and to ensure electromagnetic compatibility of radio aids, application of metamaterials, formation of required spatial-energy, and spatial-polarization distributions of electromagnetic fields in various nondissipative and dissipative media. To solve these tasks, electric and magnetic radiators, based on various impedance structures with irregular geometric or electrophysical parameters and on combined vibrator-slot structures, should be created. This chapter presents the methodological basis for application of the generalized method of induced EMMF for the analysis of electrodynamic characteristics of the combined vibrator-slot structures. Characteristic feature of the generalization to a new class of approximating functions consists in using them as a function of the current distributions along the impedance vibrator and slot elements; these distributions are derived as the asymptotic solution of integral equations for the current (key problems) by the method of averaging. It should be noted that for simple structures similar to that considered in the model problem, the proposed approach yields an analytic solution of the electrodynamic problem. For more complex structures, the method may be used to design effective numerical-analytical algorithms for their analyses.

The demonstrative simulation (the comparative analysis of all electrodynamic characteristics in the operating frequencies range) has confirmed the validity of the proposed generalized method of induced EMMF for analysis of vibrator-slot systems with rather arbitrary structure (within accepted assumptions). Here, as examples, some fragments of this comparative analysis were presented. This method retains all benefits of analytical methods as compared with direct numerical methods and allows to expand significantly the boundaries of numerical and analytical studies of practically important problems, concerning the application of single impedance vibrator, including irregular vibrator, the systems of such vibrators, and narrow slots.

**Keywords:** Waves excitation, thin impedance vibrators, narrow slots, vibrator-slot structures

## 1. Introduction

At present, linear vibrator and slot radiators, i.e. radiators of electric and magnetic type, respectively, are widely used as separate receiver and transmitter structures, elements of antenna systems, and antenna-feeder devices, including combined vibrator-slot structures [1-4]. Widespread occurrence of such radiators is an objective prerequisite for theoretical analysis of their electrodynamic characteristics. During last decades researchers have published results which make it possible to create a modern theory of thin vibrator and narrow slot radiators. This theory combines the fundamental asymptotic methods for determining the single radiator characteristics [5-7], the hybrid analytic-numerical approaches [8-10], and the direct numerical techniques for electrodynamic analysis of such radiators [11]. However, the electrodynamics of single linear electric and magnetic radiators is far from been completed. It may be explained by further development of modern antenna techniques and antenna-feeder devices which can be characterized by such features as multielement structures, integration and modification of structural units to minimize their mass and dimensions and to ensure electromagnetic compatibility of radio aids, application of metamaterials, formation of required spatial-energy and spatial-polarization distributions of electromagnetic fields in various nondissipative and dissipative media. To solve these tasks electric and magnetic radiators, based on various impedance structures with irregular geometric or electrophysical parameters, and on combined vibrator-slot structures, should be created [12-20].

Mathematical modeling of antenna-feeder devices requires multiparametric optimization of electrodynamic problem solution and, hence, effective computational resources and software. Therefore, in spite of rapid growth of computer potential, there exists a necessity to develop new effective methods of electrodynamic analysis of antenna-feeder systems, being created with linear vibrator and slot structures with arbitrary geometric and electrophysical parameters, satisfying modern versatile requirements, and widening their application in various spheres. Efficiency of mathematical modeling is defined by rigor of corresponding boundary problem definition and solution, by performance of computational algorithm, requiring minimal possible RAM space, and directly depends upon analytical formulation of the models. That is, the weightier is the analytical component of the method the grater is its efficiency. In this connection it should be noted that formation of analytical concepts of electrodynamic analysis extending the capabilities of physically correct mathematical models for new classes of boundary problems is always an important problem.

This chapter presents the methodological basis of a new approach to solving the electrodynamic problems associated with combined vibrator–slot structures, defined as a generalized method of induced electro-magneto-motive forces (EMMF). This approach is based on the classical method of induced EMMF, i.e, basis functions, approximating the currents along the vibrator and slot elements, are obtained in advance as analytical solutions of key problems, formulated as integral equations for the currents by the asymptotic averaging method. Bearing this in mind, we present here solutions of two key problems: a single impedance vibrator and slot scatterer in a waveguide, obtained by averaging method, and then solve a problem for the multielement vibrator-slot structures by generalized method of induced EMMF.

## 2. Problem formulation and initial integral equations

Let us formulate the problem of electromagnetic fields excitation (scattering, radiation) by finite-size material bodies in two electrodynamic volumes coupled by holes cut in their common boundary. Suppose that there exists some arbitrary volume $V_1$, bounded by a perfectly conducting, impedance, or partially impedance surface $S_1$, some parts of which may be infinitely distant. The volume $V_1$ is coupled with another arbitrary volume $V_2$ through holes $\Sigma_n$ $(n=1, 2...N)$, cut in the surface $S_1$. The boundary between the volumes $V_1$ and $V_2$ in the regions around the coupling holes has an infinitely small thickness. Permittivity and permeability of the medium filling volumes $V_1$ and $V_2$ are $\varepsilon_1$, $\mu_1$ and $\varepsilon_2$, $\mu_2$, respectively. Material bodies, enclosed in local volumes $V_{m_1}$ $(m_1=1, 2, ...M_1)$ and $V_{m_2}$ $(m_2=1, 2, ...M_2)$, bounded by smooth closed surfaces $S_{m_1}$ and $S_{m_2}$, are allocated in the volumes $V_1$ and $V_2$, respectively. The bodies have homogeneous material parameters: permittivity $\varepsilon_{m_1}$, $\varepsilon_{m_2}$, permeability $\mu_{m_1}$, $\mu_{m_2}$, and conductivity $\sigma_{m_1}$, $\sigma_{m_2}$. The fields of extraneous sources can be specified as the electromagnetic wave fields, incident on the bodies and the holes (scattering problem), or as fields of electromotive forces, applied to the bodies (radiation problem), or as combination of these fields. Without loss of generality, we assume that electromagnetic fields of extraneous sources $\{\vec{E}_0(\vec{r}),\ \vec{H}_0(\vec{r})\}$ exist only in the volume $V_1$. The fields $\{\vec{E}_0(\vec{r}),\ \vec{H}_0(\vec{r})\}$ depend on the time $t$ as $e^{i\omega t}$ ($\vec{r}$ is the radius vector of the observation point, $\omega=2\pi f$ is an circular frequency and $f$ is frequency, measured in Hertz). We seek the electromagnetic fields $\{\vec{E}_{V_1}(\vec{r}),\ \vec{H}_{V_1}(\vec{r})\}$ and $\{\vec{E}_{V_2}(\vec{r}),\ \vec{H}_{V_2}(\vec{r})\}$ in the volumes $V_1$ and $V_2$, satisfying Maxwell's equations and boundary conditions on the surfaces $S_{m_1}$, $S_{m_2}$, $\Sigma_n$, $S_1$ and $S_2$ (Figure 1).

To solve the above-mentioned problem we express the electromagnetic fields in volumes $V_1$ and $V_2$ in terms of the tangential fields components on the surfaces $S_{m_1}$, $S_{m_2}$ and $\Sigma_n$. In the Gaussian CGS system of units, the electromagnetic fields can be represented by the well-known Kirchhoff-Kotler integral equations [3,4]:

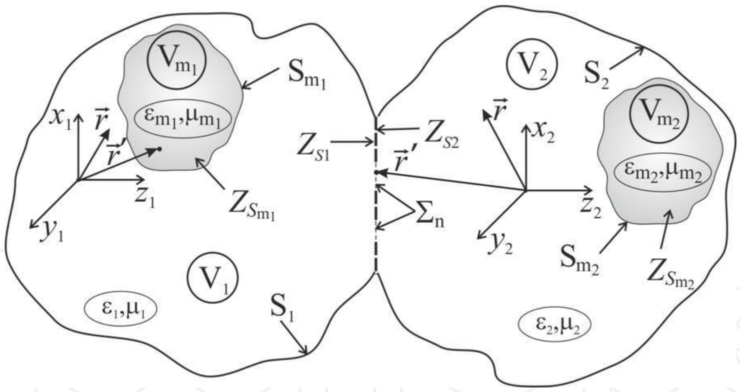


**Figure 1.** The problem geometry and notations

$$
\begin{aligned}
\vec E_{V_1}(\vec r) &= \vec E_0(\vec r) + \frac{1}{4\pi i k\varepsilon_1}(\text{graddiv} + k_1^2)\sum_{m_1=1}^{M_1}\int_{S_{m_1}} \hat G^e_{V_1}(\vec r,\vec r'_{m_1})[\vec n_{m_1},\vec H_{V_1}(\vec r'_{m_1})]\mathrm d\vec r'_{m_1} \\
&-\frac{1}{4\pi}\text{rot}\left\{\sum_{m_1=1}^{M_1}\int_{S_{m_1}} \hat G^m_{V_1}(\vec r,\vec r'_{m_1})[\vec n_{m_1},\vec E_{V_1}(\vec r'_{m_1})]\mathrm d\vec r'_{m_1} + \sum_{n=1}^{N}\int_{\Sigma_n} \hat G^m_{V_1}(\vec r,\vec r'_n)\left[\vec n_n,\vec E_{V_1}(\vec r'_n)\right]\mathrm d\vec r'_n\right\}, \\
\vec H_{V_1}(\vec r) &= \vec H_0(\vec r) + \frac{1}{4\pi i k\mu_1}(\text{graddiv} + k_1^2)\left\{\begin{aligned}&\sum_{m_1=1}^{M_1}\int_{S_{m_1}} \hat G^m_{V_1}(\vec r,\vec r'_{m_1})\left[\vec n_{m_1},\vec E_{V_1}(\vec r'_{m_1})\right]\mathrm d\vec r'_{m_1} \\ &+\sum_{n=1}^{N}\int_{\Sigma_n} \hat G^m_{V_1}(\vec r,\vec r'_n)\left[\vec n_n,\vec E_{V_1}(\vec r'_n)\right]\mathrm d\vec r'_n\end{aligned}\right\} \\
&+\frac{1}{4\pi}\text{rot}\sum_{m_1=1}^{M_1}\int_{S_{m_1}} \hat G^e_{V_1}(\vec r,\vec r'_{m_1})\left[\vec n_{m_1},\vec H_{V_1}(\vec r'_{m_1})\right]\mathrm d\vec r'_{m_1},
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\vec E_{V_2}(\vec r) &= \frac{1}{4\pi i k\varepsilon_2}(\text{graddiv} + k_2^2)\sum_{m_2=1}^{M_2}\int_{S_{m_2}} \hat G^e_{V_2}(\vec r,\vec r'_{m_2})\left[\vec n_{m_2},\vec H_{V_2}(\vec r'_{m_2})\right]\mathrm d\vec r'_{m_2} \\
&-\frac{1}{4\pi}\text{rot}\left\{\sum_{m_2=1}^{M_2}\int_{S_{m_2}} \hat G^m_{V_2}(\vec r,\vec r'_{m_2})[\vec n_{m_2},\vec E_{V_2}(\vec r'_{m_2})]\mathrm d\vec r'_{m_2} + \sum_{n=1}^{N}\int_{\Sigma_n} \hat G^m_{V_2}(\vec r,\vec r'_n)\left[\vec n_n,\vec E_{V_2}(\vec r'_n)\right]\mathrm d\vec r'_n\right\}, \\
\vec H_{V_2}(\vec r) &= \frac{1}{4\pi i k\mu_2}(\text{graddiv} + k_2^2)\left\{\begin{aligned}&\sum_{m_2=1}^{M_2}\int_{S_{m_2}} \hat G^m_{V_2}(\vec r,\vec r'_{m_2})\left[\vec n_{m_2},\vec E_{V_2}(\vec r'_{m_2})\right]\mathrm d\vec r'_{m_2} \\ &+\sum_{n=1}^{N}\int_{\Sigma_n} \hat G^m_{V_2}(\vec r,\vec r'_n)\left[\vec n_n,\vec E_{V_2}(\vec r'_n)\right]\mathrm d\vec r'_n\end{aligned}\right\} \\
&+\frac{1}{4\pi}\text{rot}\sum_{m_2=1}^{M_2}\int_{S_{m_2}} \hat G^e_{V_2}(\vec r,\vec r'_{m_2})\left[\vec n_{m_2},\vec H_{V_2}(\vec r'_{m_2})\right]\vec r'_{m_2}.
\end{aligned}
$$

Here $k = 2\pi / \lambda$ is the wave number, $\lambda$ is the free space wavelength, $k_1 = k\sqrt{\varepsilon_1\mu_1}$ and $k_2 = k\sqrt{\varepsilon_2\mu_2}$ are wave numbers in the media filling the volumes $V_1$ and $V_2$, respectively; $\vec{r}'_{m_1,m_2,n}$ are radius-vectors of sources allocated at the surfaces $S_{m_1}$, $S_{m_2}$ and $\Sigma_n$ ; $\vec{n}_{m1,m2,n}$ are unit vectors of external normals to the surfaces; $\hat{G}^e_{V_1,V_2}(\vec{r}, \vec{r}')$ and $\hat{G}^m_{V_1,V_2}(\vec{r}, \vec{r}')$ are the electric and magnetic tensor Green's functions for Hertz's vector potentials in the coupled volumes satisfying the vector Helmholtz equation and the boundary conditions on surfaces $S_1$ and $S_2$. For the infinitely distant parts of surfaces $S_1$ or $S_2$ the boundary conditions for the Green's functions are transformed to the Sommerfeld's radiation condition.

Interpretation of the fields in the left-hand side of equations (1) depends upon position of an observation point $\vec{r}$. If the observation point $\vec{r}$ belongs to the surfaces $S_{m_1}$, $S_{m_2}$ or to the apertures $\Sigma_n$, the fields $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$ represent the same fields as in the integrals in the right-hand sides of equations (1). In this case, equations (1) are non-homogeneous linear integral Fredholm equations of the second kind, which are known to have the unique solution. If the observation point lies outside areas $V_{m_1}$, $V_{m_2}$ and $\Sigma_n$, the equations (1) become the equalities determining the total electromagnetic field by the field of specified extraneous sources. These equalities solve, in general terms, the problem of electromagnetic fields excitation by finite size obstacles if fields on the objects' surfaces are known. Certainly, to find these fields, the Fredholm integral equations should be solved beforehand.

The equations (1) can be also used to solve electrodynamics problems if the fields on the material body surfaces can be defined by additional physical considerations. For example, if induced currents on well-conducting bodies ($\sigma \to \infty$) are concentrated near the body surface the skin layer thickness can be neglected and the well-known Leontovich-Shchukin approximate impedance boundary condition becomes applicable [4]

$$\left[\vec{n}, \vec{E}(\vec{r})\right] = \bar{Z}_S(\vec{r})\left[\vec{n}, \left[\vec{n}, \vec{H}(\vec{r})\right]\right], \tag{2}$$

where $\bar{Z}_S(\vec{r}) = \bar{R}_S(\vec{r}) + i\bar{X}_S(\vec{r}) = Z_S(\vec{r}) / Z_0$ is the distributed complex surface impedance, normalized to the characteristic free space impedance $Z_0 = 120\pi$ Ohm; the value of $\bar{Z}_S(\vec{r})$ may vary over the body surface. It is generally accepted that the boundary condition (2) are physically adequate under condition $|\bar{Z}_S(\vec{r})| \ll 1$. If $|\bar{Z}_S(\vec{r})| \to 0$, the boundary condition become that for the perfect conductor. In contrast to the limiting case of the perfect conductor, the impedance boundary condition allow to take into account losses in the real material. Since the relative error of (2) is of order $|\bar{Z}_S(\vec{r})|^3$, the inequality $0 \le |\bar{Z}_S(\vec{r})| \le 0.4$ must hold to obtain valid results by the mathematical model.

Using the impedance boundary condition (2) we can introduce a new unknown, density of surface currents. Let us perform such change of unknown in the equations (1). Without loss of generality, we carry the system of equations (1) the transition to the case when all the material bodies are located in volume $V_1$. By placing the observation point on the surface $S_m$ (index 1 is omitted) and using the continuity condition for the tangential components of the magnetic field on the holes $\Sigma_n$, we obtain the system of integral equations relative to the density of surface currents: electric $\vec{J}^{\,e}_m(\vec{r}_m)$ at $S_m$ and equivalent magnetic $\vec{J}^{\,m}_n(\vec{r}_n)$ at $\Sigma_n$. The system can be presented as

$$
\begin{aligned}
&Z_{Sq}(\vec{r}_q)\vec{J}^{\,e}_q(\vec{r}_q) + \frac{k}{\omega}\operatorname{rot}\sum_{n=1}^{N}\int\limits_{\Sigma_n}\hat{G}^m_{V_1}(\vec{r}_q,\vec{r}\,'_n)\vec{J}^{\,m}_n(\vec{r}\,'_n)d\vec{r}\,'_n = \\
&= \vec{E}_0(\vec{r}_q) + \frac{1}{i\omega\varepsilon_1}(\operatorname{graddiv} + k_1^2)\sum_{m=1}^{M}\int\limits_{S_m}\hat{G}^e_{V_1}(\vec{r}_q,\vec{r}\,'_m)\vec{J}^{\,e}_m(\vec{r}\,'_m)d\vec{r}\,'_m + \\
&+\frac{1}{4\pi}\operatorname{rot}\sum_{m=1}^{M}\int\limits_{S_{1_1}}\hat{G}^m_{V_1}(\vec{r}_q,\vec{r}\,'_m)Z_{Sm}(\vec{r}\,'_m)[\vec{n}_m,\vec{J}^{\,e}_m(\vec{r}\,'_m)]d\vec{r}\,'_m, \qquad \text{(a)} \\
&\vec{H}_0(\vec{r}_p) + \frac{1}{i\omega\mu_1}(\operatorname{graddiv} + k_1^2)\sum_{n=1}^{N}\int\limits_{\Sigma_n}\hat{G}^m_{V_1}(\vec{r}_p,\vec{r}\,'_n)\vec{J}^{\,m}_n(\vec{r}\,'_n)d\vec{r}\,'_n \\
&+\frac{1}{i\omega\mu_2}(\operatorname{graddiv} + k_2^2)\sum_{n=1}^{N}\int\limits_{\Sigma_n}\hat{G}^m_{V_2}(\vec{r}_p,\vec{r}\,'_n)\vec{J}^{\,m}_n(\vec{r}\,'_n)d\vec{r}\,'_n \\
&= \frac{1}{i\omega\varepsilon_1}(\operatorname{graddiv} + k_1^2)\sum_{m=1}^{M}\int\limits_{S_m}\hat{G}^m_{V_1}(\vec{r}_p,\vec{r}\,'_m)Z_{Sm}(\vec{r}\,'_m)[\vec{n}_m,\vec{J}^{\,e}_m(\vec{r}\,'_m)]d\vec{r}\,'_m - \\
&-\frac{k}{\omega}\operatorname{rot}\sum_{m=1}^{M}\int\limits_{S_m}\hat{G}^e_{V_1}(\vec{r}_p,\vec{r}\,'_m)\vec{J}^{\,e}_m(\vec{r}\,'_m)d\vec{r}\,'_m, \qquad \text{(b)}
\end{aligned}
\qquad (3)
$$

where $q = 1, 2, ..., m, ..., M$, $p = 1, 2, ..., n, ..., N$, $\vec{J}^{\,e}_m(\vec{r}_m) = \frac{c}{4\pi}[\vec{n}_m, \vec{H}(\vec{r}_m)]$, $\vec{J}^{\,m}_n(\vec{r}_n) = \frac{c}{4\pi}[\vec{n}_n, \vec{E}(\vec{r}_n)]$, $c$ is velocity of light in free space.

Thus, the problem of electromagnetic waves excitation by the impedance bodies of finite dimensions and by the coupling holes between two electrodynamic volumes is formulated as a rigorous boundary value problem of macroscopic electrodynamics, reduced to the system of integral equations for surface currents. Solution of this system is an independent problem, significant in its own right, since it often present considerable mathematical difficulties. If characteristic dimensions of an object are much greater than wavelength (high-frequency region) a solution is usually searched as series expansion in ascending power of inverse wave number. If dimensions of an object are less than wavelength (low-frequency or quasi-static region), representation of the unknown functions as series expansion in wave number powers reduces the problem to a sequence of electrostatic problems. Contrary to asymptotic cases, resonant region, where at least one dimension of an object is comparable with wavelength, is the most complex for analysis, and requires rigorous solution of field equations. It should be noted that, from the practical point of view, the resonant region is of exceptional interest for thin impedance vibrators and narrow slots.

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## 3. Integral equations for electric and magnetic currents in thin impedance vibrators and narrow slots

A straightforward solution of the system (3) for the material objects with irregular surface shape and for holes with arbitrary geometry may often be impossible due to the known mathematical difficulties. However, the solution is sufficiently simplified for thin impedance vibrators and narrow slots, i.e. cylinders, which cross-section perimeter is small as compared to their length and the wavelength in the surrounding media and for holes, which one dimension satisfy the analogous conditions [19,20]. The approach used in [19,20] for the analysis of slot-vibrator systems can be generalized for multi-element systems. In addition, the boundary condition (2) can be extended for cylindrical vibrator surfaces with an arbitrary distribution of complex impedance regardless of the exciting field structure and electrophysical characteristics of vibrator material [4].

For thin vibrators made of circular cylindrical wire and narrow straight slots the equation system (3) can be easily simplified using inequalities

$$\frac{r_m}{L_m} << 1, \frac{r_m}{\lambda_{1,2}} << 1, \frac{d_n}{2L_n} << 1, \frac{d_n}{\lambda_{1,2}} << 1, \tag{4}$$

where $r_m$ is vibrator radius, $L_m$ is vibrator length, $d_n$ is slot width, $2L_n$ is slot length, and $\lambda_{1,2}$ is wavelength in the corresponding media. The electric current induced on the vibrator surfaces and equivalent magnetic currents in the slots can be presented using the inequalities (4) as

$$\vec{J}^e_m(\vec{r}_m) = \vec{e}_{s_m} J_m(s_m)\psi_m(\rho_m, \varphi_m), \quad \vec{J}^m_n(\vec{r}_n) = \vec{e}_{s_n} J_n(s_n)\chi_n(\xi_n), \tag{5}$$

where $\vec{e}_{s_m}$ and $\vec{e}_{s_n}$ are unit vectors directed along the vibrator and slot axis, respectively; $s_m$ and $s_n$ are local coordinates related to the vibrator and slot axes; $\psi_m(\rho_m, \varphi_m)$ are functions of transverse ($\perp_m$) polar coordinates $\rho_m$, $\varphi_m$ for the vibrators; $\chi_n(\xi_n)$ are functions of transverse coordinates $\xi_n$ for the slots. The functions $\psi_m(\rho_m, \varphi_m)$ and $\chi_n(\xi_n)$ satisfy the normality conditions

$$\int_{\perp_m} \psi_m(\rho_m, \varphi_m)\rho_m \mathrm{d}\rho_m \mathrm{d}\varphi_m = 1, \quad \int_{\xi_n} \chi_n(\xi_n)\mathrm{d}\xi_n = 1, \tag{6}$$

and the unknown currents $J_m(s_m)$ and $J_n(s_n)$ must satisfy the boundary conditions

$$J_m(\pm L_m) = 0, \quad J_n(\pm L_n) = 0, \tag{7}$$

where upper indexes $e$ and $m$ are omitted.

Now we take into account that $[\vec{n}_m, \vec{J}_m(\vec{r}_m)] \ll 1$ according to inequalities (4) and project the equations (3a) and (3b) on the axes of the vibrators and slots, respectively, and arrive at a system of linear integral equations relative to the currents in the vibrators and slots

$$\begin{aligned}
&\left(\frac{\mathrm{d}^2}{\mathrm{d}s_q^2}+k_1^2\right)\sum_{m=1}^{M}\int_{-L_m}^{L_m} J_m(s'_m)G_{s_m}^{V_1}(s_q,s'_m)\mathrm{d}s'_m - ik\vec{e}_{s_q}\,\mathrm{rot}\sum_{n=1}^{N}\int_{-L_n}^{L_n} J_n(s'_n)G_{s_n}^{V_1}(s_q,s'_n)\mathrm{d}s'_n \\
&\quad = -i\omega\varepsilon_1\left[E_{0s_q}(s_q)-z_{iq}(s_q)J_q(s_q)\right], \\
&\frac{1}{\mu_1}\left(\frac{\mathrm{d}^2}{\mathrm{d}s_p^2}+k_1^2\right)\sum_{n=1}^{N}\int_{-L_n}^{L_n} J_n(s'_n)G_{s_n}^{V_1}(s_p,s'_n)\mathrm{d}s'_n + \frac{1}{\mu_2}\left(\frac{\mathrm{d}^2}{\mathrm{d}s_p^2}+k_2^2\right)\sum_{n=1}^{N}\int_{-L_n}^{L_n} J_n(s'_n)G_{s_n}^{V_2}(s_p,s'_n)\mathrm{d}s'_n \\
&\quad + ik\vec{e}_{s_p}\,\mathrm{rot}\sum_{m=1}^{M}\int_{-L_m}^{L_m} J_m(s'_m)G_{s_m}^{V_1}(s_p,s'_m)\mathrm{d}s'_m = -i\omega H_{0s_p}(s_p).
\end{aligned} \tag{8}$$

Here $z_{im}(s_m)$ are internal lineal impedance of the vibrators ($Z_{Sm}(\vec{r}_m)=2\pi r_m z_{im}(\vec{r}_m)$) measured in Ohm/m, $E_{0s_m}(s_m)$ and $H_{0s_n}(s_n)$ are projections of extraneous sources on the vibrators and slots axes, $G_{s_m}^{V_1}(s_{m,n}, s'_m)$ and $G_{s_n}^{V_{1(2)}}(s_{m,n}, s'_n)$ are components of the tensor Green's functions in the volumes $V_1$ and $V_2$.

For solitary vibrator or slot as well as for the absence of electromagnetic interaction between them, the system (8) splits into two independent equations:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}s_v^2}+k_1^2\right)\int_{-L_v}^{L_v} J_v(s'_v)G_{s_v}^{V}(s_v,s'_v)\mathrm{d}s'_v = -i\omega\varepsilon_1 E_{0s_v}(s_v) + i\omega\varepsilon_1 z_i(s_v)J_v(s_v), \tag{9}$$

$$\frac{1}{\mu_1}\left(\frac{\mathrm{d}^2}{\mathrm{d}s_{sl}^2}+k_1^2\right)\int_{-L_{sl}}^{L_{sl}} J_{sl}(s'_{sl})G_{s_{sl}}^{V_1}(s_{sl},s'_{sl})\mathrm{d}s'_{sl} + \frac{1}{\mu_2}\left(\frac{\mathrm{d}^2}{\mathrm{d}s_{sl}^2}+k_2^2\right)\int_{-L_{sl}}^{L_{sl}} J_{sl}(s'_{sl})G_{s_{sl}}^{V_2}(s_{sl},s'_{sl})\mathrm{d}s'_{sl} = -i\omega H_{0s_{sl}}(s_{sl}). \tag{10}$$

Here $\vec{e}_{s'_v}$ and $\vec{e}_{s'_{sl}}$ are unit vectors of vibrator and slot axes at the sources, and

$$G_{s_v}^{V}(s_v,s'_v) = \int_{-\pi}^{\pi}\frac{e^{-ik_1\sqrt{(s_v-s'_v)^2+[2r\sin(\varphi/2)]^2}}}{\sqrt{(s_v-s'_v)^2+[2r\sin(\varphi/2)]^2}}\psi(r,\varphi)r\,\mathrm{d}\varphi, \tag{11}$$

$$G_{s_{sl}}^{V_{1,2}}(s_{sl},s'_{sl}) = \int_{-d/2}^{d/2}\frac{e^{-ik_{1,2}\sqrt{(s_{sl}-s'_{sl})^2+(\xi)^2}}}{\sqrt{(s_{sl}-s'_{sl})^2+(\xi)^2}}\chi(\xi)\,\mathrm{d}\xi\,. \tag{12}$$

Solution of the integral equation with the exact kernel expressions (11) and (12) may be very difficult, therefore we will use approximate expressions, the so called "quasi-one-dimensional" kernels [5,15]

$$G_{s_v}^{V}(s_v,s'_v)=\frac{e^{-ik_1\sqrt{(s_v-s'_v)^2+r^2}}}{\sqrt{(s_v-s'_v)^2+r^2}}, \tag{13}$$

$$G_{s_{sl}}^{V_{1,2}}(s_{sl},s'_{sl})=\frac{e^{-ik_{1,2}\sqrt{(s_{sl}-s'_{sl})^2+(d/4)^2}}}{\sqrt{(s_{sl}-s'_{sl})^2+(d/4)^2}} \tag{14}$$

derived with the assumption that source points belong to the geometric axes of the vibrator and slot while observation points belong to vibrator surface and to slot axis, having coordinates $\{s_{sl},\ \xi/2\}$. In that case the functions $G_{s_v}^{V}(s_v,\ s'_v)$ and $G_{s_{sl}}^{V_{1,2}}(s_{sl},\ s'_{sl})$ are everywhere continuous and equations for the currents are simplified significantly.

Since the form of the Green's functions was not specified, the equations (8) are valid for any electrodynamic volumes, provided that the Green's functions for any electrodynamic volumes are known or can be constructed. Although the boundary between the volumes $V_1$ and $V_2$ initially was supposed to be of infinitesimal thickness, its actual thickness can be accounted for by introducing into the equations (8) an effective slot width, defined by the formula given in the Section 5.

## 4. Solution of integral equation for current in an impedance vibrator, located in unbounded free space

Let us use the equation (9) with the approximate kernel (13), being a quasi-one-dimensional analog of the exact integral equation with kernel (11) as starting point for the analysis. Note that impedance $z_i(s)\equiv const$, $\varepsilon_1=\mu_1=1$, and index $v$ is omitted. Thus, the equation may be written as

$$\left(\frac{d^2}{ds^2}+k^2\right)\int_{-L}^{L}J(s')\frac{e^{-ikR(s,s')}}{R(s,s')}ds'=-i\omega E_{0s}(s)+i\omega z_iJ(s), \tag{15}$$

where $R(s,\ s')=\sqrt{(s-s')^2+r^2}$. Let us isolate the logarithmic singularity in the kernel of equation (15) by identical transformation

$$\int_{-L}^{L}J(s')\frac{e^{-ikR(s,s')}}{R(s,s')}ds'=\Omega(s)J(s)+\int_{-L}^{L}\frac{J(s')e^{-ikR(s,s')}-J(s)}{R(s,s')}ds'. \tag{16}$$

Here

$$\Omega(s) = \int_{-L}^{L} \frac{\mathrm{d}s'}{\sqrt{(s-s')^2 + r^2}} = \Omega + \gamma(s), \tag{17}$$

$\gamma(s) = \ln \frac{[(L+s)+\sqrt{(L+s)^2+r^2}][(L-s)+\sqrt{(L-s)^2+r^2}]}{4L^2}$ is a function, equal to zero at the vibrator center and reaching maximal value at its ends where the current in accordance with boundary condition (7) is equal to zero, $\Omega = 2\ln\frac{2L}{r}$ is a large parameter. Then, equation (15) in view of (16) is transformed to integral equation with a small parameter

$$\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + k^2 J(s) = \alpha\left\{i\omega E_{0s}(s) + F[s, J(s)] - i\omega z_i J(s)\right\}. \tag{18}$$

Here $\alpha = -\frac{1}{\Omega} = \frac{1}{2\ln[r/(2L)]}$ is a natural small parameter of the problem ($|\alpha| \ll 1$),

$$\begin{aligned} F[s, J(s)] = &-\frac{\mathrm{d}J(s')}{\mathrm{d}s'} \frac{e^{-ikR(s,s')}}{R(s,s')}\bigg|_{-L}^{L} + \left[\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + k^2 J(s)\right]\gamma(s) \\ &+ \int_{-L}^{L} \frac{\left[\frac{\mathrm{d}^2 J(s')}{\mathrm{d}s'^2} + k^2 J(s')\right] e^{-ikR(s,s')} - \left[\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + k^2 J(s)\right]}{R(s,s')}\mathrm{d}s' \end{aligned} \tag{19}$$

is the vibrator self-field in free space.

To find the approximate analytic solution of equation (18) we will use the asymptotic averaging method. The basic principles of the method are presented in [3,4]. To reduce the equation (18) to a standard equation system with a small parameter in compliance with the method of variation of constants we will change variables

$$\begin{aligned} &J(s) = A(s)\cos ks + B(s)\sin ks, \\ &\frac{\mathrm{d}J(s)}{\mathrm{d}s} = -A(s)k\sin ks + B(s)k\cos ks, \quad \left(\frac{\mathrm{d}A(s)}{\mathrm{d}s}\cos ks + \frac{\mathrm{d}B(s)}{\mathrm{d}s}\sin ks = 0\right), \\ &\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + k^2 J(s) = -\frac{\mathrm{d}A(s)}{\mathrm{d}s}\sin ks + \frac{\mathrm{d}B(s)}{\mathrm{d}s}\cos ks, \end{aligned} \tag{20}$$

where $A(s)$ and $B(s)$ are new unknown functions. Then the equation (18) reduces to a system of integral equations

$$\frac{\mathrm{d}A(s)}{\mathrm{d}s}=-\frac{\alpha}{k}\left\{i\omega E_{0s}(s)+F\left[s,A(s),\frac{\mathrm{d}A(s)}{\mathrm{d}s},B(s),\frac{\mathrm{d}B(s)}{\mathrm{d}s}\right]-i\omega z_i[A(s)\cos ks+B(s)\sin ks]\right\}\sin ks,$$
$$\frac{\mathrm{d}B(s)}{\mathrm{d}s}=+\frac{\alpha}{k}\left\{i\omega E_{0s}(s)+F\left[s,A(s),\frac{\mathrm{d}A(s)}{\mathrm{d}s},B(s),\frac{\mathrm{d}B(s)}{\mathrm{d}s}\right]-i\omega z_i[A(s)\cos ks+B(s)\sin ks]\right\}\cos ks. \quad (21)$$

This system is equivalent to the equation (18) and represents the standard equations system unsolvable with respect to derivatives. The right-hand sides of the equations are proportional to small parameter $\alpha$, therefore, the functions $A(s)$ and $B(s)$ in the left-hand sides of the equations system (21) are slowly varying functions and the system can be solved by the asymptotic averaging method. Then, we replace the system (21) by the simplified system wherein assume $\frac{dA(s)}{ds}=0$ and $\frac{dB(s)}{ds}=0$ in rigth-hand members and carry out partial averaging over the explicit variable $s$ to obtain the equations of first approximation. The term *partial averaging* means that averaging operator acts on all terms, but containing $E_{0s}(s)$ and it may be done for the system (21). The averaged system can be written as

$$\frac{\mathrm{d}\overline{A}(s)}{\mathrm{d}s}=-\alpha\left\{\frac{i\omega}{k}E_{0s}(s)+\overline{F}[s,\overline{A}(s),\overline{B}(s)]\right\}\sin ks+\chi\overline{B}(s),$$
$$\frac{\mathrm{d}\overline{B}(s)}{\mathrm{d}s}=+\alpha\left\{\frac{i\omega}{k}E_{0s}(s)+\overline{F}[s,\overline{A}(s),\overline{B}(s)]\right\}\cos ks-\chi\overline{A}(s), \quad (22)$$

where $\chi=\alpha\frac{i\omega}{2k}z_i$,

$$\overline{F}[s,\overline{A}(s),\overline{B}(s)]=[\overline{A}(s')\sin ks'-\overline{B}(s')\cos ks']\frac{e^{-ikR(s,s')}}{R(s,s')}\bigg|_{-L}^{L} \quad (23)$$

is self-field of the vibrator (19), averaged over its length.

We will seek the solution of the equations system (22) in the form

$$\overline{A}(s)=C_1(s)\cos\chi s+C_2(s)\sin\chi s,$$
$$\overline{B}(s)=-C_1(s)\sin\chi s+C_2(s)\cos\chi s. \quad (24)$$

Then, substitution (24) into (22) gives

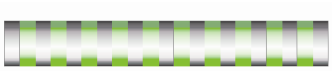
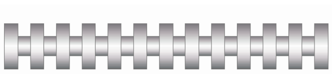
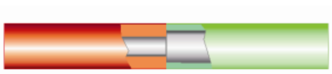
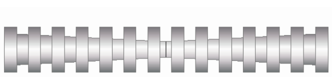
$$\frac{\mathrm{d}C_1(s)}{\mathrm{d}s}=-\alpha\left\{\frac{i\omega}{k}E_{0s}(s)+\overline{F}[s,C_1,C_2]\right\}\sin(k+\chi)s,$$
$$\frac{\mathrm{d}C_2(s)}{\mathrm{d}s}=+\alpha\left\{\frac{i\omega}{k}E_{0s}(s)+\overline{F}[s,C_1,C_2]\right\}\cos(k+\chi)s. \quad (25)$$

Then we find $C_1(s)$ and $C_2(s)$ by solving system (25), determine $\overline{A}(s)$ и $\overline{B}(s)$ from (24), and substitute them as approximating functions for the current into (20). Thus, the general asymptotic expression in parameter $\alpha$ for the current in a thin impedance vibrator under arbitrary excitation may be presented as

$$J(s)=\overline{A}(-L)\cos(\tilde{k}s+\chi L)+\overline{B}(-L)\sin(\tilde{k}s+\chi L)+\alpha\int_{-L}^{s}\left\{\frac{i\omega}{k}E_{0s}(s')+\overline{F}[s',\overline{A},\overline{B}]\right\}\sin\tilde{k}(s-s')\,ds', \qquad (26)$$

where $\tilde{k}=k+\chi=k+i(\alpha/r)\overline{Z}_S$, $\overline{Z}_S=\overline{R}_S+i\overline{X}_S$ is the normalized complex surface impedance: $\overline{Z}_S=2\pi r z_i/Z_0$.

For electrically thin vibrators ( $\left|(k\sqrt{\varepsilon\mu}r)^2 ln(k\sqrt{\varepsilon\mu}r_i)\right|$ <<1, $r_i$ is the radius of the inner conductor) with the parameters of material $\varepsilon$, $\mu$, $\sigma$, from which they are made, the formulas of the distributed surface impedance $\overline{Z}_S$ are presented in Table 1.

| № | Design type of vibrator | Breadboard view of vibrator | Formula for impedance |
|---|---|---|---|
| 1 | The solid metallic cylinder of the $r\Delta^0$ radius, $\Delta^0=\omega/k\sqrt{2\pi\sigma\omega\mu}$ is the skin-layer thickness |  | $\overline{Z}_S=\frac{1+i}{Z_0\sigma\Delta^0}$ |
| 2 | The dielectrical metalized cylinder with covering, made of the metal of the $h_R\Delta^0$ thickness |  | $\overline{Z}_S=\frac{1}{Z_0\sigma h_R+ikr(\varepsilon-1)/2}$ |
| 3 | The metal-dielectrical cylinder ($L_1$ is the thickness of the metal disk, $L_2$ is the thickness of the dielectric disc) |  | $\overline{Z}_S=-i\frac{L_2}{L_1+L_2}\frac{2}{kr\varepsilon}$ |
| 4 | The magnetodielectrical metalized cylinder with the inner conducting cylinder with the radius $r_i$ |  | $\overline{Z}_S=\frac{1}{Z_0\sigma h_R-i/kr\mu\ln(r/r_i)}$ |
| 5 | The metallic cylinder with covering, made of magnetodielectric of the $r$-$r_i$ thickness, or the corrugated cylinder |     | $\overline{Z}_S=ikr\mu\ln(r/r_i)$ <br> $\overline{Z}_S(s)=\overline{R}_S(s)+i\overline{X}_S(s)$ |

http://dx.doi.org/10.5772/61188

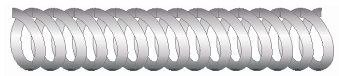
| № | Design type of vibrator | Breadboard view of vibrator | Formula for impedance |
|---|---|---|---|
| 6 | The metallic monofilar helix of the $r$ radius ($kr1$) with the $\psi$ winding angle |  | $\bar{Z}_S = (i/2)kr\,ctg^2\psi$ |

**Table 1.** The formulas of the distributed surface impedance $\bar{Z}_S$



The formulas have been obtained in the frames of impedance conception [4], and they are just for thin cylinders both of infinite and finite extension, located in free space. It is necessary to introduce the multiplier $\sqrt{\mu_1/\varepsilon_1}$ in all formulas for the vibrator in the material medium with the $\varepsilon_1$ and $\mu_1$ parameters. We note, that most of the formulas for impedances include the parameters $\varepsilon$ and $\mu$, smooth change of which (in the case of their dependence from the static electrical and magnetic fields) and the characteristics of radiation of the system, correspondingly, (at its fixed geometrical sizes) can be made, for example, by external field effects.

The constants $\bar{A}(\pm L)$ and $\bar{B}(\pm L)$ can be found employing the boundary conditions (7) and the symmetry conditions [5], unambiguously related to a method of vibrator excitation; if $E_{0s}(s)=E_{0s}^{s}(s)$, $J(s)=J(-s)=J^{s}(s)$ and $\bar{A}(-L)=\bar{A}(+L)$, $\bar{B}(-L)=-\bar{B}(+L)$ ; if $E_{0s}(s)=E_{0s}^{a}(s)$, $J(s)=-J(-s)=J^{a}(s)$ and $\bar{A}(-L)=-\bar{A}(+L)$, $\bar{B}(-L)=\bar{B}(+L)$. Then, in terms of symmetric and antisymmetric current components, marked by indexes $s$ and $a$, respectively, for arbitrary vibrator excitation by $E_{0s}(s)=E_{0s}^{s}(s)+E_{0s}^{a}(s)$ it is not difficult to show that

$$
\begin{aligned}
J(s) = J^s(s) + J^a(s) = \alpha\frac{i\omega}{k}\Bigg\{ & \int_{-L}^{s} E_{0s}(s')\sin\tilde{k}(s-s')\,\mathrm{d}s' \\
& -\frac{\sin\tilde{k}(L+s)+\alpha P^s[kr,\tilde{k}(L+s)]}{\sin 2\tilde{k}L+\alpha P^s(kr,2\tilde{k}L)}\int_{-L}^{L} E_{0s}^{s}(s')\sin\tilde{k}(L-s')\,\mathrm{d}s' \\
& -\frac{\sin\tilde{k}(L+s)+\alpha P^a[kr,\tilde{k}(L+s)]}{\sin 2\tilde{k}L+\alpha P^a(kr,2\tilde{k}L)}\int_{-L}^{L} E_{0s}^{a}(s')\sin\tilde{k}(L-s')\,\mathrm{d}s' \Bigg\},
\end{aligned}
\tag{27}
$$



where $P^s$ and $P^a$ are the functions of vibrator self-fields equal to

$$
\begin{aligned}
P^s[kr,\tilde{k}(L+s)] &= \int_{-L}^{s}\left[\frac{e^{-ikR(s',-L)}}{R(s',-L)}+\frac{e^{-ikR(s',L)}}{R(s',L)}\right]\sin\tilde{k}(s-s')\,\mathrm{d}s'\Bigg|_{s=L} = P^s(kr,2\tilde{k}L), && \text{(a)} \\
P^a[kr,\tilde{k}(L+s)] &= \int_{-L}^{s}\left[\frac{e^{-ikR(s',-L)}}{R(s',-L)}-\frac{e^{-ikR(s',L)}}{R(s',L)}\right]\sin\tilde{k}(s-s')\,\mathrm{d}s'\Bigg|_{s=L} = P^a(kr,2\tilde{k}L). && \text{(b)}
\end{aligned}
\tag{28}
$$

It is evident that if an impedance vibrator is located in restricted volume $V$, the expression for the current coincides with (27), but the functions of vibrator self-field (28) must contain components of electric Green's function for corresponding electrodynamic volume.

Let us consider a problem of vibrator excitation at its geometric center by a lumped EMF with amplitude $V_0$. The mathematical model of excitation is presented as

$$E_{0s}(s) = E_{0s}^{s}(s) = V_0\delta(s-0), \tag{29}$$

where $\delta(s-0)=\delta(s)$ is Dirac delta-function. Then the expression for the current (27) is

$$J(s) = -\alpha V_0\left(\frac{i\omega}{2\tilde{k}}\right)\frac{\sin\tilde{k}(L-|s|)+\alpha P_{\delta}^{s}(kr,\tilde{k}s)}{\cos\tilde{k}L+\alpha P_{L}^{s}(kr,\tilde{k}L)}. \tag{30}$$

Here $P_{\delta}^{s}(kr,\ \tilde{k}s)=P^{s}[kr,\ \tilde{k}(L\ +s)]-(\sin\tilde{k}s+\sin\tilde{k}\,|\,s\,|\,)\,P_{L}^{s}(kr,\ \tilde{k}L\ )$ and $P^{s}[kr,\ \tilde{k}(L\ +s)]$ are defined by the formula (28a). Explicit expressions for $P_{\delta}^{s}(kr,\ \tilde{k}s)$ and $P_{L}^{s}(kr,\ \tilde{k}L\ )$ can be expressed explicitly in terms of generalized integral functions [4,5]. Thus, $P_{L}^{s}(kr,\ \tilde{k}L\ )$ which will be needed below may be presented as

$$\begin{aligned} P_{L}^{s}(kr,\tilde{k}L) = \cos\tilde{k}L\{&2\ln 2-\gamma(L)-(1/2)[\mathrm{Cin}(2\tilde{k}L+2kL)+\mathrm{Cin}(2\tilde{k}L-2kL)] \\ &-(i/2)[\mathrm{Si}(2\tilde{k}L+2kL)-\mathrm{Si}(2\tilde{k}L-2kL)]\} \\ +\sin\tilde{k}L\{&(1/2)[\mathrm{Si}(2\tilde{k}L+2kL)+\mathrm{Si}(2\tilde{k}L-2kL)]-(i/2)[\mathrm{Cin}(2\tilde{k}L+2kL)-\mathrm{Cin}(2\tilde{k}L-2kL)]\}, \end{aligned} \tag{31}$$

where $Si(x)$ and $Cin(x)$ are sine and cosine integrals of complex argument.

Since the current distribution (30) is now known we can calculate electrodynamic characteristics of an impedance vibrator. Thus, an input impedance $Z_{in}=R_{in}+i\,X_{in}$ of vibrator in a feed point is equal

$$Z_{in}[\mathrm{Ohm}] = \frac{V_0}{J(0)} = \left(\frac{60i\tilde{k}}{\alpha k}\right)\frac{\cos\tilde{k}L+\alpha P_{L}^{s}(kr,\tilde{k}L)}{\sin\tilde{k}L+\alpha P_{\delta L}(kr,\tilde{k}L)}, \tag{32}$$

where

$$\begin{aligned} P_{\delta L}(kr,\tilde{k}L) &= \int_{-L}^{L}\frac{e^{-ikR(s,L)}}{R(s,L)}\sin\tilde{k}\,|\,s\,|\,ds \\ &= \sin\tilde{k}L\{-\gamma(L)+(1/2)[\mathrm{Cin}(2\tilde{k}L+2kL)-\mathrm{Cin}(2\tilde{k}L-2kL)]-\mathrm{Cin}(\tilde{k}L+kL)+\mathrm{Cin}(\tilde{k}L-kL) \\ &+(i/2)[\mathrm{Si}(2\tilde{k}L+2kL)-\mathrm{Si}(2\tilde{k}L-2kL)]-i[\mathrm{Si}(\tilde{k}L+kL)-\mathrm{Si}(\tilde{k}L-kL)]\} \\ &+\cos\tilde{k}L\{(1/2)[\mathrm{Si}(2\tilde{k}L+2kL)+\mathrm{Si}(2\tilde{k}L-2kL)]-\mathrm{Si}(\tilde{k}L+kL)-\mathrm{Si}(\tilde{k}L-kL) \\ &-(i/2)[\mathrm{Cin}(2\tilde{k}L+2kL)+\mathrm{Cin}(2\tilde{k}L-2kL)]+i[\mathrm{Cin}(\tilde{k}L+kL)+\mathrm{Cin}(\tilde{k}L-kL)]\}. \end{aligned} \tag{33}$$

Note, that an input admittance $Y_{in}=G_{in}+iB_{in}$ can be calculated as $Y_{in}=1/Z_{in}$.

To confirm the validity of the above analytical formulas we present the results of a comparative analysis of calculated and experimental data available in the literature. Figure 2 and Figure 3 show the graphs of the input admittance for two realizations of surface impedance: 1) metal wire (radius $r_i$=0.3175 cm), covered by dielectric ($\varepsilon$=9.0) shell (radius $r$=0.635 cm), the experimental data [21] at Figure 2 and 2) metal wire ($r_i$=0.5175 cm), covered with ferrite ($\mu$=4.7) shell ($r$=0.6 cm), the experimental data from [22] at Figure 3. The plots show that trends of the theoretical curves coincide with that of the experimental curves, especially near the resonance for $B_{in}$=0, though in absolute values some difference is observed. In our opinion, the discrepancy of theoretical curves, obtained by solving the integral equation for the current by averaging method, and the experimental curves may be caused by evident fact that vibrator self-field (19) was averaged and the current amplitude was determined with some error.

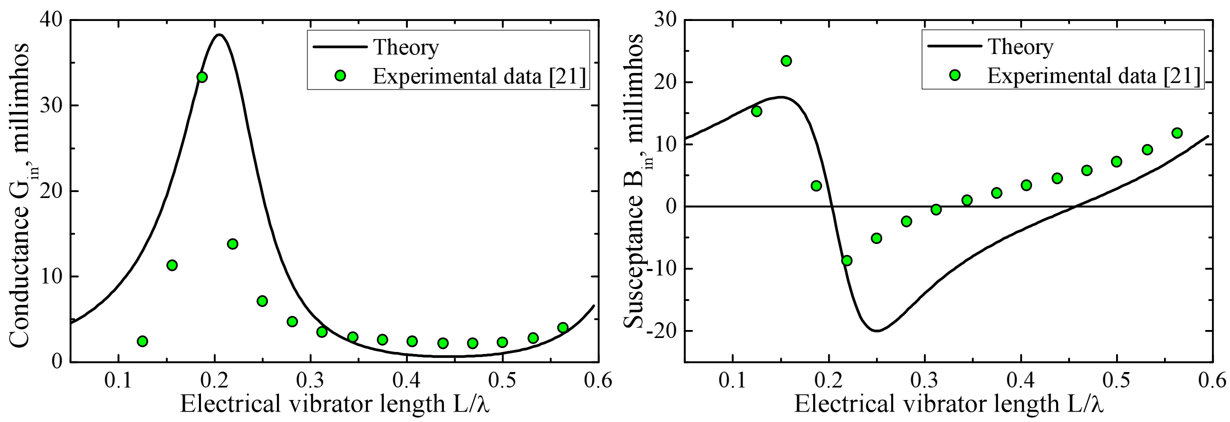



**Figure 2.** The input admittance of metal wire (radius $r_i$ =0.3175 cm), covered by dielectric shell ($\varepsilon$ =9.0, radius $r$ =0.635 cm) versus its electrical length at the frequency $f$ =600 MHz

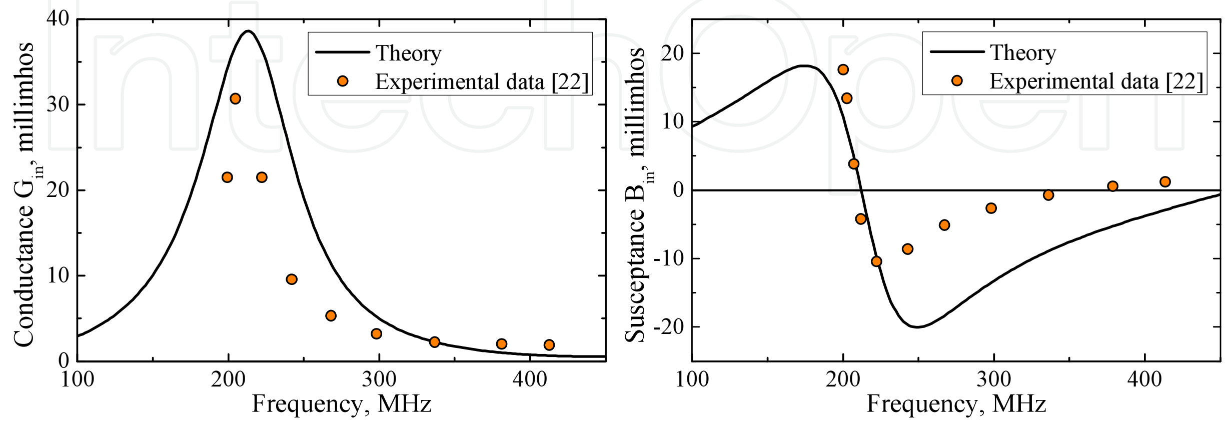



**Figure 3.** The input admittance of metal wire (radius $r_i$ =5175 cm), covered with ferrite shell ($\mu$ =4.7, $r$ =0.6 cm) versus frequency for $L$ =30.0 cm

## 5. Solution of equation for current in a slot between two semi-infinite rectangular waveguides

Now let us solve the second key problem. Let a resonant iris is placed in infinite hollow ($\varepsilon_1=\mu_1=\varepsilon_2=\mu_2=1$) rectangular waveguide so that its slot has arbitrary orientation in the plane of waveguide cross-section and has no contacts with waveguide walls (Figure 4).

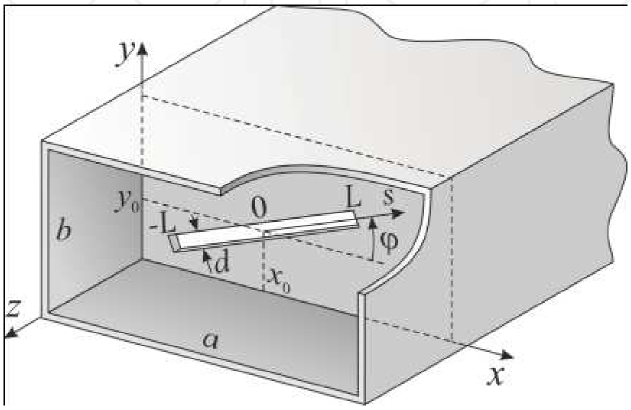



**Figure 4.** The problem geometry and notations

A starting point for the analysis is equation (10) written as (index *sl* is omitted)

$$\left(\frac{d^2}{ds^2}+k^2\right)\int_{-L}^{L} J(s')4\frac{e^{-ikR(s,s')}}{R(s,s')}ds' = -i\omega H_{0s}(s) - \left(\frac{d^2}{ds^2}+k^2\right)\int_{-L}^{L} J(s')\left[G_{0s}^{V_1}(s,s')+G_{0s}^{V_2}(s,s')\right]ds', \qquad (34)$$

where $4\frac{e^{-ikR(s,s')}}{R(s,\, s')}$ is the Green's function of the slot in infinite perfectly conducting plane, $G_{0s}^{V_{1,2}}(s,\, s')$ are the Green's functions, which takes into account multiple reflection from walls of volumes.

Isolating the logarithmic singularity in the kernel of equation (34) as in (17), we reduce the equation (34) to an integral equation with small parameter

$$\frac{d^2J(s)}{ds^2}+k^2J(s)=\alpha\left\{i\omega H_{0s}(s)+F\left[s,J(s)\right]+F_0\left[s,J(s)\right]\right\}. \qquad (35)$$

Here $\alpha=1/8\ln[d_e/(8L)]$ is the natural small parameter of the problem ($|\alpha|<<1$), $d_e=d\,e^{-\frac{\pi h}{2d}}$ is equivalent slot width which takes into account a real wall thickness $h$ ($h/\lambda<<1$) [3],

$$F\left[s,J(s)\right]=-4\frac{\mathrm{d}J(s')}{\mathrm{d}s'}\frac{e^{-ikR(s,s')}}{R(s,s')}+4\left[\frac{\mathrm{d}^2J(s)}{\mathrm{d}s^2}+k^2J(s)\right]\gamma(s)$$
$$+4\int\limits_{-L}^{L}\left\{\frac{\left[\frac{\mathrm{d}^2J(s')}{\mathrm{d}s'^2}+k^2J(s')\right]e^{-ikR(s,s')}-\left[\frac{\mathrm{d}^2J(s)}{\mathrm{d}s^2}+k^2J(s)\right]}{R(s,s')}\right\}\mathrm{d}s' \quad (36)$$

is self-field of the slot in infinite perfectly conducting plane,

$$F_0\left[s,J(s)\right]=-\frac{\mathrm{d}J(s')}{\mathrm{d}s'}\left[G_{0s}^{V_1}(s,s')+G_{0s}^{V_2}(s,s')\right]\Bigg|_{-L}^{L}+\int\limits_{-L}^{L}\left[\frac{\mathrm{d}^2J(s')}{\mathrm{d}s'^2}+k^2J(s')\right]\left[G_{0s}^{V_1}(s,s')+G_{0s}^{V_2}(s,s')\right]\mathrm{d}s' \quad (37)$$

is self-field of the slot, which takes into account multiple reflection from walls of volumes.

To solve the equation (35) by averaging method we change the variable according to (20) and obtain the standard system of integral equations relative to new unknown functions $A(s)$ and $B(s)$ which is equivalent to initial equation (35)

$$\frac{\mathrm{d}A(s)}{\mathrm{d}s}=-\frac{\alpha}{k}\left\{i\omega H_{0s}(s)+F_N\left[s,A(s),\frac{\mathrm{d}A(s)}{\mathrm{d}s},B(s),\frac{\mathrm{d}B(s)}{\mathrm{d}s}\right]\right\}\sin ks,$$
$$\frac{\mathrm{d}B(s)}{\mathrm{d}s}=+\frac{\alpha}{k}\left\{i\omega H_{0s}(s)+F_N\left[s,A(s),\frac{\mathrm{d}A(s)}{\mathrm{d}s},B(s),\frac{\mathrm{d}B(s)}{\mathrm{d}s}\right]\right\}\cos ks, \quad (38)$$

where $F_N = F + F_0$ is the total self-field of the slot.

Assuming, as in Section 4, $\frac{dA(s)}{ds}=0$ and $\frac{dB(s)}{ds}=0$ in the right-hand members of equations (38) and making partial averaging over the variable $s$, we derive the equations of the first approximation by averaging method

$$\frac{\mathrm{d}\overline{A}(s)}{\mathrm{d}s}=-\alpha\left\{\frac{i\omega}{k}H_{0s}(s)+\overline{F}_N[s,\overline{A},\overline{B}]\right\}\sin ks,$$
$$\frac{\mathrm{d}\overline{B}(s)}{\mathrm{d}s}=+\alpha\left\{\frac{i\omega}{k}H_{0s}(s)+\overline{F}_N[s,\overline{A},\overline{B}]\right\}\cos ks, \quad (39)$$

where

$$\overline{F}_N[s,\overline{A},\overline{B}]=[\overline{A}(s')\sin ks'-\overline{B}(s')\cos ks']G_s^{\Sigma}(s,s')\Big|_{-L}^{L},$$
$$G_s^{\Sigma}(s,s')=G_s^{V_1}(s,s')+G_s^{V_2}(s,s') \tag{40}$$

is the slot total self-field, averaged over the slot length.

Solving the system (39), we obtain the general asymptotic expression for the current in narrow slot, located in arbitrary position relative to the walls of coupling volumes

$$J(s)=\overline{A}(-L)\cos ks+\overline{B}(-L)\sin ks+\alpha\int_{-L}^{s}\left\{\frac{i\omega}{k}H_{0s}(s')+\overline{F}_N[s',\overline{A},\overline{B}]\right\}\sin k(s-s')\,\mathrm{d}s'. \tag{41}$$

To determine constants $\overline{A}(\pm L)$ and $\overline{B}(\pm L)$ we will use the boundary conditions (7) and the symmetry conditions, uniquely related both to slot excitation method and its position in waveguide. Then, in terms of symmetric and antisymmetric magnetic current components, marked by indexes $s$ and $a$, respectively, for arbitrary slot excitation by $H_{0s}(s)=H_{0s}^{s}(s)+H_{0s}^{a}(s)$ with an accuracy of order $\alpha^2$ we have

$$J(s)=J^s(s)+J^a(s)=\alpha\frac{i\omega}{k}\left\{\int_{-L}^{s}H_{0s}(s')\sin k(s-s')\,\mathrm{d}s'\right.$$
$$\left.-\frac{\sin k(L+s)\int_{-L}^{L}H_{0s}^{s}(s')\sin k(L-s')\,\mathrm{d}s'}{\sin 2kL+\alpha N^s(kd_e,2kL)}-\frac{\sin k(L+s)\int_{-L}^{L}H_{0s}^{a}(s')\sin k(L-s')\,\mathrm{d}s'}{\sin 2kL+\alpha N^a(kd_e,2kL)}\right\}, \tag{42}$$

where $N^s(kd_e, 2kL)$ and $N^a(kd_e, 2kL)$ are the functions of self-field which are equal

$$N^s(kd_e,2kL)=\int_{-L}^{L}[G_s^{\Sigma}(s,-L)+G_s^{\Sigma}(s,L)]\sin k(L-s)\,\mathrm{d}s,$$
$$N^a(kd_e,2kL)=\int_{-L}^{L}[G_s^{\Sigma}(s,-L)-G_s^{\Sigma}(s,L)]\sin k(L-s)\,\mathrm{d}s, \tag{43}$$

which are completely defined by the Green's functions of the coupling volumes.

Supposing that dominant wave $H_{10}$ with amplitude $H_0$ is propagated from the region $z=-\infty$, we have

$$H_{0s}(s)=2H_0\cos\varphi\left[\sin\frac{\pi x_0}{a}\cos\frac{\pi(s\cos\varphi)}{a}+\cos\frac{\pi x_0}{a}\sin\frac{\pi(s\cos\varphi)}{a}\right]. \tag{44}$$

The symmetric and antisymmetric components of the slot current, relative to the slot center $s=0$, become equal

$$J(s)=J_0 f(s)=-\alpha 2H_0\cos\varphi\frac{2i\omega/k^2}{[1-(k_\varphi/k)^2][\sin 2kL+\alpha 2W_\varphi^{sa}(kd_e,2kL)]}$$
$$\times\left\{\sin\frac{\pi x_0}{a}\sin kL(\cos ks\cos k_\varphi L-\cos kL\cos k_\varphi s)+\cos\frac{\pi x_0}{a}\cos kL(\sin ks\sin k_\varphi L-\sin kL\sin k_\varphi s)\right\}, \tag{45}$$

where $J_0$ is current amplitude, $f(s)$ is the current distribution function, $k_\varphi=\frac{\pi}{a}\cos\varphi$, $W_\varphi^{sa}(kd_e,\ 2kL\ )$ is the function of slot self-field, defined by formulas (43).

Reflection and transmission coefficients, $S_{11}$ and $S_{12}$ for the dominant wave in the slot iris are define by the current as

$$S_{11}=\left(1+S_{12}\right)e^{2i\gamma z},\ S_{12}=-\alpha\frac{16\pi k_g\cos^2\varphi\, f(k_\varphi L)}{iabk^3[1-(k_\varphi/k)^2][\cos kL+\alpha 2W_\varphi(kd_e,kL)]}, \tag{46}$$

$$f(k_\varphi L)=2\cos k_\varphi L\frac{\sin kL\cos k_\varphi L-(k_\varphi/k)\cos kL\sin k_\varphi L}{1-(k_\varphi/k)^2}-\cos kL\frac{\sin 2k_\varphi L+2k_\varphi L}{2(k_\varphi/k)},$$

where $k_g=\sqrt{k^2-(\pi/a)^2}$ is the propagation constant of $H_{10}$ wave.

Figure 5 shows the theoretical and experimental wavelength dependences of power reflection coefficient $|S_{11}|^2$ for the iris, which oriented so that the angle between slot axis $\{0s\}$ and waveguide axis $\{0x\}$ are $0^O$ and $30^O$.

Note that a comparative analysis of the analytical solution of key problems is not limited only by the examples presented above. Thus, the solution for current in the impedance vibrator, located in free space, was preliminary compared with the known approximate analytical solutions of integral equations. The adequacy of the constructed mathematical models to real physical processes and the reliability of simulation results has been also confirmed by comparative calculations, obtained by the numerical method of moments and other methods, in particular, by the finite element method implemented in the software package *Ansoft HFSS*.

## 6. Combined vibrator–slot structures

Now let us consider a problem of electromagnetic waves excitation by a narrow straight transverse slot in the broad wall of rectangular waveguide with a two passive impedance vibrators in it.

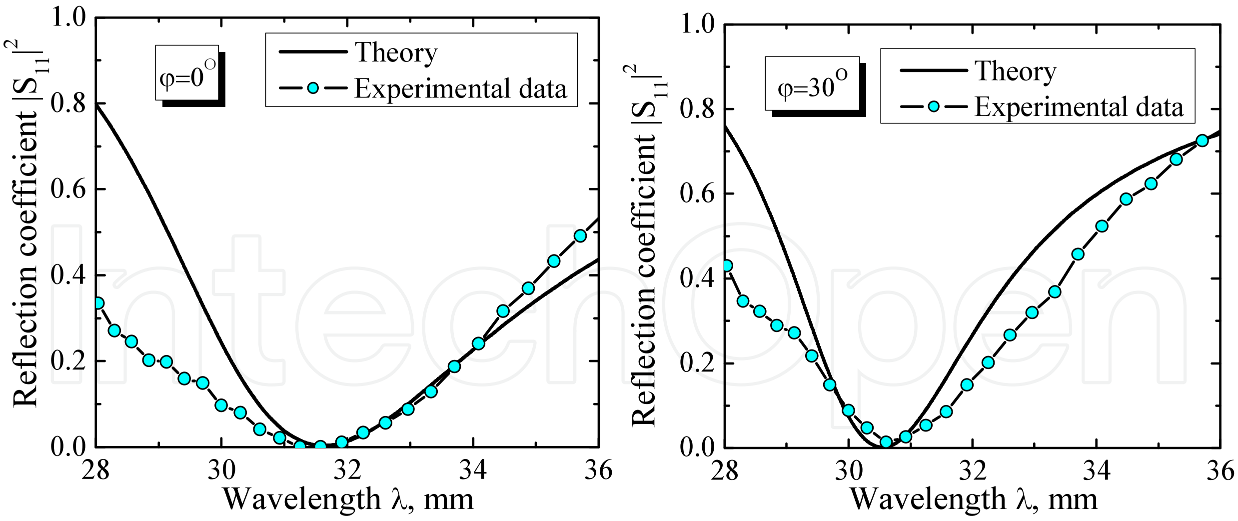



**Figure 5.** Power reflection coefficient $|S_{11}|^2$ versus wavelength for the iris at $a$ =23.0 mm, $b$ =10.0 mm, $2L$ =16.0 mm, $d$ =1.5 mm, $h$ =2.0 mm, $x_0 = a/2$, $y_0 = b/2$

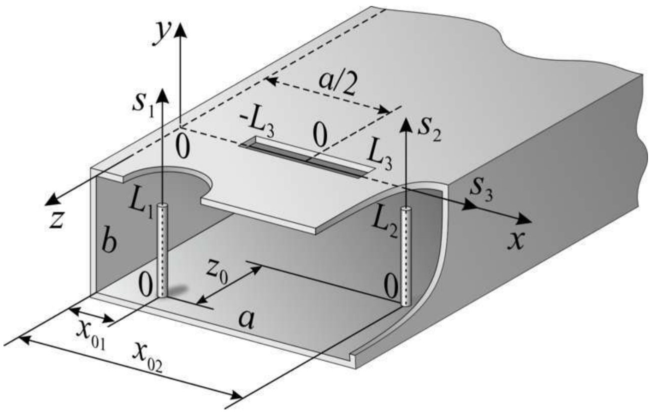



**Figure 6.** The geometry of three-element vibrator-slot system and notations

Let a fundamental wave $H_{10}$ propagates from the area $z = -\infty$ in a hollow infinite rectangular waveguide, the area index is "*Wg*". Two thin nonsymmetrical vibrators (monopoles) with variable surface impedance are located in a waveguide with cross-section $\{a \times b\}$. A narrow transverse slot cut in a broad wall of the waveguide symmetrically relative to its longitudinal axis is radiating into free half-space, the area index is "*Hs*". The vibrators radiuses and lengths are $r_{1,2}$ and $L_{1,2}$ $((r_{1,2}/L_{1,2})<<1)$, the slot width is $d$, the slot length is $2L_3$ $((d/L_3)<<1)$ and the waveguide wall thickness is $h$. One vibrator is located in the plane $\{x0y\}$ and the second vibrator may be shifted along the axis $\{0z\}$ at the distance $z_0$ (Figure 6).

For this configuration the system of integral equations relative to electrical currents at the vibrators $J_{1,2}(s_{1,2})$ and equivalent magnetic current in the slot $J_3(s_3)$ in accordance with (8) may be represented as

$$\left(\frac{d^2}{ds_1^2}+k^2\right)\left\{\int_{-L_1}^{L_1} J_1(s_1')G_{s_1}^{Wg}(s_1,s_1')ds_1' + \int_{-L_2}^{L_2} J_2(s_2')G_{s_2}^{Wg}(s_1,s_2')ds_2'\right\}-$$
$$-ik\int_{-L_3}^{L_3} J_3(s_3')\tilde{G}_{s_3}^{Wg}(s_1,s_3')ds_3'$$
$$=-i\omega\left[E_{0s_1}(s_1)-z_{i1}(s_1)J_1(s_1)\right], \qquad \text{(a)}$$
$$\left(\frac{d^2}{ds_2^2}+k^2\right)\left\{\int_{-L_2}^{L_2} J_2(s_2')G_{s_2}^{Wg}(s_2,s_2')ds_2' + \int_{-L_1}^{L_1} J_1(s_1')G_{s_1}^{Wg}(s_2,s_1')ds_1'\right\}= \qquad (47)$$
$$=-i\omega\left[E_{0s_2}(s_2)-z_{i2}(s_2)J_2(s_2)\right], \qquad \text{(b)}$$
$$\left(\frac{d^2}{ds_3^2}+k_1^2\right)\int_{-L_3}^{L_3} J_3(s_3')\left[G_{s_3}^{Wg}(s_3,s_3')+G_{s_3}^{Hs}(s_3,s_3')\right]ds_3' -$$
$$-ik\int_{-L_1}^{L_1} J_1(s_1')\tilde{G}_{s_1}^{Wg}(s_3,s_1')ds_1' = -i\omega H_{0s_3}(s_3). \qquad \text{(c)}$$

Here $G_{s_{1,2}}^{Wg}(s_{1,2}, s'_{1,2})$ and $G_{s_3}^{Wg,Hs}(s_3, s'_3)$ are components of the Green's functions of the rectangular waveguide and the half-space over the plane [3,4], $\tilde{G}_{s_1}^{Wg}(s_3, s'_1)=\frac{\partial}{\partial z}G_{s_1}^{Wg}[x(s_3), 0, z; x'(s'_1), y'(s'_1), z_0]$ and $\tilde{G}_{s_3}^{Wg}(s_1, s'_3)=\frac{\partial}{\partial z}G_{s_3}^{Wg}[x(s_1), y(s_1), z; x'(s'_3), 0, 0]$ after substitution $z=0$ into $\tilde{G}_{s_1}^{Wg}$ and $z=z_0$ into $\tilde{G}_{s_3}^{Wg}$ after first derivation, $z_{i1,2}(s_{1,2})$ is the internal impedance per unit length of the vibrators ([Ohm/m]), $E_{0s_{1,2}}(s_{1,2})$ and $H_{0s_3}(s_3)$ are projections of impressed sources fields on the vibrators and the slot axes, $s_1=-L_1$ and $s_2=-L_2$ are end coordinates of mirror vibrator images relative to the lower broad wall of the waveguide [4]

We will seek the solution of equations system (47) by a generalized method of induced EMMF [19,20], using functions $J_{1(2)}(s_{1(2)})=J_{1(2)}^0 f_{1(2)}(s_{1(2)})$ and $J_3(s_3)=J_3^0 f_3(s_3)$ as approximating expressions for the currents. Here $J_{1(2)}^0$ and $J_3^0$ are unknown current amplitudes, $f_{1(2)}(s_{1(2)})$ and $f_3(s_3)$ are predetermined functions of the current distributions. In accordance with (27) and (42) for the vibrator-slot structure excited by the fundamental wave $H_{10}$ we have $f_{1(2)}(s_{1(2)})=\cos\tilde{k}_{1(2)}s_{1(2)}-\cos\tilde{k}_{1(2)}L_{1(2)}$, $f_3(s_3)=\cos ks_3-\cos kL_3$, $\tilde{k}_{1(2)}=k-\frac{i2\pi z_{i1(2)}^{av}}{Z_0\Omega_{1(2)}}$, $z_{i1(2)}^{av}=\frac{1}{L_{1(2)}}\int_0^{L_{1(2)}} z_{i1(2)}(s_{1(2)})ds_{1(2)}$ are average values [4] of internal impedances, $\Omega_{1(2)}=2\ln(2L_{1(2)}/r_{1(2)})$.

In accordance with the generalized method of induced EMMF, we multiply equation (47a) by the function $f_1(s_1)$, equation (47b) by the function $f_2(s_2)$, and the equation (47c) by the function

$f_3(s_3)$ and integrate the equations (47a) and (47b) over the length of the vibrators, and the equation (47c) over the length of the slot. As a result, we obtain a system of linear algebraic equations relative to the current amplitudes $J_{1,2,3}^0$

$$\begin{aligned}
&J_1^0 Z_{11}^{\Sigma} + J_2^0 Z_{12} + J_3^0 Z_{13} = -\frac{i\omega}{2k}\int_{-L_1}^{L_1} f_1(s_1)E_{0s_1}(s_1)ds_1,\\
&J_2^0 Z_{22}^{\Sigma} + J_1^0 Z_{21} = -\frac{i\omega}{2k}\int_{-L_2}^{L_2} f_2(s_2)E_{0s_2}(s_2)ds_2,\\
&J_3^0 Z_{33}^{\Sigma} + J_1^0 Z_{31} = -\frac{i\omega}{2k}\int_{-L_3}^{L_3} f_3(s_3)H_{0s_3}(s_3)ds_3.
\end{aligned} \tag{48}$$

Here

$$\begin{aligned}
Z_{11(22)} = &\frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\left\{\frac{\varepsilon_n (k^2-k_y^2)\tilde{k}_{1(2)}^2}{kk_z(\tilde{k}_{1(2)}^2-k_y^2)^2} e^{-k_z r_{1(2)}} \sin^2 k_x x_{01(02)}\right.\\
&\left.\times\left[\sin\tilde{k}_{1(2)}L_{1(2)}\cos k_y L_{1(2)} - \frac{\tilde{k}_{1(2)}}{k_y}\cos\tilde{k}_{1(2)}L_{1(2)}\sin k_y L_{1(2)}\right]^2\right\},
\end{aligned} \tag{49}$$

$$F_{1(2)}^z = -\frac{i}{r_{1(2)}}\int_0^{L_{1(2)}} f_{1(2)}^2(s_{1(2)})\bar{Z}_{S1(2)}(s_{1(2)})\mathrm{d}s_{1(2)}, \tag{50}$$

$$\begin{aligned}
Z_{12} = Z_{21} = &\frac{4\pi}{ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\left\{\frac{\varepsilon_n (k^2-k_y^2)\tilde{k}_1\tilde{k}_2 e^{-k_z z_0}}{kk_z(\tilde{k}_1^2-k_y^2)(\tilde{k}_2^2-k_y^2)}\sin k_x x_{01}\right.\\
&\times\sin k_x x_{02}\left[\sin\tilde{k}_1 L_1\cos k_y L_1 - (\tilde{k}_1/k_y)\cos\tilde{k}_1 L_1 \sin k_y L_1\right]\\
&\left.\times\left[\sin\tilde{k}_2 L_2\cos k_y L_2 - (\tilde{k}_2/k_y)\cos\tilde{k}_2 L_2 \sin k_y L_2\right]\right\},
\end{aligned} \tag{51}$$

$$\begin{aligned}
Z_{33}^{Hs} = &\,\mathrm{Si}4kL_3 - i\mathrm{Cin}4kL_3 - 2\cos kL_3\left[2\left(\sin kL_3 - kL_3\cos kL_3\right)\right.\\
&\left.\times\left(\ln\frac{16L_3}{d_e} - \mathrm{Cin}2kL_3 - i\mathrm{Si}2kL_3\right) + \sin 2kL_3 e^{-ikL_3}\right],
\end{aligned} \tag{52}$$

$$Z_{33}^{Wg} = \frac{8\pi}{ab}\sum_{m=1,3..}^{\infty}\sum_{n=0,1..}^{\infty}\left\{\frac{\varepsilon_n k}{k_z(k^2-k_x^2)} e^{-k_z\frac{d_e}{4}}\left[\sin kL_3\cos k_x L_3 - (k/k_x)\cos kL_3\sin k_x L_3\right]^2\right\}, \tag{53}$$

$$
\begin{aligned}
&Z_{13} = -Z_{31} = \\
&= \frac{4\pi}{ab} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \left\{ \frac{\varepsilon_n k \tilde{k}_1 e^{-k_z z_0}}{i(\tilde{k}_1^2 - k_y^2)(k^2 - k_x^2)} \sin k_x x_{01} \sin \frac{k_x a}{2} \left[ \sin \tilde{k}_1 L_1 \cos k_y L_1 - \frac{\tilde{k}_1}{k_y} \cos \tilde{k}_1 L_1 \sin k_y L_1 \right] \right. \\
&\left. \times \left[ \sin k L_3 \cos k_x L_3 - \frac{k}{k_x} \cos k L_3 \sin k_x L_3 \right] \right\}, \\
&Z_{11(22)}^{\Sigma} = Z_{11(22)} + F_{1(2)}^{z},\ Z_{33}^{\Sigma} = Z_{33}^{Hs} + Z_{33}^{Wg},
\end{aligned}
\tag{54}
$$

where $\varepsilon_n = \begin{cases} 1, & n=0 \\ 2, & n \neq 0 \end{cases}$, $k_{x(y)} = \frac{m(n)\pi}{a(b)}$, $k_z = \sqrt{k_x^2 + k_y^2 - k^2}$, $m$, $n$ are integers; $Si$ and $Cin$ are integral sine and cosine.

The energy characteristics of the vibrator-slot system: the reflection and transmission coefficients, $S_{11}$ and $S_{12}$, and power radiation coefficient $|S_{\Sigma}|^2$, are defined by the expressions

$$
S_{11} = \frac{4\pi i}{abkk_g} \left\{ J_3 \frac{2k_g^2}{k^2} f(kL_3) - J_1 \frac{k_g}{\tilde{k}_1} \sin\left(\frac{\pi x_{01}}{a}\right) f(\tilde{k}_1 L_1) e^{-ik_g z_0} - J_2 \frac{k_g}{\tilde{k}_2} \sin\left(\frac{\pi x_{02}}{a}\right) f(\tilde{k}_2 L_2) \right\} e^{2ik_g z}, \tag{55}
$$

$$
S_{12} = 1 + \frac{4\pi i}{abkk_g} \left\{ J_3 \frac{2k_g^2}{k^2} f(kL_3) + J_1 \frac{k_g}{\tilde{k}_1} \sin\left(\frac{\pi x_{01}}{a}\right) f(\tilde{k}_1 L_1) e^{ik_g z_0} + J_2 \frac{k_g}{\tilde{k}_2} \sin\left(\frac{\pi x_{02}}{a}\right) f(\tilde{k}_2 L_2) \right\}, \tag{56}
$$

$$
|S_{\Sigma}|^2 = 1 - |S_{11}|^2 - |S_{12}|^2 \tag{57}
$$

In expressions (55)-(57)

$$
\begin{aligned}
J_1 &= 1 \Big/ \left( Z_{11}^{\Sigma} Z_{22}^{\Sigma} Z_{33}^{\Sigma} - Z_{21} Z_{12} Z_{33}^{\Sigma} - Z_{31} Z_{13} Z_{22}^{\Sigma} \right) \\
&\times \left[ \frac{k^2}{k_g \tilde{k}_1} \sin \frac{\pi x_{01}}{a} f_1(\tilde{k}_1 L_1) e^{-ik_g z_0} Z_{22}^{\Sigma} Z_{33}^{\Sigma} - \frac{k^2}{k_g \tilde{k}_2} \sin \frac{\pi x_{02}}{a} f_2(\tilde{k}_2 L_2) Z_{12} Z_{33}^{\Sigma} - f_3(kL_3) Z_{13} Z_{22}^{\Sigma} \right],
\end{aligned}
$$

$$
\begin{aligned}
J_2 &= 1 \Big/ \left( Z_{11}^{\Sigma} Z_{22}^{\Sigma} Z_{33}^{\Sigma} - Z_{21} Z_{12} Z_{33}^{\Sigma} - Z_{31} Z_{13} Z_{22}^{\Sigma} \right) \\
&\times \left[ \frac{k^2}{k_g \tilde{k}_2} \sin \frac{\pi x_{02}}{a} f_2(\tilde{k}_2 L_2) (Z_{11}^{\Sigma} Z_{33}^{\Sigma} - Z_{31} Z_{13}) - \frac{k^2}{k_g \tilde{k}_1} \sin \frac{\pi x_{01}}{a} f_1(\tilde{k}_1 L_1) e^{-ik_g z_0} Z_{21} Z_{33}^{\Sigma} + f_3(kL_3) Z_{13} Z_{21} \right],
\end{aligned}
$$

$$J_3 = 1\Big/\left(Z_{11}^{\Sigma}Z_{22}^{\Sigma}Z_{33}^{\Sigma} - Z_{21}Z_{12}Z_{33}^{\Sigma} - Z_{31}Z_{13}Z_{22}^{\Sigma}\right)$$
$$\times\left[f_3(kL_3)(Z_{11}^{\Sigma}Z_{22}^{\Sigma} - Z_{21}Z_{12}) + \frac{k^2}{k_g\tilde{k}_2}\sin\frac{\pi x_{02}}{a}f_2(\tilde{k}_2L_2)Z_{12}Z_{31} - \frac{k^2}{k_g\tilde{k}_1}\sin\frac{\pi x_{01}}{a}f_1(\tilde{k}_1L_1)e^{-ik_g z_0}Z_{31}Z_{22}^{\Sigma}\right],$$

$$f_{1(2)}(\tilde{k}_{1(2)}L_{1(2)}) = \sin\tilde{k}_{1(2)}L_{1(2)} - \tilde{k}_{1(2)}L_{1(2)}\cos\tilde{k}_{1(2)}L_{1(2)},$$

$$f_3(kL_3) = \frac{\sin kL_3\cos(\pi L_3/a) - (ka/\pi)\cos kL_3\sin(\pi L_3/a)}{1-[\pi/(ka)]^2}.$$

Let us consider several distribution functions for the surface impedance along the vibrator, namely: 1) $\phi_0(s_{1(2)})=1$, the constant distribution, 2) $\phi_1(s_{1(2)})=2[1-(s_{1(2)}/L_{1(2)})]$, the triangular distribution linear decreasing to the vibrator end, and 3) $\phi_2(s_{1(2)})=2(s_{1(2)}/L_{1(2)})$, the triangular linear increasing distribution. All distribution have equal average values $\overline{\phi_{0,1,2}(s_{1(2)})}=1$. The expression for $F_{1(2)}^{z0}$ with the distribution function 1), in accordance with (50), can be presented as

$$F_{1(2)}^{z0} = -\frac{2i(\bar{R}_{S1(2)} + i\bar{X}_{S1(2)})}{\tilde{k}_{1(2)}^2L_{1(2)}r_{1(2)}}\left[\left(\frac{\tilde{k}_{1(2)}L_{1(2)}}{2}\right)^2\left(2+\cos2\tilde{k}_{1(2)}L_{1(2)}\right) - \frac{3}{8}\tilde{k}_{1(2)}L_{1(2)}\sin2\tilde{k}_{1(2)}L_{1(2)}\right]$$
$$= \tilde{F}_{1(2)}^{z}(\bar{R}_{S1(2)} + i\bar{X}_{S1(2)})\Phi_{1(2)} \quad (58)$$

with the distribution function 2) as

$$F_{1(2)}^{z1} = \tilde{F}_{1(2)}^{z}$$
$$\times\left\{\bar{R}_{S1(2)}\Phi_{1(2)} + i\bar{X}_{S1(2)}\left[\left(\frac{\tilde{k}_{1(2)}L_{1(2)}}{2}\right)^2\left(2+\cos2\tilde{k}_{1(2)}L_{1(2)}\right) - \frac{7}{4}\sin^2\tilde{k}_{1(2)}L_{1(2)} - 2\left(\cos\tilde{k}_{1(2)}L_{1(2)} - 1\right)\right]\right\} \quad (59)$$

and with the distribution function 3) as

$$F_{1(2)}^{z2} = \tilde{F}_{1(2)}^{z}\left\{\bar{R}_{S1(2)}\Phi_{1(2)}\right.$$
$$+ i\bar{X}_{S1(2)}\left[\left(\frac{\tilde{k}_{1(2)}L_{1(2)}}{2}\right)^2\left(2+\cos2\tilde{k}_{1(2)}L_{1(2)}\right) + \frac{7}{4}\sin^2\tilde{k}_{1(2)}L_{1(2)}\right. \quad (60)$$
$$\left.\left. - \frac{3}{4}\tilde{k}_{1(2)}L_{1(2)}\sin2\tilde{k}_{1(2)}L_{1(2)} + 2\left(\cos\tilde{k}_{1(2)}L_{1(2)} - 1\right)\right]\right\}$$

Since the formulas for $F_{1(2)}^{z0,1,2}$ differ from one another, in spite of equal average values of functions $\phi_{0,1,2}(s_{1(2)})$ and identical functional dependences in formulas for currents, the current amplitudes and, hence, energy characteristics will be substantially different.

Figures 7, 8 shows the wavelength dependences of the radiation coefficient, modules of the reflection and transmission coefficients in the wavelength range of the waveguide single-mode regime, obtained using the following common parameters: $a$ =58.0 mm, $b$ =25.0 mm, $h$ =0.5 mm, $r_{1,2}$ =2.0 mm, $L_{1,2}$ =15.0 mm, $\bar{R}_{S1(2)}$=0, $x_{01}=a/8$, $x_{02}=7a/8$, $d$ =4.0 mm and $2L_3$ =40.0 mm.

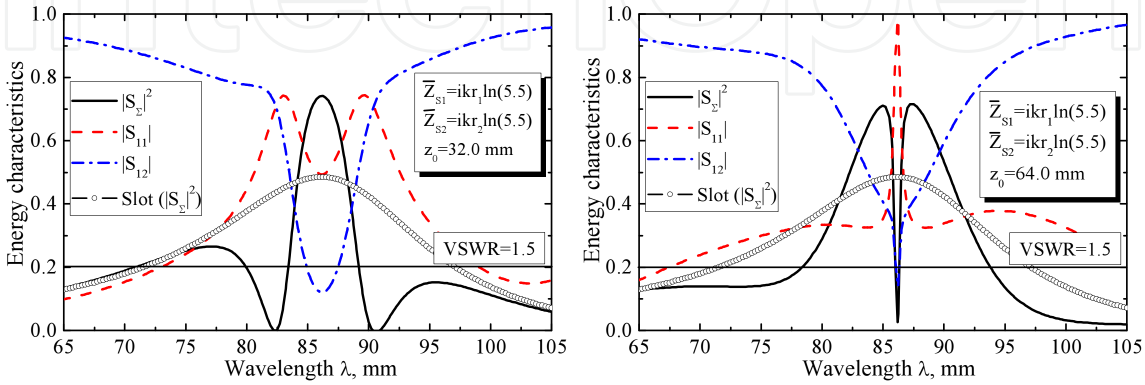



**Figure 7.** The energy characteristics versus wavelength at $\lambda_{v1,2}^{res}=\lambda_{sl}^{res}$, $\bar{Z}_{S1}=\bar{Z}_{S2}$

The choice of slot dimensions was stipulated by its natural resonance at the average wavelength of the waveguide frequency range $\lambda_3^{res}$ =86.0 mm. The dimensions of the vibrators have been selected so that their resonant wavelength was within the waveguide operating range. Here we present the results only for vibrators with inductive impedances ($\bar{X}_{S1(2)}$ >0), known to increase the vibrator electrical length, i.e. to increase $\lambda_{1,2}^{res}$ as compared to case $\bar{Z}_{S1(2)}$=0, without decreasing a distance between the vibrators ends and the upper broad wall of the waveguide. This is very important for increasing the breakdown power for waveguide device as a whole.

As might be expected from physical considerations, displacement of the impedance vibrator along the longitudinal axis of the waveguide at a distance $z_0$ from the centre of the slot, where the maximum mutual influence between elements of the structure is observed, are multiple of $\lambda_G/4$ (Fig. 7: $z_0=\lambda_G/4$ =32.0 mm and $z_0=\lambda_G/2$ =64.0 mm). Here $\lambda_G=2\pi\Big/\sqrt{(2\pi/\lambda_{sl}^{res})^2-(\pi/a)^2}$ is resonant wavelength of the slot in the waveguide, and $\lambda_{sl}^{res}$ is the resonant wavelength of the slot in the free half-space over the plane. As seen from Figure 7, an acceptable reflection coefficient $|S_{11}|$ and high level of radiation could not be achieved if the monopoles have the equal distributed impedances $\bar{Z}_{S1}=\bar{Z}_{S2}$. The maximum of radiation coefficient $|S_\Sigma|^2$ and almost perfect agreement with the feed line, as well as tuning to other resonant wavelengths can be achieved by changing the distribution functions of impedance along the monopoles axes (Figure 8). Fig. 8 also shows that the results of mathematical modeling are confirmed by the experimental data. Experimental models have been made in the form of corrugated brass rods (see photo in Figure 8).

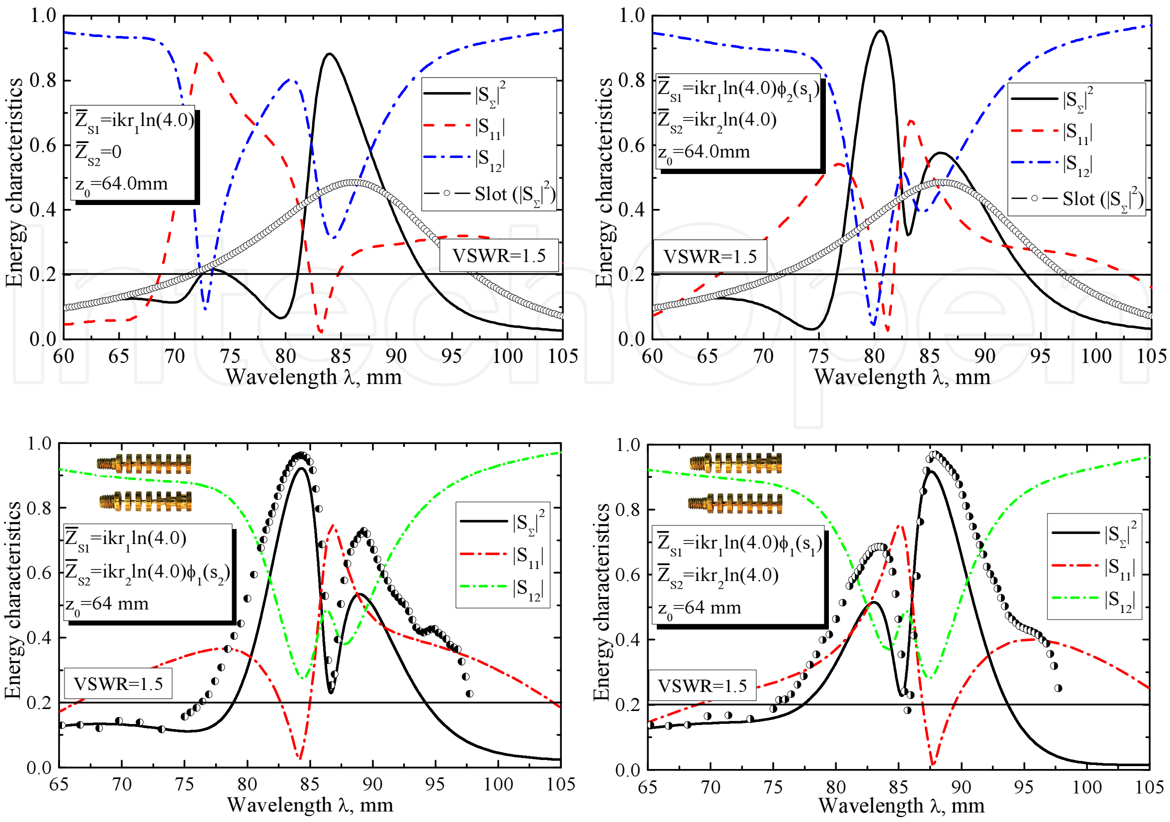



**Figure 8.** The energy characteristics versus wavelength at $\lambda_{v1,2}^{res} \neq \lambda_{sl}^{res}$, $\bar{Z}_{S1} \neq \bar{Z}_{S2}$, experimental data are marked by circles

For the arbitrary vibrator-slot structures and coupled electrodynamic volumes expressions for $f_v^{s,a}(s_v)$ and $f_{sl}^{s,a}(s_{sl})$ (the subscripts $s$, $a$ denote the symmetric and antisymmetric components of the currents with respect to the vibrator ($s_v=0$) and slot ($s_{sl}=0$) centers, respectively), in accordance with the results, presented in Sections 4 and 5 (see formulas (27) and (42)), can be obtained from the following relations

$$
\begin{aligned}
f_v^{s,a}(s_v) &\sim \left\{ \begin{aligned} &\sin\tilde{k}(L_v - s_v)\int_{-L_v}^{s_v} E_{0s_v}^{s,a}(s'_v)\sin\tilde{k}(L_v + s'_v)ds'_v \\ &+\sin\tilde{k}(L_v + s_v)\int_{s_v}^{L_v} E_{0s_v}^{s,a}(s'_v)\sin\tilde{k}(L_v - s'_v)ds'_v \end{aligned} \right\}, \quad \text{(a)} \\
f_{sl}^{s,a}(s_{sl}) &\sim \left\{ \begin{aligned} &\sin k(L_{sl} - s_{sl})\int_{-L_{sl}}^{s_{sl}} H_{0s_{sl}}^{s,a}(s'_{sl})\sin k(L_{sl} + s'_{sl})ds'_{sl} \\ &+\sin k(L_{sl} + s_{sl})\int_{s_{sl}}^{L_{sl}} H_{0s_{sl}}^{s,a}(s'_{sl})\sin k(L_{sl} - s'_{sl})ds'_{sl} \end{aligned} \right\}, \quad \text{(b)}
\end{aligned}
\qquad (61)
$$

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where $E^{s,a}_{0s_v}(s_v)$ and $H^{s,a}_{0s_{sl}}(s_{sl})$ are projections of symmetric and antisymmetric components of impressed sources on the vibrator and the slot axes. Here the sign ~ means that after integration in expressions (61) only multipliers, depending upon coordinates $s_v$ and $s_{sl}$, are left.

Note once more that for arbitrary orientations of the vibrator, or the slot relative to the waveguide walls, or for another impressed field sources, the expressions (61) should be used to determine the distribution functions of electric and magnetic currents in the vibrator and slot. For example, for the longitudinal slot in the broad wall of waveguide, i.e. if axes $\{0s_{sl}\}$ and $\{0z\}$ coincide, we obtain

$$\begin{aligned} f^s_{sl}(s_{sl}) &= \cos ks_{sl} \cos k_g L_{sl} - \cos kL_{sl} \cos k_g s_{sl}, \\ f^a_{sl}(s_{sl}) &= \sin ks_{sl} \sin k_g L_{sl} - \sin kL_{sl} \sin k_g s_{sl}. \end{aligned} \quad (62)$$

If vibrator is excited at its base by voltage $\delta$ -generator as in a waveguide-to-coaxial adapter we have

$$f_v(s_v) = \sin \tilde{k}(L_v - s_v). \quad (63)$$

## 7. Conclusion

This chapter presents the methodological basis for application of the generalized method of induced EMMF for the analysis of electrodynamic characteristics of the combined vibrator-slot structures. Characteristic feature of the generalization to a new class of approximating functions consists in using them as a function of the current distributions along the impedance vibrator and slot elements; these distributions are derived as the asymptotic solution of integral equations for the current (key problems) by the method of averaging. Comparison of theoretical and experimental curves indicates that the solution of integral equations for combined vibrator-slot structures by the generalized method of induced EMMF with approximating functions for the currents in the impedance vibrator and the slot, obtained by averaging method is quite legitimate. It should be noted that for simple structures similar to that considered in the model problem, the proposed approach yields an analytic solution of the electrodynamic problem. For more complex structures, the method may be used to design effective numerical-analytical algorithms for their analyses.

The demonstrative simulation (the comparative analysis of all electrodynamic characteristics in the operating frequencies range) has confirmed the validity of the proposed generalized method of induced EMMF for analysis of vibrator-slot systems with rather arbitrary structure (within accepted assumptions). Here, as examples, some fragments of this comparative analysis were presented. This method retains all benefits of analytical methods as compared with direct numerical methods and allows to expand significantly the boundaries of numerical and analytical studies of practically important problems, concerning the application of single impedance vibrator, including irregular vibrator, the systems of such vibrators and narrow slots. And this is a natural step in the further development of the general fundamental theory of linear radiators of electric and magnetic types.

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