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Use of Fast Fourier Transform for Sensitivity Analysis

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1. Introduction

The uncertainty quantification of code calculations is typically accompanied by a sensitivity analysis, in which the influence of the individual contributors to the uncertainty is determined. In the sensitivity analysis, the basic step is to perform sensitivity calculations varying the input parameters. One or more input parameters could be varied at a time. The typical statistical methods for the sensitivity analysis used in uncertainty methods are for example Pearson's Correlation Coefficient, Standardized Regression Coefficient, Partial Correlation Coefficient and others [1]. The output results are time domain signals. The objective of this study was to use fast Fourier transform (FFT) based approaches to determine the sensitivity of output parameters. In the reference [2], the FFT based approaches have been used for the accuracy quantification. The difference between the accuracy quantification and the sensitivity analysis is that in the accuracy quantification the experimental data are compared to the code calculated data, while in the sensitivity analysis the reference calculation signal is compared to the sensitivity run calculation signal. To do this comparison, first the fast Fourier transform is used to transform time domain signals into frequency domain signals. Then, the average amplitude is calculated, which is the sum of the amplitudes of the frequency domain difference signal (between the sensitivity run calculation signal and the reference calculation signal) normalized by the sum of the amplitudes of the frequency domain reference signal. Finally, the figures of merit based on the average amplitude are used to judge the sensitivity.

Such a FFT based approach is different from the typical sensitivity analysis using a statistical procedure to determine the influence of sensitive input parameters on the output parameter. Namely, the influence of the sensitive input parameters is represented by the average amplitude which remembers the previous history.

In this study it is shown that the proposed FFT based tool is complementary and a good alternative to the mentioned typical statistical methods, if one parameter is varied at a time.

The advantage of the proposed method is that the results of the sensitivity analysis obtained by the FFT based tool could be ranked. It provides a consistent ranking of sensitive input parameters according to their influence to the output parameter when one parameter is varied at a time, and based on the fact that the same method can be used for more participants performing calculations. For example, there is no need to have ranking levels like it was proposed in the Phase III of the Best Estimate Methods – Uncertainty and Sensitivity Evaluation (BEMUSE) programme [3] for the qualitative judgement, because in this approach the figure of merit is a quantitative value and therefore can be directly ranked. The zero value of the figure of merit means a not relevant sensitive parameter and the larger the figure of merit is the more influential the input parameter is.

The difference between statistical methods and the FFT based approach is that in the case of statistical methods the influence of each varied input parameter on the output result can be obtained even if more parameters are varied at a time. This cannot be done by FFT based tools when more parameters are varied at a time. Rather, the total influence of sensitive parameters on the result is given by the FFT based tool. But the good thing is that the same measure is used for both single and multiple variations and in this way the compensation effects of the influence of different sensitive parameters could be studied. As was already mentioned the FFT based approaches are complementary to statistical methods.

In this Chapter, first the original fast Fourier transform based method (FFTBM) approach is described. The average amplitude, the signal mirroring, the FFTBM Add-in tool and the time dependent accuracy measures are introduced. Based on signal mirroring the improved FFTBM by signal mirroring (FFTBM-SM) was developed. By calculating the time dependent average amplitude it can be answered, which discrepancy due to the parameter variation contributes to the sensitivity and how much is its contribution. The past application of the FFT based tool for the accuracy quantification showed that the original FFTBM gave an unrealistic judgment of the average amplitude for monotonically increasing or decreasing functions, causing problems in the FFTBM results interpretation. It was found out [4] that the reason for such an unrealistic calculated accuracy of increasing/decreasing signals is the edge between the first and last data point of the investigated signal, when the signal is periodically extended. Namely, if the values of the first and last data point of the investigated signal differ, then there are discontinuities present in the periodically extended signal seen by the discrete Fourier transform, which views the finite domain signal as an infinite periodic signal. The discontinuities give several harmonic components in the frequency domain, thus increasing the sum of the amplitudes, on which FFTBM is based, and by this influencing the accuracy. The influence of the edge due to the periodically extended signal is for clarity reasons called the edge effect.

Then the methods used for the sensitivity analysis are described. For the demonstration of the sensitivity study using FFT based tools the L2-5 test, which simulates the large break loss of coolant accident in the Loss of Fluid test (LOFT) facility, was used. The signals used were obtained from the Organisation for Economic Co-operation and Development (OECD) BEMUSE project. In the BEMUSE project there were 14 participants, each performing a reference calculation and 15 sensitivity runs of the LOFT L2-5 test. Three output parameters

were provided: the upper plenum pressure, the primary side mass inventory and the rod surface temperature.

Finally, the application of the FFT based approaches to the sensitivity analysis is described. Both FFTBM and FFTBM-SM were used.

2. Fast fourier transform based method description

The FFT based method is proposed for the sensitivity analysis, which is analogous to the FFT based code-accuracy assessment described in ref. [5]. The FFT based approach for code accuracy consists of three steps: a) selection of the test case (experimental or plant measured time trends to compare), b) qualitative analysis, and c) quantitative analysis. The qualitative analysis is necessary before quantifying the discrepancies between the measured and calcu‐ lated trends. The qualitative analysis includes also the visual observation of plots and the evaluation of the discrepancies between the measured and calculated trends, which should be predictable and understood. For the sensitivity analysis the same FFT based approach is used for the quantitative analysis as used for the code-accuracy. However, the signals compared now are the output signal obtained with the reference value of the input parameter and the output signal as the result of the sensitive input parameter variation.

In the quantitative analysis, the influence of the sensitive input parameter variation is judged in the frequency domain. Therefore the time domain signals used in the sensitivity analysis have to be transformed in the frequency domain signals. The addressed time domain signals assume values different from zero only in the interval [0, T_d], where T_d is the duration of the signal. Also the digital computers can only work with information that is discrete and finite in length (e.g. N points) and there is no version of the Fourier transform that uses finite length signals [6]. The way around this is to make the finite data look like an infinite length signal. This is done by imagining that the signal has an infinite number of samples on the left and right of the actual points. The imagined samples can be a duplication of the actual data points. In this case, the signal looks discrete and periodic. This calls for the discrete Fourier transform (DFT) to be used. There are several ways to calculate DFT. One method is FFT. While it produces the same results as the other approaches, it is incredibly more efficient. The key point to understand the FFTBM is that the periodicity is invoked in order to be able to use a mathematical tool, i.e., the DFT. It seems that the developers of the original FFTBM have not been sufficiently aware of this fact.

The discrete Fourier transform views both, the time domain and the frequency domain, as periodic [6]. However, the signals to be used for the comparison are not periodic and the user must conform to the DFT's view of the world. When a new period starts, the N samples on the left side are not related to the samples on the right side. However, DFT views these N points to be a single period of an infinitely long periodic signal. This means that the left side of the signal is connected to the right side of the signal, and vice versa. The most serious consequence of the time domain periodicity is the occurrence of the edge, where the signals are glued. When the signal spectrum is calculated with DFT, the edge is taken into account, despite the fact that

the edge has no physical meaning for the comparison, since it was introduced artificially by the applied numerical method. It is known that the edge produces a variegated spectrum of frequencies due to the discontinuity of the edge. These frequencies originating from the artificially introduced edge may overshadow the frequency spectrum of the investigated signal. Therefore an improved version of FFTBM by signal mirroring has been proposed, which is described in detail in [4]. Both the original FFTBM and the improved FFTBM by signal mirroring have been used in the demonstration application. The same equations are used for the calculation of the average amplitude, like for the original FFTBM, except that, instead of the original signals, the symmetrized signals are used (for further details see ref. [4]). In the following it is first described how the average amplitude is calculated.

2.1. Average amplitude

FFT is another method for calculating the DFT. While it produces the same result as the other approaches, it significantly reduces the computation time. FFT usually operates with a number of values N that is a power of two. Typically, N is selected between 32 and 4096 [6]. In addition, the sampling theorem must be fulfilled to avoid the distortion of sampled signals due to the aliasing occurrence. The sampling theorem says: "a signal that varies continuously with time is completely determined by its values at an infinite sequence of equally spaced times if the frequency of these sampling times is greater than twice the highest frequency component of the signal" [7]. Thus if the number of points defining the function in the time domain is $N=2^{m+1}$, then according to the sampling theorem the sampling frequency is:

$$\frac{1}{\tau} = f_s = 2f_{max} = \frac{N}{t_d} = \frac{2^{m+1}}{t_d} \tag{1}$$

where τ is the sampling interval, t_d is the transient time duration of the sampled signal and f_{max} is the highest (maximum) frequency component of the signal. The sampling theorem does not hold beyond f_{max}. From the relation in Eq. (1) it is seen that the number of points selection is strictly connected to the sampling frequency. The FFT algorithm requires the number of points, equally spaced, which is a power with base 2. Generally an interpolation is necessary to satisfy this requirement. The original FFTBM is done so that the default value of the exponent m ranges from 8 to 11. This gives N ranging from 512 to 4096. The final number of points used by FFTBM is determined depending on the value of f_{fix}, which is the minimum requested maximum frequency and is input value. If f_{max} is not larger than f_{fix}, the number of points is doubled (exponent m is increased for 1) until the criterion is satisfied or the exponent m equals to 11. Please note, that the minimum value of the exponent m is 8. The FFTBM application implies the following input values: the fixed frequency f_{fix} (minimum maximum frequency of the analysis, this determines the number of points N), the cut off frequency (f_{cut}), the start time t_s and the end time t_e of the analysed window (determines the analysis window $t_d = t_s - t_e$). A cut off frequency has been introduced to cut off spurious contributions, generally negligible. When f_{cut} is equal or larger than f_{fix}, all frequency components are considered.

For the calculation of the differences between the output signal obtained with the reference value of the input parameter (reference signal $F_{ref}(t)$) and the output signal as the result of the sensitive input parameter variation (sensitive signal $F_{sen}(t)$), the reference signal ($F_{ref}(t)$) and the difference signal $\Delta F(t)$ are needed. The difference signal in the time domain is defined as:

$$\Delta F(t) = F_{ref}(t) - F_{sen}(t). \tag{2}$$

After performing the fast Fourier transform the obtained spectra of amplitudes are used for the calculation of the average amplitude (AA):

$$AA = \frac{\sum_{n=0}^{2^m} \left|\tilde{\Delta}F(f_n)\right|}{\sum_{n=0}^{2^m} \left|\tilde{F}_{ref}(f_n)\right|}, \tag{3}$$

where $\left|\tilde{\Delta}F(f_n)\right|$ is the difference signal amplitude at frequency f_n and $\left|\tilde{F}_{ref}(f_n)\right|$ is the reference signal amplitude at frequency f_n. The AA factor can be considered as a sort of average fractional difference and the closer the AA value is to zero, the smaller is the sensitivity (influence). In our specific application, the larger the sensitivity is the larger is the difference between the signals, normally resulting in a larger AA value. Typically, based on the previous experience in the accuracy quantification the values of AA below 0.3 indicate a small influence (in the case of pressure below 0.1), while the values above 0.5 indicate a large influence.

The above Eq. (3) can be also viewed as:

$$AA = \frac{AA_{dif}}{AA_{ref}}, \tag{4}$$

where AA_{dif} is the average amplitude of the difference signal and AA_{ref} is the average amplitude of the reference signal:

$$AA_{dif} = \frac{1}{2^m + 1} \sum_{n=0}^{2^m} \left|\tilde{\Delta}F(f_n)\right|, \tag{5}$$

$$AA_{ref} = \frac{1}{2^m + 1} \sum_{n=0}^{2^m} \left|\tilde{F}_{ref}(f_n)\right|. \tag{6}$$

If the reference and sensitive signals are the same, the difference signal is zero. The larger the difference is, the larger is AA_{dif} (in principle). On the other hand, AA_{ref} normalizes the average amplitude AA and the higher the sum of amplitudes is, the lower is the average amplitude AA. This means that for ranking the sensitivities of the selected output parameter only AA_{dif} has an influence. On the other hand, for judging the influence of a single sensitive parameter variation on different output parameters, besides AA_{dif} also AA_{ref} influences the ranking due to the different AA_{ref} the output parameters have.

$$fraction\ A0 = \frac{\left|\tilde{\Delta}F(f_0)\right|}{\sum\limits_{n=0}^{2^m}\left|\tilde{\Delta}F(f_n)\right|},$$

(7)

where $|\tilde{\Delta}F(f_0)|$ is the mean value of the difference signal and it is equivalent to the mean error (ME) in the time domain as defined in ref. [8]. The measure *fraction A0* shows when the frequency amplitudes are dominating or when the mean values (like constant differences) are dominating the sum of the amplitudes.

2.2. Signal mirroring

If we have a function $F(t)$ where $0 \le t \le t_d$ and t_d is the transient time duration, its mirrored function is defined as $F_{mir}(t) = F(-t)$, where $-t_d \le t \le 0$. From these functions a new function is composed which is symmetrical in regard to the y-axis: $F_m(t)$, where $-t_d \le t \le t_d$. This is illustrated in Figure 1. By composing the original signal (shown in Figure 1(a)) and its mirrored signal (signal mirroring), a signal without the edge between the first and the last data sample is obtained, and it is called symmetrized signal (shown in Figure 1(b)). It has the double number of points in order not to lose any information. Also it should be noted that the edge is not present in the original time domain signal (see Figure 1(a)). However, when performing FFT, the aperiodic original signal is treated as a periodic original signal as mentioned before and therefore the edge is part of the periodic original signal, what is not physical. In the case of the symmetrized signal the edge is not present even when treating the signal as periodic.

For the calculation of the average amplitude by signal mirroring AA_m the Eq. (3) is used like for the calculation of AA, except that, instead of the original signals, the symmetrized signals are used. This may be efficiently done by signal mirroring, where the investigated signal is mirrored before the original FFTBM is applied. By composing the original signal and its mirrored signal (signal mirroring), a symmetric signal (also called symmetrized signal) with the same characteristics is obtained, but without introducing the edge when viewed as an infinite periodic signal (for details refer to refs. [4, 9]). FFTBM using the symmetrized signals is called FFTBM-SM.
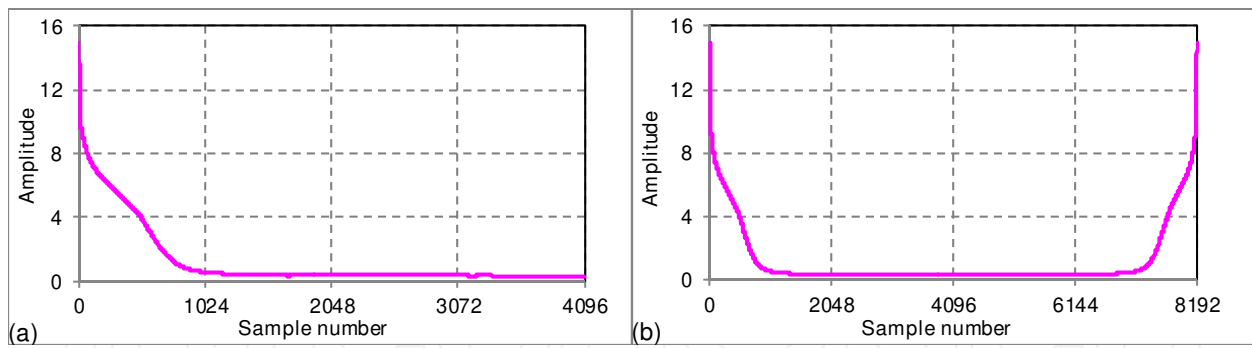
Figure 1. Examples of (a) original digital signal and (b) symmetrized digital signal

2.3. FFTBM Add-in tool

To take the advantages of spread sheets in preparing input forms, analysing data (including analysis of values), modifying graphs and the capability to store time recorded data, plots, input forms and results, the Jožef Stefan Institute (JSI) in-house Microsoft Excel Add-in for the accuracy evaluation of thermal-hydraulic code calculations with FFTBM has been developed in 2003 [10]. Later the tool was upgraded with the capability to symmetrize the signals, and some other improvements. The upgraded tool is called JSI FFTBM Add-in 2007 [11] and it has been used for the sensitivity study, described in this chapter. It includes both FFTBM and FFTBM-SM. As already mentioned, the difference between FFTBM and FFTBM-SM is that in the latter the signals are symmetrized to eliminate the edge effect in calculating the average amplitude by signal mirroring (AA_m).

JSI FFTBM Add-in 2007 provides additional information on interpolated data of the signals used, the difference signals, the amplitude spectra and the AA dependency on the cut frequency. The user can use the interpolated data for visual checking about the agreement of the original signal and the interpolated signal. The amplitude spectra give the possibility to compare the spectra between different signals. Information on the AA dependency on the cut frequency is used to check if the cut frequency is selected properly. Usually the dependency is not so big, therefore by default AA at the selected f_{cut} frequencies is calculated:

- minimal AA (AA_{min}) at frequency (when $f_{cut} > 0.05 f_{max}$) which gives the minimum AA,

- average AA (AA_{avg}) is calculated as the average AA at all cut frequencies,

- maximal AA (AA_{max}) at the frequency (when $f_{cut} > 0.05 f_{max}$) which gives the maximum AA,

- 5 percentile AA (AA_{05}) at frequency $f_{cut} = 0.05 f_{max}$,

- 50 percentile AA (AA_{50}) at frequency $f_{cut} = 0.5 f_{max}$,

- 100 percentile AA (AA_{100}) considering all amplitudes (for $f_{cut} = f_{max}$).

This gives an indication if AA is significantly dependent on the cut frequency. In principle, AA should not be much different when half or all frequencies from the amplitude spectrum are considered for the AA calculation. If this is not the case, deeper insight into AA is needed

to judge if there is a spurious contribution present in the signal. For example, in the case of measured data, noise may be present in the signal. In the case of code calculated digital data, noise is not present in the signals, therefore the whole amplitude spectrum is recommended for the AA calculation. Nevertheless, the user should be aware that AA depends on the cut frequency and that the result may change when not all frequencies are considered. Typically higher frequency components have lower amplitudes than lower frequencies, therefore the lower frequency content is always used for the AA calculation (only higher frequency components are filtered).

2.4. Time dependent average amplitude

In the ref. [12] the influence of the time window selection was studied. Instead of a few phenomenological windows a series of narrow windows (phases) could be selected. This gives the possibility to get the time dependency of the average amplitude. The increasing time interval was defined as a set of time intervals each increased for the duration of one narrow time window and the last time interval being the whole transient duration time. By increasing the time interval we see how the average amplitude changes with the time progression as it is shown in Figure 2. The average amplitude was calculated by the original FFTBM not considering the edge effect. Therefore the average amplitude shown in Figure 2(b) first increases and then partly decreases in spite of the discrepancy present all this time during the temperature increase shown in Figure 2(a).

The time dependent average amplitude is also indispensable for the sensitivity analysis. From such a time dependant average amplitude it can be easily seen when the largest influence occurs on the output parameter due to the sensitive parameter variation.
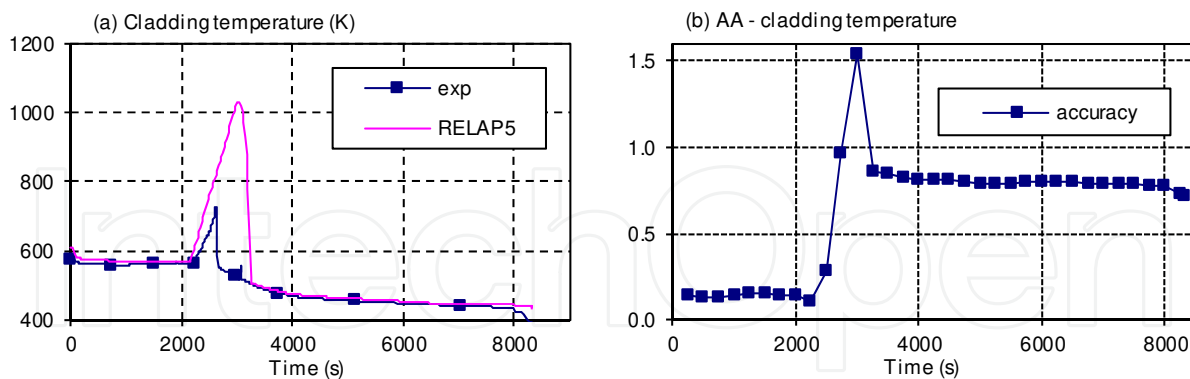

Figure 2. Time trend of (a) primary pressure and (b) the corresponding average amplitude

3. Methods used for sensitivity analysis

In BEMUSE, Phase II, single parameter sensitivity analyses have been proposed and performed by the participants to study the influence of different parameters (break area, gap conductivity,

core pressure drops, time of scram etc.) upon the predicted large-break loss-of-coolant accident (LBLOCA) evolution [13]. The performed sensitivity studies were intended to be used as guidance for deriving uncertainties of relevant input parameters like for phase III of the BEMUSE programme.

The sensitivity analysis is concerned, generally, with the influence of inputs on the output and output variability. Generally, sensitivity analyses are conducted by defining the model and its independent and dependent variables, assigning probability density functions to each input parameter, generating an input matrix through an appropriate random sampling method, performing calculations, and assessing the influences and relative importance of each input/output relationship.

In our demonstration case, the calculated data obtained in the Phase II of BEMUSE provided by the host organization have been used. The input matrix consists of the single parameter variation. The range of variations has been proposed by the host organization. For sensitivity calculations, only one value (minimal or maximal) of the input parameter was proposed when the range of the parameter variation was specified for selected sensitive parameter. For each single parameter variation the calculation was performed. The influence of the sensitive parameter variation has been estimated through the application of FFT based approaches.

We will call sensitivity (of the output parameter Y versus the i-th input parameter X_i) a measure having the dimension of $\dfrac{\partial Y}{\partial X_i}$ that is independent of the range of variation of the parameter X_i. Sensitivity is related to output variability.

We will call influence a measure of the effect of the variation of the parameter X_i on its full range ($\varDelta X_i$) having the dimension of $\dfrac{\partial Y}{\partial X_i} \varDelta X_i$ (same as Y) or more often dimensionless form

$$\dfrac{\partial Y}{\partial X_i} \dfrac{\varDelta Y}{\varDelta X_i}.$$

In our sensitivity analysis the influences were determined. Please note that classical measures of influences are: Pearson's or Spearman's Correlation Coefficients, Standardised Regression Coefficients, etc. [14]. In FFT based approaches the AA is a dimensionless number, showing influences in terms of the average amplitude obtained in the frequency domain which represents the physical influence (e.g. temperature or pressure change).

3.1. Test description

The LOFT L2-5 test was selected for this demonstration because a huge amount of data was available [15]. The reference and sensitivity calculations of the LOFT L2-5 test were performed in the phase II of the BEMUSE research program. The nuclear LOFT integral test facility is a scale model of a pressurized water reactor. The objective of the test was to simulate a loss of coolant accident (LOCA) caused by a double-ended, off-shear guillotine cold leg rupture coupled with a loss of off-site power. The experiment was initiated by opening the quick opening blowdown valves in the broken loop hot and cold legs. The reactor scrammed and emergency core cooling systems started their injection. After initial heatup the core was

quenched at 65 s, following the core reflood. The low pressure injection system (LPIS) was stopped at 107.1 s, after the experiment was considered complete. In total 14 calculations from 13 organizations were performed. For more detailed information on the calculations the reader is referred to [4, 15].

3.2. Sensitivity calculations description

The series of sensitivity calculations with assigned parameters was proposed to participants. For each parameter the host organization recommended the value to be used. The short description of the cases to be analysed is given in Table 1.

ID	Parameter	Recommended values (RV)	Description
S-1	Break Area	RC x 1.15	Tube diameter from reactor pressure vessel to break point shall be modified in respect to RC.
S-2	Gap Conductivity	RC x 0.2	Only in the hot rod in the hot channel (zone 4).
S-3	Gap Thickness	RC x 3	Only in the hot rod in the hot channel (zone 4).
S-4	Presence of Crud	0.15 mm	Consideration of 0.15 mm of crud in hot rod in hot channel with thermal conductivity that is characteristic of ceramic material, e.g. Al2O3.
S-5	Fuel Conductivity	RC x 0.4	Only in the hot rod in the hot channel (zone 4).
S-6	Core Pressure Drop	RC + D: $DP_{tot}=(DP_{tot})_{RC}$ x 1.3	The pressure drop across the core shall increase (decrease) of an amount D to obtain a total pressure drop that is 30% bigger than the total pressure drop of the reference case.
S-7	CCFL at Upper Tie Plate (UTP) and/or connection upper plenum (UP)	Range not assigned	Counter courant flow limitation (CCFL) is nodalization dependent. Each participant can propose a solution.
S-8	Decay Power	RC x 1.25	The decay power has to be 25% bigger than in the reference case.
S-9	Time of Scram	RC + 1 s	The power curve shall follow the imposed trend that implies full power till RC + 1 sec and after that shall followed the decay power.
S-10	Maximum Linear Power	RC x 1.5	Only in the hot rod in the hot channel (zone 4).
S-11	Accumulator Pressure	RC - 0.5 MPa	Set point of accumulator pressure 0.5 MPa lower than the set point in the base case (= 4.29 MPa).
S-12	Accumulator Liquid Mass	RC x 0.7	Accumulator liquid mass shall be 0.7 times the value in the reference case.
S-13	Pressurizer Level	RC - 0.5 m	Pressurizer level shall be 0.5 m lower than in the reference case.
S-14	HPIS	Failure	Failure of HPIS.
S-15	LPIS injection initiated	RC + 30 s	Delay in starting LPIS injection.

RC: value used in Reference Case

Table 1. List of sensitivity analyses and proposed parameter variations (adapted per Table 6 in ref. [15])

In Table 2 sensitivity calculations performed by 14 participants are shown. Each row presents sensitivity calculations S-1 to S-15. If the calculation is performed with recommended values, the sign √ is used. If another value has been used, the value of the sensitive parameter is indicated. If the sensitivity calculation has not been performed, the cell is shaded grey.

Name	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11	S-12	S-13	S-14	S-15
P-1	RC × 1.15 (area)	RC × 0.5	√	√	RC × 0.5	√	CCFL at UTP	√	√	√	√	√	√	√	√
P-2	√*	√*	√*		√*	√*		√*	√*	√*	√*	√	√*	√*	√*
P-3	RC × 0.7	RC × 2	√*	√*	√*	√*		√*	√*	√*	√*	√*	√*	√*	√*
P-4	√		√		√	√		√	√	√	√	√	√	√	√
P-5	Adjusted to pressure	√*					RC × 2 (resistance)	√	√	√					
P-6	RC × 1.15 (open size)	√	√	√*	√*	√*	√*	√*	√*	√*	√*	√*	√*	√*	√*
P-7	√*					√*		√*		√*	√*				
P-8	√	RC × 0.4	RC × 2.4	√	RC × 0.6	√	√	√	√	RC × 1.2	√	√	√	√	√
P-9	√*	√*	√*	√*	√*	√*	CCFL at UP	√*	√*	√*	√*	√*	√*	√*	√*
P-10	√	√	√		√	√		√	√	√	√	√	√	√	√
P-11	√*	√*	√*	√*	√*	√*		√*	√*	√*	√*	√*	√*	√*	√*
P-12	√	√	√	√	√	√	turned off	√	√	√	√	√	√	√	√
P-13	RC × 1.15 (area)	√	√	√	√	√	default	√	√	√	√	√	√	√	√
P-14	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

* no information given in Ref. [15], but high certainty that recommended values were used;

√ - recommended value used

Table 2. Sensitivity calculations performed by participants

3.3. Figures of merit for sensitivity analysis

Same figures of merit were proposed for the original FFTBM and improved FFTBM-SM. In the case of signal mirroring the additional index m is used to distinguish the FFTBM-SM from FFTBM. The first figure of merit is the average amplitude (AA or AA_m), which tells how the single input parameter variation (or combination of input parameter variations) influences the

output parameter. As there are several sensitive input parameters, several participants performing sensitivity runs and more selected output parameters, three additional figures of merit were proposed. The average amplitude of the participant sensitivity runs (AA_p or AA_{mp}) is used to judge which sensitivity runs set is more influential to the input parameter variations. AA_p or AA_{mp} is calculated as the average of AAs for the participant sensitivity runs (15 in our specific case). The average amplitude of the sensitivity runs for the same sensitive parameter (AA_s or AA_{ms}) is used to judge how influential (in average) the sensitive parameter is in calculations performed by different participants. AA_s or AA_{ms} is calculated as the average of AAs of participants for the same sensitivity run (14 in our specific case). Finally, the total average amplitude (AA_t or AA_{mt}) is the average AA or average AA_m obtained from all sensitivity runs performed by all participants.

4. Application of FFT based approches to sensitivity analysis

In this section the application of the original FFTBM and FFTBM-SM is demonstrated. As was explained in Subsection 2.1, two digital signals (reference and sensitive) of the same duration are used for calculating the average amplitude, which is a measure of the influence of the sensitive parameter variation. An example of calculating AA_m is given first. Then the single value influence (based on the average amplitude) for the whole time window is given. Finally, the time dependent influence is presented for the sensitivity runs. We conclude this section with the discussion.

4.1. AA_m figure of merit example

In the example the influence of sensitive parameters on three output parameters for the interval 0-119.5 s (whole transient time) is shown for calculations of participant P-14. The participant P-14 provided for each time trend 241 samples. This means that the sampling frequency was 2 Hz. The input values for FFTBM-SM were therefore the following: f_{fix}=2 Hz, f_{cut}=2 Hz, t_b=0 s and t_e=119.5 s (time 119.5 s was selected because some users provided the last data point at a value slightly smaller than 120 s). Per sampling theorem (Eq. (1)) at least 478 data points are needed. However, the minimum number of points per FFTBM-SM is 512. Table 3 shows that requesting more samples than required has a minor influence on the results.

To get some qualitative impression on the influence of input parameters on output parameters judged by FFTBM-SM, Figure 3 shows time trends of P-14 upper plenum pressure, primary side mass and cladding temperature. Visually it may be indicated that the upper plenum pressure is the most influenced by the break flow (S-1). On the other hand, hot rod parameters (S-2, S-3, S-4 and S-5) and accumulator initial mass (S-12) do not have significant influence on upper plenum pressure (see Figures 3(a2), 3(a3), 3(a4) and 3(a5)). The primary side mass inventory is the most influenced by the gap conductivity (S-2), fuel conductivity (S-5), and the accumulator liquid mass (S-12). When comparing S-2 and S-5 calculations, one may indicate that the S-5 calculation is a bit closer to the reference calculation. On the other hand, comparing the S-2 and S-12 calculation, in the case of S-2 the difference is absolutely larger than S-12,

	Upper plenum pressure				Primary side mass				Rod surface temperature			
	$N=2^9$	$N=2^{10}$	$N=2^{11}$	$N=2^{12}$	$N=2^9$	$N=2^{10}$	$N=2^{11}$	$N=2^{12}$	$N=2^9$	$N=2^{10}$	$N=2^{11}$	$N=2^{12}$
S-1	0.085	0.088	0.089	0.089	0.102	0.103	0.103	0.104	0.330	0.335	0.338	0.339
S-2	0.032	0.033	0.034	0.034	0.124	0.125	0.125	0.125	0.799	0.806	0.805	0.807
S-3	0.015	0.016	0.016	0.016	0.056	0.056	0.057	0.057	0.488	0.491	0.494	0.495
S-4	0.015	0.016	0.016	0.016	0.056	0.057	0.057	0.057	0.295	0.301	0.301	0.303
S-5	0.026	0.027	0.027	0.027	0.108	0.108	0.109	0.109	0.937	0.944	0.948	0.948
S-6	0.019	0.019	0.020	0.020	0.046	0.046	0.046	0.046	0.256	0.258	0.261	0.263
S-7	0.023	0.024	0.024	0.024	0.051	0.051	0.052	0.052	0.258	0.269	0.270	0.272
S-8	0.016	0.016	0.017	0.017	0.041	0.041	0.042	0.042	0.143	0.145	0.147	0.148
S-9	0.018	0.019	0.019	0.019	0.039	0.040	0.040	0.040	0.205	0.208	0.209	0.210
S-10	0.015	0.016	0.016	0.017	0.043	0.043	0.044	0.044	0.449	0.460	0.460	0.461
S-11	0.021	0.022	0.022	0.022	0.090	0.090	0.091	0.091	0.239	0.240	0.243	0.245
S-12	0.029	0.030	0.030	0.031	0.117	0.118	0.119	0.119	0.480	0.487	0.488	0.490
S-13	0.045	0.046	0.046	0.047	0.065	0.066	0.066	0.066	0.302	0.308	0.310	0.311
S-14	0.015	0.016	0.016	0.016	0.052	0.053	0.053	0.053	0.255	0.256	0.259	0.261
S-15	0.016	0.016	0.017	0.017	0.059	0.059	0.060	0.060	0.147	0.150	0.151	0.152

Table 3. Average amplitude AAm for the whole time interval for participant P-14 calculated by FFFTB-SM as a function of the number of samples N

however in case of S-12 the difference is present a longer time than in the case of S-2. The calculated AA_m is comparable.

To see this in more detail, Figure 4 shows part of the magnitude difference signal spectra $|\widetilde{\Delta}F(f_n)|$ for S-2 and S-12, which are used for the calculation of AA_{dif} per Eq. (5). Please note that f_{max} is 2.14 Hz. However, summing of amplitudes up to 0.2 Hz contributes more than 90% to total AA_{dif} (for S-2 the sum is 11.26 out of 12.43 and for S-12 the sum is 10.83 out of 11.79). Summing amplitudes up to 0.05 Hz (representing 13 samples out of 513) contributes more than 80% to total AA_{dif} (for S-2 the sum is 10.16 out of 12.43 and for S-12 the sum is 9.57 out of 11.79). One may see that the zero frequency component (mean value of difference signal in the time domain) is larger for S-2 than S-12 and that due to this contribution finally S-2 is judged as more influential than S-12.

4.2. Single value influence for whole time window

In the example presented in Section 4.1, AA_m for the P-14 calculation was determined. In this section all fourteen calculations are considered, and both FFTBM and FFTBM-SM are used for calculating all figures of merit presented in Section 3.3. The obtained results for the sensitive parameter influence on the three output parameters (upper plenum pressure, primary side mass inventory and rod surface temperature) are shown in Tables 4 through 6.

The qualitative comparison between FFTBM and FFTBM-SM results showed that the agreement is quite good. This is expected as at the end of the transient the influence of the sensitive parameter is generally insignificant, resulting that in the difference signal the edge is very
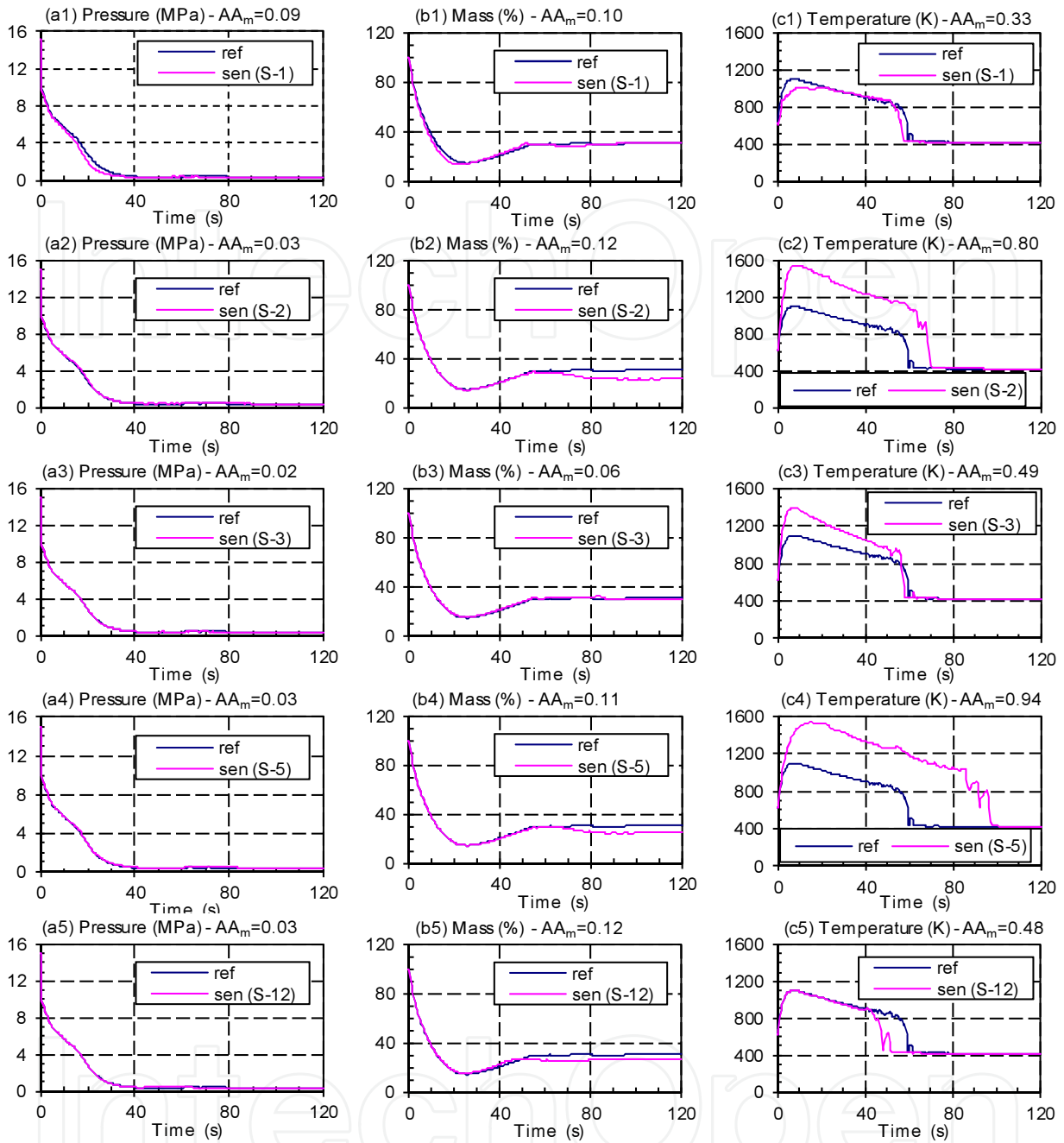
Figure 3. Participant P-14 time trends of (a) primary pressure, (b) primary side mass and (c) rod surface temperature for S-1, S-2, S-3, S-5 and S-12 with AA_m

small or not present at all. The edge is still present in the reference signal, but because it is used for the normalization it has no impact on the ranking of parameters and so the qualitative agreement between FFTBM and FFTBM-SM is good. This is not the case for the quantitative agreement as the normalization directly impacts the average amplitude. The average amplitudes obtained by both FFTBM and FFTBM-SM suggest that the most influential parameter for the upper plenum pressure is in all calculations the break flow area (S-1). To judge how influential the parameter is, the average amplitude of the sensitivity runs for the same sensitive

parameter (AA_s or AA_{ms}) obtained both by FFTBM and FFTBM-SM show that the variations of the break flow area (S-1), pressurizer level (S-13), core pressure drop (S-6) and presence of crud (S-4) the most influence the output parameter upper plenum pressure. The only difference between the FFTBM and FFTBM-SM results is that ranks for S-6 and S-4 are changed.

(a) Application of FFTBM - upper plenum pressure, time interval 0 - 119.5 s

ID-S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11	S-12	S-13	S-14	S-15	
ID-P	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA_p
P-1	0.051	0.011	0.010	0.006	0.009	0.011	0.008	0.013	0.011	0.019	0.019	0.015	0.036	0.011	0.010	● 0.016
P-2	0.042	0.008	0.010		0.008	0.005		0.002	0.006	0.001	0.006	0.015	0.013	0.001	0.002	● 0.009
P-3	0.070	0.025	0.035	0.030	0.034	0.108		0.045	0.028	0.042	0.043	0.045	0.060	0.010	0.008	◆ 0.042
P-4	0.110		0.014		0.013	0.015		0.011	0.011	0.012	0.030	0.033	0.027	0.010	0.012	▲ 0.025
P-5	0.089	0.014				0.016		0.023	0.018	0.019						◆ 0.030
P-6	0.029	0.010	0.012	0.013	0.010	0.077	0.015	0.009	0.011	0.010	0.011	0.010	0.017	0.008	0.005	● 0.016
P-7	0.030					0.008		0.006		0.005	0.009					● 0.012
P-8	0.068	0.013	0.009	0.010	0.012	0.015	0.018	0.014	0.015	0.015	0.016	0.018	0.027	0.012	0.010	▲ 0.018
P-9	0.074	0.007	0.009	0.008	0.008	0.010	0.009	0.009	0.010	0.008	0.011	0.015	0.027	0.000	0.008	● 0.014
P-10	0.059	0.030	0.018		0.008	0.023		0.006	0.007	0.029	0.039	0.009	0.052	0.004	0.012	▲ 0.023
P-11	0.035	0.008	0.012	0.117	0.016	0.012		0.012	0.013	0.011	0.019	0.019	0.028	0.012	0.016	▲ 0.024
P-12	0.035	0.006	0.009	0.007	0.008	0.009	0.010	0.004	0.009	0.008	0.009	0.006	0.021	0.004	0.009	● 0.010
P-13	0.041	0.014	0.015	0.011	0.016	0.132	0.016	0.014	0.013	0.015	0.017	0.011	0.024	0.006	0.005	▲ 0.023
P-14	0.044	0.017	0.008	0.008	0.013	0.010	0.012	0.008	0.009	0.008	0.011	0.015	0.023	0.008	0.008	● 0.013
AA_s	0.055	0.014	0.013	0.023	0.013	0.032	0.013	0.012	0.012	0.014	0.018	0.017	0.030	0.007	0.009	AA_t=0.019

Legend: [ ] 0.02-0.04   [ ] >0.04

(b) Application of FFTBM-SM - upper plenum pressure, time interval 0 - 119.5 s

ID-S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11	S-12	S-13	S-14	S-15	
ID-P	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_{mp}
P-1	0.098	0.021	0.020	0.012	0.018	0.021	0.016	0.026	0.022	0.038	0.038	0.030	0.070	0.021	0.019	● 0.031
P-2	0.080	0.016	0.018		0.014	0.009		0.003	0.011	0.002	0.012	0.029	0.026	0.002	0.004	● 0.017
P-3	0.145	0.048	0.071	0.055	0.065	0.127		0.090	0.054	0.084	0.086	0.090	0.120	0.019	0.015	◆ 0.076
P-4	0.212		0.026		0.025	0.028		0.021	0.023	0.023	0.059	0.064	0.051	0.019	0.023	▲ 0.048
P-5	0.160	0.024				0.028		0.039	0.032	0.034						◆ 0.053
P-6	0.054	0.019	0.021	0.023	0.018	0.130	0.028	0.016	0.020	0.018	0.021	0.018	0.032	0.014	0.010	● 0.029
P-7	0.056					0.016		0.010		0.008	0.016					● 0.021
P-8	0.127	0.024	0.017	0.018	0.024	0.029	0.034	0.026	0.029	0.028	0.031	0.036	0.050	0.024	0.018	▲ 0.034
P-9	0.139	0.013	0.017	0.015	0.015	0.018	0.017	0.017	0.019	0.015	0.022	0.028	0.053	0.000	0.014	● 0.027
P-10	0.118	0.060	0.035		0.015	0.043		0.012	0.013	0.056	0.073	0.017	0.098	0.009	0.024	▲ 0.044
P-11	0.068	0.017	0.024	0.234	0.030	0.023		0.024	0.025	0.023	0.036	0.036	0.052	0.024	0.033	▲ 0.046
P-12	0.065	0.011	0.017	0.014	0.016	0.017	0.019	0.008	0.016	0.014	0.018	0.010	0.039	0.007	0.017	● 0.019
P-13	0.072	0.024	0.026	0.019	0.027	0.225	0.029	0.024	0.023	0.026	0.030	0.018	0.042	0.010	0.007	▲ 0.040
P-14	0.085	0.032	0.015	0.015	0.026	0.019	0.023	0.016	0.018	0.015	0.021	0.029	0.045	0.015	0.016	● 0.026
AA_{ms}	0.106	0.026	0.026	0.045	0.024	0.052	0.024	0.024	0.023	0.027	0.036	0.034	0.056	0.014	0.017	AA_{mt}=0.036

Legend: [ ] 0.04-0.08   [ ] >0.08

Table 4. Influence of sensitive parameters on upper plenum pressure in time interval 0-119.5 s as judged by (a) original FFTBM and (b) improved FFTBM by signal mirroring

When looking calculations, the most influenced upper plenum calculation is P-3. To judge how the calculation is influenced by the sensitive parameters variation, the average amplitude of the participant sensitivity runs (AA_p or AA_{mp}) is used. Besides P-3 the P-5 calculation was also judged as the much influenced by the sensitive parameter variations. Both methods qualitatively give the same results for the average amplitude of the participant sensitivity runs. The total average amplitude (AA_t or AA_{mt}) show the overall influence of the sensitive parameters variations of all calculations on the output upper plenum pressure. The higher the value is the higher is the influence. The ratio between AA_{mt} and AA_t to be 1.88 also tells what the contribution of the edge effect is in average.
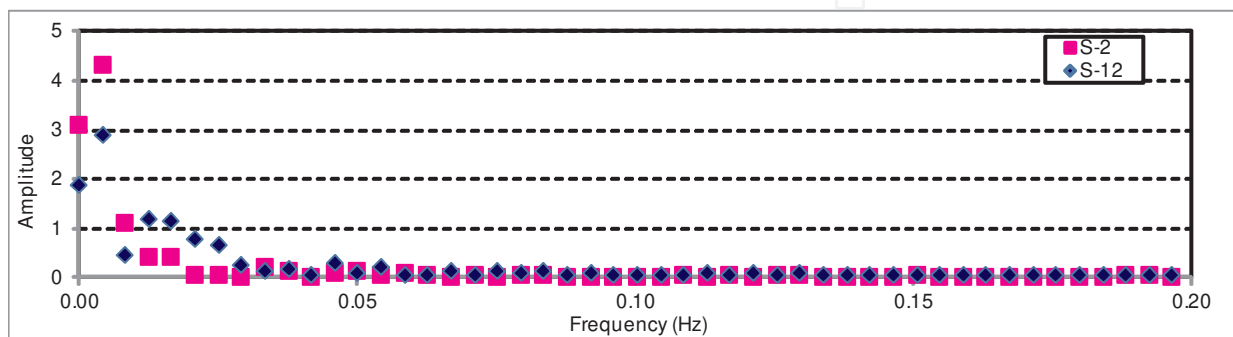

Figure 4. Magnitude difference signal spectra for S-2 and S-12 runs of P-14 participants

The average amplitudes shown in Table 5, obtained by both FFTBM and FFTBM-SM suggest that the most influential parameter for the primary side mass inventory when considering all calculations is the accumulator liquid mass (S-12). Both the AA_s and AA_{ms} indicate the accumulator liquid mass (S-12), break flow area (S-1), accumulator pressure (S-11) and pressurizer level (S-13) the most influential sensitive parameters on the primary side mass inventory.

When looking the calculations, the most influenced primary side mass inventory calculation is P-3. The AA_p and AA_{mp} indicate as the second most influenced the P-13 calculation. Again both methods qualitatively give very similar results for the average amplitude of the participant sensitivity runs. The AA_t and AA_{mt} show that the overall influence of the sensitive parameters variations of all calculations on the output primary side mass inventory is higher than on the upper plenum pressure. The ratio between AA_{mt} and AA_t is 1.55, indicating that the primary side mass inventory is less influenced by the edge effect (see also Figure3).

The average amplitudes shown in Table 6, obtained by both FFTBM and FFTBM-SM suggest that the most influential parameter for the rod surface temperature when considering all calculations is the fuel conductivity (S-5). Both AA_s and AA_{ms} indicate the fuel conductivity (S-5) and gap conductivity (S-2) as the most influential. Significant influences have also the gap thickness (S-3), maximum linear power (S-10), break flow area (S-1) and the accumulator liquid mass (S-12) and a few others. The only difference between the FFTBM and FFTBM-SM results is that the ranks for S-1 and S-12 are changed.

(a) Application of FFTBM - primary side mass inventory, time interval 0 - 119.5 s															
ID-S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11	S-12	S-13	S-14	S-15	
ID-P	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA_p
P-1	0.052	0.026	0.019	0.018	0.015	0.026	0.018	0.017	0.015	0.017	0.048	0.052	0.032	0.014	0.02	● 0.026
P-2	0.046	0.014	0.014		0.013	0.015		0.014	0.015	0.016	0.017	0.084	0.027	0.012	0.02	● 0.023
P-3	0.102	0.083	0.071	0.060	0.092	0.180		0.118	0.064	0.086	0.124	0.137	0.124	0.013	0.01	◆ 0.091
P-4	0.125		0.022		0.021	0.042		0.018	0.037	0.023	0.054	0.091	0.036	0.025	0.03	▲ 0.043
P-5	0.071	0.027				0.022		0.051	0.038	0.033						▲ 0.040
P-6	0.066	0.036	0.056	0.020	0.052	0.045	0.061	0.035	0.035	0.036	0.034	0.158	0.044	0.040	0.07	▲ 0.052
P-7	0.035					0.035		0.017		0.007	0.026					● 0.024
P-8	0.083	0.053	0.016	0.046	0.025	0.051	0.042	0.022	0.038	0.033	0.042	0.073	0.069	0.031	0.03	▲ 0.043
P-9	0.073	0.014	0.013	0.015	0.011	0.012	0.012	0.022	0.020	0.008	0.038	0.093	0.033	0.000	0.06	● 0.028
P-10	0.089	0.010	0.003		0.059	0.019		0.039	0.034	0.019	0.153	0.095	0.041	0.038	0.04	▲ 0.049
P-11	0.041	0.019	0.024	0.145	0.041	0.020		0.048	0.020	0.033	0.040	0.053	0.032	0.039	0.07	▲ 0.045
P-12	0.039	0.010	0.033	0.018	0.022	0.022	0.024	0.007	0.037	0.025	0.024	0.203	0.039	0.029	0.09	▲ 0.042
P-13	0.072	0.047	0.035	0.045	0.066	0.077	0.000	0.050	0.077	0.044	0.092	0.179	0.075	0.049	0.07	◆ 0.065
P-14	0.049	0.136	0.035	0.034	0.104	0.032	0.026	0.022	0.023	0.023	0.049	0.090	0.060	0.041	0.05	▲ 0.052
AA_s	0.067	0.040	0.028	0.044	0.043	0.043	0.026	0.034	0.035	0.029	0.057	0.109	0.051	0.028	0.05	AA_t=0.046

Legend: 0.05-0.1 >0.1

(b) Application of FFTBM-SM - primary side mass inventory, time interval 0 - 119.5 s															
ID-S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11	S-12	S-13	S-14	S-15	
ID-P	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_{mp}
P-1	0.096	0.043	0.034	0.030	0.028	0.034	0.030	0.033	0.028	0.032	0.071	0.084	0.062	0.025	0.037	● 0.045
P-2	0.096	0.028	0.028		0.025	0.028		0.025	0.028	0.029	0.035	0.139	0.047	0.024	0.033	● 0.044
P-3	0.171	0.146	0.142	0.107	0.158	0.215		0.212	0.124	0.168	0.202	0.226	0.229	0.027	0.027	◆ 0.154
P-4	0.227		0.041		0.040	0.076		0.036	0.063	0.045	0.107	0.160	0.071	0.043	0.048	▲ 0.080
P-5	0.141	0.049				0.041		0.086	0.072	0.063						▲ 0.076
P-6	0.096	0.058	0.076	0.035	0.070	0.074	0.081	0.050	0.055	0.055	0.060	0.159	0.085	0.055	0.069	▲ 0.072
P-7	0.060					0.051		0.020		0.014	0.048					● 0.039
P-8	0.152	0.073	0.032	0.091	0.047	0.094	0.081	0.042	0.074	0.067	0.077	0.133	0.084	0.062	0.053	▲ 0.078
P-9	0.136	0.024	0.019	0.022	0.020	0.022	0.021	0.026	0.029	0.014	0.052	0.134	0.057	0.000	0.062	● 0.043
P-10	0.111	0.017	0.006		0.073	0.025		0.048	0.043	0.034	0.161	0.101	0.065	0.062	0.062	▲ 0.062
P-11	0.070	0.038	0.044	0.290	0.064	0.038		0.077	0.035	0.058	0.068	0.091	0.060	0.057	0.094	▲ 0.077
P-12	0.057	0.011	0.035	0.019	0.025	0.025	0.030	0.007	0.037	0.027	0.035	0.135	0.050	0.019	0.066	● 0.039
P-13	0.133	0.085	0.063	0.085	0.102	0.129	0.000	0.092	0.128	0.076	0.144	0.235	0.092	0.091	0.124	◆ 0.105
P-14	0.102	0.124	0.056	0.056	0.108	0.046	0.051	0.041	0.039	0.043	0.090	0.117	0.065	0.052	0.059	▲ 0.070
AA_{ms}	0.118	0.058	0.048	0.082	0.063	0.064	0.042	0.057	0.058	0.052	0.088	0.143	0.081	0.043	0.061	AA_{mt}=0.071

Legend: 0.07-0.14 >0.14

Table 5. Influence of sensitive parameters on primary side mass inventory in time interval 0-119.5 s as judged by (a) original FFTBM and (b) improved FFTBM by signal mirroring

When looking the calculations, the most influenced rod surface rod temperature calculation is P-10. The AA_p and AA_{mp} indicate that the next two most influenced are the P-2 and P-3 calculation. Again both methods qualitatively give pretty similar results for the average amplitude of the participant sensitivity runs. The AA_t and AA_{mt} show that the overall influence of the sensitive parameters variations of all calculations on the output rod surface temperature

(a) Application of FFTBM - rod surface temperature, time interval 0 - 119.5 s															
ID-S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11	S-12	S-13	S-14	S-15	
ID-P	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA	AA_p
P-1	0.323	0.498	0.424	0.085	0.318	0.189	0.122	0.194	0.253	0.413	0.283	0.342	0.204	0.104	0.129	● 0.259
P-2	0.323	0.403	0.608		0.493	0.312		0.301	0.282	0.495	0.220	1.679	0.097	0.099	0.174	◆ 0.422
P-3	0.432	0.451	0.476	0.406	0.703	0.496		0.415	0.386	0.266	0.426	0.198	0.212	0.247	0.184	◆ 0.378
P-4	0.368		0.252		0.445	0.185		0.210	0.289	0.527	0.195	0.388	0.207	0.328	0.107	▲ 0.292
P-5	0.184	0.284				0.174		0.330	0.308	0.328						● 0.268
P-6	0.622	0.379	0.402	0.167	0.543	0.415	0.444	0.349	0.157	0.483	0.392	0.210	0.360	0.172	0.166	▲ 0.351
P-7	0.340					0.207		0.239		0.358	0.261					▲ 0.281
P-8	0.446	0.396	0.305	0.150	0.351	0.227	0.283	0.329	0.337	0.360	0.117	0.434	0.149	0.171	0.133	▲ 0.279
P-9	0.229	0.407	0.343	0.135	0.386	0.208	0.089	0.152	0.255	0.207	0.208	0.212	0.160	0.000	0.072	● 0.204
P-10	0.345	0.848	0.555		0.762	0.264		0.349	0.339	0.683	0.311	0.238	0.401	0.080	0.307	◆ 0.422
P-11	0.207	0.507	0.639	0.335	0.243	0.358		0.681	0.204	0.437	0.224	0.239	0.279	0.104	0.142	▲ 0.328
P-12	0.344	0.598	0.419	0.216	0.543	0.192	0.110	0.252	0.339	0.825	0.232	0.187	0.245	0.145	0.211	▲ 0.324
P-13	0.253	0.535	0.339	0.124	0.464	0.343	0.238	0.197	0.349	0.221	0.290	0.256	0.111	0.090	0.160	● 0.265
P-14	0.257	0.668	0.420	0.267	0.738	0.206	0.205	0.113	0.173	0.386	0.197	0.380	0.236	0.204	0.114	▲ 0.304
AA_s	0.334	0.498	0.432	0.209	0.499	0.270	0.213	0.294	0.282	0.428	0.258	0.397	0.222	0.145	0.158	AA_t=0.314

0.25-0.45   >0.45

(b) Application of FFTBM-SM - rod surface temperature, time interval 0 - 119.5 s															
ID-S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11	S-12	S-13	S-14	S-15	
ID-P	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_m	AA_{mp}
P-1	0.396	0.605	0.499	0.103	0.383	0.229	0.149	0.231	0.302	0.490	0.338	0.415	0.247	0.127	0.155	● 0.311
P-2	0.389	0.475	0.717		0.605	0.368		0.358	0.348	0.542	0.265	0.913	0.121	0.120	0.214	◆ 0.418
P-3	0.508	0.515	0.516	0.481	0.861	0.580		0.486	0.461	0.305	0.489	0.234	0.250	0.285	0.217	◆ 0.442
P-4	0.438		0.285		0.504	0.226		0.255	0.350	0.557	0.230	0.459	0.249	0.402	0.131	▲ 0.340
P-5	0.227	0.360				0.213		0.402	0.385	0.400						▲ 0.331
P-6	0.736	0.467	0.444	0.200	0.656	0.481	0.526	0.412	0.183	0.525	0.452	0.247	0.423	0.201	0.196	▲ 0.410
P-7	0.411					0.251		0.288		0.416	0.316					▲ 0.336
P-8	0.528	0.456	0.372	0.188	0.432	0.274	0.342	0.404	0.414	0.449	0.149	0.483	0.189	0.209	0.165	▲ 0.337
P-9	0.293	0.492	0.418	0.172	0.506	0.264	0.114	0.194	0.311	0.238	0.264	0.270	0.202	0.000	0.091	● 0.255
P-10	0.410	0.982	0.625		0.930	0.307		0.402	0.391	0.768	0.367	0.284	0.472	0.094	0.353	◆ 0.491
P-11	0.244	0.579	0.727	0.412	0.270	0.442		0.829	0.243	0.523	0.278	0.286	0.342	0.126	0.170	▲ 0.391
P-12	0.414	0.680	0.487	0.260	0.667	0.231	0.128	0.309	0.383	0.637	0.282	0.227	0.293	0.170	0.251	▲ 0.361
P-13	0.302	0.604	0.401	0.148	0.563	0.411	0.291	0.238	0.412	0.266	0.351	0.292	0.134	0.109	0.177	● 0.313
P-14	0.330	0.799	0.488	0.295	0.937	0.256	0.258	0.143	0.205	0.449	0.239	0.480	0.302	0.255	0.147	▲ 0.372
AA_{ms}	0.402	0.585	0.498	0.251	0.609	0.324	0.258	0.353	0.338	0.469	0.309	0.382	0.269	0.175	0.189	AA_{mt}=0.366

Legend: 0.3-0.5   >0.5

Table 6. Influence of sensitive parameters on rod surface temperature in time interval 0-119.5 s as judged by (a) original FFTBM and (b) improved FFTBM by signal mirroring

is higher than on the primary side mass inventory. The ratio between AA_{mt} and AA_t is 1.17, indicating that the rod surface temperature is the least influenced by the edge effect (see also Figure 3).

4.3. Time dependent influence

The results presented in Section 4.2 give information on the accumulated influence of sensitive parameters for the whole transient duration (single value figures of merit). Additional insight into the results is obtained from the time dependent average amplitudes for each single variation of parameters. They provide information how the influence changes during the transient progression.

Figure 5 shows the comparison between the FFTBM and FFTBM-SM results for the P-14 sensitive calculations shown in Figure 3. It is shown how the sensitive parameter influence changes during the transient progression. The judged quantitative influence in Figure 5 reflects well what is seen during the visual observation of Figure 3, in which 5 out of 15 sensitive parameter variations for the three output parameters for the P-14 calculation are shown. Please note that the FFT based approaches are especially to be used when there are several calculations (fourteen in our case) with several sensitive parameter variations (fifteen in our case) to judge the influence of the sensitive parameters in an uniform way.

When looking the output parameter upper plenum pressure, both FFTBM (excluding period when edge effect significantly contributes to average amplitude) and FFTBM-SM clearly show when during the transient the parameter was influential. For all parameters shown in Figure 5 the major influence was during the first 30 seconds when the pressure was dropping. In the case of the S-1 parameter the influence was the largest (see Figure 5(a1)), but still not extremely significant. For parameters S-2, S-3, S-5 and S-12 the total influence is small. The values of average amplitudes up to 0.03 are small. This is confirmed by Figures 3(a2), 3(a3), 3 (a4) and 3 (a5) which show that the reference and sensitive signals for the upper plenum pressure practically match each other.

When looking the output parameter primary side mass inventory, the influence of the sensitive parameters is also quite small. Parameter S-1 is the most influential in the beginning of the transient (see also Figure 3(b1)), while all other shown sensitive parameters (S-2, S-3, S-5 and S-12) become more influential later into the transient. This is in agreement with the Figures 3(b2), 3(b3), 3(b4) and 3(b5), in which the differences in the first 20 seconds are practically not visible.

Finally, when looking the output parameter rod surface temperature, the influence of the sensitive parameters is the largest among the selected output parameters as shown by the plots and the average amplitude trends. The variation of the break flow area (S-1) having the largest influence on the upper plenum pressure has a lower influence on the rod surface temperature than the sensitive parameters S-2, S-3, S-5 and S-12. This is logically as the break area size directly impacts the upper plenum pressure.

From Figure 5(c1) it can be seen that the influence of S-1 on the rod surface temperature is judged to be in the beginning of the transient and at around 60 s. When comparing the sensitive signal to the reference signal in Figure 3(c1), in the beginning for the sensitive signal a slower temperature increase with under predicted peak and earlier temperature decrease (rod quench) at around 60 s can be seen. At other times the trends are similar. In the case of S-2 the temperature is over predicted and the quench is delayed. Therefore besides the initial jump
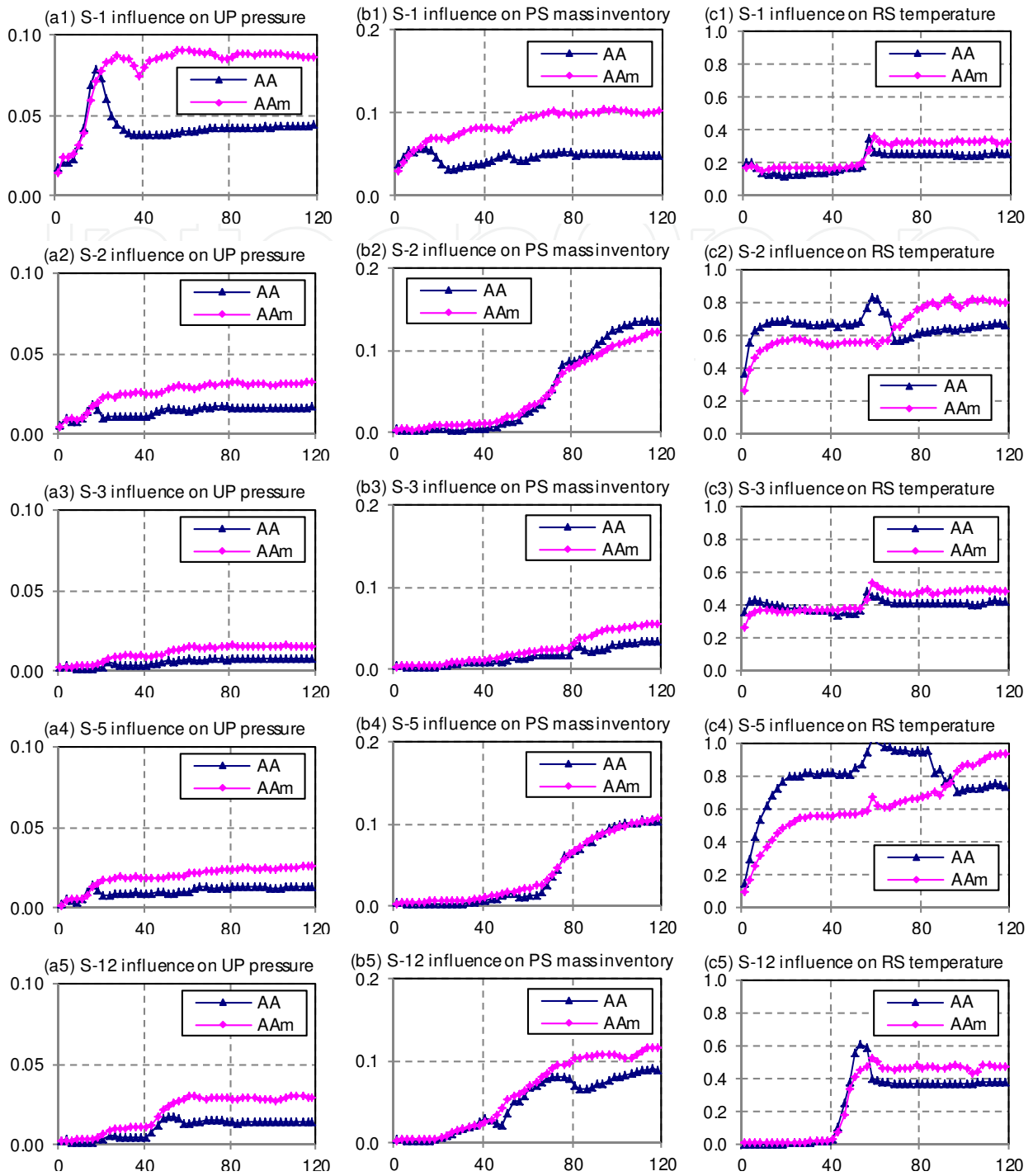
Figure 5. Participant P-14 influence of S-1, S-2, S-3, S-5, S-11 and S-12 sensitive parameter variations on (a) primary pressure and (b) primary side mass and (c) cladding temperature

the AA_m is still increasing till 20 s and for the time duration of the rod quench delay. For parameter S-3 it can be seen that its influence is between the S-1 and the S-2 influence, what can be confirmed from Figures 3(c1), 3(c2) and 3(c3). The influence of S-5 on the rod surface

temperature is the largest what can be easily confirmed when comparing Figure 3(c4) and Figure 5(c4). The influence of S-11 is smaller than S-1 because S-11 influences only the time of rod quenching. Finally, S-12 shows that the larger discrepancy in the times when the rod quench starts causes also larger values of average amplitudes. Also when comparing the average amplitudes obtained by FFTBM and FFTBM-SM it can be seen that they agree pretty well for the output parameters primary side mass inventory and the rod surface temperature, because the edge present in the periodic signal is relatively smaller from the edge present in the upper plenum pressure periodic signal.

Figure 6(a) shows the comparison of the rod surface temperature reference calculations to experimental data. One may see that the calculations differ and that they do not exactly match the experimental data. Therefore the reader should always keep in mind that the direct comparison of sensitive signals (see Figure 6(b)) obtained by different participants could not answer in which calculation the sensitive parameter is the most influential.
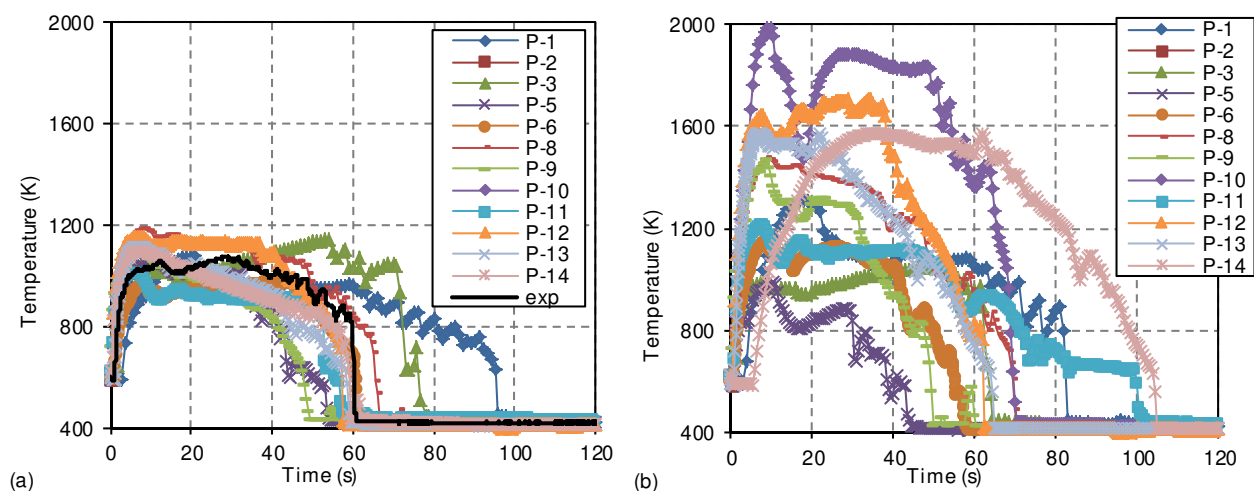

Figure 6. Comparison of (a) reference calculations with experimental value and (b) sensitive runs for S-2 sensitive parameter variation

However, by calculating the average amplitude the sensitive runs by different participants could be compared as it is shown in the Figures 7, 8 and 9 for the output parameters upper plenum pressure, primary side mass inventory and rod surface temperature, respectively. For each of the output parameters the most influential parameters are shown as identified from Tables 4, 5 and 6. Figure 7 for the upper plenum pressure shows that for the majority of calculations the influences of the same sensitive parameter variation are similar. There are only a few calculations significantly deviating, for example P-4 for S-1 variation, P-3 and P-10 for S-13 variation, P-13 for S-6 variation and P-11 for S-4 variation. Figure 8 for the primary side mass inventory shows that the P3 calculation was more sensitive to variations than other

calculations. In the case of the S-4 sensitive parameter variation the P-11 calculation significantly deviates from other calculations and the reason may be that the code model is used outside its validation range [13].

Figure 9 shows that some parameters are more influential in the beginning of the transient (e.g. S-5), some in the middle of the transient (e.g. S-12) and that in the last part normally there is no significant influence (exception is the calculation P-2 for the S-12 sensitive parameter in which the rod surface temperature did not quench due to no accumulator injection when it was supposed to inject).
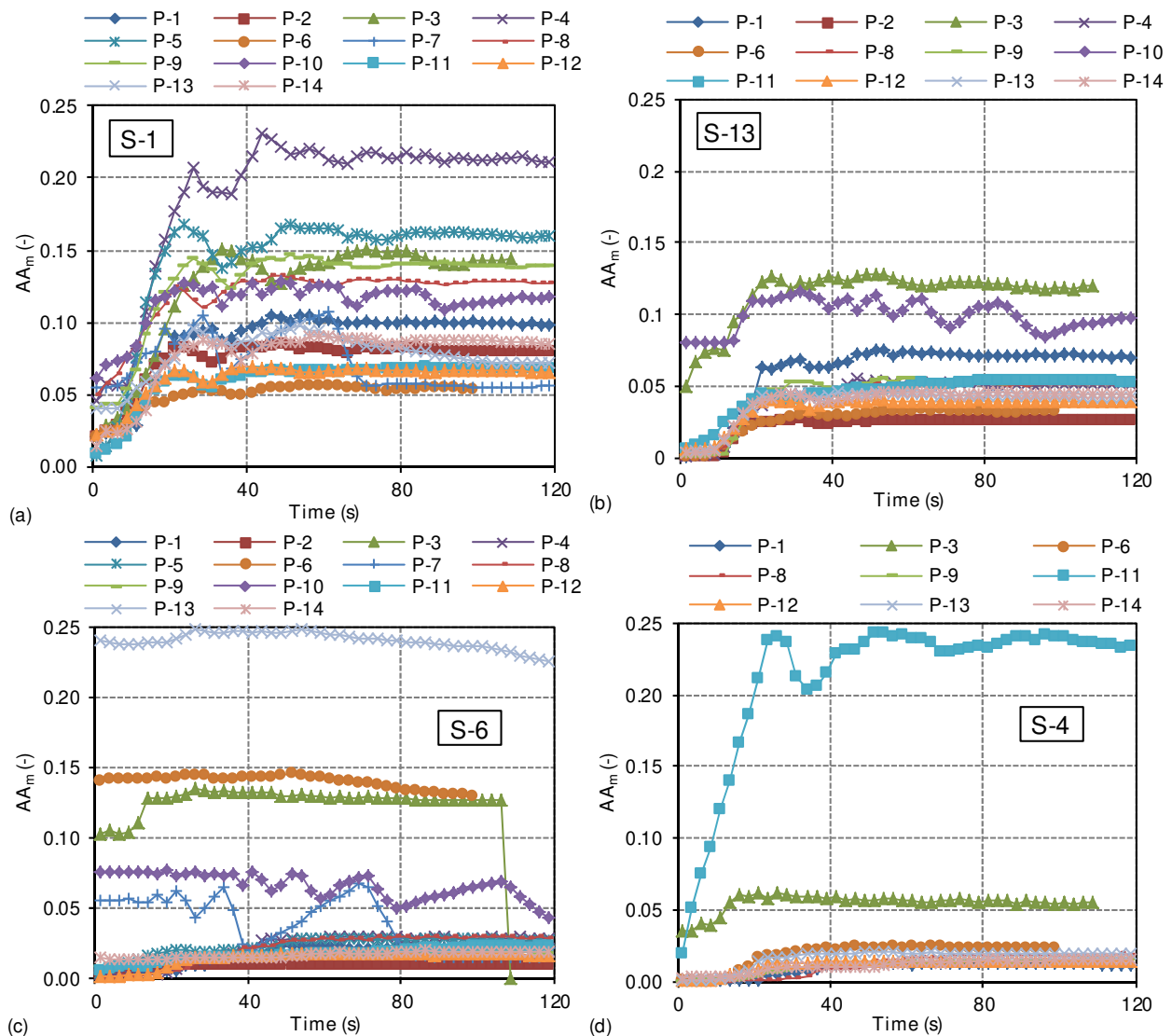
Figure 7. Upper plenum pressure time dependent AA_m of participants for (a) S-1, (b) S-13, (c) S-6 and (d) S-4 sensitive parameter variations
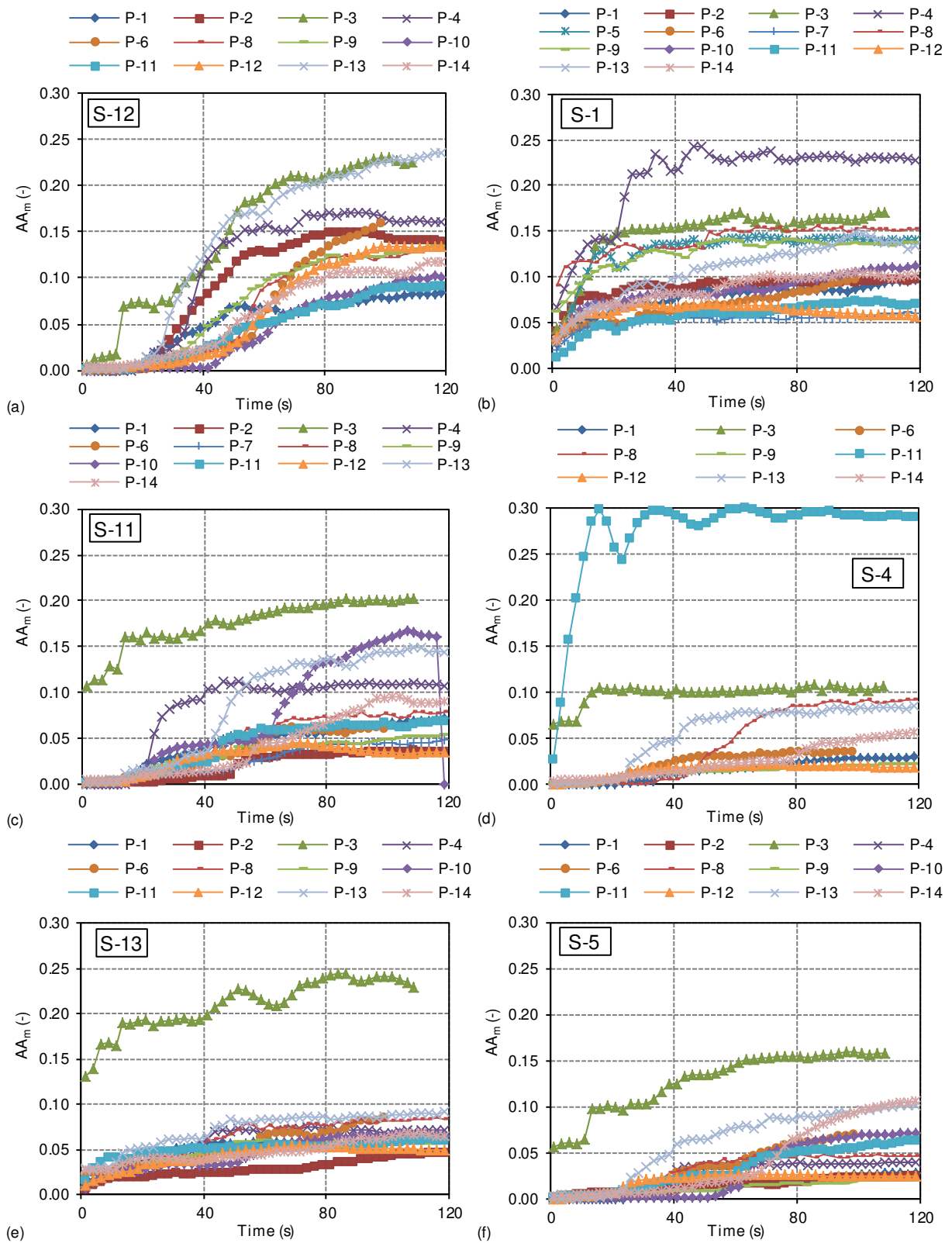
Figure 8. Primary side mass inventory time dependent AA_m of participants for (a) S-12, (b) S-1, (c) S-11, (d) S-4, (e) S-13 and (f) S-5 sensitive parameter variations
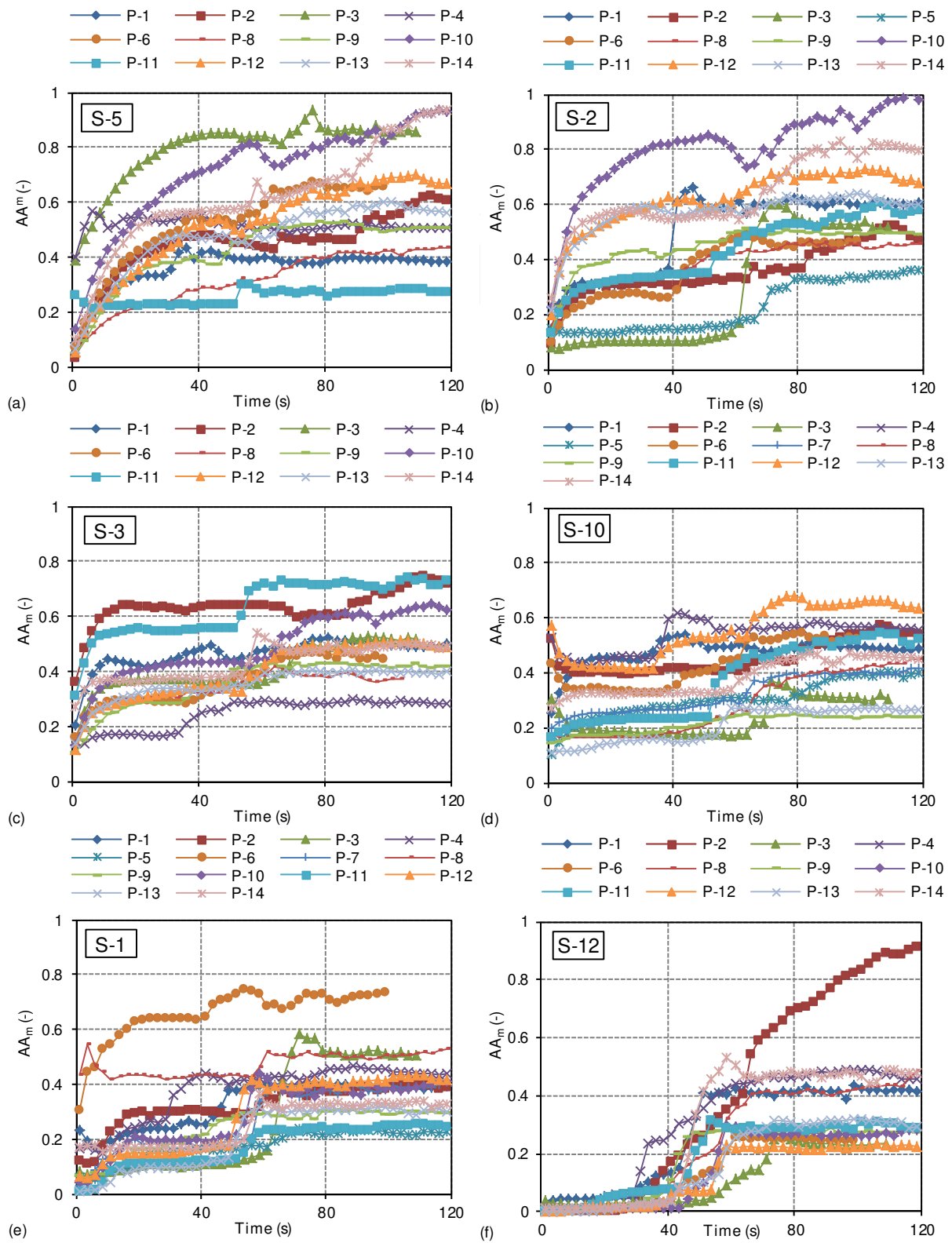
Figure 9. Rod surface temperature time dependent AA_m of participants for (a) S-5, (b) S-2, (c) S-3, (d) S-10, (e) S-1 and (f) S-12 sensitive parameter variations

4.4. Discussion

In the application for each participant each sensitivity run in the set was compared to his reference calculation and the average amplitude, which is the figure of merit for FFT based approaches, was used to judge how each output parameter is sensitive to the selected input parameter. For brevity reasons, only some examples were given in Figures 6 through 9. Nevertheless, these examples are sufficient to demonstrate that the results for the whole transient interval, shown in Tables 4, 5 and 6 may not be always sufficient for judging the sensitivity. For example, for the output parameter rod surface temperature there is higher interest in the value of the maximum rod surface temperature rather the average influence on the rod surface temperature, therefore it is important to know how the influence changes with time. Also, some parameters are more influential at the beginning and the others later in the calculation.

The results suggest that the FFT based approaches are especially appropriate for a quick sensitivity analysis in which several calculations need to be compared. It is very appropriate also due to the inherent feature, which integrates the contribution of the parameter variation with progressing transient time.

In addition, the average amplitude of participant sensitivity runs for the participants (AA_p or AA_{mp}) and the average amplitude for the same sensitive parameter (AA_s or AA_{ms}) were calculated. These measures could be used for ranking purposes. In this way information on the most influential input parameter and which participant calculation is the most sensitive to variations is obtained. Finally, different output parameters could be compared between each other regarding the influence of input parameters of all participants (AA_t or AA_{mt}). Quantitatively it is judged that the most influenced output parameter is the rod surface temperature and the least influenced the upper plenum pressure.

5. Conclusions

The study using FFTBM-SM and FFTBM was performed to show that the FFT based approaches could be used for sensitivity analyses. The LOFT L2-5 test, which simulates the large break loss of coolant accident, was used in the frame of the BEMUSE programme for sensitivity runs. In total 15 sensitivity runs were performed by 14 participants.

It can be concluded that with FFTBM-SM the analyst can get a good picture of the influence of the single parameter variation to the results throughout the transient. Some sensitive parameters are more influential at the beginning and the others later in the calculation. Due to the edge effect FFTBM-SM is advantageous for time dependent sensitivity calculations with respect to FFTBM, while for the whole transient duration (average sensitivity during whole transient) in general also FFTBM gives consistent results. FFT based approaches could be also used to quantify the influence of several parameter variations on the results. However, the influential parameters could not be identified nor the direction of the influence.

The results suggest that the FFT based approaches are especially appropriate for a quick assessment of a sensitivity analysis in which several calculations need to be compared or the influence of single sensitive parameters needs to be ranked. Such a sensitivity analysis could provide information which are the most influential parameters and how influential the input parameters are on the selected output parameters and when they influence during a transient.

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