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# Polarization Effects in Optical Fiber Links 

Krzysztof Perlicki

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## 1. Introduction

Polarization effects were already observed in the first optical fiber transmission experiments. Initially, polarization effects in an optical fiber were a pure laboratory curiosity. During telecom expansion in 1990s these effects became the focus of many research groups. Optical fiber polarization effects and interaction between them become particular important as bit rate of a single optical channel increases. These effects must be overcome to implement more than $10 \mathrm{~Gb} / \mathrm{s}$ transmission in a single wavelength over fiber plants in long haul optical systems. It can be a seriously limiting factor in systems in which the fiber plants were installed by 1998. Because, these old fibers are characterised by high internal birefringence such as core asymmetry and built in stress. The current fiber plants are characterised by low internal birefringence. However, external birefringence such as twists and external stress applied to optical fiber, significantly contributes to polarization effects.

Polarization effects are now a fundamental requirement to understand the signal propagation in modern long haul lighwave communication networks. The present chapter is designed to cover: description of polarized light, polarization phenomena in optical fiber links, modeling of polarization phenomena and polarizing component.

## 2. Description of polarized light

### 2.1. Polarization ellipse equation

All the important features of light wave follow from a detailed examination of Maxwell equations. Electromagnetic waves have two polarization along the $x$ axis and along the $y$ axis. The general form of polarized light wave propagating in $z$ direction can be derived from two linear polarized components in the x and y directions [1]:

$$
\begin{align*}
& \mathrm{E}_{\mathrm{x}}(\mathrm{z}, \mathrm{t})=\mathrm{E}_{0 \mathrm{x}} \cos \left(\tau_{\omega}+\phi_{\mathrm{x}}\right),  \tag{1}\\
& \mathrm{E}_{\mathrm{y}}(\mathrm{z}, \mathrm{t})=\mathrm{E}_{0 \mathrm{y}} \cos \left(\tau_{\omega}+\phi_{\mathrm{y}}\right), \tag{2}
\end{align*}
$$

where: x and y refers to the components in the x and y directions, $\mathrm{E}_{0 \mathrm{x}}$ and $\mathrm{E}_{0 \mathrm{y}}$ are the real maximum amplitudes of electric field, $\phi_{x}$ and $\phi_{y}$ are the phases and $\tau_{\omega}$ is so called propagator, which describes the propagation of the signal component in the $z$-direction.

Next,equations (1) and (2) can be written as: [1]:

$$
\begin{align*}
& \frac{E_{x}(z, t)}{E_{0 x}}=\cos \left(\tau_{\omega}\right) \cos \left(\phi_{x}\right)-\sin \left(\tau_{\omega}\right) \sin \left(\phi_{x}\right),  \tag{3}\\
& \frac{E_{y}(z, t)}{E_{0 y}}=\cos \left(\tau_{\omega}\right) \cos \left(\phi_{y}\right)-\sin \left(\tau_{\omega}\right) \sin \left(\phi_{y}\right) . \tag{4}
\end{align*}
$$

Squaring and adding (3) and (4) then yields:

$$
\begin{equation*}
\frac{E_{x}^{2}(z, t)}{E_{0 x}^{2}}+\frac{E_{y}^{2}(z, t)}{E_{0 y}^{2}}-2 \frac{E_{x}(z, t)}{E_{0 x}} \frac{E_{y}(z, t)}{E_{0 y}} \cos (\phi)=\sin ^{2}(\phi), \tag{5}
\end{equation*}
$$

where: $\phi=\phi_{y}-\phi_{x}$.
Equation (5) is an ellipse equation. This equation is called the polarization ellipse.
Figure 1 shows the polarization ellipse for optical field.
The polarization ellipse presents some important parameters enabling the characterization of the state of light polarization (SOP) [2]:

1. Axis $x$ and $y$ are the initial, unrotated axes, $\xi$ and $\eta$ are a new set of axes along the rotated.
2. The area of polarization ellipse depends on the lengths of major and minor axes, amplitudes $\mathrm{E}_{0 \mathrm{x}}$ and $\mathrm{E}_{0 \mathrm{y}}$ and phase shift $\phi$.
3. The angle $\beta_{\mathrm{p}}=\operatorname{arctg}\left(\mathrm{E}_{0 y} / \mathrm{E}_{0 \mathrm{x}}\right)$ is called the auxiliary angle $(0 \leq \psi \leq \pi / 2)$.
4. The rotation angle $\psi\left(-\beta_{\mathrm{p}} \leq \psi \leq \beta_{\mathrm{p}}\right)$ is the angle between axis x and major axis $\xi$. This angle is called the azimuth angle.
5. The ellipticity is the major axis to minor axis ratio (b/a).


Figure 1. Polarization ellipse for optical field
6. The angle equals to $v=\operatorname{arctg}(b / a)$ is called ellipticity angle. For linearly polarized light $v=0^{\circ}$; for circularly polarized light $|v|=45^{\circ}$. In turn, for right polarized light: $0^{\circ}<v \leq 45^{\circ}$ and for left polarized light:-45 $5^{\circ} \leq v<0^{\circ}$.

Figure 2 presents some polarization states; The phase shift $\phi$ is only changed.
$\phi=0$ or $2 \pi$
$0<\phi<\pi / 2$
$\phi=\pi / 2$

$\phi=3 \pi / 2$

$$
\pi / 2<\phi<\pi
$$


$3 \pi / 2<\phi<2 \pi$

Figure 2. Different shapes of the polarization ellipse as a function of phase shift

### 2.2. Jones notation

The light wave components in terms of complex quantities can be expressed by means of the Jones vector [1]:

$$
\left[\begin{array}{c}
E_{x}  \tag{6}\\
E_{y}
\end{array}\right]=\left[\begin{array}{l}
E_{0 x} e^{i \mathrm{e}_{x}} \\
E_{0 y} \mathrm{e}^{j \phi_{y}}
\end{array}\right] .
$$

The Jones vector representation is suited to all problems related to the totally polarized light.
Table 1 gives the Jones vectors corresponding to the fundamental SOPs.

| State of polarization | Jones vector |
| :--- | :--- |
| Linear horizontal | $\left[\begin{array}{l}1 \\ 0\end{array}\right]$ |
| $\mathrm{E}_{0 \mathrm{y}}=0 ; E_{0 x}^{2}=1$ | $\left[\begin{array}{l}0 \\ 1\end{array}\right]$ |
| Linear vertical |  |
| $\mathrm{E}_{0 \mathrm{x}}=0 ; E_{0 y}^{2}=1$ | $\frac{1}{\sqrt{2}}\left[\begin{array}{l}1 \\ 1\end{array}\right]$ |
| Linear $45^{\underline{o}}$ | $\frac{1}{\sqrt{2}}\left[\begin{array}{c}1 \\ -1\end{array}\right]$ |
| $\mathrm{E}_{0 \mathrm{x}}=\mathrm{E}_{0 \mathrm{y}} ; 2 E_{0 x}^{2}=1$ | $\frac{1}{\sqrt{2}}\left[\begin{array}{l}1 \\ j\end{array}\right]$ |
| Linear $-45^{o}$ | $\frac{1}{\sqrt{2}}\left[\begin{array}{c}1 \\ -j\end{array}\right]$ |
| $\mathrm{E}_{0 \mathrm{x}}=-\mathrm{E}_{0 \mathrm{y}} ; 2 E_{0 x}^{2}=1$ | $\mathrm{E}_{0 \mathrm{x}}=\mathrm{E}_{0 y}, \phi=\pi / 2 ; 2 E_{0 x}^{2}=1$ |

Table 1. Jones vectors of the fundamental SOPs
The Jones matrices for some polarization components are $2 \times 2$ matrices.
The relationship between the both output and input Jones vectors can be written as:

$$
\left[\begin{array}{l}
E_{x, \text { out }}  \tag{7}\\
E_{y, \text { out }}
\end{array}\right]=\left[\begin{array}{ll}
j_{x x} & j_{x y} \\
j_{y x} & j_{y y}
\end{array}\right] \cdot\left[\begin{array}{l}
E_{x, \text { in }} \\
E_{y, \text { in }}
\end{array}\right] .
$$

Where $\left[\begin{array}{ll}j_{x x} & j_{x y} \\ j_{y x} & j_{y y}\end{array}\right]$ is the Jones matrix of a polarization component.
We now describe the matrix forms for the retarder (wave plate), rotator and polarizer (diattenuator), respectively.

1. Retarder

The retarder causes a phase shift of $\phi / 2$ along the fast (i.e. $x$ ) axis and a phase shift of- $\phi / 2$ along slow (i.e. y) axis. This behavior is described by [3]:

$$
\left[\begin{array}{l}
E_{x, \text { out }}  \tag{8}\\
E_{y, \text { out }}
\end{array}\right]=\left[\begin{array}{cc}
e^{j \frac{\phi}{2}} & 0 \\
0 & e^{-j \frac{\phi}{2}}
\end{array}\right] \cdot\left[\begin{array}{c}
E_{x, \text { in }} \\
E_{y, \text { in }}
\end{array}\right] .
$$

For quarter-wave plate $\phi$ is $\pi / 2$ and for half-wave plate $\phi$ is $\pi$.

## 2. Rotator

If the angle of rotation is $\Theta$ then the components of light emerging from rotation are written as [3]:

$$
\left[\begin{array}{l}
E_{x, \text { out }}  \tag{9}\\
E_{y, \text { out }}
\end{array}\right]=\left[\begin{array}{cc}
\cos (\Theta) & \sin (\Theta) \\
-\sin (\Theta) & \cos (\Theta)
\end{array}\right] \cdot\left[\begin{array}{l}
E_{x, \text { in }} \\
E_{y, \text { in }}
\end{array}\right],
$$

3. Polarizer

The polarizer behavior is characterized by the transmission factor $p_{x}$ and $p_{y}$. Here, for complete transmission $\mathrm{p}_{\mathrm{x}}=\mathrm{p}_{\mathrm{y}}=1$ and for complete attenuation $\mathrm{p}_{\mathrm{x}}=\mathrm{p}_{\mathrm{y}}=0$.

The output Jones vector for a polarizer is given by [3]:

$$
\left[\begin{array}{c}
E_{x, \text { out }}  \tag{10}\\
E_{y, \text { out }}
\end{array}\right]=\left[\begin{array}{cc}
p_{x} & 0 \\
0 & p_{y}
\end{array}\right] \cdot\left[\begin{array}{l}
E_{x, \text { in }} \\
E_{y, \text { in }}
\end{array}\right] .
$$

The Jones matrix $\left(\mathrm{J}_{\mathrm{ar}}\right)$ a polarization component (J) rotated through an angle $\Theta$ is:

$$
\begin{equation*}
\mathrm{J}_{\mathrm{ar}}=\mathrm{J}_{\mathrm{R}}(-\Theta) \cdot \mathrm{J} \cdot \mathrm{~J}_{\mathrm{R}}(\Theta), \tag{11}
\end{equation*}
$$

where $J_{R}(\Theta)$ is the rotation matrix.

### 2.3. Stokes parameters

Let us introduce the $S_{0}, S_{1}, S_{2}$ i $S_{3}$ real quantities defined by the following relations [4]:

$$
\begin{gather*}
\mathrm{S}_{0}=\mathrm{E}_{0 \mathrm{x}}^{2}+\mathrm{E}_{0 \mathrm{y}}^{2},  \tag{12}\\
\mathrm{~S}_{1}=\mathrm{E}_{0 \mathrm{x}}^{2}-\mathrm{E}_{0 \mathrm{y}}^{2},  \tag{13}\\
\mathrm{~S}_{2}=2 \mathrm{E}_{0 \mathrm{x}} \mathrm{E}_{0 \mathrm{y}} \cos (\phi),  \tag{14}\\
\mathrm{S}_{3}=2 \mathrm{E}_{0 \mathrm{x}} \mathrm{E}_{0 \mathrm{y}} \sin (\phi), \tag{15}
\end{gather*}
$$

where $\mathrm{E}_{0 x}, \mathrm{E}_{0 \mathrm{y}}$ are the real maximum amplitudes and $\phi$ is the phase difference.
These quantities are called the Stokes parameters. The Stokes parameters have a physical meaning in terms of intensity. The parameter $S_{0}$ represents the total intensity of light. The second parameter $S_{1}$ describes the difference in the intensities of the linearly horizontal polarized light and the linearly vertical polarized light. The third parameter $S_{2}$ represents the difference in the intensities of the linearly $45^{\circ}$ polarized light and linearly- $45^{\circ}$ polarized light. The last parameter $\mathrm{S}_{3}$ represents the difference in the intensities of the right circularly polarized light and the left circularly polarized light. The Stokes parameters are real values. The Stokes representation is the most adequate representation in treating partially polarized and unpolarized light problems. Moreover, Stokes representation is well suited to the definition of the Degree Of Polarization (DOP). This parameter is equal to [2]:

$$
\begin{equation*}
\mathrm{DOP}=\frac{\sqrt{\mathrm{S}_{1}^{2}+\mathrm{S}_{2}^{2}+\mathrm{S}_{3}^{2}}}{\mathrm{~S}_{0}} \tag{16}
\end{equation*}
$$

with value between 0 (unpolarized light) and 1 (totally polarized light).
Often the normalized Stokes parameters are used to describe the light polarization: $\frac{S_{1}}{S_{0}}, \frac{S_{2}}{S_{0}}, \frac{S_{3}}{S_{0}}$; with value between-1 and 1.

Table 2 shows some Stokes vectors corresponding to the fundamental SOPs.

| State of polarization | Stokes vector |
| :--- | :--- |
| Linear horizontal | $\left[\begin{array}{l}1 \\ 1 \\ E_{0 y}=0, E_{0 x}^{2}=1 \\ 0\end{array}\right]$ |
| Linear vertical |  |
| $\mathrm{E}_{0 \mathrm{x}}=0, E_{0 y}^{2}=1$ | $\left[\begin{array}{c}1 \\ -1 \\ 0 \\ 0\end{array}\right]$ |
| $\mathrm{Linear}^{2} 45^{\mathrm{o}}$ | $\left[\begin{array}{c}1 \\ 0 \\ \mathrm{E}_{0 \mathrm{x}}=\mathrm{E}_{0 \mathrm{y}}=\mathrm{E}_{0}, \phi=0,2 E_{0}^{2}=1 \\ 1 \\ 0\end{array}\right]$ |
| $\mathrm{Linear}_{0 \mathrm{x}}=\mathrm{E}_{0 \mathrm{y}}=\mathrm{E}_{0}, \phi=\pi, 2 E_{0}^{2}=1$ | $\left[\begin{array}{l}1 \\ 0 \\ -1 \\ 0\end{array}\right]$ |


| State of polarization | Stokes vector |
| :--- | :--- |
| Right circular | $\left[\begin{array}{l}1 \\ 0 \\ 0 \\ 1\end{array}\right]$ |
| $\mathrm{E}_{0 \mathrm{x}}=\mathrm{E}_{0 \mathrm{y}}=\mathrm{E}_{0}, \phi=\pi / 2,2 E_{0}^{2}=1$ |  |
| Left circular | $\left[\begin{array}{c}1 \\ 0 \\ 0 \\ -1\end{array}\right]$ |
| $\mathrm{E}_{0 \mathrm{x}}=\mathrm{E}_{0 \mathrm{y}}=\mathrm{E}_{0}, \phi=3 \pi / 2,2 E_{0}^{2}=1$ |  |

Table 2. Stokes vectors of the fundamental SOPs

The Poincaré sphere (Figure 3) is a very useful graphical tool representation of polarization in real three-dimensional space.


Figure 3. The Poincaré sphere and fundamental SOPs on this sphere

Each polarization is represented by a point on the Poincaré sphere (totally polarized light) or within the Poincaré sphere (partially polarized light) centered on rectangular coordinate system. Center of the Poincaré sphere represents unpolarized light. The coordinates of the point are normalized Stokes parameters. All linear SOPs lie on the equator. The right circular SOP and left one is located at the North and South Pole, respectively. Elliptically polarized states are represented everywhere else on the surface of the Poincaré sphere. The two orthogonal polarizations are located diametrically opposite on the Poincaré sphere. A continuous evolution of SOP is represented on the Poincare sphere as a continuous path on this sphere (Figure 4).


Figure 4. Example of a continuous evolution of SOP on the Poincaré sphere
Figure 5 presents changing the SOPs by means of the retarder and rotator.


Figure 5. The effect of changing the SOPs by the retarder (a) and the rotator (b)

On the Poincaré sphere the phase shift causes that the intial SOP moves to a new SOP along the same longitude line (Figure 5a). The linear, elliptically and circular SOPs can be achieved by means of a single retarder. In turn, on the Poincaré sphere the rotation by a rotator causes that the intial SOP moves to a new SOP along the same latitude line (Figure 5b).

### 2.4. Mueller notation

Impact of an optical component (or optical system) properties on the polarization of light can be determined by constructing the Stokes vector for the input light and applying Mueller calculus, to obtain the Stokes vector of the light leaving the component:

$$
\begin{equation*}
\overrightarrow{\mathrm{S}}_{\text {out }}=\mathrm{M} \cdot \overrightarrow{\mathrm{~S}}_{\text {in }}, \tag{17}
\end{equation*}
$$

where $\vec{S}_{\text {in }}$ and $\vec{S}_{\text {out }}$ is the input and output Stokes vector, respectively, M is the Mueller matrix of an optical component.

We now describe the Mueller matrix forms for the retarder, rotator and polarizer, respectively.

## 1. Retarder

The retarder causes a total phase shift $\phi$ between fast (x) and slow (y) axis. The Mueller matrix of the retarder is seen to be [3]:

$$
\mathrm{M}_{\text {Ret }}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0  \tag{18}\\
0 & 1 & 0 & 0 \\
0 & 0 & \cos (\phi) & -\sin (\phi) \\
0 & 0 & \sin (\phi) & \cos (\phi)
\end{array}\right]
$$

2. Rotator

The Mueller matrix of the rotator is given [3]:

$$
\mathrm{M}_{\text {Rot }}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0  \tag{19}\\
0 & \cos (2 \Theta) & \sin (2 \Theta) & 0 \\
0 & -\sin (2 \Theta) & \cos (2 \Theta) & 0 \\
0 & 0 & 0 & 1
\end{array}\right] .
$$

Because polarization effects are described in the intensity domain the physical rotation through an angle $\Theta$ leeds to the appearance of $2 \Theta$.
3. Polarizer

The Mueller matrix for polarizer is [3]:

$$
M_{\text {polar }}=\frac{1}{2}\left[\begin{array}{cccc}
p_{x}^{2}+p_{y}^{2} & p_{x}^{2}-p_{y}^{2} & 0 & 0  \tag{20}\\
p_{x}^{2}-p_{y}^{2} & p_{x}^{2}+p_{y}^{2} & 0 & 0 \\
0 & 0 & 2 p_{x} p_{y} & 0 \\
0 & 0 & 0 & 2 p_{x} p_{y}
\end{array}\right],
$$

where $p_{x}$ and $p_{y}$ are so called transmission factors.
Here, the transmission factors are $p_{x}=1$ and $p_{y}=0$ for the linear horizontal polarizer. In turn, the transmission factors $p_{x}=1$ and $p_{y}=0$ for the linear vertical polarizer.

The Mueller matrix $\left(\mathrm{M}_{\mathrm{ar}}\right)$ for a polarization component $(\mathrm{M})$ rotated through an angle $\Theta$ is:

$$
\begin{equation*}
M_{a r}=M_{R}(-2 \Theta) \cdot M \cdot M_{R}(2 \Theta), \tag{21}
\end{equation*}
$$

where $\mathrm{M}_{\mathrm{R}}(\Theta)$ is the rotation matrix.

## 3. Polarization phenomena in optical fiber links

### 3.1. Polarization mode dispersion

The optical fiber transmission systems are exposed to some polarization effects. Changing of tranmsission quality (e.g. transmission capacity) of an optical fiber links during high bit rate transmission is caused by Polarization Mode Dispersion (PMD), Polarization Dependent Loss (PDL), Polarization Dependent Gain (PDG).

Polarization Mode Dispersion is impairment phenomenon that limits the transmission speed and distance in high bit rate optical fiber communication systems. The impairment results from PMD is similar to chromatic dispersion impairment.

According to [5]: There always exists an orthogonal pair of polarization states output a birefringent concatenation which are stationary to first order in frequency. These two states are called Principle States of Polarization (PSP).

A diffrential delay exists between signals launched along one PSP and its orthogonal complement. This effect is quantified by Differential Group Delay (DGD). There are many ways in which an optical fiber can become birefringent. Birefringence can arise due to an asymmetric fiber core or asymmetric fiber refractive index or can be introduced through internal stresses during fiber manufacture or through external stresses during cabling and installation. Polarization Mode Dispersion of an optical fiber link is proportional to the square root of the fiber link length (strong coupling between the orthogonally polarized signal components) or to the fiber link length (weak coupling between ones). The frequency dependent evolution of SOPs in an optical fiber link (Figure 6) is described by the following equation [6]:

$$
\begin{equation*}
\frac{\mathrm{d} \overrightarrow{\mathrm{~S}}}{\mathrm{~d} \omega}=\vec{\Omega} \times \overrightarrow{\mathrm{S}}, \tag{22}
\end{equation*}
$$

where $\vec{S}$ is the Stokesa vector, $\omega$ is angular frequency and $\vec{\Omega}$ is the PMD vector.


Figure 6. State of polarization transformation through the PMD vector
The pointing direction of the PMD vector is aligned to the slow PSP. The length of the PMD vector is the DGD value between the slow and fast PSP.

The Probability Density Function for DGD $\left(\tau_{g, r}\right)$ is given by [7]:

$$
\begin{equation*}
\operatorname{PDF}\left(\tau_{\mathrm{g}, \mathrm{r}}\right)=\frac{8}{\pi^{2}\left\langle\tau_{\mathrm{g}, \mathrm{r}}\right\rangle}\left(\frac{2 \tau_{\mathrm{g}, \mathrm{r}}}{\left\langle\tau_{\mathrm{g}, \mathrm{r}}\right\rangle}\right)^{2} \mathrm{e}^{-\frac{\left(\frac{2 \tau_{\mathrm{g}, \mathrm{r}}}{\left\langle\tau_{\mathrm{g}, \mathrm{r}}\right\rangle}\right)^{2}}{\pi}}, \tag{23}
\end{equation*}
$$

where $\left\langle\tau_{g, r}\right\rangle$ is average value of DGD.
Figure 7 shows Probability Density Function for DGD.
Differential Group Delay distribution is Maxwellian distribution. Differential Group Delay can be also expressed as [8]:

$$
\begin{equation*}
\left\langle\tau_{\mathrm{g}, \mathrm{r}}\right\rangle^{2}=\frac{1}{3}\left(\frac{\lambda \mathrm{~L}_{\mathrm{c}}}{\mathrm{cL}_{\mathrm{B}}}\right)^{2}\left(\frac{\mathrm{~L}}{\mathrm{~L}_{\mathrm{c}}}-1+\mathrm{e}^{-\frac{\mathrm{L}}{\mathrm{~L}_{\mathrm{c}}}}\right), \tag{24}
\end{equation*}
$$

where: $\lambda$ is wavelength, c is light wave velocity in vacuum, $\mathrm{L}_{\mathrm{B}}$ is beat length and $\mathrm{L}_{\mathrm{c}}$ is correlation length, L is optical fiber length.


Figure 7. Probability Density Function (PDF) for DGD; average value of DGD is 40 ps
The beat length describes the length required for SOP to rotate $2 \pi$ ( 360 degrees). In turn, the correlation length is defined to be length at which the difference between average power of orthogonally polarized signal components is within $1 / \mathrm{e}^{2}$.

Second order PMD is generated by a change of the PMD with frequency (wavelength) [9]:

$$
\begin{equation*}
\frac{\mathrm{d} \vec{\Omega}}{\mathrm{~d} \omega}=\frac{\mathrm{d} \tau_{\mathrm{g}, \mathrm{r}}}{\mathrm{~d} \omega} \overrightarrow{\mathrm{p}}+\tau_{\mathrm{g}, \mathrm{r}} \frac{\mathrm{~d} \overrightarrow{\mathrm{p}}}{\mathrm{~d} \omega}, \tag{25}
\end{equation*}
$$

where $\vec{p}$ is the Stokes vector pointing in the direction of the fast PSP.
Differentiating the PMD vector with respect to frequency gives two components of second order PMD. The first term on the right side of equation (25) is so called polarization dependent chromatic dispersion, it is known to cause polarization-dependent pulse compression and broadening, while the second term causes depolarization. Figure 8 illustrates changing the SOP with frequency - second order PMD.

The Probability Density Function of second order PMD is given by [10]:

$$
\begin{equation*}
\operatorname{PDF}\left(\left|\vec{\Omega}_{\omega}\right|\right)=\frac{32\left|\vec{\Omega}_{\omega}\right|}{\pi\langle | \vec{\Omega}_{\omega}| |^{4}} \tanh \left(\frac{4\left|\vec{\Omega}_{\omega}\right|}{\langle | \vec{\Omega}_{\omega}| \rangle^{2}}\right) \operatorname{sech}\left(\frac{4\left|\vec{\Omega}_{\omega}\right|}{\langle | \vec{\Omega}_{\omega}| \rangle^{2}}\right) \tag{26}
\end{equation*}
$$



Figure 8. The effect of changing the SOP with frequency
Figure 9 shows Probability Density Function of second order PMD.


Figure 9. Probability Density Function (PDF) of second order PMD; average value of second order PMD is $10,0 \mathrm{ps}^{2}$
It should be note that concatenation of two birefringent optical components (e.g. two sections of polarization-maintaining optical fiber) generates only first order PMD (Figure 10a). These birefringent sections are orientated randomly relative to each other. In turn, concatenation of three (and more) birefringent optical components generates high order PMD (Figure 10b).
a

b


Figure 10. State of polarization evolution through rotating two birefringent optical components (a) and three birefringent optical components (b) which are orientated randomly relative to each other

### 3.2. Polarization Dependent Loss

Polarization Dependent Loss is defined as absolute value or the relative difference between an optical component maximium and minimum transmission loss given all possible input SOPs [3]. Dichroism phenomenon is responsible for the PDL effect. Dichroism can be achived by optical fiber bending or interaction between optical beam and tilted glass plate. The PDL value can be given by the following relationship [11]:

$$
\begin{equation*}
\operatorname{PDL}[\mathrm{dB}]=10 \log _{10}\left(\frac{\mathrm{~T}_{\mathrm{r}, \text { max }}}{\mathrm{T}_{\mathrm{r}, \text { min }}}\right), \tag{27}
\end{equation*}
$$

where $T_{r, \text { max }}$ and $T_{r, \text { min }}$ are the maximum and minimum transmission intensities through an optical component.

The PDL can be also written as [11]:

$$
\begin{equation*}
\operatorname{PDL}[\mathrm{dB}]=10 \log _{10}\left(\frac{1+|\vec{\Gamma}|}{1-|\vec{\Gamma}|}\right) \tag{28}
\end{equation*}
$$

where $\vec{\Gamma}$ is the PDL vector.
This vector is equal to: $\frac{T_{r, \text { max }}-T_{r, \text { min }}}{T_{r, \text { max }}+T_{r, \text { min }}}$. The pointing direction of the PDL vector is aligned to maximum transmission direction. In other words, this vector is aligned to a polarization vector that imparts the least PDL value. The cumulative PDL vector over concatenate two optical components with the PDL vectors $\left(\vec{\Gamma}_{1}, \vec{\Gamma}_{2}\right)$ is [12]:

$$
\begin{equation*}
\overline{\Gamma_{12}}=\frac{\sqrt{1-\Gamma_{2}^{2}}}{1+\overrightarrow{\Gamma_{1}} \overline{\Gamma_{2}}} \overrightarrow{\Gamma_{1}}+\frac{1+\overrightarrow{\Gamma_{1}} \overline{\Gamma_{2}}\left(\frac{1-\sqrt{1-\Gamma_{2}^{2}}}{\Gamma_{2}^{2}}\right)}{1+\overrightarrow{\Gamma_{1}} \overrightarrow{\Gamma_{2}}} \vec{\Gamma}_{2} \tag{29}
\end{equation*}
$$

If we want to calculate the resulting PDL value from PDL of each optical component we need to average over all possible orientation between $\vec{\Gamma}_{1}$ and $\vec{\Gamma}_{2}$ vectors [12]:

$$
\begin{equation*}
\left\langle\Gamma_{12}\right\rangle=\frac{1}{2} \int_{-1}^{1} \sqrt{\frac{\Gamma_{1}^{2}+\Gamma_{2}^{2}-\Gamma_{1}^{2} \Gamma_{2}^{2}+2 \Gamma_{1} \Gamma_{2} \eta_{\mathrm{a}}+\Gamma_{1}^{2} \Gamma_{2}^{2} \eta_{\mathrm{a}}^{2}}{\left(1+\Gamma_{1} \Gamma_{2} \eta_{\mathrm{a}}\right)^{2}}} d \eta_{\mathrm{a}^{\prime}} \tag{30}
\end{equation*}
$$

where $\eta_{a}$ is angle between $\vec{\Gamma}_{1}$ and $\vec{\Gamma}_{2}$ vectors.
Concatenation of N optical components with PDL gives the following result:

$$
\begin{equation*}
\vec{\Gamma}=\sum_{\mathrm{j}=1}^{\mathrm{N}} \vec{\Gamma}_{\mathrm{j}}+0\left(\Gamma^{2}\right) . \tag{31}
\end{equation*}
$$

Polarization Dependent Loss distribution is Rayleigh distribution (Figure 11).


Figure 11. Probability density function (PDF) of PDL; average value of PDL is 0.5 dB
In the presence of PMD the PDL distribution is closed to Maxwellian distribution. It is important to note that in the case of single mode fiber the orthogonal SOPs pairs at the input lead to orthogonal output SOPs pairs, although the input SOP is not maintained in general. But, when the optical fiber link includes PDL the SOPs are no longer orthogonal. Moreover,
polarization effects due to interaction between PMD and PDL can significantly impair optical fiber transmission systems. The accumulative PMD and PDL impairment is more dangerous for lightwave communication systems than a pure PMD or PDL impairment.

### 3.3. Polarization Dependent Gain

Another polarization effect which is closely related to PDL is PDG.This phenomenon is present in optical amplifiers (first of all in Semiconductor Optical Amplifier). Polarization dependent gain can be defined as absolute value or the relative difference between an optical amplifier maximium gain $\left(\mathrm{G}_{\max }\right)$ and minimum one $\left(\mathrm{G}_{\min }\right)$ :

$$
\begin{equation*}
\operatorname{PDG}[\mathrm{dB}]=10 \log _{10}\left(\frac{\mathrm{G}_{\max }}{\mathrm{G}_{\min }}\right), \tag{32}
\end{equation*}
$$

Polarization Hole Burning phenomenon is responsible for the PDG effect. It is important to know that PDG effect is observed for linear polarized optical signals which are amplified. Polarization Dependent Gain for circular polarization can be neglected [13].

## 4. Modeling of polarization phenomena

The analysis of impact of optical fiber polarization properties on optical signal transmissions requires a detailed description of polarization effects. The most popular approaches are using an optical fiber links modeling based on homogeneous polarization segments and electromagnetic wave propagation equations.

In general, an optical fiber exhibits axially-varing birefringence and can be represented by a series of short and homogeneous polarization segments. Each polarization is described as the randomly rotated polarization elements. These polarization elements are characterised by birefringence (i.e. PMD) or dichroism (i.e. PDL).

The birefringent element can be represented by retarder (phase shifter). In terms of the Jones matrix this element is described by:

$$
J_{\text {DGD }}=\left[\begin{array}{cc}
\mathrm{e}^{\mathrm{j} \frac{\phi}{2}} & 0  \tag{33}\\
0 & \mathrm{e}^{-\mathrm{j} \frac{\phi}{2}}
\end{array}\right],
$$

where $\phi$ is is the total phase shift between the polarization signal components (two polarization modes).

In terms of the Mueller matrix the birefringent element is given by:

$$
\mathrm{M}_{\mathrm{DGD}}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0  \tag{34}\\
0 & 1 & 0 & 0 \\
0 & 0 & \cos (\phi) & -\sin (\phi) \\
0 & 0 & \sin (\phi) & \cos (\phi)
\end{array}\right]
$$

The value of phase shift can be given by:

$$
\begin{equation*}
\phi=\tau_{g, r} \cdot \omega, \tag{35}
\end{equation*}
$$

where $\tau_{\mathrm{g}, \mathrm{r}}$ is DGD and $\omega$ is angular frequency.
The phase shift between the two polarization signal components (two polarization modes) can be also expressed as:

$$
\begin{equation*}
\phi=\mathrm{b} \cdot \mathrm{~L}_{\mathrm{el}}, \tag{36}
\end{equation*}
$$

where $b$ is birefringence of birefringent element and $L_{e l}$ is birefringent element length.
Then PDL element is described by the following Jones matrix [14]:

$$
\mathrm{J}_{\mathrm{PDL}}=\left[\begin{array}{cc}
\mathrm{e}^{-\frac{\alpha_{1}}{2}} & 0  \tag{37}\\
0 & \mathrm{e}^{\frac{\alpha_{1}}{2}}
\end{array}\right]
$$

where $\alpha_{1}$ is defined as: $\operatorname{PDL}[\mathrm{dB}]=10 \log _{10}\left(\exp \left(2 \alpha_{1}\right)\right)$.
The Mueller matrix corresponding to equation (37) is equal to:

$$
\mathrm{M}_{\mathrm{PDL}}=\left[\begin{array}{cccc}
\frac{1+\alpha^{2}}{2} & \frac{1-\alpha^{2}}{2} & 0 & 0  \tag{38}\\
\frac{1-\alpha^{2}}{2} & \frac{1+\alpha^{2}}{2} & 0 & 0 \\
0 & 0 & \alpha & 0 \\
0 & 0 & 0 & \alpha
\end{array}\right]
$$

Here, value of $\alpha$ equals to: $\alpha=10^{-\frac{P D L[\alpha B]}{10}}$.
Figure 12 illustrates an optical fiber link which is split into some polarization segments (rotated polarization elements).


Figure 12. Optical fiber link model consists of $N$ polarization segments; $M_{s, 1}, M_{s, 2}, M_{s, n}$ - matrix of polarization elements, $\Theta_{1}, \Theta_{2}, \Theta_{\mathrm{n}}$ - angle of rotation

If we take into account only the PMD effect then the matrix of a single polarization segment $\mathrm{M}_{\mathrm{s}, \mathrm{n}}$ is given by:

$$
\begin{equation*}
M_{s, n}=M_{R, n}\left(-2 \Theta_{n}\right) \cdot M_{D G D, n} \cdot M_{R, n}\left(2 \Theta_{n}\right) \tag{39}
\end{equation*}
$$

When considering the PMD and PDL effect, matrix of a single polarization segment is can be writing as:

$$
\begin{equation*}
M_{s, n}=M_{R, n}\left(-2 \Theta_{n}\right) \cdot M_{P D L, n} \cdot M_{D G D, n} \cdot M_{R, n}\left(2 \Theta_{n}\right) . \tag{40}
\end{equation*}
$$

The matrix $\mathrm{M}_{\mathrm{T}}$ of the whole optical fiber link which consists of N polarization segments is equal to:

$$
\begin{equation*}
\mathrm{M}_{\mathrm{T}}=\mathrm{M}_{\mathrm{s}, \mathrm{~N}} \cdot \ldots \cdot \mathrm{M}_{\mathrm{s}, 3} \cdot \mathrm{M}_{\mathrm{s}, 2} \cdot \mathrm{M}_{\mathrm{s}, 1^{\prime}} \tag{41}
\end{equation*}
$$

Furthermore, the PDG element matrix $\left(\mathrm{M}_{\mathrm{PDG}}\right)$ can be described by the following equation:

$$
\left[\begin{array}{cccc}
\frac{\mathrm{g}^{2}+1}{2} & \frac{\mathrm{~g}^{2}-1}{2} & 0 & 0  \tag{42}\\
\frac{\mathrm{~g}^{2}-1}{2} & \frac{\mathrm{~g}^{2}+1}{2} & 0 & 0 \\
0 & 0 & \mathrm{~g} & 0 \\
0 & 0 & 0 & \mathrm{~g}
\end{array}\right]
$$

where $g$ is the PDG coefficient equals to $g=10^{\frac{P D C[A B]}{10}}$.
Moreover, to describe the backscattering process we treat an optical fiber link as a cascade of backscattering elements. We treat Rayleigh backscattering as many small reflections distributed over the optical fiber link. For a single element the matrix representing the round-trip propagation (fiber in forward direction, reflector, fiber in backward direction) is computed by:

$$
\begin{equation*}
\mathrm{M}_{\mathrm{Rs}, 1}=\mathrm{M}_{\mathrm{s}, 1}^{\mathrm{T}} \cdot \mathrm{M}_{\mathrm{R}} \cdot \mathrm{M}_{\mathrm{s}, 1}, \tag{43}
\end{equation*}
$$

where $M_{s, 1}^{T}$ is the transpose of $\mathrm{M}_{\mathrm{s}, 1}$, and $\mathrm{M}_{\mathrm{R}}$ is the Mueller matrix of a reflection:

$$
\mathrm{M}_{\mathrm{R}}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0  \tag{44}\\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{array}\right]
$$

For light propagating to the end of the N -th element the round-trip Mueller matrix has the following form [4]:

$$
\begin{equation*}
\mathrm{M}_{\mathrm{Rs}, \mathrm{~N}}=\mathrm{M}_{\mathrm{s}, 1}^{\mathrm{T}} \cdot \mathrm{M}_{\mathrm{s}, 2}^{\mathrm{T}} \cdot \ldots \cdot \mathrm{M}_{\mathrm{s}, \mathrm{~N}}^{\mathrm{T}} \cdot \mathrm{M}_{\mathrm{R}} \cdot \mathrm{M}_{\mathrm{s}, \mathrm{~N}} \cdot \ldots \cdot \mathrm{M}_{\mathrm{s}, 2} \cdot \mathrm{M}_{\mathrm{s}, 1} \cdot \tag{45}
\end{equation*}
$$

We can use polarization segments model for the DGD and PDL values calculation. We should take into account Jones and Muller notation.

### 4.1. Jones notation

Differential Group Delay value can be found by the following relationship [15]:

$$
\begin{equation*}
\tau_{\mathrm{g}, \mathrm{r}}=\frac{\left|\operatorname{Arg}\left(\frac{\lambda_{\tau, 1}}{\lambda_{\tau, 2}}\right)\right|}{\mathrm{d} \omega}, \tag{46}
\end{equation*}
$$

where: $\operatorname{Arg}$ denotes the argument function, $\lambda_{\tau, 1}$ and $\lambda_{\tau, 2}$ are two eigenvalues of matrix $M_{T}(\omega+d \omega) \cdot M_{T}^{-1}(\omega) ; M_{T}^{-1}$ is inverse matrix.

Polarization Dependent Loss in the unit of dB at angular frequency $\omega$ is given by [16]:

$$
\begin{equation*}
\operatorname{PDL}[\mathrm{dB}]=10 \log _{10}\left(\frac{\lambda_{\alpha, 1}}{\lambda_{\alpha, 2}}\right), \tag{47}
\end{equation*}
$$

where: $\lambda_{\alpha, 1}$ and $\lambda_{\alpha, 2}$ are two eigenvalues of matrix $M_{T}^{\mathrm{T}}(\omega) \cdot M_{T}(\omega)$, where $M_{T}^{\mathrm{T}}$ is transpose matrix.

### 4.2. Mueller notation

Differential Group Delay value can be expressed as the length of the PMD vector $\vec{\Omega}$ [6]:

$$
\begin{equation*}
\tau_{\mathrm{g}, \mathrm{r}}=|\vec{\Omega}|=\sqrt{\Omega_{\mathrm{x}}^{2}+\Omega_{\mathrm{y}}^{2}+\Omega_{\mathrm{z}}^{2}}, \tag{48}
\end{equation*}
$$

The PMD vector after $(\mathrm{n}+1)$-th polarization segment may be written as [6]:

$$
\begin{equation*}
\vec{\Omega}_{\mathrm{n}+1}=\mathrm{D} \vec{\Omega}_{\mathrm{n}+1}+\mathrm{MB} \vec{\Omega}_{\mathrm{n}}, \tag{49}
\end{equation*}
$$

where $\vec{\Omega}_{\mathrm{n}}$ is the PMD dispersion vector of the first n polarization segments, $\Delta \vec{\Omega}_{\mathrm{n}+1}$ is the PMD vector of the $(\mathrm{n}+1)$-th polarization segments, matrix MB represents a transformation of the PMD vector caused by the propagation through the $(\mathrm{n}+1)$-th polarization segment.

The recursive relation for the PMD vector is given by:

$$
\left[\begin{array}{l}
\Omega_{\mathrm{x}, \mathrm{n}+1}  \tag{50}\\
\Omega_{\mathrm{y}, \mathrm{n}+1} \\
\Omega_{\mathrm{z}, \mathrm{n}+1}
\end{array}\right]=\left[\begin{array}{c}
\tau_{\mathrm{g}, \mathrm{r}, \mathrm{n}+1} \\
0 \\
0
\end{array}\right]+\left[\begin{array}{lll}
\mathrm{m}_{11} & \mathrm{~m}_{12} & \mathrm{~m}_{13} \\
\mathrm{~m}_{21} & \mathrm{~m}_{22} & \mathrm{~m}_{23} \\
\mathrm{~m}_{31} & \mathrm{~m}_{32} & \mathrm{~m}_{33}
\end{array}\right] \cdot\left[\begin{array}{c}
\Omega_{\mathrm{x}, \mathrm{n}} \\
\Omega_{\mathrm{y}, \mathrm{n}} \\
\Omega_{\mathrm{z}, \mathrm{n}}
\end{array}\right] .
$$

We use $3 \times 3$ matrix in equation (50). Because we assume that $\mathrm{SOP}=1$.
To calculation the PDL value of an optical component or optical fiber link, one must determine the minimum and maximum transmission. Because of this we should take into account $4 \times 4$ matrix:

$$
\left[\begin{array}{llll}
\mathrm{m}_{00} & \mathrm{~m}_{01} & \mathrm{~m}_{02} & \mathrm{~m}_{03}  \tag{51}\\
\mathrm{~m}_{10} & \mathrm{~m}_{11} & \mathrm{~m}_{12} & \mathrm{~m}_{13} \\
\mathrm{~m}_{20} & \mathrm{~m}_{21} & \mathrm{~m}_{22} & \mathrm{~m}_{23} \\
\mathrm{~m}_{30} & \mathrm{~m}_{31} & \mathrm{~m}_{32} & \mathrm{~m}_{33}
\end{array}\right],
$$

Polarization Dependent Loss in the unit of dB is [17]:

$$
\begin{equation*}
\mathrm{PDL}[\mathrm{~dB}]=10 \log _{10}\left(\frac{\mathrm{~m}_{00}+\sqrt{\mathrm{m}_{01}^{2}+\mathrm{m}_{02}^{2}+\mathrm{m}_{03}^{2}}}{\mathrm{~m}_{00}-\sqrt{\mathrm{m}_{01}^{2}+\mathrm{m}_{02}^{2}+\mathrm{m}_{03}^{2}}}\right) \tag{52}
\end{equation*}
$$

For an understanding of linear and, first of all, nonlinear optical effects in optical fiber links it is necessary to consider the electromagnetic wave propagation. The linear and nonlinear optical effects in an optical fiber are described by so called nonlinear Schroedinger propagation equation. The nonlinear coupled Schroedinger propagation equations governing evolution of an optical pulse consisiting of the two polarization components along a fiber link $(\mathrm{z})$ are given by [18]:

$$
\begin{align*}
& \frac{\partial E_{x}}{\partial z}=-\frac{\alpha_{x}}{2} E_{x}+j \frac{\beta_{2}}{2} \frac{\partial^{2} E_{x}}{\partial t^{2}}+j \gamma\left(\left|E_{x}\right|^{2}+\frac{2}{3}\left|E_{y}\right|^{2}\right) E_{x}+j \frac{\gamma}{3} E_{x}^{*} E_{y}^{2}  \tag{53}\\
& \frac{\partial E_{y}}{\partial z}=-\frac{\alpha_{y}}{2} E_{y}+j \frac{\beta_{2}}{2} \frac{\partial^{2} E_{y}}{\partial t^{2}}+j \gamma\left(\left|E_{y}\right|^{2}+\frac{2}{3}\left|E_{x}\right|^{2}\right) E_{y}+j \frac{\gamma}{3} E_{y}^{*} E_{x}^{2}, \tag{54}
\end{align*}
$$

Where $\mathrm{E}_{\mathrm{x}}, \mathrm{E}_{\mathrm{y}}$ are slowly varying amplitudes, $\alpha_{\mathrm{x}}$ and $\alpha_{\mathrm{y}}$ is attenuation coefficient for $\mathrm{E}_{\mathrm{x}}$ and $\mathrm{E}_{\mathrm{y}}$, respectively. Moreover, $\beta_{2}$ is second-order term of the expansion of the propagation constant (the group velocity dispersion parameter), $\gamma$ is the nonlinear parameter,* designates complex conjugation, j is imaginary unit.

A numerical approach is necessary for the polarization and nonlinear propagation equations solution.

The most popular numerical method is Split-Step Fourier Method. There is useful to write equations (53) and (54) formally in the following form [19]:

$$
\begin{equation*}
\frac{\partial \overrightarrow{\mathrm{E}}(\mathrm{~T}, \mathrm{z})}{\partial \mathrm{z}}=\left[\ell_{1}(\mathrm{~T})+\ell_{2}(\mathrm{~T})+\aleph(\mathrm{z})\right] \overrightarrow{\mathrm{E}}(\mathrm{~T}, \mathrm{z}) \tag{55}
\end{equation*}
$$

where: $\vec{E}=\left[\begin{array}{l}E_{x} \\ E_{y}\end{array}\right]$ and $\mathrm{T}=\mathrm{t}-\beta_{1} \mathrm{z}$; t is time, $\beta_{1}$ is first-order term of the expansion of the propagation constant (differential coefficient of the propagation constant with respect to optical frequency).

The operators on the right side of equation (55) are linear $\ell_{1}(T), \ell_{2}(T)$ and nonlinear $\aleph(z)$. These operators have the following definitions [19]:

$$
\begin{gather*}
\ell_{1}(T)=\frac{1}{2}\left[\begin{array}{cc}
\beta_{1} & 0 \\
0 & -\beta_{1}
\end{array}\right] \cdot \frac{\partial}{\partial \mathrm{T}},  \tag{56}\\
\ell_{2}(T)=-\frac{1}{2} I\left(j \beta_{2} \frac{\partial^{2}}{\partial T^{2}}+\frac{1}{3} \beta_{3} \frac{\partial^{3}}{\partial T^{3}}\right),  \tag{57}\\
\aleph(z)=-j \frac{\gamma}{3}\left[\begin{array}{cc}
3\left|E_{x}\right|^{2}+2\left|E_{y}\right|^{2} & E_{x}^{*} E_{y} \\
E_{y}^{*} E_{x} & 3\left|E_{y}\right|^{2}+2\left|E_{x}\right|^{2}
\end{array}\right]+,  \tag{58}\\
j \frac{1}{2}\left[\begin{array}{cc}
-j \alpha_{1}+\beta_{0} & 0 \\
0 & j \alpha_{1}-\beta_{0}
\end{array}\right]
\end{gather*}
$$

Where $\beta_{0}$ is zeroth-order term of the expansion of the propagation constant, $\beta_{3}$ is third-order term of the expansion of the propagation constant (third differential coefficient of the propagation constant with respect to optical frequency). The symbol I stands for the identity matrix.

The linear operators describe first-order and high-order PMD effect. It is a function of T alone. The nonlinear operator includes phenomena that do not depend on T i.e. PMD, nonlinear effects. It is a function of z alone.

The Split-Step Fourier Method obtains an aproximate solution by assuming that in propagating an optical pulse over a small distance $h$ optical effects are independent [19].

$$
\begin{equation*}
\mathrm{E}(\mathrm{~T}, \mathrm{z}+\mathrm{h}) \approx \mathrm{NL}_{1} \mathrm{~L}_{2} \mathrm{E}(\mathrm{~T}, \mathrm{z}), \tag{59}
\end{equation*}
$$

where:

$$
\begin{gather*}
\mathrm{L}_{1}=\exp \left(\ell_{1} \mathrm{~h}\right),  \tag{60}\\
\mathrm{L}_{2}=\exp \left(\ell_{2} \mathrm{~h}\right),  \tag{61}\\
\mathrm{N}=\exp \left(\int_{\mathrm{z}}^{\mathrm{z}+\mathrm{h}} \aleph\left(z^{\prime}\right) d z^{\prime}\right) \approx \exp \left(\mathrm{h} \frac{\aleph(z+h)+\aleph(z)}{2}\right) . \tag{62}
\end{gather*}
$$

Propagation from z to $\mathrm{z}+\mathrm{h}$ is carried out in two steps. In the first step linear effects only ( $\mathrm{L} 1 \neq 0$, $\mathrm{L} 2 \neq 0, \mathrm{~N}=0$ ) are taken into account. In the first step vice versa ( $\mathrm{L} 1=0, \mathrm{~L} 2=0, \mathrm{~N} \neq 0$ ).

Figure 13 shows schematic illustration of the Split-Step Fourier Method. Fiber length is split into a large number of small sygments of width $h$.


Figure 13. Schematic illustration of the Split-Step Fourier Method
It is important to know, that the linear operators are evaluated on the Fourier domain. In turn, nonlinear operator is evaluated on the time domain.

## 5. Polarizing components

Optical polarizing components belong to a class of optical components characterized by the modyfication of some polarization properties of light wave. Optical polarizing components are very useful for optical fiber communication technologies. Some of them are used for PMD and PDL compensating, Polarization Division Multiplexing transmission technique and measurement procedures. Ones of the most important optical polarizing components for modern, high capacity optical communication solutions are: polarization controller, polarization attractor, polarization scrambler and polarization effects emulator.

### 5.1. Polarization controller

The polarization controller is an optical component which allows one to modify the polarization state of light. The polarization controller is used to change polarized (or unpolarized) light into any well-defined SOP. Typically, the polarization controller consists of rotated retarders (wave plates). We can distingush the polarization controllers which are based on: two rotated quarter-wave plates, two rotated quarter-wave plates and one rotated half-wave plate or one rotated quarter-wave plate and one rotated half-wave plate. Figure 14 presents structure of polarization controller which is based on two rotated quarter-wave plates and distribution of SOPs at the polarization controller output port.

b


Figure 14. Polarization controller based on two rotated quarter-wave plates; structure of polarization controller (a), output SOPs distribution on the Poincaré sphere (b)

This polarization controller changes an arbitrary SOP into the other arbitrary SOP.
Figure 15 shows structure of polarization controller which is based on two rotated quarterwave plates, one rotated half-wave plate and distribution of SOPs at the polarization controller output port.

This polarization controller is similar to above one. It transforms an arbitrary SOP into the other arbitrary SOP. Finally, Figure 16 presents structure of polarization controller which is based on one rotated quarter-wave plate, one rotated half-wave plate and distribution of SOPs at the polarization controller output port.


Figure 15. Polarization controller based on two rotated quarter-wave plates and one rotated half-wave plate; structure of polarization controller (a), output SOPs distribution on the Poincaré sphere (b)


Figure 16. Polarization controller based on one rotated quarter-wave plate and one rotated half-wave plate; structure of polarization controller (a), output SOPs distribution on the Poincaré sphere (b)

This type of polarization controller only transforms linear polarization into an arbitrary SOP. We would expect flowed operation of this polarization controller with the other input SOP. This case is shown in Figure 17.


Figure 17. Distribution of SOPs at the polarization controller output port for circular input SOP

### 5.2. Polarization attractor

In real fibers the SOPs are not preserved because of the random birefringence. The uncontrolled SOPs variable can dramatically affect the performances of telecommunication systems. This phenomenon is very important, especially for demultiplexing process for Polarization Division Multiplexing transmission system. Possibility of polarization controlling is key issue for modern optical fiber communication technologies. The optical component which can stabilize an arbitrary polarized optical signal by lossless and instantaneous interaction is polarization attractor. This type of controlling the optical signal polarization can be based on: stimulated Brillouin scattering, stimulated Raman scattering or four wave mixing phenomenon. Here we focuse on the stimulated Raman scattering for polarization attraction effect [2, 3]. An arbitrary input SOP of the optical signal is pulled (attracted) by the SOP of the propagating pump (Raman pump), so that at the fiber output the signal SOP is matched the pump SOP. The power evolution of the pump $(\overrightarrow{\mathrm{P}})$ and signal $(\overrightarrow{\mathrm{S}})$ for copumped configuration along the optical fiber link can be modeled by means of coupled equations, respectively [20]:

$$
\begin{align*}
& \frac{d \overrightarrow{\mathrm{P}}}{\mathrm{dz}}=-\alpha_{\mathrm{p}} \overrightarrow{\mathrm{P}}-\frac{\omega_{\mathrm{p}}}{2 \omega_{\mathrm{s}}} \mathrm{~g}_{\mathrm{R}}\left(\mathrm{P}_{0} \overrightarrow{\mathrm{~S}}+\mathrm{S}_{0} \overrightarrow{\mathrm{P}}\right)+\left(\omega_{\mathrm{p}} \overrightarrow{\mathrm{~b}}+\overrightarrow{\mathrm{W}_{\mathrm{p}}^{\mathrm{NL}}}\right) \times \overrightarrow{\mathrm{P}},  \tag{63}\\
& \frac{\mathrm{~d} \overrightarrow{\mathrm{~S}}}{\mathrm{dz}}=-\alpha_{\mathrm{s}} \overrightarrow{\mathrm{~S}}+\frac{1}{2} \mathrm{~g}_{\mathrm{R}}\left(\mathrm{~S}_{0} \overrightarrow{\mathrm{P}}+\mathrm{P}_{0} \overrightarrow{\mathrm{~S}}\right)+\left(\omega_{\mathrm{s}} \overrightarrow{\mathrm{~b}}+\overrightarrow{\mathrm{W}_{\mathrm{s}}^{N L}}\right) \times \overrightarrow{\mathrm{S}}, \tag{64}
\end{align*}
$$

where $\omega_{\mathrm{p}}$ and $\omega_{\mathrm{s}}$ are pump and signal carrier angular frequencies, $\alpha_{\mathrm{p}}$ and $\alpha_{\mathrm{s}}$ are optical fiber attenuation coefficients for the pump and signal wavelengths, respectively. The $\mathrm{g}_{\mathrm{R}}$ component is the Raman gain coefficient. The vector lengths $\mathrm{P}_{0}=|\overrightarrow{\mathrm{P}}|$ and $\mathrm{S}_{0}=|\overrightarrow{\mathrm{S}}|$ represent the pump and signal powers, respectively. The vector $\vec{b}$ is the local linear birefringence vector for optical fiber. The vectors $W_{p}^{N L}$ and $W_{s}^{N L}$ are given by [20]:

$$
\begin{align*}
\overline{\mathrm{W}_{\mathrm{p}}^{\mathrm{NL}}} & =\frac{2}{3} \gamma_{\mathrm{p}}\left(-2 \mathrm{~S}_{\mathrm{S}, 1},-2 \mathrm{~S}_{\mathrm{S}, 2}, \mathrm{~S}_{\mathrm{P}, 3}\right),  \tag{65}\\
\overline{\mathrm{W}_{\mathrm{s}}^{\mathrm{NL}}} & =\frac{2}{3} \gamma_{\mathrm{s}}\left(-2 \mathrm{~S}_{\mathrm{P}, 1},-2 \mathrm{~S}_{\mathrm{P}, 2}, \mathrm{~S}_{\mathrm{S}, 3}\right), \tag{66}
\end{align*}
$$

where $\gamma_{\mathrm{p}}$ and $\gamma_{\mathrm{s}}$ are the nonlinear coefficients, $\mathrm{S}_{\mathrm{P}, 1}, \mathrm{~S}_{\mathrm{P}, 2} \mathrm{~S}_{\mathrm{P}, 3}, \mathrm{~S}_{\mathrm{S}, 1} \mathrm{~S}_{\mathrm{S}, 2} \mathrm{~S}_{\mathrm{S}, 3}$ are the Stokes parameters for the pump and signal, respectively.

The values of polarization attractor parameters (i.e.: pump power, pump SOP) should be accurately selected depending on expected polarization pulling.

Figure 18 shows scheme of polarization attractor based on stimulated Raman scattering.


Figure 18. Polarization attractor based on Raman scattering
Figures 19 and 20 demonstrate simulated examples of polarization pulling effect for pump power equals to $1 \mathrm{~W}, 2 \mathrm{~W}$ and 5 W . The simulated polarization attractor is based on standard single mode optical fiber [21].


Figure 19. Simulated examples of polarization pulling; distribution of polarized signals at the attractor input port (a), distribution of output signal SOPs at the output attractor port for pump power $1 \mathrm{~W}(\mathrm{~b})$


Figure 20. Simulated examples of polarization pulling; distribution of output signal SOPs at the output attractor port for pump power 2 W (a), distribution of output signal SOPs at the output attractor port for pump power 5 W (b)

It should be note that for stimulated Raman scattering the proper Raman polarization pulling and amplification for optical fiber communication systems may be simultaneously achieved.

### 5.3. Polarization scrambler

It is known that, d ue to the random nature of polarization mode coupling in an optical fiber several polarization effects (PMD, PDL, PDG) may occur that lead to impairments in long haul and high bit rate optical fiber transmission systems. Polarization scrambling the states of polarization has been shown to be technique that can reduction polarization impairments or the reduction of measurement uncertainly. A polarization scrambler actively changes the SOPs using polarization modulation method. In generally, the polarization scrambler configuration consists of rotating retarders (wave plates) or phase shifting elements. Furthermore, it is often necessary that the scrambler output SOPs are distributed uniformly on the entire Poincarè sphere. The spherical radial distribution function is very useful tool for the SOPs distribution analysis on the Poincarè sphere [22]. The spherical radial distribution function is the modified form of the well known plane radial distribution function. The spherical radial distribution function is defined as follows [22]:

$$
\begin{equation*}
\mathrm{g}(\mathrm{~d})=\frac{\mathrm{K}(\mathrm{~d}) \mathrm{A}_{\mathrm{T}}}{\mathrm{~A}(\mathrm{~d}) \mathrm{K}_{\mathrm{T}}} \tag{67}
\end{equation*}
$$

where $K(d)$ is the total number of pairs of the points separated by a given range of radial distances ( $d, d+\Delta d$ ), $A(d)$ is area of the sphere between two circles $c_{d}$ and $c_{d+\Delta d}, K_{T}$ is the total number of pairs of the points on the sphere; $\mathrm{K}_{\mathrm{T}}$ is equal to $\mathrm{N}^{2}-\mathrm{N}$, where N is number of points on the sphere, $\mathrm{A}_{\mathrm{T}}$ is area of the sphere (Figure 21).


Figure 21. Example of spherical radial distribution function calculation for one reference point
The value of radial distance $d$ changes from 0 to $\pi-\Delta d$, step is equal to $\Delta d$. The "great circle" distance between two points $n_{n}$ and $n_{k}$ (Figure 21), whose coordinates are $\left(\Theta_{n}, \phi_{\mathrm{n}}\right)$ and ( $\left.\Theta_{\mathrm{k}}, \phi_{\mathrm{k}}\right)$, is given by so called Haversine formula:

$$
\begin{equation*}
d_{n, k}=2 R \arcsin \left(\sqrt{\sin ^{2}\left(\frac{\Theta_{\mathrm{n}}-\Theta_{\mathrm{k}}}{2}\right)+\sin \left(\Theta_{\mathrm{n}}\right) \sin \left(\Theta_{\mathrm{k}}\right) \sin ^{2}\left(\frac{\phi_{\mathrm{k}}-\phi_{\mathrm{n}}}{2}\right)}\right) \tag{68}
\end{equation*}
$$

where $R$ is the sphere radius.
We can distinguish three typical theoretical distributions: uniform (Figure 22a), random (Figure 23a) and clustered (Figure 24a). In turn, Figures 22b, 23b and 24b show the spherical radial distribution as a function of the distance d for the uniform, random and clustered distribution, respectively [22].


Figure 22. Theoretical uniform distribution; SOPs distribution on the Poincarè sphere (a), spherical radial distribution function versus radial distance (b)


Figure 23. Theoretical random distribution; SOPs distribution on the Poincarè sphere (a), spherical radial distribution function versus radial distance (b)
a

b


Figure 24. Theoretical clustered distribution; SOPs distribution on the Poincarè sphere (a), spherical radial distribution function versus radial distance (b)

For the uniform distribution (Figure 22b) the peaks on the $g(d)$ curve provides information about the mean distance of the following neighbouring points (SOPs) on the Poincarè sphere. In the case of the random distribution (Figure 23b) the value of $g(d)$ is close to 1 . For the clustered distribution (Figure 24b) the localization of the first minimum on the $g(d)$ curve provide information about the mean dimension (diameter) of the clusters. The location of the first lower peaks on the curve indicates the mean distance between clusters.

The analysis of the distribution of SOPs generated by polarization scramblers shows that SOPs distribution is clustered for three (and less) rotating retarders and for four (and less) phase shifting elements. For four and more rotating retarders and for five and more phase shifting elements random distribution is obtained [22].

### 5.4. Polarization effects emulator

You know well that polarization effects due to interaction between PMD and PDL can significantly impair optical fiber transmission systems. When PMD and PDL are both present they interact must be studied together. Emulating of PMD and PDL is one way to test and verify new transmission systems in the presence of PMD and PDL effects. Polarization effects emulation play a useful role, since it is possible to examine a large ensemble of system states far more rapidly than in a test bed with commercially available fiber optics. The polarization effects emulation devices can be split into two groups: statistical and deterministic emulators. Devices which are intended to mimic the random statistical behavior of a long single mode fiber links are termed as statistical emulators. Statistical polarization effect emulators should accurately reproduce the statistics of the polarization effect that a signal would see on a real link, as well as have good stability and repeatability. Devices that map the polarization effect space to the emulator settings and predictably generate the desired values are generally termed as deterministic emulators.

We typically have PMD and PDL statistical emulators and PDL deterministic emulators. Through pure statistical nature of the PMD effect, the PMD deterministic emulators should not be used for an optical communication systems testing.

Each statistical emulator that realistically simulates real optical fiber links should fulfil two criteria [6]:

1. Differential Group Delay should be Maxwellian distributed at any fixed optical frequency. This condition is also valid for the PDL effect. In the absence of the PMD the PDL distribution is Rayleigh distribution. But in the presence of PMD the PDL distribution is closed to Maxwellian distribution. Thus the PDL distribution in real optical fiber links can be also approximated by a Maxwellian function.
2. Frequency AutoCorrelation Function (ACF) of PMD and PDL vectors should tend toward zero as the frequency separation increases; so called Autocorrelation Function Background (BAC) should be lower than 10 \%. Autocorrelation Function Background is defined as the mean absolute deviation of the ACF from the expected (mean) value of ACF for the frequencies larger than the autocorrelation bandwidth where the frequency autocorrelation bandwidth of the ACF is the frequency at the half of the variation of the ACF.

In [23] is demonstrated that 15 rotated polarization elements (e.g. sections of polarization maintaining fiber) realistically simulate the DGD and PDL distribution and BAC of real optical fiber links. Below, results for statistical emulator consisting of 15 rotated polarization elements. Figure 25 shows the histograms of DGD and PDL. For the statistical PMD emulator the DGD distribution is always indistinguishable from theoretical Maxwellian distribution. In turn, the PDL distribution (in the presence of PMD) is similar to Maxwellian distribution [24].


Figure 25. Statistical distribution of DGD values (a) and PDL values (b)
The theoretical and normalized ACF for both PMD and PDL vectors is shown in Figure 26.
Now, coming to to the PDL emulator, the simplest deterministic PDL emulator consists of tilted glass plate. The transmission coeffcients of a dielectric surface between two media were derived by Fresnel. They field the orthogonal component i.e. perpendicular coeffcient ( $\mathrm{T}_{\mathrm{s}}$ ) and parallel coeffcient $\left(T_{p}\right)$ to the plane of incidence [25]:


Figure 26. Theoretical normalized frequency autocorrelation function for PMD

$$
\begin{align*}
& \mathrm{T}_{\mathrm{s}}=\frac{\mathrm{n}_{\mathrm{s}} \cdot \cos \left(\xi_{\mathrm{t}}\right)}{\cos \left(\xi_{\mathrm{i}}\right)} \cdot\left|\mathrm{t}_{\mathrm{s}}\right|^{2},  \tag{69}\\
& \mathrm{~T}_{\mathrm{p}}=\frac{\mathrm{n}_{\mathrm{s}} \cdot \cos \left(\xi_{\mathrm{t}}\right)}{\cos \left(\xi_{\mathrm{i}}\right)} \cdot\left|\mathrm{t}_{\mathrm{p}}\right|^{2}, \tag{70}
\end{align*}
$$

where:

$$
\begin{align*}
\mathrm{t}_{\mathrm{s}} & =\frac{2 \cos \left(\xi_{\mathrm{i}}\right)}{\cos \left(\xi_{\mathrm{i}}\right)+\mathrm{n}_{\mathrm{s}} \cdot \cos \left(\xi_{\mathrm{t}}\right)^{\prime}},  \tag{71}\\
\mathrm{t}_{\mathrm{p}} & =\frac{2 \cos \left(\xi_{\mathrm{i}}\right)}{\cos \left(\xi_{\mathrm{t}}\right)+\mathrm{n}_{\mathrm{s}} \cdot \cos \left(\xi_{\mathrm{i}}\right)^{\prime}},  \tag{72}\\
\quad \xi_{\mathrm{t}} & =\arcsin \left(\frac{\sin \left(\xi_{\mathrm{i}}\right)}{\mathrm{n}_{\mathrm{s}}}\right) \tag{73}
\end{align*}
$$

For above equations (69-73): angle $\xi_{\mathrm{i}}$ is the angle of incidence, angle $\xi_{\mathrm{t}}$ is the angle of refraction, $\mathrm{n}_{\mathrm{s}}$ is the index of glass refraction. We assume that the index of air refraction is equal to 1 .

Next, the PDL value is given by the following relation:

$$
\begin{equation*}
\mathrm{PDL}[\mathrm{~dB}]=10 \log _{10}\left(\frac{\mathrm{~T}_{\mathrm{s}}}{\mathrm{~T}_{\mathrm{p}}}\right) \tag{74}
\end{equation*}
$$

The PDL value is strong dependent on the angle of incidence. Figure 27 presents PDL in function of this angle. These PDL values are typically for some optical components which are used for optical fiber communication technologies.

PDL [dB]


Figure 27. Polarization Dependent Loss value versus the angle of incidence; $\mathrm{n}_{\mathrm{s}}=1.75$
In conclusion, polarization issues become very important especially for long haul and high bit rate lightwave communication systems for which polarization effects, first of all, polarization mode dispersion and polarization dependent loss, become limiting factor. Optical fiber polarization phenomena must be taken into account during planning, installing and monitoring optical fiber communication systems. Additionally, the fast evolution of optical fiber transmission technologies requires powerful analysis and testing tools that must provide information about all relevant polarization phenomena in optical fiber links.

## Author details

Krzysztof Perlicki ${ }^{1,2^{*}}$
Address all correspondence to: perlicki@tele.pw.edu.pl
1 Institute of Telecommunications, Warsaw University of Technology, Warsaw, Poland
2 Orange Labs, Orange Polska, Warsaw, Poland

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