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## Chapter 18

# Eigenvalue Analysis in Mode Domain <br> Considering a Three-Phase System with two Ground Wires 

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Additional information is available at the end of the chapter
http://dx.doi.org/10.5772/58383

## 1. Introduction

The analysis of a transmission line allows a complex study that can be used to measure losses in line, to understand the behavior of them in case of a voltage surge and other kinds of phenomena.

This chapter presents the development of a routine that evaluates a method for determining real transformation matrices in three-phase systems considering the presence of ground-wires. Thus, for Z (longitudinal impedance) and $Y$ (transversal admittance) matrices that represent the transmission line, the ground wires are considered not implicit in the values of the phases. This routine was developed using the mathematical tool Matlab ${ }^{\mathrm{TM}}$.

As a proposal, the routine uses a real transformation matrix throughout the frequency range of analysis. This transformation matrix is an approximation of the exact transformation matrix. For elements related to the phase of the system considered, the transformation matrix is composed from the elements of the Clarke's matrix [18].

In parts related to ground wires, the elements of the transformation matrix must establish a relationship with the elements of the phases considering the establishment of a unique, single homopolar reference in mode domain.

In case of three-phase transmission lines with the presence of two ground wires, it is not possible to obtain the complete diagonalization of Y and Z matrices in mode domain. Finally, a correction routine is applied with the goal of minimizing errors obtained for the eigenvalues.

## 2. Background

Different methods can be used in order to analyze electromagnetic transient phenomena in transmission lines. Many mathematical tools can be used. The main tools are: circuit analysis using Laplace transform and Fourier transform, state variables and also differential equations. These tools can be included in a numerical routine in order to obtain the values of voltage and current in electromagnetic transient simulation for any point in the circuit.

The EMTP (ElectroMagnetic Transient Program) identifies a type of program, considering its various versions, which performs simulations of transients in electrical networks [1]. The prototype was developed in the 1960s by professionals in power systems, led by Dr. Hermann Dommel (University of British Columbia, Vancouver, BC, Canada), and Dr. Scott Meyer (Bonneville Power Administration in Portland, Oregon, USA). Currently, the EMTP is the basis of simulations of electromagnetic transients in power systems.

With EMTP-type programs, the following tests may be performed: simulation by switching and lightning surges, transient and temporary overvoltages, transients in electric machines, resonance phenomena, harmonics, power quality and power electronics applications. The most popular type programs are EMTP: MicroTran, PSCAD and ATP.

In analysis of transmission systems, there are simulators that represent different types of systems, from generation, transmission and distribution.

Because it is practically impossible to perform the simulation of electromagnetic transients on real transmission lines, simulations by digital models become useful tools. However, these tools do not provide satisfactory performance as regards the correct representation of the electrical line parameters, as these are dependent on the frequency.

In modal domain, $\mathrm{it}^{\prime}$ 's possible to represent the transmission line circuits using simple circuits and easily entering frequency dependence of longitudinal parameters.

In general, a system composed of $n$-phases can be transformed into independent modes using a real and unique transformation matrix, if transposition applies to all phases for the frequency range used (ideal transposition). If the analyzed system is not transposed, a mode is obtained for each phase using the frequency dependent phase-mode transformation matrix.

Applying real and unique transformation matrix for the nontransposed lines, approximate results can be obtained. Thus, there is an approximate representation of frequency dependence using a real phase-mode transformation matrix [2]-[3]. One possible simplification is to consider the transformation matrix frequency independent, obtaining insignificant errors related to the eigenvalues that represent the line. Using the mentioned simplification, the obtained numerical routine may be faster because it avoids using a convolution method [4]-[28].

The objective of this chapter is to analyze the application of a real transformation matrix that is frequency independent in three-phase lines considering the presence of two ground wires. Errors are presented in relation to the exact values obtained from the matrix of eigenvalues.

The proposed model is based on an approximate modal transformation performed by a single real phase-mode transformation matrix, and frequency independent. This matrix is obtained by linear combination of Clarke's matrix elements. With the implementation of the transformation matrix frequency independent, it's obtained a diagonal matrix for transposed lines. In case of a three-phase transmission line not transposed the line parameters matrix can't be diagonalized, with application of a single real phase-mode transformation matrix mentioned. For these cases, the goal is to analyze the relative errors obtained by the establishment circumstances to use a transformation matrix frequency independent.

For the proposed method in this chapter, a similar mathematical basis is used to homopolar hypothesis of a single reference for all phases of the system regardless of the geometrical distribution and organization of the three-phase circuit. Thus, the development is based on the analysis of eigenvectors and eigenvalues, using a linear combination of Clarke's matrix elements, and assuming a unique homopolar reference.

It's presented two different proposals for real matrices and frequency independent in order to replace the modal transformation matrix of a typical three-phase transmission line in the presence of two ground-wires.

## 3. Mathematical model

A transmission line is represented by a longitudinal impedance Z and the transversal admittance Y matrices, the characteristics of ground wires are not implied in the values related to the phases. Thus, for a line with two ground wires, the Z and Y matrices are 5-order ones.

For the mentioned systems, the three-phase circuit configurations are considered and the transposed case can be described by a system where each three-phase circuit is ideally transposed and there are coupling impedances and admittances among the three-phase circuits. For EMTP type programs, if each three-phase circuit is considered independently, the transformation matrix is frequency dependent for general cases. For the method proposed in this chapter, a similar mathematical base is used for all considered cases: the assumption of unique ground reference for all phases of the system independently of the geometrical distribution and the organization of the three-phase circuits. The unique ground reference leads to a unique homopolar mode in mode domain.

The relationships between transversal voltages $u_{F}$ and the longitudinal currents $i_{F}$ can be expressed by the following equations, where Z is the longitudinal impedance matrix per unit length and $Y$ is the transversal admittance matrix per unit length in phase domain.

$$
\begin{equation*}
-\frac{d u_{F}}{d x}=Z \cdot i_{F} \text { and }-\frac{d i_{F}}{d x}=Y \cdot u_{F} \tag{1}
\end{equation*}
$$

Applying the eigenvector and eigenvalue analyses for YZ and ZY product matrices, the $\lambda$ diagonal eigenvalue matrix and the eigenvector matrices are determined. The eigenvector
matrices, $T_{V}$ and $T_{I}$, correspond to voltage and current mathematical relationships, respectively. The $T_{V}$ and $T_{I}$ matrices are related to $\lambda$ based on the following equation:

$$
\begin{equation*}
\lambda=T_{V} \cdot Z \cdot Y \cdot T_{V}^{-1}=T_{I} \cdot Y \cdot Z \cdot T_{I}^{-1} \tag{2}
\end{equation*}
$$

If the $T_{V}$ and $T_{I}$ transformation matrices are used, the eigenvalues can be obtained in mode domain using (1). The per unit length longitudinal impedance matrix $\left(Z_{M D}\right)$ and transversal admittance matrix $\left(Y_{M D}\right)$ are:

$$
\begin{equation*}
Z_{M D}=T_{V} \cdot Z \cdot T_{I}^{-1} \text { and } Y_{M D}=T_{I} \cdot Y \cdot T_{V}^{-1} \tag{3}
\end{equation*}
$$

In general, these frequency dependent transformation matrices ( $T_{V}$ and $T_{I}$ ) are different and have complex elements. Using the proposed methodology, the transformation matrices are changed into a single real transformation matrix $\left(T_{S R}\right)$. The $T_{S R}$ matrix is determined from linear combinations of the Clarke's matrix elements [4]-[7]. The determination of exact eigenvalues is approximated and changed into the following:

$$
\begin{equation*}
\lambda_{S R}=T_{S R} \cdot Z \cdot Y \cdot T_{S R}^{-1}=T_{S R} \cdot Y \cdot Z \cdot T_{S R}^{-1} \tag{4}
\end{equation*}
$$

In case of the EMTP type programs, the transformation matrices are real, if the system is ideally transposed. For this, there is only one self-impedance value for all phase interactions. Considering the admittance values, a similar structure to the impedance values is obtained. Applying the EMTP type programs, a system composed by three-phase circuits is analyzed as a non-transposed case, if the each three-phase circuit is considered transposed independently of the ground wires.

Using a single homopolar mode reference, the $\lambda_{S R}$ matrix is equal to the exact eigenvalue matrix ( $\lambda$ ) [8] as well as $T_{V}$ and $T_{I}$ being equal to a single real transformation matrix for transposed cases [9]-[11]. So, with a single homopolar mode reference, there is a link between the three-phase circuit and the ground wires of the system. With this technique, a transformation matrix $\left(T_{S R}\right)$ is obtained which has interesting characteristics: single, real, frequency independent, line parameter independent and identical to voltages and currents. With these characteristics, phase-mode transformations are carried out using only one matrix multiplication.

The homopolar or zero sequence components ( $V_{a 0}, V_{b 0}$ and $V_{c 0}$ ) for a three-phase system are equal and they make the unique ground reference for the three-order phasor system. Using the homopolar reference phasor concept, the application of a single mode reference to the single real phase-mode transformation matrix is proposed. So, the homopolar mode is used as the only mode reference for the analyzed transmission line systems. To compose the $T_{S R}$ matrix, each mode must have a unitary modulus. Because of this, each homopolar mode
element value depends on the number of phase conductors. If this number is identified by $n$, the homopolar mode elements are described by (6).

$$
\begin{equation*}
T_{S R-n}=\frac{1}{\sqrt{n}} \tag{5}
\end{equation*}
$$

## 4. Ground wires in three-phase transmission line

Considering two ground wires in a three-phase lines system, these matrices are 5-order ones. Therefore, the single real phase-mode transformation matrix has the following structure presented in (6).

$$
T_{S R 5}=\left[\begin{array}{ccccc}
-\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0  \tag{6}\\
\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\
\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\
\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\
0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{array}\right]
$$

In case of a three-phase lines system is ideally transposed, it creates only one coupling impedance between the lines. The average self-phase impedance value is represented by A. The average coupling impedances are represented by B, within a circuit, and C, between the line and ground wires or other circuits. The average ground wires impedance value is represented by D. The average coupling impedance between the both ground wires is represented by E. For the case of a single three-phase line in the presence of two ground wires (Fig. 1), the structure of the impedance matrix is shown in (7).

$$
Z_{5}=\left[\begin{array}{lllll}
A & B & B & C & C  \tag{7}\\
B & A & B & C & C \\
B & B & A & C & C \\
C & C & C & D & E \\
C & C & C & E & D
\end{array}\right]
$$

The three-phase transmission line circuit tower (Fig. 2) has a height of 36.0 m and is the structure used in this chapter. This is a 400 km length line that operates in 440 kV . It is a system whose conductors are disposed in such way that there is a vertical symmetry plane.


Figure 1. Coupling impedances for a three-phase transmission line with two ground wires for transposed cases.


Figure 2. Three phase line tower with two ground wires
The result determined through (4) is a diagonal matrix and the matrix elements are the exact eigenvalues, for the cases where the ground wires are implicit in the phase values and the line is transposed. The impedance matrix in mode domain $\left(Z_{M}\right)$ can be calculated as:

$$
\begin{equation*}
Z_{M}=T_{S R} \cdot Z \cdot T_{S R}^{-1} \tag{8}
\end{equation*}
$$

Considering ground wires, the single real phase-mode transformation matrix does not perfectly diagonalize the impedance matrix.


A routine with initial configuration was developed in order to set parameters that would be used for other routines. This routine was developed using Matlab and is presented below:

```
initial_conf.m
%Three-phase line with vertical symmetry
ntpc = 1; %three-phase circuit amount
ngw = 2; %ground wires amount
ncond = ntpc*3+ngw; %conductors amount
frequency %subroutine to call an array of frequencies from 0 to 1 GHz
%Conductors' position in x axis (in meters)
xc(1)= 0;
xc(2)= 9.27;
xc(3)= 18.54;
xc(4)= 1.76; %ground wire 1
xc(5)= 16.78; %ground wire 2
%Conductors' position in y axis (in meters)
yc(1)= 24.07-0.7*13.43; %the subtracted value is the sag of each wire
yc(2)= 27.67-0.7*13.43;
yc(3)=24.07-0.7*13.43;
yc(4)=36.00-0.7*6.40;
yc(5)= 36.00-0.7*6.40;
%Radius (in meters)
rc = 2.52e-2; %total radius
rin = 0.93e-2; %internal radius
r_dist = 0.4; %distance between conductors
nph = 4; %number of conductors per phase
Arg = rc*r_dist^3*sqrt(2); %argument of root
GMR = power(Arg,1/nph); %geometric mean radius
radius = GMR;
sk_radius = rc-rin; %for skin effect procedure
gw_radius = 0.9144e-2; %ground wire radius
resist = 1000; %Earth resistance (ohms x meters)
mizero = 4*pi*1e-7; %magnetic permeability (H/m)
epszero = 8.8542*1e-9; %dielectric permittivity (F/km)
sigma= 3.82*1e7; %conductor's conductivity (mho/m)
```


## 6. Eigenvalue analyses for non-transposed three-phase transmission line

Considering the three-phase line tower with two ground wires shown in Fig. 2, the total longitudinal impedance value is composed by earth effect, calculated by Carson`s method [28], external effects and skin effect. For phase 1, the longitudinal resistance is shown in Fig. 3 (a), the longitudinal inductance in Fig. 3 (b) and capacitance in Fig. 3 (c).


Figure 3. (a) Longitudinal Resistance, (b) Longitudinal inductance, (c) Capacitance : phase 1.
The longitudinal resistance and inductance and the capacitance of phase 1 were obtained using a routine developed in MatLab, which is commented and shown below:

```
external_impedance.m
clear all
'Calculating Z due to external effect'
%File Reading (calling subroutine of initial configuration)
```

```
initial_conf
for j = 1:ncond,
xi(j) = xc(j);
yi(j) = -yc(j);
for k = j:ncond
    str = ['Zext' int2str(j) '_' int2str(k) '.m'];
    fid(j,k) = fopen(str,'w');
        if j==k
    str = ['%% External impedance of phase ' int2str(j) ' \n clear x\n x = [\n'];
    fprintf(fid(j,k),str);
    else
    str = ['%% External impedance between phase ' int2str(j) ' and ' int2str(k) '
\n clear x\n x = [\n'];
    fprintf(fid(j,k),str);
    end
end
end
%Image conductor coordinates
for j = 1: ncond,
    xi(j) = xc(j);
    yi(j) = -yc(j);
end
%External inductance (H/km)
for j = 1:ncond,
    for k = j:ncond,
        if j == k
            if j < ntpc*3+1
    ld = radius;
    else
    ld = gw_radius;
    end
        else
        dx = xc(j) - xc(k);
    dy = yc(j) - yc(k);
    ld = sqrt(dx^2 + dy^2);
        end
        dx = xcondut(j) - xi(k);
    dy = ycondut(j) - yi(k);
    bd = sqrt(dx^2 + dy^2);
        induct(j,k) = 1000*(1/(2*pi))*mizero*log(dezao/dezinho);
    end
end
for j = 1:length(freq),
        for k = 1:ncond
            for m = k:ncond
                z = i*2*pi*freq(j)*induct (k,m);
        fprintf(fid(k,m), '%30.20f %30.20f\n',real(z),imag(z));
            end
        end
```

```
end
for j = 1:ncond
    for k = j:ncond
    fprintf(fid(j,k), ']; \n');
    str = ['ze' int2str(j) '_' int2str(k) ' = x(:,1) + i*x(:,2);'];
    fprintf(fid(j,k), str);
        end
end
fclose('all');
z_skins.m
%Internal impedance (skin effect)
clear all
'Calculatin Z due to skin effect'
%File reading
initial_conf
fid10 = fopen('zskin.m','w')
fprintf(fid10, '%% Internal impedance \n clear x\n x = [\n');
%=====================================================================
%Bessel Formula
%===============================================================
%mi = mizero/1000 ;
%radiuso = raio/1000 ;
for j = 1:length(freq),
    m = sqrt(i*2*pi*freq(j)*mizero*sigma);
    mr = sk_radius * sqrt(i*2*pi*freq(j)*mizero*sigma);
    IO = BESSELI(0,(mr),1);
        I1 = BESSELI(1, (mr),1);
        %Impedance calculus (number 4 appears because there are 4 subconctors)
    z = (1/4)*1000*((1/sigma)*m)/(2*pi*sk_radius)*(I0/I1);
    fprintf(fid10, '%30.20f %30.20f\n',real(z),imag(z));
end
fprintf(fidl0, ']; \n');
fprintf(fid10, 'zskin = x(:,1) + x(:,2)*i;');
fclose('all');
z_carson.m
%Impedance due to earth effect (Carson's method)
clear all
'Calculating Z due to earth effect - Carson's method
%File reading
initial_conf
nt = 120; %amount of terms to be used (number multiple of 4)
v = -1; %variable used for signal
```

```
for j = 1:ncond,
    xi(j) = xc(j);
    yi(j) = -yc(j);
        for k = j:ncond
    str = ['Zcarson' int2str(j) '_' int2str(k) '.m'];
        fid(j,k) = fopen(str,'w');
    if j==k
    str = ['%% Phase impedance ' int2str(j)' due to earth effect \n clear x\n x
= [\n'];
    fprintf(fid(j,k),str);
    else
    str = ['%% Impedance between phases ' int2str(j) ' and ' int2str(k) ' due to
Earth effect \n clear x\n x = [\n'];
    fprintf(fid(j,k),str);
        end
        end
end
for j = 1:ncond,
        for k = j:ncond,
            dx = xc(j) - xi(k);
    dy = yc(j) - yi(k);
    bd(j,k) = sqre( dx^2 + dy^2);
        cat = yc(j) + yc(k);
    cossine = cat/bd(j,k);
    ang(j,k) = acos(cossine);
    end
end
% Calculus of terms b, c and d of Carson's series
%signal change each 4 terms
n = 0;
for j = 1:nt/4,
        v}=-\textrm{v}
        for k = 1:4,
    n = n+1;
    signal_b(n) = v;
        end
end
%Calculus of bi element
b(1) = sqrt (2)/6;
b(2) = 1/16;
for j = 3:nt,
    b(j)= abs(b(j-2))*(1/(j*(j+2)))*signal_b(j);
end
%Calculus of ci element
c(2) = 1.3659315;
```

```
for j = 4:nt,
    c(j) = c(j-2) + (1/j) + (1/(j+2));
end
%Calculus of di element
d = (pi/4)*b;
%*******************************************************************************
for f = 1:length(freq),
        for j = 1:ncond,
            for k = j:ncond,
                phi = ang(j,k);
    a = sqrt(mizero*2*pi*freq(f)/resist)*bd(j,k);
                subrotine_carson_delta_r;
                subrotine_carson_delta_x;
                fprintf(fid(j,k), '%30.20f %30.20f\n',delta_r(j,k), delta_x(j,k));
    end
    end
end
for j = 1:ncond,
        for k = j:ncond,
    fprintf(fid(j,k), ']; \n');
    fprintf(fid(j,k), ['zsolo' int2str(j) '_' int2str(k) ' = x(:,1) + i*x(:,
2);']);
        end
end
fclose('all');
subrotine_carson_delta_r.m
if a < 5,
%'ok'
    r1 = b(1)*a*cos(phi);
    for nj = 1:(nt/4) -1,
        term1 = b (4*nj +1)* (a^(4*nj +1))* cos((4*nj + 1)*phi);
    r1 = term1 + r1;
end
    parc1 = (c(2) - log(a))*(a^2)* cos(2*phi);
parc2 = (phi*(a^2)*sin(2*phi));
    r2 = b(2)* (parc1 + parc2);
        for nj = 1:(nt/4) -1,
            parc1 = (c(4*nj +2) - log(a))*(a^(4*nj +2))* cos((4*nj +2)*phi);
parc2 = (phi*(a^(4*nj +2))*sin((4*nj +2)*phi));
    r2 = r2 + b(4*nj +2)*(parc1 + parc2);
end
    r3 = b (3)* (a^3)* cos(3*phi);
    for nj = 1:(nt/4) -1,
            term3 = b(4*nj + 3)*(a^(4*nj + 3))* cos((4*nj + 3)*phi);
r3 = r3 + term3;
    end
r4 = d(4)*(a^4)*\operatorname{cos(4*phi);}
for nj = 1:(nt/4) -1,
        term4 = d(4*nj + 4)*(a^(4*nj + 4))*\operatorname{cos}((4*nj + 4)*phi);
```

```
    r4 = r4 + term4;
    end
    delta_r(j,k) = 4*2*pi*freq(f)*(1e-4)*((pi/8) - r1 + r2 + r3 - r4);
else
    t1 = cos(phi)/a;
    t2 = sqrt(2)* cos(2*phi)/(a^2);
    t3 = cos(3*phi)/(a^3);
    t4 = 3*}\operatorname{cos(5*phi)/(a^5);
    t5 = 5* cos(7*phi)/(a^7);
    delta_r (j,k) = (4*2*pi*freq(f)*(1e-4)/sqrt(2))*(t1 -t2 + t3 +t4 +t5);
end
subrotine_carson_delta_r.m
%Carson's series to calculate reactance of conductors due to Earth effect
if a < 5,
    x1 = b(1)*a*cos(phi);
    for nj = 1:(nt/4) -1,
        term1 = b (4*nj +1)* (a^(4*nj +1))*\operatorname{cos}((4*nj + 1)*phi);
    x1 = term1 + x1;
    end
    x2 = d(2)* (a^2)* cos(2*phi);
    for nj = 1:(nt/4) -1,
        term2 = d(4*nj + 2)*(a^(4*nj + 2))* cos((4*nj + 2)*phi);
        x2 = x2 + term2;
    end
    x3 = b (3)* (a^3)* cos(3*phi);
    for nj = 1:(nt/4) -1,
        term3 = b(4*nj + 3)*(a^(4*nj + 3))* cos((4*nj + 3)*phi);
    x3 = x3 + term3;
    end
    term4 = (c(4) - log(a))*(a^4)* cos(4*phi) + (phi)*(a^4)*sin(4*phi);
    x4 = b(4)*term4;
    for nj = 1:(nt/4) -1,
            term4 = (c(4*nj + 4) - log(a))*(a^(4*nj + 4))*\operatorname{cos}((4*nj + 4)*phi) +
(phi)*(a^(4*nj + 4))*sin((4*nj + 4)*phi);
            x4 = x4 + b(4*nj + 4)*term4;
    end
    delta_x(j,k) = 4*2*pi*freq(f)* (1e-4)*(0.5*(0.6159315 - log(a)) + x1 - x2 + x3
- x4);
else
    t1 = cos(phi)/a;
    t2 = sqrt (2)* cos(2*phi)/(a^2);
    t3 = cos(3*phi)/(a^3);
    t4 = 3* cos(5*phi)/(a^5);
    t5 = 5* cos(7*phi)/(a^7);
    delta_x(j,k) = (4*2*pi*freq(f)*(1e-4)/sqrt(2))*(t1 - t3 + t4 + t5);
end
```

The theoretical procedure of routines presented above can be fully found in [29]-[30]. The above routines show how useful Matlab is in order to perform calculus and link routines. It's easy to note that the routines are simple to implement and to understand.

The principal point to be observed is that all the data generated by the routines are stored into m -files to be used by other routines. To perform this action, first it's necessary to open a file (if doesn't exist, it'll be automatically created) and link with a variable, for this operation, function fopen shall be used, the arguments of this function are the name of file and the kind of action to be performed by the file, in this case, $w$ was used in order to write in the file. To write in the file, the function fprintf must be used, the arguments are the name of file, the text (which can be variables of decimal point (\%d), float point (\%f), and so forth, followed by the variables name. Finally, it's necessary to close the file, with the function fclose. In this case the argument all is used in order to close all opened files.

As the procedure for calculus in all impedances cases shown above are the same, a lasso function can be used in order to make the routine shorter. Thus, structure for is implemented together structure if in order to make a loop and to decide what kind of operation shall be performed.

In the z_skins.m file it's noted the use of Bessel function; this is a special function which can be found in any advanced mathematical calculus. This function is used in order to calculate the impedance due to skin effect, as described in [29].

```
calc_capacitance.m
clear all
'Calculating capacitances'
%File reading
initial_conf
fid10 = fopen('capacitance.m','w');
fprintf(fid10, '%% Capacitances \n clear x\n cap = [\n');
%Image conductors coordinates
for j = 1: ncond,
    xi(j) = xc(j);
    yi(j) = -yc(j);
end
%Potential matrix coefficients
for j = 1:ncond,
    for k = j:ncond,
        if j == k
            if j < ntpc*3+1
        ld = radius;
            else
    ld = gw_radius;
    end
        else
            dx = xc(j) - xc(k);
dy = yc(j) - yc(k);
ld = sqrt(dx^2 + dy^2);
    end
        dx = xc(j) - xi(k);
dy = yc(j) - yi(k);
bd = sqrt(dx^2 + dy^2);
        pot(j,k) = log(bd/ld);
```

```
            pot(k,j) = pot(j,k);
        end
end
cap = 2*pi*epslonzero*(inv(pot));
for j = 1:ncond,
    for k = 1:ncond,
    fprintf(fid10, '%30.20f ',cap(j,k));
        end
    fprintf(fid10, '\n');
end
fprintf(fidl0, ']; \n');
fclose('all');
```

Evaluation of proposed real transformation matrix
A first analysis is based on Eq. (9). Through iterative process the exact eigenvectors and eigenvalues, and also the eigenvectors and eigenvalues, from Clark`s matrix are calculated. At the end of process the relative difference is calculated for each mode with Eq. (10).

$$
\begin{equation*}
\operatorname{err}(\%)=\frac{\lambda_{c l}-\lambda_{e x}}{\lambda_{e x}} \cdot 100 \tag{10}
\end{equation*}
$$



Figure 4. Relative differences between the exact modes and the quasi-modes.
The relative difference between modes $\alpha, \beta, 0$ and $\delta$ is relatively low (Fig. 4 (a)), however, the relative difference of mode $\gamma$ is high (Fig. 4 (b)). To minimize the error shown for mode 4 , a correction procedure for non-transposed three-phase transmission line cases [31]-[35] shall be used in a future work.

In order to verify the limits of this method, a frequency range from $10[\mathrm{~Hz}]$ to $1[\mathrm{GHz}]$ was applied, from the results of resistance, inductance and capacitance obtained, it was verified that the method could converge until $1[\mathrm{MHz}]$, after this range the method is not valid and need a new approach will be requested.

The numeric routine used to obtain the results shown above was developed with Matlab and is described below in details:

```
external_impedance.m
clear all
'Calculating Z due to external effect'
%File Reading (calling subroutine of initial configuration)
initial_conf
for j = 1:ncond,
xi(j) = xc(j);
yi(j) = -yc(j);
for k = j:ncond
    str = ['Zext' int2str(j) '_' int2str(k) '.m'];
    fid(j,k) = fopen(str,'w');
    if j==k
    str = ['%% External impedance of phase ' int2str(j) ' \n clear x\n x = [\n'];
    fprintf(fid(j,k),str);
        else
    str = ['%% External impedance between phase ' int2str(j) ' and ' int2str(k) '
\n clear x\n x = [\n'];
    fprintf(fid(j,k),str);
    end
end
end
%Image conductor coordinates
for j = 1: ncond,
    xi(j) = xc(j);
    yi(j) = -yc(j);
end
%External inductance (H/km)
for j = 1:ncond,
    for k = j:ncond,
        if j == k
        if j < ntpc*3+1
ld = radius;
else
    ld = gw_radius;
    end
        else
            dx = xc(j) - xc(k);
dy = yc(j) - yc(k);
ld = sqrt(dx^2 + dy^2);
    end
    dx = xcondut(j) - xi(k);
dy = ycondut(j) - yi(k);
bd = sqrt(dx^2 + dy^2);
    induct(j,k) = 1000*(1/(2*pi))*mizero*log(dezao/dezinho);
end
```

```
end
for j = 1:length(freq),
        for k = 1:ncond
        for m = k:ncond
            z = i*2*pi*freq(j)*induct (k,m);
    fprintf(fid(k,m), '%30.20f %30.20f\n',real(z),imag(z));
        end
        end
end
for j = 1:ncond
    for k = j:ncond
    fprintf(fid(j,k), ']; \n');
    str = ['ze' int2str(j) '_' int2str(k) ' = x(:,1) + i*x(:,2);'];
    fprintf(fid(j,k), str);
        end
end
fclose('all');
z_skins.m
%Internal impedance (skin effect)
clear all
'Calculatin Z due to skin effect'
%File reading
initial_conf
fid10 = fopen('zskin.m','w');
fprintf(fid10, '%% Internal impedance \n clear x\n x = [\n');
%================================================================
%Bessel Formula
%=============================================================
%mi = mizero/1000 ;
%radiuso = raio/1000 ;
for j = 1:length(freq),
    m = sqrt(i*2*pi*freq(j)*mizero*sigma);
mr = sk_radius * sqrt(i*2*pi*freq(j)*mizero*sigma);
    IO = BESSELI (0,(mr),1);
        I1 = BESSELI (1, (mr),1);
        %Impedance calculus (number 4 appears because there are 4 subconctors)
    z = (1/4)*1000*((1/sigma)*m)/(2*pi*sk_radius)*(I0/I1);
    fprintf(fid10, '%30.20f %30.20f\n',real(z),imag(z));
end
fprintf(fid10, ']; \n');
fprintf(fid10, 'zskin = x(:,1) + x(:,2)*i;');
fclose('all');
z_carson.m
%Impedance due to earth effect (Carson's method)
clear all
```

```
'Calculating Z due to earth effect - Carson's method
%File reading
initial_conf
nt = 120; %amount of terms to be used (number multiple of 4)
v = -1; %variable used for signal
%Image conductors coordinates
for j = 1:ncond,
    xi(j) = xc(j);
yi(j) = -yc(j);
    for k = j:ncond
    str = ['Zcarson' int2str(j) '_' int2str(k) '.m'];
            fid(j,k) = fopen(str,'w');
    if j==k
    str = ['%% Phase impedance ' int2str(j) ' due to earth effect \n clear x\n x
= [\n'];
    fprintf(fid(j,k),str);
    else
    str = ['%% Impedance between phases ' int2str(j) ' and ' int2str(k) ' due to
Earth effect \n clear x\n x = [\n'];
    fprintf(fid(j,k),str);
                end
        end
end
for j = 1:ncond,
        for k = j:ncond,
            dx = xc(j) - xi(k);
    dy = yc(j) - yi(k);
    bd(j,k)= sqrt(dx^2 + dy^2);
        cat = yc(j) + yc(k);
    cossine = cat/bd(j,k);
    ang(j,k) = acos(cossine);
    end
end
%*****************************************************************************
% Calculus of terms b, c and d of Carson's series
%signal change each 4 terms
n = 0;
for j = 1:nt/4,
        v = - v;
        for k = 1:4,
    n = n+1;
    signal_b(n) = v;
        end
end
%Calculus of bi element
```

```
b(1)=sqrt(2)/6;
b (2) = 1/16;
for j = 3:nt,
    b(j) = abs(b(j-2))*(1/(j*(j+2)))*signal_b(j);
end
%Calculus of ci element
c(2) = 1.3659315;
for j = 4:nt,
    c(j) =c(j-2)+(1/j)+(1/(j+2));
end
%Calculus of di element
d = (pi/4)*b;
%********************************************************************************
for f = 1:length(freq),
    for j = 1:ncond,
        for k = j:ncond,
            phi = ang(j,k);
    a = sqrt(mizero*2*pi*freq(f)/resist)*bd(j,k);
                subrotine_carson_delta_r;
                subrotine_carson_delta_x;
                fprintf(fid(j,k), '%30.20f %30.20f\n',delta_r(j,k), delta_x(j,k));
    end
        end
end
for j = 1:ncond,
        for k = j:ncond,
    fprintf(fid(j,k), ']; \n');
    fprintf(fid(j,k), ['zsolo' int2str(j) '_' int2str(k) ' = x(:,1) + i*x(:,
2);']);
        end
end
fclose('all');
subrotine_carson_delta_r.m
if a< 5,
%'ok'
    r1 = b(1)*a*cos(phi);
    for nj = 1:(nt/4) -1,
        term1 = b(4*nj +1)*(a^(4*nj +1))*cos((4*nj + 1)*phi);
    r1 = term1 + r1;
    end
    parc1 = (c(2) - log(a))*(a^2)*cos(2*phi);
    parc2 = (phi*(a^2)*sin(2*phi));
    r2 = b(2)*(parc1 + parc2);
        for nj = 1:(nt/4) -1,
            parc1 = (c(4*nj +2) - log(a))*(a^(4*nj +2))*cos((4*nj +2)*phi);
parc2 = (phi*(a^(4*nj +2))*sin((4*nj +2)*phi));
    r2 = r2 + b(4*nj +2)*(parc1 + parc2);
    end
```

```
    r3 = b (3)* (a^3)* cos(3*phi);
    for nj = 1:(nt/4) -1,
    term3 = b(4*nj + 3)*(a^(4*nj + 3))* cos((4*nj + 3)*phi);
r3 = r3 + term3;
    end
r4 = d(4)*(a^4)*\operatorname{cos(4*phi);}
for nj = 1:(nt/4) -1,
    term4 = d(4*nj + 4)*(a^(4*nj + 4))* cos((4*nj + 4)*phi);
r4 = r4 + term4;
end
    delta_r(j,k) = 4*2*pi*freq(f)*(1e-4)*((pi/8) - r1 + r2 + r3 - r4);
else
    t1 = cos(phi)/a;
    t2 = sqrt(2)* cos(2*phi)/(a^2);
    t3 = cos(3*phi)/(a^3);
    t4 = 3* cos(5*phi)/(a^5);
    t5 = 5* cos(7*phi)/(a^7);
    delta_r(j,k) = (4*2*pi*freq(f)*(1e-4)/sqrt(2))*(t1 -t2 + t3 +t4 +t5);
end
subrotine_carson_delta_r.m
%Carson's series to calculate reactance of conductors due to Earth effect
if a < 5,
    x1 = b(1)*a*cos(phi);
    for nj = 1:(nt/4) -1,
        term1 = b (4*nj +1)* (a^(4*nj +1))* cos((4*nj + 1)*phi);
    x1 = term1 + x1;
    end
    x2 = d(2)* (a^2)* cos(2*phi);
    for nj = 1:(nt/4) -1,
        term2 = d(4*nj + 2)*(a^(4*nj + 2))* cos((4*nj + 2)*phi);
        x2 = x2 + term2;
    end
    x3 = b (3)* (a^3)* cos(3*phi);
    for nj = 1:(nt/4) -1,
            term3 = b (4*nj + 3)* (a^(4*nj + 3))* cos((4*nj + 3)*phi);
    x3 = x3 + term3;
    end
    term4 = (c(4) - log(a))*(a^4)* cos(4*phi) + (phi)*(a^4)*sin(4*phi);
    x4 = b(4)*term4;
    for nj = 1:(nt/4) -1,
            term4 = (c(4*nj + 4) - log(a))*(a^(4*nj + 4))* cos((4*nj + 4)*phi) +
(phi)*(a^(4*nj + 4))*sin((4*nj + 4)*phi);
            x4 = x4 + b(4*nj + 4)*term4;
    end
    delta_x(j,k) = 4*2*pi*freq(f)*(1e-4)*(0.5*(0.6159315 - log(a)) + x1 - x2 + x3
- x4);
else
    t1 = cos(phi)/a;
    t2 = sqrt(2)*\operatorname{cos (2*phi)/(a^2);}
    t3 = cos(3*phi)/(a^3);
    t4 = 3* cos(5*phi)/(a^5);
```

```
    t5 = 5* cos(7*phi)/(a^7);
    delta_x(j,k) = (4*2*pi*freq(f)*(1e-4)/sqrt(2))*(t1 - t3 + t4 + t5);
end
```

The above routines show the procedure to calculate the correct and proposed method values. The first routine call subroutines capacitance and Zfull (which is the sum of impedances shown before), thus for the correct value it uses the eigenvalue function (eig) and for the proposed method it performs calculus as shown before.

The second routine gets all processed data and plot the information considering the relative difference between the correct value and the proposed one as could be seen in Fig. 4.

## 7. Conclusion

The objective of this project was to analyze the application of modal transformation matrix that is independent of frequency in analyses of three-phase lines considering the presence of 2 ground wires. Through analysis, both the limits of this approach and the possible errors in relation to the exact values obtained from eigenvalues and eigenvectors.

The model proposed in this project uses approximate modal transformation, accomplished through a transformation matrix independent of frequency. This matrix is obtained by linear combination of elements of Clarke's matrix. With application of this transformation matrix independent of frequency, it obtains diagonal matrices for the cases of transposed three-phase lines. For non-transposed three-phase lines, matrices of parameters are not diagonal with the application of the transformation matrix mentioned. For those cases not implemented, the proposal is to analyze the relative errors obtained by establishing circumstances in which one can use a transformation matrix independent of frequency.

This chapter presented a method that can be used for analyzing electromagnetic transients using real transformation matrices in three-phase systems considering the presence of ground wires. This method was implemented using Matlab, and then the routines used to develop it were presented and commented. The proposal analyzed used a real transformation matrix for the entire frequency range considered in this case. For those elements related to the phases of the considered system, the transformation matrix was composed of the elements of Clarke's matrix. In part related to the ground wires, the elements of the transformation matrix had to establish a relationship with the elements of the phases considering the establishment of a single homopolar reference in the mode domain. In the case of three-phase lines with the presence of two ground wires, it was unable to get the full diagonalization of the matrices Z and $Y$ in the mode domain. The relative errors between the proposed routine and the correct values of eigenvalues were shown by graphs plotted using Matlab. Thus, for a future work, a correction routine will be used for non-transposed three-phase transmission line cases for the transformation matrix presented.

## Acknowledgements

This work was supported by FAPESP.

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