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Modeling of Control Systems

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Additional information is available at the end of the chapter

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1. Introduction

In studying control systems the reader must be able to model dynamical systems in mathematical terms and analyze their dynamic characteristics. This section provides an introduction to basic concepts and methodologies on modeling control systems. It also introduces some fundamentals to solve realistic models used basically in mechanical, electrical, thermal, economic, biological, and so on. A mathematical model is composed by a set of equations that describe a real system accurately, or at least fairly well. However a mathematical model is not unique for a given system, and the system under study can be represented in many different ways, in the same way a mathematical model can be used to represent different systems. Algorithms used to solve the set of equations that represent a control system require a great amount of programming instructions. Matlab is a tool that simplifies and accelerates such algorithms allowing to modeling a great variety of control systems in a very elegant way [1]. There are special Matlab toolbox useful to solve different control systems, in particular Control System Toolbox (included in MATLAB Version 7.8.0.347 (R2009a)): Creating linear models, data extraction, time-domain analysis, frequency-domain analysis, state space models, etc. Some of these are used throughout the chapter to facilitate algorithm development.

This chapter is organized as follows; the section 1 is an introduction to modeling control systems. In section 2, some applications using electrical circuits for series and parallel circuits are given. Section 3, a second order analysis is presented. In section 4, control systems for mechanical vibrations are analyzed. Optical control systems are given in section 5, given a perspective with two basic systems: laser diode, and optical fiber. Some conclusions are presented in section 6.

2. Electrical circuits

Undoubtedly, the most classic circuit in literature is the RLC circuit. Due to its great usefulness in the study of linear systems, the model of the RLC circuit helps to understand some of the behaviors of an electrical control system. Thus, the next subsections will address to the modeling of series and parallel RLC circuits. Some behaviors will be analyzed using some tools from Matlab.

Case I: RLC series circuit

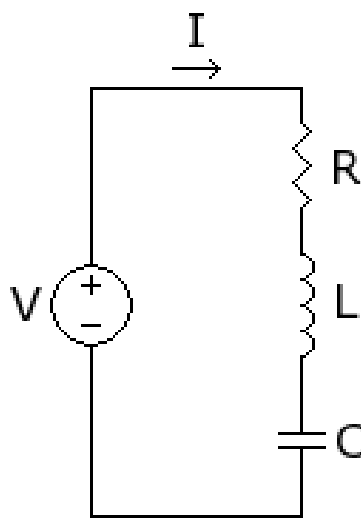


Figure 1. RLC series circuit.

In Figure 1, a RLC series circuit is shown. The modeling consists in to express a number of equations such that all kinds of moves are expressed in those equations. Thus, the input equation is given by

$$v_i(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int_0^t i(\tau) d\tau. \quad (1)$$

Let be $v_o(t)$ the output in capacitor C. Then, the output equation is represented by

$$v_o(t) = \frac{1}{C} \int_0^t i(\tau) d\tau. \quad (2)$$

In order to find a transfer function, the Laplace transform for both equations is applied obtaining

$$V_i(s) = RI(s) + LsI(s) + \frac{1}{Cs}I(s), \quad (3)$$

and

$$V_o(s) = \frac{1}{Cs}I(s). \quad (4)$$

The transfer function becomes

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}. \quad (5)$$

In order to compute a general solution and to analyze the behavior of this RLC series circuit, consider an input constant force $v_i(t) = V_i$. Then $V_i(s) = V_i/s$, therefore the output signal in the Laplace domain is given by

$$V_o(s) = \frac{V_i}{s(LCs^2 + RCs + 1)}, \quad (6)$$

therefore, the output signal results

$$v_o(t) = V_i \left(1 - e^{-\frac{R}{2L}t} \cos \sqrt{\frac{4L + R^2C}{4L^2C}}t - \sqrt{\frac{R^2C}{4L + R^2C}} e^{-\frac{R}{2L}t} \sin \sqrt{\frac{4L + R^2C}{4L^2C}}t \right). \quad (7)$$

Figure 2, shows the block diagram for the RLC series circuit.

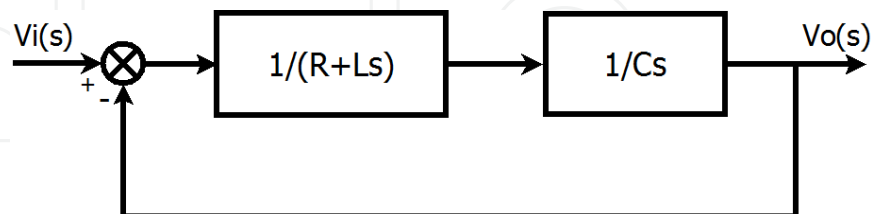


Figure 2. Block diagram for the RLC series circuit.

Suppose a particular case for the block diagram with $R = 2k\Omega$, $L = 1mH$ and $C = 1\mu F$. An easy way to obtain a step response for a RLC series circuit is to draw a block diagram in Simulink, from Matlab, using a source Step and a Scope to observe the response as shown in Figure 3. In Figure 4, the step response for the RLC series circuit is plotted.

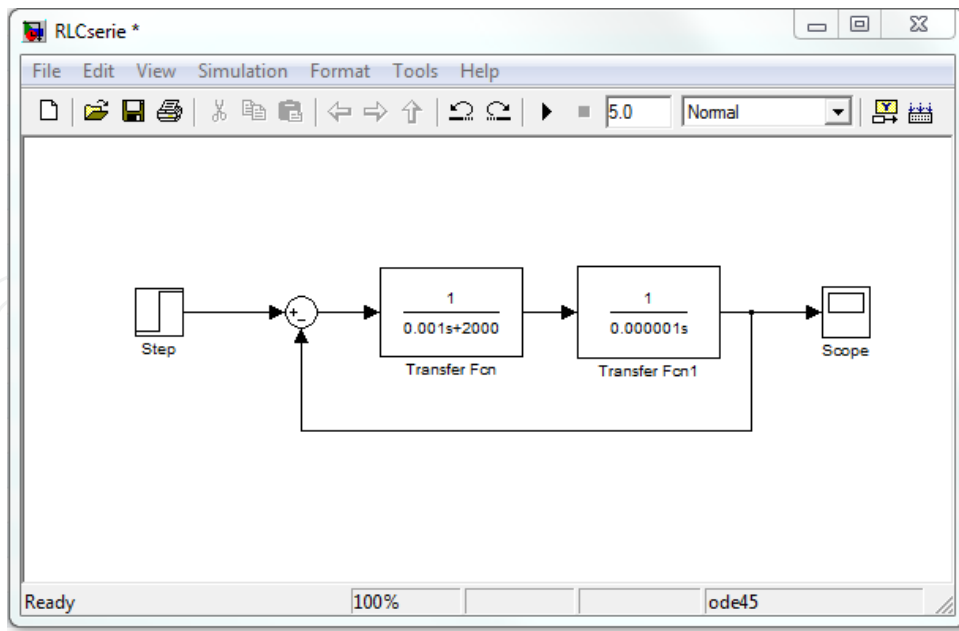


Figure 3. Block diagram using Simulink for a step response.

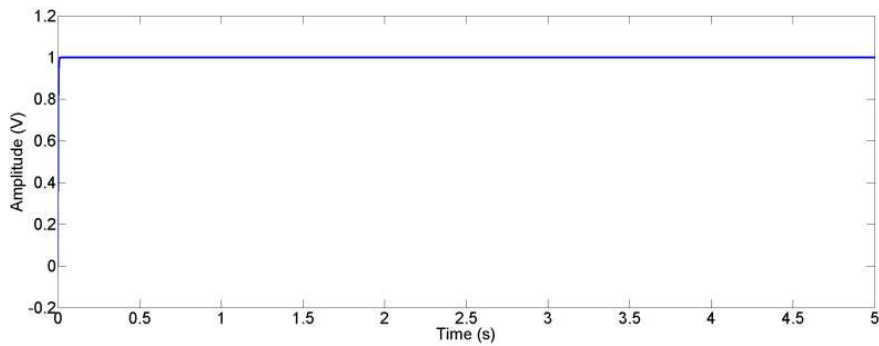


Figure 4. Step response for RLC series circuit using Simulink.

Case 2: RLC Parallel circuit

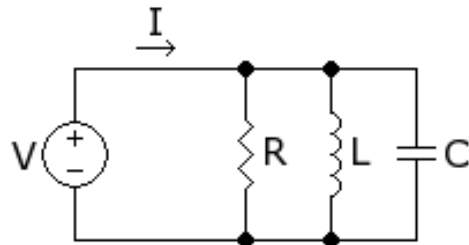


Figure 5. RLC parallel circuit.

In Figure 5, a RLC parallel circuit is shown. The equation of current could be represented by

$$i(t) = \frac{v_R(t)}{R} + \frac{1}{L} \int_0^t v_L(\tau) d\tau + C \frac{dv_C(t)}{dt}. \quad (8)$$

The output voltage through capacitor C, establishes the equalities

$$v_o(t) = v_R(t) = v_L(t) = v_C(t), \quad (9)$$

such that the mathematical model can be written in only one equation

$$i(t) = \frac{v_o(t)}{R} + \frac{1}{L} \int_0^t v_o(\tau) d\tau + C \frac{dv_o(t)}{dt}. \quad (10)$$

In order to find the transfer function, the Laplace transform is

$$I(s) = \frac{V_o(s)}{R} + \frac{1}{Ls} V_o(s) + Cs V_o(s). \quad (11)$$

Then, the transfer function results

$$\frac{V_o(s)}{I(s)} = \frac{RLs}{RLCs^2 + Ls + R}. \quad (12)$$

To compute a general solution and to analyze the behavior of this RLC parallel circuit, consider an input force of constant current $i(t) = I$. Then $I(s) = I/s$, therefore the output signal in the Laplace domain is given by

$$V_o(s) = \frac{RL}{RLCs^2 + Ls + R} I, \quad (13)$$

and the solution signal results

$$v_o(t) = 2IR \sqrt{\frac{L}{4R^2C - L}} e^{-\frac{1}{2RC}t} \sin \sqrt{\frac{4R^2C - L}{4R^2L C^2}} t. \quad (14)$$

Figure 6, shows the corresponding block diagram for the RLC parallel circuit.

In order to analyze the stability using Matlab, consider a RLC parallel circuit with $R = 1k\Omega$, $L = 1mH$ and $C = 1\mu F$. The aim is to obtain the transfer function, a state space representation, the eigenvalues, and the solution to state space equation.

The coefficients of the transfer function are stored in two vectors *num* and *den* for the numerator and denominator, respectively. The following solution uses the instructions *tf*, *tf2ss*, from Control Systems Toolbox. Window a) shows this algorithm obtaining the transfer function

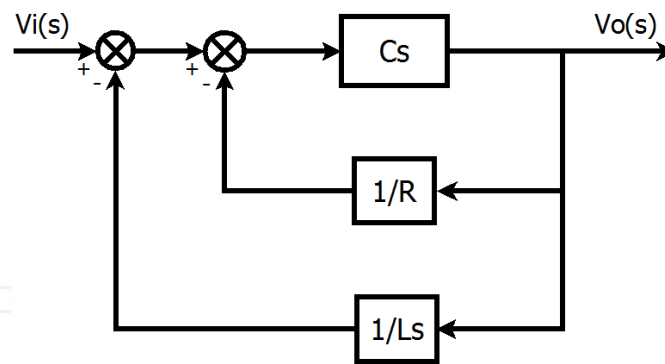


Figure 6. Block diagram for the RLC parallel circuit.

$G(s)$. Window b), uses the instruction `tf2ss` to convert from transfer function to state space, getting arrays A, B, C and D. Window c) illustrate the stability of the system, computing the eigenvalues from array A. Note that the eigenvalues from the system in state space is equivalent to compute the poles of transfer function $G(s)$. Thus, the poles are placed at the negative place of the complex plane, which means that the system is stable.

<p>a)</p> <pre>>> num=[1 0]; >> den=[0.000001 0.001 1000]; >> G=tf(num,den) Transfer function: s -----</pre>
<p>b)</p> <pre>>> [A,B,C,D]=tf2ss(num,den) A = 1.0e+003 * -1 -1000000 0.001 0 B = 1 0 C = 1000000 0 D = 0</pre>
<p>c)</p> <pre>>> eig(A) ans = 1.0e+004 * -0.0500 + 3.1619i -0.0500 - 3.1619i</pre>

To obtain the solution to the state space equation using Matlab, it is necessary to construct an object S , of class 'sym', from A , using the expression $S=sym(A)$, in order of using the instruction *ilaplace*, from Symbolic Math Toolbox. It is well known that the solution to state equation is given by

$$x(t)=L[(sI - A)^{-1}]x(0). \quad (15)$$

Then, window d) gives a solution using symbolic mathematics and the instruction *ilaplace*, with $x(0)=[1 \ 1]^T$.

```

d)
>> s=sym('s');
>> f=ilaplace(inv(s*eye(2)-A))
f =
[ (cos(500*3999^(1/2)*t) - (3999^(1/2)*sin(500*3999^(1/2)*t))/3999)/exp(500*t),
-(2000000*3999^(1/2)*sin(500*3999^(1/2)*t))/(3999*exp(500*t))]
[
(3999^(1/2)*sin(500*3999^(1/2)*t))/(1999500*exp(500*t)),
(cos(500*3999^(1/2)*t) + (3999^(1/2)*sin(500*3999^(1/2)*t))/3999)/exp(500*t)]

```

Note that, all solutions have a negative exponential or they are written explicitly as denominators with positive exponential, which mean that the system is stable. This kind of results could be useful for the next sections for mechanical vibrations and optical control systems.

3. Second order analysis

In the Nature there are many examples that can be modeled with second-degree equations. The general form is represented by a homogeneous linear differential equation with constant coefficients a , b and, c as shown below

$$a \frac{d^2x}{dt^2} + b \frac{dx}{dt} + cx = 0, \quad (16)$$

Laplace transform of the second-order equation is

$$s^2X(s) - sx(0) - \dot{x}(0) + \frac{b}{a}sX(s) - \frac{b}{a}x(0) + \frac{c}{a}X(s) = 0, \quad (17)$$

and solving for the dependent variable

$$X(s) = \frac{sx(0) + \frac{b}{a}x(0) + \dot{x}(0)}{s^2 + \frac{b}{a}s + \frac{c}{a}}. \quad (18)$$

The response for a unitary step input is given by the next equation

$$X(s) = \frac{s^2 x(0) + s \left[\frac{b}{a} x(0) + \dot{x}(0) \right]}{s^2 + \frac{b}{a} s + \frac{c}{a}} \bullet \frac{1}{s} . \quad (19)$$

There are four possible behaviors of equation (19) [2]: overdamped, underdamped, undamped and critically damped. These results can be predicted by analyzing the characteristic equation, as shown below:

$$b^2 - 4ac = \begin{cases} \text{positive, overdamped} \\ \text{negative, underdamped} \\ -4ac, \text{ undamped} \\ \text{zero, critically damped} \end{cases} \quad (20)$$

Where the initial conditions $x(0)$ and $\dot{x}(0)$ represent the initial state and the point of convergence of the response signal, respectively. The next results uses a step signal as input signal, with the instruction *step*, from Control Systems Toolbox. Thus, the graphical representation of a second-order analysis using Matlab as shown in Figure 7, can be obtained with the next code:

```
clear all;
x0=0.8;
xp0=0.35;
%%% b*b>4*a*c
a=1;b=5;c=1;
num1=[x0 x0*b/a xp0];den1=[1 b/a c/a];
signal1=step(num1,den1);
%%% b*b<4*a*c
a=1;b=1;c=1;
num2=[x0 x0*b/a xp0];den2=[1 b/a c/a];
signal2=step(num2,den2);
%%% b=0
a=1;b=0;c=1;
num3=[x0 x0*b/a xp0];den3=[1 b/a c/a];
signal3=step(num3,den3);
%%% b*b=4*a*c
a=1;b=5;c=1;
num4=[x0 x0*b/a xp0];den4=[1 b/a c/a];
signal4=step(num4,den4);
subplot(2,2,1);plot(signal1);grid;title('b*b>4*a*c, overdamped');
subplot(2,2,2);plot(signal2);grid;title('b*b<4*a*c, underdamped');
subplot(2,2,3);plot(signal3);grid;title('b=0, undamped');
subplot(2,2,4);plot(signal4);grid;title('b*b=4*a*c, critically damped');
```

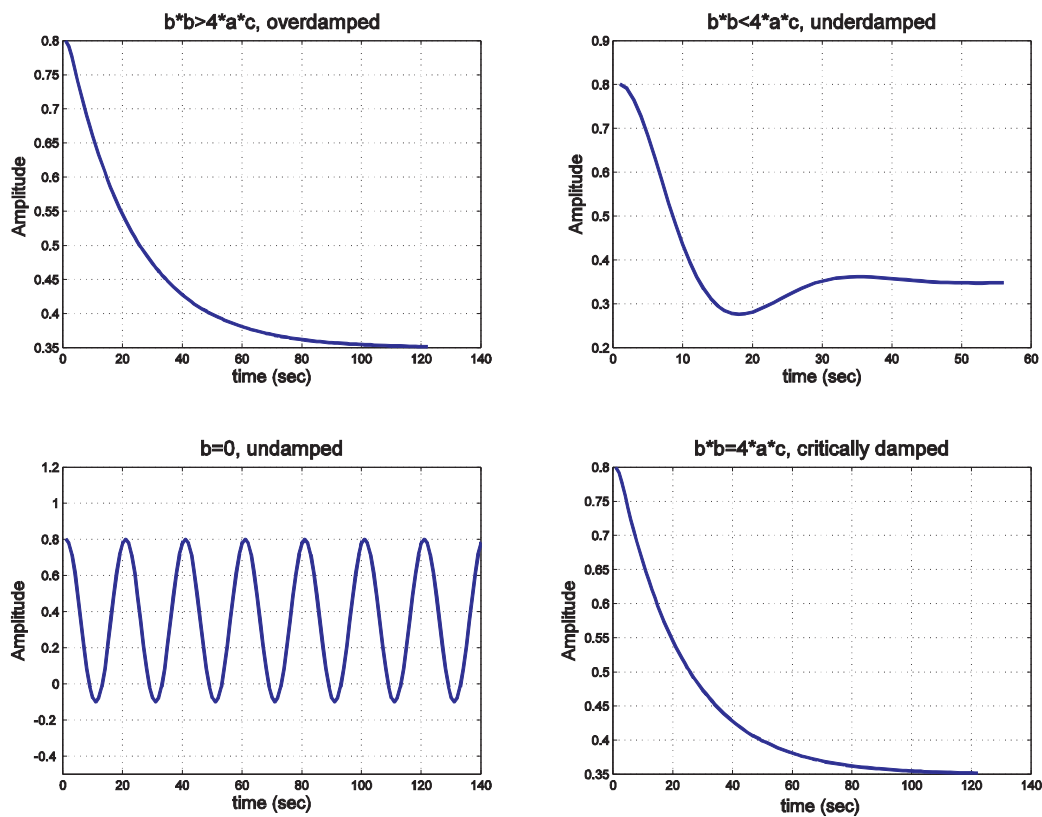


Figure 7. Analysis of second-order system.

4. Mechanical vibrations

Structural vibrations in most cases are undesirable because they can negatively affect the appropriate operation of mechanical systems because of the increasing stresses and energy losses by dissipation especially at resonant frequencies. For these reasons, it is very important the prediction of resonances in the design of mechanical elements to prevent structural failures as fatigue after long period of time [3,4]. The main purpose of this section is to analyze and design a simple mechanical model well known as one-degree of freedom (ODOF) system and calculate with Matlab the frequency response to an external sinusoidal force.

In order to understand the dynamic response, in Figure 8 is illustrated the schematic of a linear damped ODOF system subjected to a harmonic excitation input.

The system consists in a mass m connected to a spring of stiffness k . The other side of the spring is connected to the support. In a damped ODOF system, the coefficient c is the viscous damping

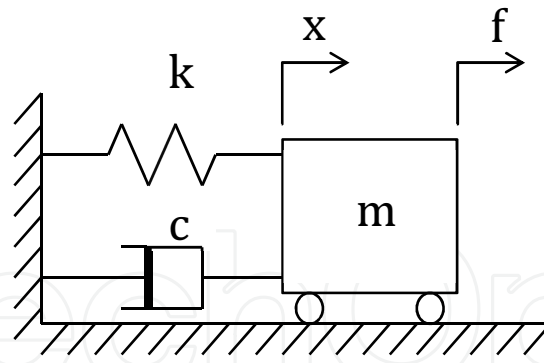


Figure 8. Damped ODOF system

related with the loss of energy in the vibrating system. Considering a force varying harmonically as $f(t) = A \sin(\omega t)$; where A is the amplitude oscillating at an angular frequency ω in rad/sec; the equation of motion along the x direction can be written as:

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (21)$$

With the initial conditions equal to zero, the steady state solution of the differential equation can be obtained applying the Laplace transform of equation 21.

$$ms^2x(s) + csx(s) + kx(s) = F(s) \quad (22)$$

Reordering the terms, the second order transfer function $G(s)$ is obtained as:

$$G(s) = \frac{x(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \quad (23)$$

Replacing the terms $\omega_n = \sqrt{\frac{k}{m}}$ and $\xi = \frac{c}{2\sqrt{mk}}$, that represent the natural frequency and the relative damping factor of the system respectively; then $G(s)$ can be written as:

$$G(s) = \frac{\frac{1}{m}}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (24)$$

When a sinusoidal input force is applied to a linear ODOF system, the response is also sinusoidal with the same oscillation frequency but with differences in amplitude and phase. In order to know the frequency response, s is replaced with $j\omega$, where $j = \sqrt{-1}$ is the imaginary unit

$$G(j\omega) = \frac{\frac{1}{m}}{-\omega^2 + j2\xi\omega_n\omega + \omega_n^2} \quad (25)$$

Defining r as the frequency ratio of the natural frequency ω_n to the angular frequency of the sinusoidal input ω and multiplying numerator and denominator by $1/\omega^2$, the above equation gives:

$$G(j\omega) = \frac{1}{m\omega^2} \frac{1}{[(r^2 - 1) + j2\xi r]} \quad (26)$$

Thus, it is clear that the frequency response of a damped ODOF represented by a complex number depends on ω of the sinusoidal input and the damping ratio ξ . Hence, the magnitude ratio and the phase angle of the frequency response can be expressed as:

$$|G(j\omega)| = \frac{1/(m\omega^2)}{\sqrt{(r^2 - 1)^2 + (2\xi r)^2}}, \quad (27)$$

and

$$\phi(j\omega) = -\tan^{-1} \frac{2\xi r}{r^2 - 1}. \quad (28)$$

By inspection of equation 26 we can make some interpretations [5]:

- for $r \approx 0$; that is, when $\omega \ll \omega_n$; the problem can be considered as static, since the frequency of the excitation force is very small in comparison with the natural frequency obtaining also a very small phase shift.
- for $r=1$; namely $\omega=\omega_n$; the system is at resonance, the amplitude of the output increases in function of the damping and the phase shift is 90° for any value of damping.
- for $r > 1$; that is, $\omega \gg \omega_n$ the amplitude of the response approaches to zero and the phase angle to 180° .

Dynamic analysis of a cantilever

The schematic in Figure 9 is cantilever beam with a lumped mass at its free end and is used to detect mechanical resonances; the beam is fixed perpendicular to the support of a machine that can operate at different velocities, generating angular frequencies that move harmonically along the x direction.

Ignoring the mass of the beam, consider the following model to calculate the amplitude and phase angle of the dynamic displacement for angular frequencies of $0 < \omega < 2$ rad/sec.

$$3\ddot{x} + 2\dot{x} + 3x = \sin(\omega t). \quad (29)$$

The following Matlab code plots the magnitude ratio and the phase angle of the frequency response in function of the angular frequency of the sinusoidal input.

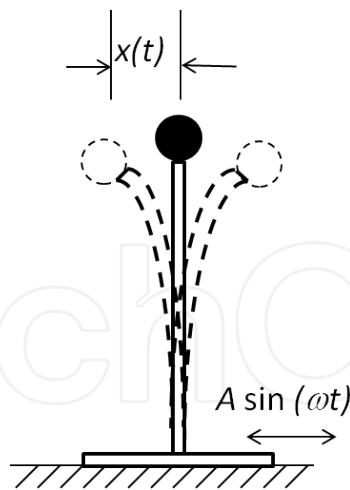


Figure 9. Cantilever beam

```

m=3; c=2; k=3;
wn=sqrt(k/m);
w=[0:0.1:2];
zeta=c/(2*sqrt(m*k));
r=wn./w;
M=abs((1./(m*w.^2))./sqrt((r.^2-1).^2+(2*zeta*r).^2));
phase=-atan2((2*zeta*r),(r.^2-1))*180/pi;
subplot(2,1,1);plot(r,M)
subplot(2,1,2);plot(r,phase)

```

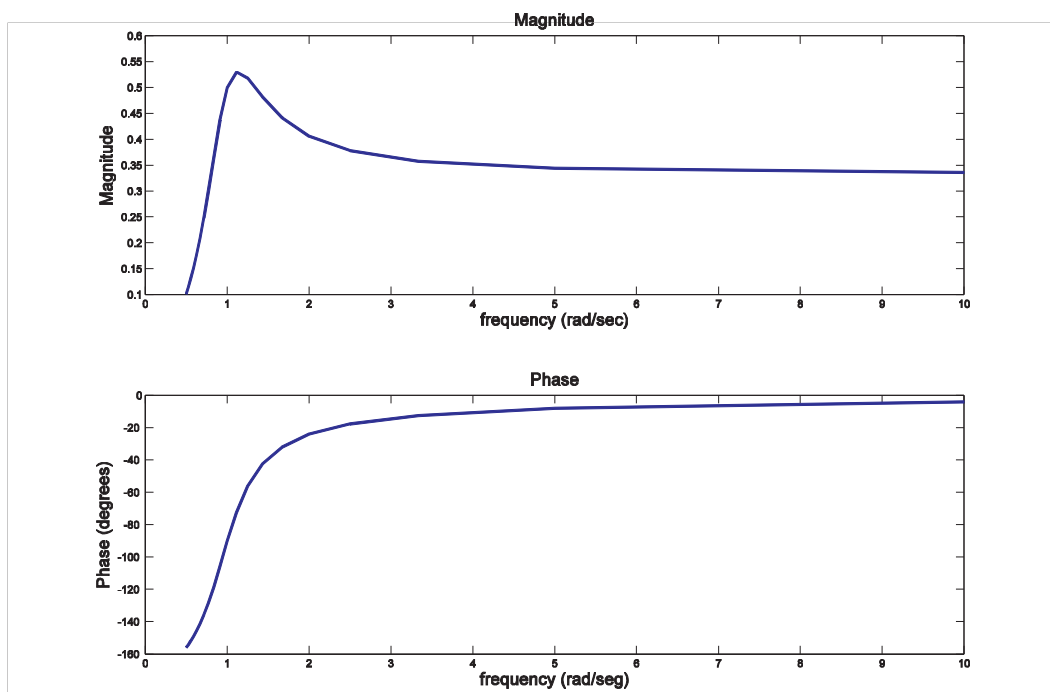


Figure 10. Frequency response.

5. Optical control systems

5.1. Laser diode

Laser diode (LD) are perhaps one of the most used optical systems, in particular for those applications where the high frequency modulation is involved. A LD can be described as a device in which an electric current is converted to photons. The time dependent relation between the input electric current and the output photons are commonly described by the following pair of equations describing the time evolution of photon and carrier densities inside the laser medium.

$$\frac{\partial N_e}{\partial t} = \frac{I}{qad} - A(N_e - N_{om})N_{ph} - \frac{N_e}{\tau_s}, \quad (30)$$

and

$$\frac{\partial N_{ph}}{\partial t} = A(N_e - N_{om})N_{ph} - \frac{N_{ph}}{\tau_{ph}} + \beta \frac{N_e}{\tau_s}, \quad (31)$$

where N_{ph} is the photon density. A is a proportionality constant, N_{om} is the minimum electron density required to obtain a positive gain, and β is the fraction of the spontaneous emission that is coupled the lasing mode, τ_s is the spontaneous carrier life time, τ_{ph} is the photon life time, and N_e is the carrier density, I is the input current, a is the diode area, d is the thickness of the active region, q is the electron charge. These pair of equations can be modeled using the RCL parallel electric model proposed in [6].

Here the LD can be modeled using the parallel circuit whose elements are:

$$R = R_d \left(\frac{I_{th}}{I^0} \right) \quad (32)$$

$$L = \frac{R_d \tau_{ph}}{\left[\left(\frac{I^0}{I_{th}} \right) - 1 \right]}, \quad (33)$$

$$C = \frac{\tau_s}{R_d}, \quad (34)$$

and

$$R_d = \frac{2kT}{q} \left(\frac{1}{I_d} \right). \quad (35)$$

If we consider a sinusoidal modulation in small signal around the quiescent point I_p in the form $I = I_p + i_p \sin \omega t$ the equations (30) and (31) can be linearized as follows:

$$Z_0(s) = \frac{V_0(s)}{I_0(s)} = \frac{RLs}{RLCs^2 + Ls + R} \tag{36}$$

or

$$Z_0(s) = \frac{V_0(s)}{I_0(s)} = \frac{R_d \left(\frac{is}{\tau_s} + \frac{1 + n_{om} - n_e^0}{\tau_s \tau_{ph}} \right)}{-s^2 + i \frac{s}{\tau_s} \left[n_{ph}^0 + 1 + \frac{\tau_s}{\tau_{ph} (1 + n_{om} - n_e^0)} \right] + \frac{n_{ph}^0 + \beta \left[n_e^0 \left(1 + \frac{1}{n_{ph}^0} \right) - n_{om} \right]}{\tau_s \tau_{ph}}} \tag{37}$$

Figure 11, shows a comparative curve for a sinusoidal input and the photon density output, here; we observe a phase difference between the input and output.

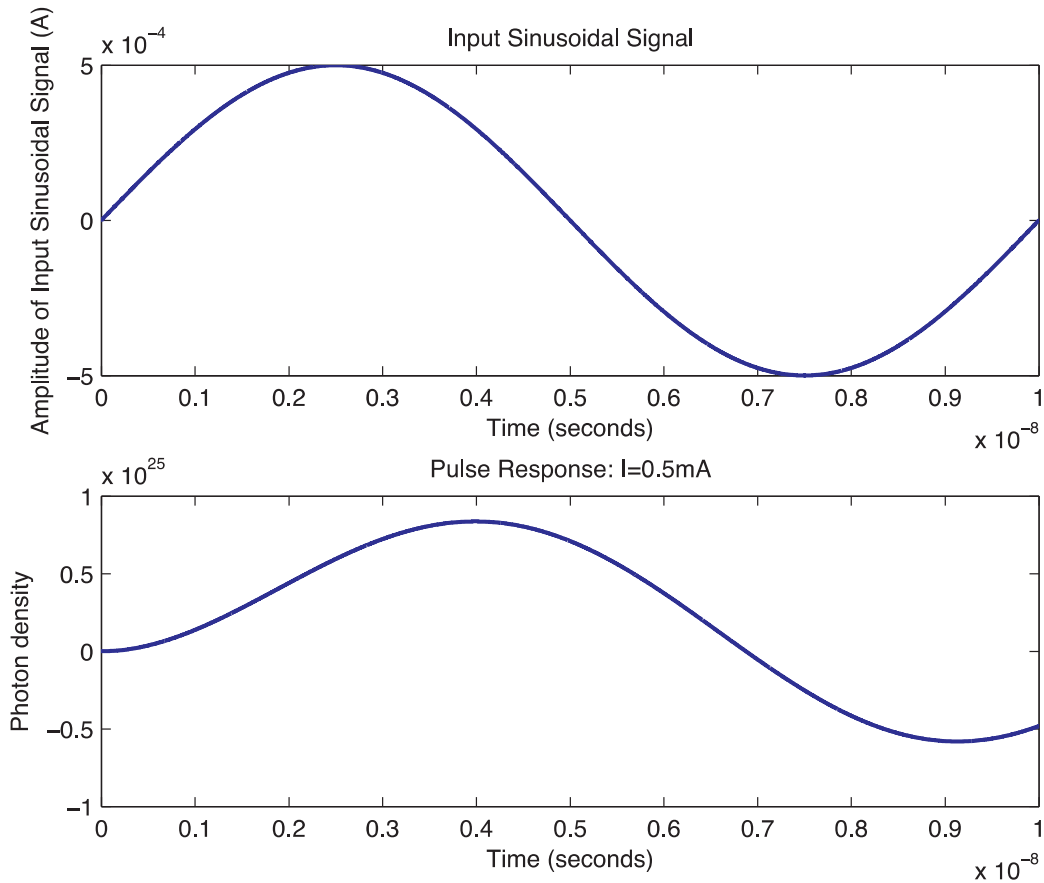


Figure 11. Comparative curve for sinusoidal Input current vs photon density.

Figure 12, shows the photon density response for different values of input signal.

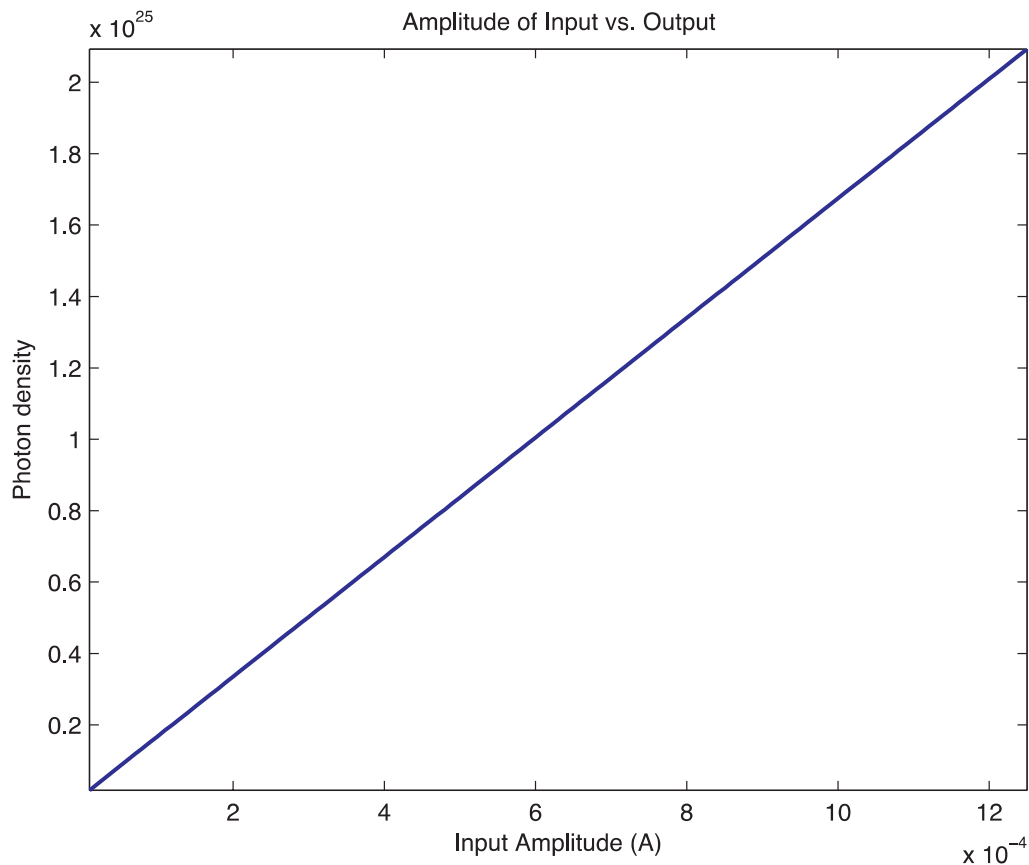


Figure 12. Photon density response for different input values.

The code that was used to generate and plot the curves showed in Figures 11 and 12, is placed in Appendix A.

5.2. Temperature control system for a furnace in a fiber optic drawing process.

The manufacturing process of optical fiber usually requires several control systems, for example, the temperature control system of the furnace for melting the preform, the control system to maintain the fiber diameter constant, the system for deposit control and ultraviolet light curing of the fiber coating system and finally the traction control and fiber winding on the drum (Figure 13).

The problem of modeling a furnace for drawing fiber requires the consideration of variables such as the speed of stretching of the fiber, the rate of purge gas into the furnace, the furnace temperature, the diameter of the preform, etcetera. Moreover, variables such as the flow of gas within the furnace, radiative heat transport between the preform and the interior of the furnace is actually not so simple, this problem has been analyzed for a graphite furnace [7]. However, only for demonstration purposes, we will treat the problem for the linear case and without considering the phenomena mentioned above.

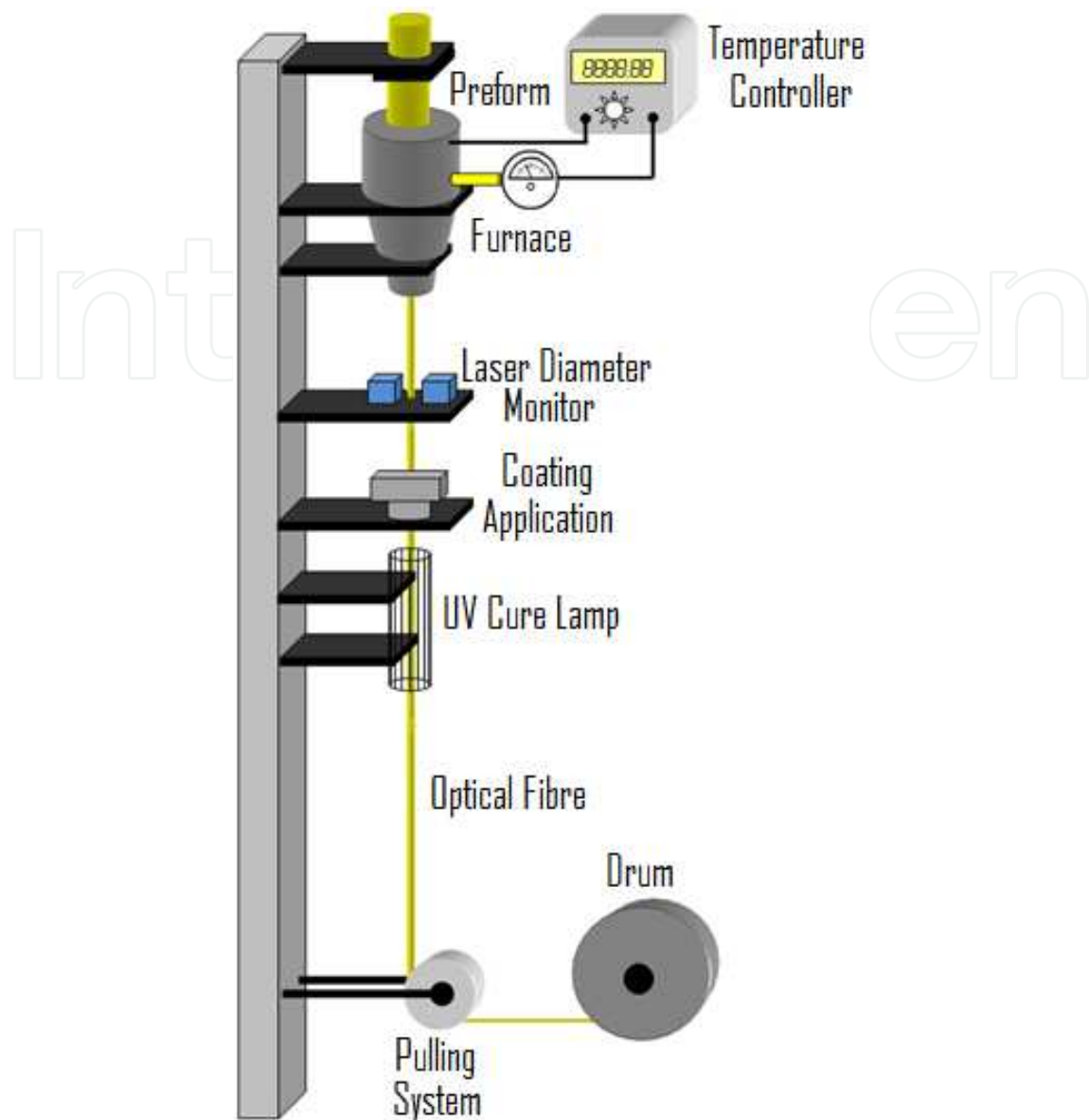


Figure 13. Fiber drawing process system

In the next section, we will demonstrate the use of Simulink of Matlab for modeling a simple temperature control system for a furnace in a fiber optic drawing process.

5.3. Mathematical model

Uniform heating of the preform is due in large part to effective radiative transport inside the furnace and the control of the changes in the temperature distribution. The above with the purpose to achieve fiber softening and stretching at high speed.

Making a simple energy balance of the furnace, we have the variation of energy per unit of time (ΔQ_H) given by

$$\Delta Q_H = Q_C - Q_F = K_T \frac{dT_i}{dt} \quad (38)$$

Where Q_C is the energy provided by the controller, Q_F is the gas heat flow into the furnace, K_T is the thermal capacity and T_i is the internal temperature.

We estimate the thermal capacity K_T for a preform of 100 mm of diameter and 450 mm long. We propose the active cavity of the furnace as a cylindrical shape with 120 mm in diameter and 300 mm long.

Then, knowing the mass of the perform M_p and silicon specific heat C_p , the heat capacity calculated for the silicon preform is

$$K_T = M_p \times C_p = 8.2313[\text{Kg}] \times 2.575 \left[\frac{\text{Joule}}{\text{Kg} \cdot ^\circ\text{C}} \right] = 21.19 \left[\frac{\text{joule}}{^\circ\text{C}} \right]. \quad (39)$$

Simplifying the heat flux in the furnace to

$$Q_F = \frac{T_i - T_o}{R_{eq}}. \quad (40)$$

Where R_{eq} is the equivalent thermal resistance of the inner walls of the furnace.

The equivalent thermal resistance of the internal cylindrical wall of the furnace made of graphite with a thickness of 5mm is calculated with

$$R_{eq} = 5 \times \frac{10^{-3}[\text{m}]}{(0.476 \left[\frac{\text{W}}{\text{m} \cdot ^\circ\text{C}} \right] \cdot 0.1131[\text{m}^2])} = 92.87 \times 10^{-3} \left[^\circ\frac{\text{C}}{\text{W}} \right]. \quad (41)$$

Then, applying the Laplace transform to Equation 38 we obtain

$$T_i = \frac{Q_C \cdot Q_F}{K_T \cdot s}. \quad (42)$$

Using as reference the Thermo demo of Simulink, the furnace temperature control is sketched in Figure 14

To run the program `Fiber_Furnace_System.mdl` is necessary to preload the characteristics of the furnace in the file `Data_furnace.m` (Appendix A). The graph of the system behavior is show in Figure 15.

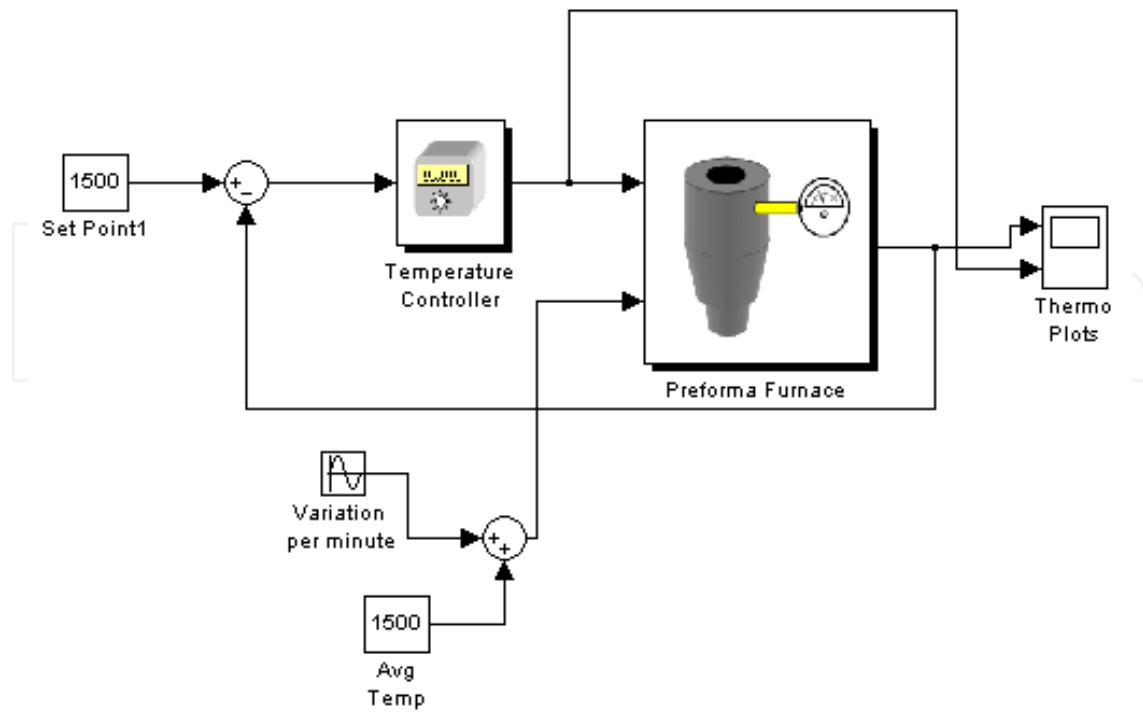


Figure 14. Simulink model of the perform furnace.

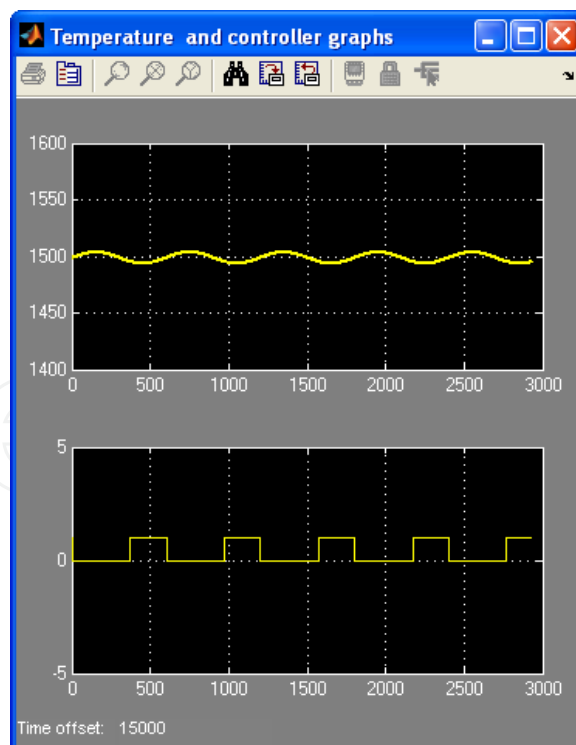


Figure 15. Graph of the system behavior. Upper: Temperature around 1500°C; Lower Controller activation for each cycle.

6. Conclusions

In order to study and analyze physical systems, along of this chapter, several systems were treated, and different Matlab codes were proposed to solve, analyze and, simulate those systems. Matlab is a powerful tool, and can help to students, engineers and, scientific to develop very simple, elegant and, powerful algorithms, that allows achieving successfully the study and analysis of control systems.

Appendix A

The following Matlab code was used to generate and plot the curves showed in Figures 11 and 12.

```

1      %program for the simulation of laserdiode system using
2      %RLC equivalent circuit
3
4      FontName = 'arial'; %Define the font type
5      FontSize = 9;      %define the font size
6      LineWidth = 1.5;  %define the line width
7
8      %parameters
9      a = 3.5e-3*1.3e-3;%area diode
10     d = 2000e-6;      %thickness of active region
11     q = .3e-6;       %Electron charge
12     Ne= 3e-5;        %DC value for the injectec electron density
13     Ith = 10e-4;
14     Io = 10e-5;      %input current
15     Tau_ph=.23e-6;   %spontaneous photon life time
16     Tau_s=.25e-9;   %spontaneous carrier life time
17     T = 77;         %temperature
18     Id = (Ne/Tau_s)*a*q*d; %normalized current
19     k = 8.61e-5;     %Boltzman constant
20     Rd = ((2*k*T)/q)*(1/Id);
21
22
23     R = Rd*(Ith/Io);
24     L = Rd*Tau_ph/((Io/Ith)-1);
25     C = Tau_s/Rd;
26     [u,t] = gensig('sin',1e-8,1e-8,0.001e-9);
27     u = u*5e-4;
28     gs = tf([R*L 0],[R*L*C L R]);
29     s_10 = 1.9221e13;
30     y=s_10*lsim(gs,u,t,'zoh');
31
32     subplot(2,1,1)
33     plot(t,u,'linewidth',LineWidth)
34     set(gca,'fontname',FontName,'fontsize',FontSize)
35     xlabel('Time (seconds)','fontname',FontName,'fontsize',FontSize);
36     ylabel('Amplitude of Input Sinusoidal Signal (A)','fontname',FontName,'fontsize',FontSize);
37     title('Input Sinusoidal Signal','fontname',FontName,'fontsize',FontSize)
38     subplot(2,1,2)
39     plot(t,y,'linewidth',LineWidth)
40     set(gca,'fontname',FontName,'fontsize',FontSize)
41     xlabel('Time (seconds)','fontname',FontName,'fontsize',FontSize);
42     ylabel('Photon density','fontname',FontName,'fontsize',FontSize);
43     title('Pulse Response: I=0.5mA','fontname',FontName,'fontsize',FontSize);
44

```

```

45 - [u,t] = gensig('sin', 1e-8, 1e-8, 0.001e-9);
46 - count=1;
47 - for i=1:1:125
48 -     I_step = i*1e-5;
49 -     u = u*I_step;
50 -     y = s_l0*lsim(gs,u,t);
51 -     u_A(1,count) = max(u);
52 -     y_A(1,count) = max(y);
53 -     ytemp = y(1:round(length(y)/2),1);
54 -     y_P(1,count) = t(find(ytemp==max(ytemp)))-1e-8/4;
55 -     count = count + 1;
56 -     [u,t] = gensig('sin', 1e-8, 1e-8, 0.001e-9);
57 - end
58 - % Plot Amplitude Response
59 - figure
60 - plot(u_A,y_A,'linewidth',LineWidth)
61 - axis tight
62 - set(gca,'fontname',FontName,'fontsize',FontSize)
63 - xlabel('Input Amplitude (A)','fontname',FontName,'fontsize',FontSize);
64 - ylabel('Photon density','fontname',FontName,'fontsize',FontSize);
65 - title('Amplitude of Input vs. Output','fontname',FontName,'fontsize',FontSize);

```

Program Data_furnace.m. The results are showed in Fig. 15.

```

% DATA_furnace
% This script runs in conjunction with the%"Fiber_Furnace_System"
% Loading images
A= imread('Preforma_Furnace.bmp');
B= imread('Controller.bmp');
% -----
% Define the furnace geometry
% -----
% internal furnace height = 0.3 m
inhFurnace = 0.30;
% internal furnace radio = 0.16 m
inrFurnace = 0.16;
% internal wall area
inwallArea = 2*pi*inrFurnace*inhFurnace;
% -----
% Determine the equivalent thermal
% resistance for graphite wall
% -----
kg = 0.476;
LWall = .005;
Req = LWall/(kg*inwallArea);
% c = cp of silice = 2.575 J/Kg-C
c = 2.575;
% -----
% Temperature Set Controller = 1500 deg C = 1773 K
% -----
TSC = 1773;
% ha = enthalpy of air at 40 deg C = 311400 J/kg
ha = TSC*c;
% Air flow rate Mdot = 0.1 kg/sec
Mdot = 0.1;
% -----

```

```
% Determine total internal glass mass = M
% -----
% Density of preforma silice glass = 2329 kg/m^3
densglass = 2329;
Mg = pi*inrFurnace*inhFurnace;
```

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