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# Lattice Reduction-Based User Selection in Multiuser MIMO Systems

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Additional information is available at the end of the chapter

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## 1. Introduction

In wireless communications, the spectral efficiency can be improved by exploiting the space domain when antenna arrays are used. In particular, space-division multiple access (SDMA) [1–3] can be adopted with various beamforming techniques. If both the transmitter and the receiver are equipped with multiple antennas, the resulting channel becomes a multiple-input-multiple-output (MIMO) channel, which can provide a rich spatial diversity gain. In MIMO systems, it is often desirable to use the maximum likelihood (ML) detection to jointly detect received signals for optimal performance and full receive diversity. However, since the complexity of the ML detection exponentially grows with the number of transmit antennas, the ML detection approach becomes impractical for high-dimensional detection problems. To derive low-complexity suboptimal MIMO detectors, various approaches based on the properties of lattice are considered. For example, using the Lenstra-Lenstra-Lovász (LLL) algorithm in [4], the lattice reduction (LR)-based low-complexity detectors are proposed in [5–8], which can provide a full receive diversity gain with a near-ML performance. The basic idea of the LR-based MIMO detection is to generate a nearly orthogonal basis for a given channel matrix to mitigate the effect of (multiple antenna) interference.

Due to users' different locations and channel conditions, it is possible to exploit another diversity gain in a multiuser system, where the throughput can be maximized by choosing the user of the strongest channel gain at a time. The resulting diversity gain is called the multiuser diversity gain [9]. Multiuser systems can be extended to the case of MIMO systems [10], where the multiuser MIMO user selection plays a key role in increasing the throughput of downlink channels [11]. It is noteworthy that, by viewing the multiuser MIMO system as virtual antennas in a single-user MIMO system, various antenna selection techniques can be applied to user selection [12, 13]. A mutual information-based criterion is proposed in [12] to

select the antenna subset that maximizes the mutual information. In [13], a geometry-based criterion is developed with an LR-based linear detector to minimize the error probability. In general, user selection problems are combinatorial problems, and the complexity required to solve the problems could be prohibitively high for a large multiuser MIMO system. Thus, low-complexity suboptimal selection strategies are considered in [14–21], at the expense of degraded performance. In [14–17], a single antenna is selected at a time to maximize the throughput based on greedy selection schemes.

Although the achievable rate or related signal-to-noise ratio (SNR) can be used for the user selection criterion, it would be more practical to use a certain performance measure that is directly related to the performance of the actual detector or decoder employed. Therefore, it is desirable to derive a user selection criterion that can maximize the performance of the MIMO detector that is *actually* employed in a multiuser MIMO system.

In this chapter, for the user selection in uplink channels of a cellular system, where a single user is selected to transmit signals to a base station (BS) at a time, the error probability is used for the user selection criterion to choose the user with the smallest error probability for given MIMO detectors. Various user selection criteria will be derived with the ML detector, LR-based detectors and other low-complexity suboptimal detectors. It will be shown that a near-optimal performance with a full diversity gain (i.e., multiuser diversity and multiple antenna diversity) can be achieved using the proposed user selection criteria in this chapter with LR-based detectors.

Based on the single user selection criteria derived, we will extend them to support multiple users at a time. This extension of the user selection (i.e., multiple user selection) is not straightforward, because the multiple-user selection problem becomes a combinatorial problem. If an exhaustive search is used for multiple user selection when an LR-based MIMO detector is employed, LR needs to be performed for all the possible channel matrices composited by a group of subchannel matrices of the selected users. Unfortunately, this results in a high computational complexity, because the number of user combinations is large. Therefore, we will propose a greedy user selection algorithm to reduce the computational complexity at the expense of degraded performance when LR-based detectors are used. Moreover, to further reduce the computational complexity, an iterative LR updating algorithm will be investigated. Based on a theoretical analysis in this chapter, we can show that, with the combinatorial user selection, the LR-based detection can achieve the same diversity as the ML detector. Through simulations, we will compare the performance obtained by our selection criteria (i.e., combinatorial and greedy ones) to other existing approaches.

With the LR-based detection employed, simulation results will confirm that our combinatorial user selection can provide the best performance, whereas the performance of the greedy user selection scheme could approach that of the combinatorial approach as the correlation between possible composite channel matrices decreases. It will also be shown that our greedy user selection provides a better performance and a significantly reduced complexity compared with other approaches.

## 2. System model

In this section, we introduce the model of multiuser MIMO system together with several MIMO detection techniques.

### 2.1. Multiuser MIMO system

Consider the multiuser MIMO system with  $K$  users in uplink channels, where each user is equipped with  $P$  transmit antennas, and the base station (BS) is equipped with  $N$  receive antennas. Each user has an  $N \times P$  channel matrix and a  $P \times L$  signal matrix, which are denoted by  $\mathbf{H}_k$  and  $\mathbf{S}_k$ , respectively, where  $k \in \{1, 2, \dots, K\}$ . Here,  $L$  is the number of symbols transmitted by a user. It is assumed that all the users share a common uplink channel and that  $M$  users can access the channel at a time, where  $M = \lfloor N/P \rfloor$ . The channel is assumed to be a quasi-static block fading channel, with its channel matrix not varying over a time slot duration of  $L$  symbols. Here, a set of the  $M$  users who can access the channel could be updated for every time slot interval. Note that this selection problem can also be regarded as that with virtual antennas in a single-user MIMO system, where  $MP$  antennas are selected out of  $KP$  available antennas. Let  $k_{(m)}$  be the  $m$ th selected user's index. For convenience, define the set of the selected users' indexes as  $\mathcal{K} = \{k_{(1)}, k_{(2)}, \dots, k_{(M)}\}$ . Then, over a slot duration, the received signal at the BS is given by

$$\mathbf{Y}_{\mathcal{K}} = \mathbf{H}_{\mathcal{K}} \mathbf{S}_{\mathcal{K}} + \mathbf{N}, \quad (1)$$

where  $\mathbf{H}_{\mathcal{K}}$ ,  $\mathbf{S}_{\mathcal{K}}$ , and  $\mathbf{N}$  are the  $N \times MP$  composite channel matrix, the  $MP \times L$  transmitted signal matrix, and the  $N \times L$  background noise matrix, respectively. We assume that each column vector of  $\mathbf{N}$  is an independent zero-mean circularly symmetric complex Gaussian (CSCG) random vector with  $\mathbb{E} \{\mathbf{n}_l \mathbf{n}_l^H\} = N_0 \mathbf{I}$ , where  $\mathbf{n}_l$  denotes the  $l$ th column of  $\mathbf{N}$ . Note that  $\mathbf{H}_{\mathcal{K}} = [\mathbf{H}_{k_{(1)}}, \dots, \mathbf{H}_{k_{(M)}}]$  and that  $\mathbf{S}_{\mathcal{K}} = [\mathbf{S}_{k_{(1)}}, \dots, \mathbf{S}_{k_{(M)}}]$ .

Throughout this chapter, we assume that the channel state information (CSI) is perfectly known at the receiver. Furthermore, the following assumptions are used to derive user selection methods.

- A1) The elements of  $\mathbf{S}_{\mathcal{K}}$  have a common signal alphabet, denoted by  $\mathcal{S}$ , and  $\mathcal{S} \subset \mathbb{Z} + j\mathbb{Z}$ , where  $\mathbb{Z}$  denotes the set of integer numbers and  $j = \sqrt{-1}$ . Furthermore, let  $\mathcal{S}^A$  represent the  $A$ -dimensional Cartesian product of  $\mathcal{S}$ .
- A2) The transmitted signals are uncoded. This implies that the user selection criteria in this chapter are based on uncoded bit error rate (BER). For uncoded signals, we can assume  $L = 1$  (Note that this assumption is used to simplify the derivation of user selection criteria, while the length of slot can be any number). Thus,  $\mathbf{Y}_{\mathcal{K}}$ ,  $\mathbf{S}_{\mathcal{K}}$ , and  $\mathbf{N}$  are vectors and will be denoted by  $\mathbf{y}_{\mathcal{K}}$ ,  $\mathbf{s}_{\mathcal{K}}$ , and  $\mathbf{n}$ , respectively.

### 2.2. MIMO detection

MIMO detection plays an important role in MIMO receivers. Within this chapter, several well known MIMO detectors including the ML detector, linear detectors, and successive interference cancellation (SIC) detectors, together with LR are considered.

### 2.2.1. ML and linear detection

For the sake of convenience, we omit the index set  $\mathcal{K}$ . The ML detection is given by

$$\hat{\mathbf{s}}_{\text{ml}} = \arg \min_{\mathbf{s} \in \mathcal{S}^{MP}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2, \quad (2)$$

where the complexity grows exponentially with  $MP$ .

Alternatively, an estimate of  $\mathbf{s}$  can be obtained by a linear transformation as follows:

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{y}, \quad (3)$$

where  $\mathbf{W}$  is a linear filter that is given by  $\mathbf{W} = (\mathbf{H}^H\mathbf{H} + c\mathbf{I})^{-1}\mathbf{H}$ . If  $c = 0$ , the linear detector corresponds to the zero-forcing (ZF) detector, while the minimum mean square error (MMSE) detector is obtained if  $c = N_0/E_s$ . Here,  $E_s$  is the symbol energy and it is assumed that  $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = E_s\mathbf{I}$ .

To improve the performance of the detector, the LR is performed in the LR-based detection. A complex valued matrix can be converted into a real valued matrix for the LR as in [7]. Alternatively, the LR can be directly performed with a complex valued matrix as in [6], [8]. For convenience, in this chapter, we assume that the LR is performed with complex valued matrices.

For a given channel matrix  $\mathbf{H}$ , the LR basis can be found as follows:

$$\mathbf{H} = \mathbf{G}\mathbf{U}, \quad (4)$$

where  $\mathbf{U}$  is an (complex) integer unimodular matrix and  $\mathbf{G}$  is a matrix whose column vectors are nearly orthogonal. The received signal can be rewritten as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{G}\mathbf{c} + \mathbf{n}, \quad (5)$$

Under the MMSE criteria, the linear filter of LR-based MMSE linear detector is given by  $\mathbf{W} = \left(\mathbf{G}^H\mathbf{G} + \frac{N_0}{E_s}\mathbf{U}^H\mathbf{U}\right)^{-1}\mathbf{G}^H$ .

### 2.2.2. SIC detection

An SIC detector is not a linear detector due to its cancellation operation. In [7], the LR-based SIC detectors are proposed. To generalize the LR-based SIC detector, define the extended channel matrix as  $\mathbf{H}_{\text{ex}} = [\mathbf{H}^T \sqrt{c}\mathbf{I}]^T$ . The LR basis can be found as

$$\mathbf{H}_{\text{ex}} = \mathbf{G}_{\text{ex}}\mathbf{U}_{\text{ex}}, \quad (6)$$

where  $\mathbf{U}_{\text{ex}}$  is a complex integer unimodular matrix and  $\mathbf{G}_{\text{ex}}$  is a matrix whose column vectors are nearly orthogonal. If the LR basis is not used,  $\mathbf{U}_{\text{ex}} = \mathbf{I}$  (i.e.,  $\mathbf{G}_{\text{ex}} = \mathbf{H}_{\text{ex}}$ ).

Note that the size of  $\mathbf{H}_{\text{ex}}$  is the same as that of  $\mathbf{G}_{\text{ex}}$  which is  $2N \times MP$ . Let the QR factorization of  $\mathbf{G}_{\text{ex}}$  be  $\mathbf{G}_{\text{ex}} = \mathbf{Q}\mathbf{R}$ , where  $\mathbf{Q}$  is a matrix whose column vectors are orthonormal and  $\mathbf{R}$  is upper triangular. Let  $\mathbf{y}_{\text{ex}} = [\mathbf{y}^T \mathbf{0}]^T$  and  $\mathbf{n}_{\text{ex}} = [\mathbf{n}^T - \sqrt{c}\mathbf{s}^T]^T$ . This results in  $\mathbf{y}_{\text{ex}} = \mathbf{H}_{\text{ex}}\mathbf{s} + \mathbf{n}_{\text{ex}}$ . Then, the LR-based SIC detection can be carried out with the following signal:

$$\mathbf{Q}^H \mathbf{y}_{\text{ex}} = \mathbf{Q}^H \mathbf{G}_{\text{ex}} \mathbf{U}_{\text{ex}} \mathbf{s} + \mathbf{Q}^H \mathbf{n}_{\text{ex}} = \mathbf{R}\mathbf{c} + \bar{\mathbf{n}}, \quad (7)$$

where  $\mathbf{c} = \mathbf{U}_{\text{ex}}\mathbf{s}$  and  $\bar{\mathbf{n}} = \mathbf{Q}_{\text{ex}}^H \mathbf{n}_{\text{ex}}$ . Since the statistical properties of  $\bar{\mathbf{n}}$  and  $\mathbf{n}$  are the same, we will use  $\mathbf{n}$  to denote  $\bar{\mathbf{n}}$ . Note that  $\mathbf{n}$  also includes the self-interference as mentioned in [7].

The SIC detection can be carried out with (7). The elements of the last row, the  $MP$ th layer, are detected first. Then, their contributions in the second last row are canceled and the signals of the  $(MP - 1)$ th row are detected. This operation is repeated up to the first row.

### 3. Single user selection criteria

In this section, we derive user selection criteria depending on the type of actually employed MIMO detector, where a single user is selected to transmit signals to a BS at a time. Suppose that user  $k$  is chosen, the system model in (1) is simplified as

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{S}_k + \mathbf{N}, \quad (8)$$

For detection method, the ML detector and two suboptimal detectors will be considered: one is the linear detector and the other is the SIC detector. As for the two suboptimal detectors, the LR is applied for better performance [6][7].

#### 3.1. ML detector

Assuming that user  $k$  is selected, we omit the user index  $k$  for the sake of simplicity. To derive the selection criterion, we can consider the pairwise error probability (PEP). Suppose that  $\mathbf{s}_{(1)}$  is transmitted, while  $\mathbf{s}_{(2)}$  is erroneously detected. Then, the PEP is given by

$$\begin{aligned} P(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)}) &= \Pr\left(\|\mathbf{y} - \mathbf{H}\mathbf{s}_{(2)}\|^2 \leq \|\mathbf{y} - \mathbf{H}\mathbf{s}_{(1)}\|^2\right) \\ &= \mathcal{Q}\left(\sqrt{\frac{\|\mathbf{H}\Delta\|}{2N_0}}\right), \end{aligned} \quad (9)$$

where  $\mathcal{Q}(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$  and  $\Delta = \mathbf{s}_{(1)} - \mathbf{s}_{(2)}$ . Then, the following upper bound can be obtained as

$$P(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)}) \leq \mathcal{Q}\left(\sqrt{\frac{\|\mathbf{H}\bar{\mathbf{d}}\|^2}{2N_0}}\right), \quad (10)$$

where

$$\bar{\mathbf{d}} = \arg \min_{\mathbf{d} \in \mathcal{D}, \mathbf{d} \neq \mathbf{0}} \|\mathbf{H}\mathbf{d}\|^2. \quad (11)$$

Here,  $\mathcal{D} = \{\mathbf{d} = \mathbf{s} - \mathbf{s}' \mid \mathbf{s}, \mathbf{s}' \in \mathcal{S}^P\} \subset \mathbb{Z}^P + j\mathbb{Z}^P$ . For convenience, denote by  $S(\mathbf{H})$  the length of the shortest non-zero vector of the lattice generated by  $\mathbf{H}$ . Then, we can see that  $S(\mathbf{H}) = \|\mathbf{H}\bar{\mathbf{d}}\|$ . From (10), if the ML detector is employed, the user selection criterion to minimize the error probability becomes

$$k^* = \arg \max_k S(\mathbf{H}_k). \quad (12)$$

Throughout this chapter, the user selection criterion in (12) is referred to as the max-min distance (MDist) criterion as  $S(\mathbf{H})$  is the minimum distance of the lattice generated by  $\mathbf{H}$ .

The problem to find a non-zero shortest vector in a lattice is called the shortest vector problem (SVP) and known to be NP-hard. For an approximation, the LLL algorithm in [4], which has a polynomial time complexity, can be used.

Another approximation can be considered by relaxing the constraint on  $\Delta$ . We have

$$\|\mathbf{H}\Delta\|^2 = \Delta^H \mathbf{H}^H \mathbf{H} \Delta \geq \|\Delta\|^2 \lambda_{\min}(\mathbf{H}^H \mathbf{H}), \quad (13)$$

where  $\lambda_{\min}(\mathbf{A})$  stands for the minimum eigenvalue of  $\mathbf{A}$ . This shows that the selection criterion can be based on the minimum eigenvalue of the channel matrix, i.e.,

$$k^* = \arg \max_k \lambda_{\min}(\mathbf{H}_k^H \mathbf{H}_k). \quad (14)$$

Thus, each user can feed back its minimum eigenvalue of the channel matrix and the user who has the maximum  $\lambda_{\min}(\mathbf{H}_k^H \mathbf{H}_k)$  can be selected to access the channel. This selection criterion is referred to as the max-min eigenvalue (ME) criterion throughout this chapter.

### 3.2. Linear detectors

As the SNR increases, we have  $c \rightarrow 0$  (in this case, the MMSE detector becomes the ZF detector) and the PEP has the following upper bound:

$$\begin{aligned} P(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)}) &= Q\left(\frac{\|\Delta\|^2}{\sqrt{2N_0 \Delta^H (\mathbf{H}^H \mathbf{H})^{-1} \Delta}}\right) \\ &\leq Q\left(\sqrt{\frac{\lambda_{\min}(\mathbf{H}^H \mathbf{H})}{2N_0}} \|\Delta\|^2\right), \end{aligned} \quad (15)$$



because  $Q(\cdot)$  is a decreasing function and  $\Delta^H(\mathbf{H}^H\mathbf{H})^{-1}\Delta \leq \lambda_{\max}(\mathbf{H}^H\mathbf{H})^{-1}\|\Delta\|^2 = \frac{\|\Delta\|^2}{\lambda_{\min}(\mathbf{H}^H\mathbf{H})}$ . Therefore, the ME criterion in (14) can be used for the user selection criterion.

It is important to note that this ME criterion is valid for the LR-based linear detectors [6], [7]. Let  $\mathbf{c}_{(i)} = \mathbf{U}\mathbf{s}_{(i)}$ ,  $i = 1, 2$ . Then, from (15), the PEP is bounded as

$$P(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)}) \leq Q\left(\sqrt{\frac{\lambda_{\min}(\mathbf{G}^H\mathbf{G})\|\Delta_{\mathbf{U}}\|^2}{2N_0}}\right), \quad (16)$$

where  $\Delta_{\mathbf{U}} = \mathbf{c}_{(1)} - \mathbf{c}_{(2)} = \mathbf{U}(\mathbf{s}_{(1)} - \mathbf{s}_{(2)})$ . From (16), the selection criterion becomes

$$k^* = \arg \max_k \lambda_{\min}(\mathbf{G}_k^H\mathbf{G}_k), \quad (17)$$

where  $\mathbf{G}_k$  is the reduced basis from  $\mathbf{H}_k$ . This ME criterion is the same as that in (14) except that the channel matrix  $\mathbf{H}_k$  is replaced by its reduced one  $\mathbf{G}_k$ .

### 3.3. SIC detectors

As the LR is performed, the column vectors of  $\mathbf{G}_{\text{ex}}$  would be nearly orthogonal. In other words, the upper off-diagonal elements of  $\mathbf{R}$  would be small. Thus the SIC detection performance would mainly depend on the diagonal elements of  $\mathbf{R}$ . For convenience, let  $c = 0$  (this is the case when  $N_0 \rightarrow 0$  or high SNR). Let  $r_{p,p}^{(k)}$  denote the  $(p, p)$ th element of  $\mathbf{R}$  from the  $k$ th user's channel  $\mathbf{H}_k$ . Then, ignoring the interference terms (as they are canceled when the detection of the lower layers is successfully carried out with no error), the SNR of the  $p$ th layer of  $\mathbf{H}_k$  becomes  $\gamma_p^{(k)} = \frac{|r_{p,p}^{(k)}|^2}{N_0}$ . From this, the selection criterion can be given by

$$k^* = \arg \max_k \left\{ \min_p |r_{p,p}^{(k)}| \right\}. \quad (18)$$

This selection criterion is referred to as the max-min diagonal term (MD) criterion.

The MD criterion is also closely related to the minimum error probability criterion when the SNR is high. For convenience, let  $\mathbf{x} = \mathbf{Q}^H\mathbf{y}$ . Then, (18) is rewritten as

$$\mathbf{x} = \mathbf{R}\mathbf{c} + \mathbf{n}. \quad (19)$$

Let  $n_p$  denote the  $p$ th element of  $\mathbf{n}$ . Then, the LR-based SIC detection at the  $P$ th layer does not have error if  $\frac{|n_p|}{|r_{p,p}|} < \frac{1}{2}$  or  $|n_p|^2 < \frac{|r_{p,p}|^2}{4}$ . Thus, the LR-based SIC detection would have no error across all the layers if  $|n_p|^2 < \frac{|r_{p,p}|^2}{4}$ , for all  $p$ . The probability of no error can be lower



bounded as

$$\begin{aligned} \Pr(\text{no error}) &\geq \Pr\left(|n_p|^2 < \frac{|r_{p,1}|^2}{4}, \forall p\right) \\ &= \prod_{p=1}^P \Pr\left(|n_p|^2 < \frac{|r_{p,p}|^2}{4}\right). \end{aligned} \quad (20)$$

Since  $|n_p|^2$  is a chi-square random variable with 2 degrees of freedom (or an exponential random variable), we have  $\Pr\left(|n_p|^2 < \frac{|r_{p,p}|^2}{4}\right) = 1 - \exp\left(-\frac{|r_{p,p}|^2}{4N_0}\right)$ . Thus, from (20), the probability of error can be given by

$$\begin{aligned} \Pr(\text{error}) &\leq 1 - \prod_{p=1}^Q \left(1 - \exp\left(-\frac{|r_{p,p}|^2}{4N_0}\right)\right) \\ &\simeq \exp\left(-\min_p \frac{|r_{p,p}|^2}{4N_0}\right) \text{ as } N_0 \rightarrow 0. \end{aligned} \quad (21)$$

Therefore, to minimize the probability of error, the user who has the maximum  $\min_p |r_{p,p}|$  can be selected.

## 4. Selection criteria with multiple users

To maximize the performance, if  $M = 1$ , the user who can have the minimum PEP is chosen for a given MIMO detector. In Section 3, a few user selection criteria are derived, depending on the types of actually employed MIMO detectors. Note that only one user is selected (i.e.,  $M = 1$ ) in Section 3. To extend the user selection criteria to the case of  $M > 1$  here and in the consecutive sections, we consider the combinatorial and greedy user selection criteria.

### 4.1. ML and MMSE selection criteria

For a given  $M > 1$ , the set of the users who can access the channel can be found using the MDist or ME user selection criterion as follows:

$$\mathcal{K}_{\text{MDist}} = \arg \max_{\mathcal{K}} \mathcal{S}(\mathbf{H}_{\mathcal{K}}) \quad (22)$$

or

$$\mathcal{K}_{\text{ME}} = \arg \max_{\mathcal{K}} \lambda_{\min}(\mathbf{H}_{\mathcal{K}}^H \mathbf{H}_{\mathcal{K}}) \quad (23)$$

respectively. If the ML detector is employed, the MDist user selection criterion can be used to choose the  $M$  users who can have the lowest BER, whereas the ME criterion is used to choose the  $M$  users with the highest worst SNR (i.e., max-min SNR).

## 4.2. LR-based MMSE and MMSE-SIC selection criteria

In this subsection, the user selection criteria with LR-based detectors in Section 3 are extended to the case of  $M > 1$ , where the number of transmit layers are extended to  $MP$ , compared to  $P$  in the case of  $M = 1$ .

The MD criterion derived in Section 3, with  $M = 1$  for the LR-based MMSE-SIC detection, can be extended to the case with  $M > 1$  as follows:

$$\mathcal{K}_{\text{MD}} = \arg \max_{\mathcal{K}} \left\{ \min_q \left| r_{q,q}^{(\mathcal{K})} \right| \right\} \quad (24)$$

and the ME criterion for the LR-based MMSE detection can also be modified as

$$\mathcal{K}_{\text{ME}} = \arg \max_{\mathcal{K}} \lambda_{\min} \left( \mathbf{G}_{\mathcal{K}}^H \mathbf{G}_{\mathcal{K}} \right). \quad (25)$$

The user selection based on (22), (23), (24), and (25) is called the combinatorial user selection, because the users have to be selected by combinatorial (or exhaustive) search.

## 5. LR-based greedy user selection using an updating method

The computational complexity of the user selection under the criteria derived in Section 4 grows rapidly with  $M$  or  $K$  as they are all combinatorial optimization problems. Thus, it is desirable to derive low complexity approaches for the user selection. In this section, we propose low complexity greedy approaches for the user selection. Note that we focus on the greedy user selection with a LR-based MIMO detector only as its performance is comparable to that of the ML detector and, more importantly, we can derive a computationally efficient LR updating method in conjunction with greedy user selection.

### 5.1. LR-based greedy user selection

The user selection approaches in Section 4 have the complexity that becomes prohibitively high as  $M$  or  $K$  increases, because there are  $U = \prod_{i=0}^{M-1} (K - i)$  possible user index sets. For each user index set, an LR of an  $N \times MP$  complex channel matrix is to be performed. For example, when  $K = 10$ ,  $M = N = 4$  and  $P = 1$ ,  $10 \times 9 \times 8 \times 7 = 5040$  LRs of  $4 \times 4$  complex-valued channel matrices should be carried out.

To reduce the computational complexity in the user selection, we consider a greedy approach when a LR-based MIMO detector is employed. The resulting approach is called the LR-based greedy (LRG) user selection, which is of course suboptimal. The LRG user selection algorithm is summarized as follows:

1. Let  $m = 1$  and  $\tilde{\mathcal{K}} = \{1, \dots, K\}$ . In order to select the first user, we can use any criterion. For example, if the ME criterion is used, we have

$$k_{(1)} = \arg \max_{k \in \tilde{\mathcal{K}}} \lambda_{\min} \left( \mathbf{G}_k^H \mathbf{G}_k \right), \quad (26)$$

where  $\mathbf{G}_k$  represents the LBR matrix of  $\mathbf{H}_k$  or  $\mathbf{H}_{\text{ex},k} = \left[ \mathbf{H}_k^T \sqrt{\frac{N_0}{E_s}} \mathbf{I}_N \right]^T$  (for the LR-based MMSE detector). Once the first user is chosen, we update  $\bar{\mathcal{K}}$  as  $\bar{\mathcal{K}} \leftarrow \bar{\mathcal{K}} \setminus \{k_{(1)}\}$ . In addition, we let  $\mathbf{H}_{(1)} = \mathbf{H}_{k_{(1)}}$ .

2. Let  $m \leftarrow m + 1$  and  $\mathbf{H}_{(m),k} = \left[ \mathbf{H}_{(m-1)} \ \mathbf{H}_k \right]$ ,  $k \in \bar{\mathcal{K}}$ . The  $m$ th user can be chosen if the ME criterion is used as

$$k_{(m)} = \arg \max_{k \in \bar{\mathcal{K}}} \lambda_{\min} \left( \mathbf{G}_{(m),k}^H \mathbf{G}_{(m),k} \right), \quad (27)$$

where  $\mathbf{G}_{(m),k}$  is the LBR matrix of  $\mathbf{H}_{(m),k}$  or  $\mathbf{H}_{\text{ex},(m),k} = \left[ \mathbf{H}_{(m),k}^T \sqrt{\frac{N_0}{E_s}} \mathbf{I}_N \right]^T$ . Once the  $m$ th user is found, we update as follows:

$$\begin{aligned} &\text{add } k_{(m)} \text{ to the index set of the selected users, } \mathcal{K}, \\ &\bar{\mathcal{K}} \leftarrow \bar{\mathcal{K}} \setminus k_{(m)}, \\ &\mathbf{H}_{(m)} = \mathbf{H}_{(m),k_{(m)}}. \end{aligned} \quad (28)$$

3. If  $m = M$ , stop. Otherwise, go to 2).

Note that in this algorithm, the  $N \times mP$  complex-valued matrix  $\mathbf{H}_{(m)}$  denotes the channel matrix for the first  $m$  selected users, while the  $N \times P$  complex-valued matrix  $\mathbf{H}_{k_{(m)}}$  represents the channel matrix for the selected user in the  $m$ th selection with the index  $k_{(m)}$ , where  $k_{(m)} \in \bar{\mathcal{K}}$  and  $\bar{\mathcal{K}} = \{1, \dots, K\} \setminus \{k_{(1)}, \dots, k_{(m-1)}\}$ .

In the LRG user selection, the number of required LR operations is  $\sum_{i=1}^M (K - i + 1)$  and the matrix size for LR in selecting the  $m$ th user is  $N \times mP$ . Using the upper bound on the average complexity of LR studied in [8], we can show that the complexity of LRG is upper-bounded as  $\sum_{i=1}^M (K - i + 1) O((iP)^3 N \log(iP))$  (Note that when  $P = 1$ , no LR is required for the first user selection, where the complexity of LRG reduces to  $\sum_{i=2}^M (K - i + 1) O((iP)^3 N \log(iP))$ ). On the other hand, the number of required LR operations in the combinatorial user selection according to (24) or (25) is  $\prod_{i=1}^M (K - i + 1)$  and the matrix size for LR is always  $N \times MP$ , which leads to its complexity that is upper-bounded as  $\prod_{i=1}^M (K - i + 1) O((MP)^3 N \log(MP))$ . This shows a significant computational complexity reduction. However, since the LRG user selection does not jointly select  $M$  users, there will be performance loss.

Note that the ME criterion is used in above for illustration purposes. The MD criterion can also be used for the LRG user selection with the LR-based MMSE-SIC detector.

## 5.2. A complexity efficient method for LR updating

We note that in the LRG user selection, the LR operation is repeatedly performed for each updated channel matrix. For instance, at the  $m$ th user selection, a LR is carried out with the complex-valued channel matrix  $\mathbf{H}_{(m)} = \left[ \mathbf{H}_{(m-1)} \ \mathbf{H}_k \right]$  as shown in (27), where  $\mathbf{H}_k$  contains  $P$  newly added column vectors and the other  $(m - 1)P$  column vectors in  $\mathbf{H}_{(m)}$  are already

chosen and LBR. Instead of performing a new LR on all of the  $mP$  column vectors in  $\mathbf{H}_{(m)}$ , by utilizing the established  $(m-1)P$  LBR vectors, we can derive a computationally efficient LR updating method with new  $P$  column vectors, which is referred to as the Updated Basis LR (UBLR) in this paper. The resulting user selection scheme is referred to as the UBLR-based greedy (UBLRG)<sup>1</sup> user selection.

The UBLR algorithm is based on the complex-LLL(CLLL) algorithm [8]. Suppose that LR is performed by the CLLL algorithm in order to transform a given basis (a complex-valued channel matrix  $N \times mP$   $\mathbf{H}_{(m)}$ ) into a new  $N \times mP$  basis  $\mathbf{G}_{(m)}$  consisting of nearly orthogonal basis vectors. A basis  $\mathbf{G}_{(m)}$  is called a reduced basis of a lattice with parameter  $\delta$  if  $\mathbf{G}_{(m)}$  is QR factorized as  $\mathbf{G}_{(m)} = \mathbf{Q}_{(m)}\mathbf{R}_{(m)}$ , where  $\mathbf{Q}_{(m)}$  is unitary,  $\mathbf{R}_{(m)}$  is upper triangular, and the elements of  $\mathbf{R}_{(m)}$  satisfy the following inequalities:

$$|\Re(r_{\ell,\rho})| \leq \frac{1}{2} |r_{\ell,\ell}| \quad \text{and} \quad |\Im(r_{\ell,\rho})| \leq \frac{1}{2} |r_{\ell,\ell}|$$

for  $1 \leq \ell < \rho \leq mP$  (29)

and

$$\delta |r_{\rho-1,\rho-1}|^2 \leq |r_{\rho,\rho}|^2 + |r_{\rho-1,\rho}|^2 \quad \text{for } \rho = 2, \dots, mP. \quad (30)$$

Here,  $r_{p,q}$  denotes the  $(p,q)$ -th element of  $\mathbf{R}_{(m)}$ . The parameter  $\delta$  is closely related to the quality of the reduced basis. In this paper, we assume  $\delta = 3/4^2$  which is usually chosen for complexity and performance trade-off. For the initialization, let  $\mathcal{A}'_{(m)} = \{\mathbf{Q}'_{(m)}, \mathbf{R}'_{(m)}, \mathbf{U}'_{(m)}\}$ , where the QR factorization  $\mathbf{H}_{(m)} = \mathbf{Q}'_{(m)}\mathbf{R}'_{(m)}$  and  $\mathbf{U}'_{(m)} = \mathbf{I}_{mP}$ . With  $\{\mathbf{Q}_{(m)}, \mathbf{R}_{(m)}, \mathbf{U}_{(m)}\} = \{\mathbf{Q}'_{(m)}, \mathbf{R}'_{(m)}, \mathbf{U}'_{(m)}\}$  and  $\rho = 2$ , a version of CLLL algorithm is summarized as follows (note that since CLLL is used in UBLR, in Table 1, CLLL becomes part of UBLR).

- a) To fulfill (29), a size-reduction is performed with the 1<sup>st</sup> to  $\rho^{\text{th}}$  columns of  $\mathbf{R}_{(m)}$  and  $\mathbf{U}_{(m)}$  (see rows (15)-(21) in Table 1).
- b) As the basis of  $\mathbf{R}_{(m)}$  is size-reduced according to (29), let  $\rho \leftarrow \rho + 1$  and go to step a) if (30) is fulfilled. Swap the  $(\rho-1)^{\text{th}}$  and  $\rho^{\text{th}}$  columns in  $\mathbf{R}_{(m)}$  and  $\mathbf{U}_{(m)}$  if (14) is not satisfied and update  $\{\mathbf{R}_{(m)}, \mathbf{Q}_{(m)}\}$ . Let  $\rho \leftarrow \max(\rho-1, 2)$  and go to step a) (see rows (22)-(32) in Table 1).
- c) The algorithm is terminated if  $\rho = mP$ . The output of the CLLL reduced matrix  $\mathbf{G}_{(m)}$  is given by the updated  $\mathcal{A}_{(m)} = \{\mathbf{Q}_{(m)}, \mathbf{R}_{(m)}, \mathbf{U}_{(m)}\}$ , i.e.,  $\mathbf{G}_{(m)} = \mathbf{Q}_{(m)}\mathbf{R}_{(m)} = \mathbf{H}_{(m)}\mathbf{U}_{(m)}$ .

In our LRG user selection, at the  $m$ th user selection, the channel matrix of size  $N \times P(m-1)$  (denoted by  $\mathbf{H}_{(m-1)}$ ) is obtained from the previous user selections. Under the assumption that the CLLL has been performed with  $\mathbf{H}_{(m-1)}$  and its CLLL reduced matrix  $\mathbf{G}_{(m-1)}$  is

<sup>1</sup> Since the performance of the LRG and UBLRG user selection schemes are the same (in fact, UBLRG is a computationally efficient version of LRG), we now only consider UBLRG and assume that LRG and UBLRG are interchangeable.

<sup>2</sup> Here,  $\delta$  is a factor selected to achieve a good quality-complexity trade-off [4]. We note that  $\delta$  can be chose from  $(\frac{1}{4}, 1)$  and  $(\frac{1}{2}, 1)$  for the real and complex LLL algorithms, respectively.

---

INPUT:  $\{\mathcal{A}_{(m-1)}, \mathcal{B}_{(m-1)}, \mathbf{H}_{(m-1)}, \mathbf{H}_{k(m)}\}$ .  
OUTPUT:  $\{\mathcal{A}_{(m)}, \mathcal{B}_{(m)}\}$ .

- (1)  $\mathbf{H}_{(m)} \leftarrow \begin{bmatrix} \mathbf{H}_{(m-1)} & \mathbf{H}_{k(m)} \end{bmatrix}$
- (2)  $\omega \leftarrow \text{size}(\mathbf{H}_{(m-1)}, 2)$
- (3)  $\zeta \leftarrow \text{size}(\mathbf{H}_{(m)}, 2)$
- (4)  $[\mathbf{Q}_{(m)} \ \mathbf{R}_{(m)}] \leftarrow \text{qr}(\mathbf{H}_{(m)})$
- (5)  $\mathbf{U}_{(m)} \leftarrow \mathbf{I}_\zeta$
- (6)  $\mathbf{U}_{(m)}(1 : \omega, 1 : \omega) \leftarrow \mathbf{U}_{(m-1)}$
- (7)  $\mathbf{Q}_{(m)} \leftarrow \mathbf{Q}_{(m-1)}$
- (8)  $\mathbf{R}_{(m)}(:, 1 : \omega) \leftarrow \mathbf{R}_{(m-1)}$
- (9) for  $\ell = 1 : \eta_{(m-1)}$
- (10)  $\mathbf{R}_{(m)}(\mathbf{fl}_{(m-1, \ell)} - 1 : \mathbf{fl}_{(m-1, \ell)}, \omega + 1 : \zeta)$   
 $\leftarrow \Theta_{(m-1, \ell)} \mathbf{R}_{(m)}(\mathbf{fl}_{(m-1, \ell)} - 1 : \mathbf{fl}_{(m-1, \ell)}, \omega + 1 : \zeta)$
- (11) end for
- (14)  $\rho \leftarrow \omega + 1$
- (13)  $\eta_{(m)} \leftarrow 0$
- (14) while  $\rho \leq \zeta$
- (15) for  $\ell = 1 : \rho - 1$
- (16)  $\mu \leftarrow \lceil \mathbf{R}_{(m)}(\rho - \ell, \rho) / \mathbf{R}_{(m)}(\rho - \ell, \rho - \ell) \rceil$
- (17) if  $\mu \neq 0$
- (18)  $\mathbf{R}_{(m)}(1 : \rho - \ell, \rho) \leftarrow \mathbf{R}_{(m)}(1 : \rho - \ell, \rho) - \mu \mathbf{R}_{(m)}(1 : \rho - \ell, \rho - \ell)$
- (19)  $\mathbf{U}_{(m)}(:, \rho) \leftarrow \mathbf{U}_{(m)}(:, \rho) - \mu \mathbf{U}_{(m)}(:, \rho - \ell)$
- (20) end if
- (21) end for
- (22) if  $\delta |(\mathbf{R}_{(m)}(\rho - 1, \rho - 1))|^2 > |\mathbf{R}_{(m)}(\rho, \rho)|^2 + |\mathbf{R}_{(m)}(\rho - 1, \rho)|^2$
- (23)  $\eta_{(m)} \leftarrow \eta_{(m)} + 1$
- (24) Swap the  $(\rho - 1)$ -th and  $\rho$ th columns in  $\mathbf{R}_{(m)}$  and  $\mathbf{U}_{(m)}$
- (25)  $\Theta_{(m, \eta_{(m)})} = \begin{bmatrix} \alpha^* & \beta \\ -\beta & \alpha \end{bmatrix}$  with  $\alpha = \frac{\mathbf{R}_{(m)}(\rho-1, \rho-1)}{\|\mathbf{R}_{(m)}(\rho-1: \rho, \rho-1)\|}$   
 $\beta = \frac{\mathbf{R}_{(m)}(\rho, \rho-1)}{\|\mathbf{R}_{(m)}(\rho-1: \rho, \rho-1)\|}$
- (26)  $\mathbf{fl}_{(m, \eta_{(m)})} \leftarrow \rho$
- (27)  $\mathbf{R}_{(m)}(\rho - 1 : \rho, \rho - 1 : \zeta) \leftarrow \Theta_{(m, \eta_{(m)})} \mathbf{R}_{(m)}(\rho - 1 : \rho, \rho - 1 : \zeta)$
- (28)  $\mathbf{Q}_{(m)}(:, \rho - 1 : \rho) \leftarrow \mathbf{Q}_{(m)}(:, \rho - 1 : \rho) \Theta_{(m, \eta_{(m)})}^T$
- (29)  $\rho \leftarrow \max\{\rho - 1, 2\}$
- (30) else
- (31)  $\rho \leftarrow \rho + 1$
- (32) end if
- (33) end while

---

**Table 1.** The UBLR (based on the CLLL) algorithm at the  $m$ th user selection

available, we have  $\mathbf{H}_{(m)} = \begin{bmatrix} \mathbf{H}_{(m-1)} & \mathbf{H}_{k(m)} \end{bmatrix}$  which is the channel matrix for the first  $m$  selected users.

The UBLR algorithm is carried out to transform  $\mathbf{H}_{(m)}$  into a reduced basis  $\mathbf{G}_{(m)}$  by utilizing a given set of already available matrices  $\mathcal{A}_{(m-1)} = \{\mathbf{Q}_{(m-1)}, \mathbf{R}_{(m-1)}, \mathbf{U}_{(m-1)}\}$  associated with the CLLL reduced matrix  $\mathbf{G}_{(m-1)}$  in the previous  $m - 1$  users selection, where  $\mathbf{G}_{(m-1)} = \mathbf{Q}_{(m-1)} \mathbf{R}_{(m-1)} = \mathbf{H}_{(m-1)} \mathbf{U}_{(m-1)}$ . The unimodular matrix  $\mathbf{U}_{(m-1)}$  is employed to represent the column swaps in the CLLL, while  $\mathbf{R}_{(m-1)}$  satisfies (29) and (30). The transformation algorithm for generating  $\mathbf{G}_{(m)}$  in UBLR is summarized as follows.

Instead of starting the size-reduction of  $\mathbf{R}'_{(m)}$  with the first two columns (the 1<sup>st</sup> to  $\rho^{\text{th}}$  columns, where  $\rho = 2$  in a)), UBLR reduces the iteration by starting the size-reduction with  $\rho = (m - 1)P + 1$ . In this case, the iteration of size-reduction from that with  $\rho = 2$  to that with  $\rho = (m - 1)P + 1$  need to be obtained by updating  $\mathcal{A}'_{(m)}$  from  $\mathcal{A}_{(m-1)}$ .

Since  $\mathbf{R}'_{(m-1)}$  of size  $N \times P(m - 1)$  and  $\mathbf{R}'_{(m)}$  of size  $N \times Pm$  are upper triangular, it is straightforward to obtain that  $\mathbf{R}'_{(m-1)} = \mathbf{R}'_{(m)}(:, 1 : P(m - 1))$ , which results in that the size reduction and column swapping performed on the first  $P(m - 1)$  columns of  $\mathbf{R}'_{(m)}$  are the same as those on  $\mathbf{R}'_{(m-1)}$ . Using  $\mathbf{R}_{(m-1)}$ , let  $\mathcal{A}_{(m)} = \mathcal{A}'_{(m)}$  and  $\mathbf{R}_{(m)}(:, 1 : P(m - 1)) = \mathbf{R}_{(m-1)}$ . Then, we have the 1<sup>st</sup> to  $P(m - 1)$ -th column vectors of  $\mathbf{R}_{(m)}$  satisfying (29) and (30). From this, we can see that CLLL is partially performed on  $\mathbf{R}_{(m)}$  by employing UBLR. Similarly, with  $\mathbf{Q}_{(m)} = \mathbf{Q}_{(m-1)}$  and  $\mathbf{U}_{(m)}(1 : P(m - 1), 1 : P(m - 1)) = \mathbf{U}_{(m-1)}$ ,  $\{\mathbf{Q}_{(m)}, \mathbf{U}_{(m)}\}$  can be updated with low computational complexity from  $\{\mathbf{Q}_{(m-1)}, \mathbf{U}_{(m-1)}\}$ . Thus, from  $\mathcal{A}_{(m-1)}$ , UBLR is carried out to update the elements in  $\mathcal{A}_{(m)}$  as shown in rows (6)-(8) in Table 1.

In addition, we note that, in row (8) of Table 1, we do not consider updating  $\mathbf{R}_{(m)}(1 : P(m - 1), P(m - 1) + 1 : Pm)$  in  $\mathcal{A}_{(m)}$ . It can be observed that when we perform a CLLL on  $\mathbf{H}_{(m)}$  with the same operations of the CLLL for previous user selections,  $\mathbf{R}_{(m)}(1 : P(m - 1), P(m - 1) + 1 : Pm)$  will also be influenced. Hence, extra processing is necessary to recover  $\mathbf{R}_{(m)}(1 : P(m - 1), P(m - 1) + 1 : Pm)$  in  $\mathcal{A}_{(m)}$ . To this end, we define that  $\mathcal{B}_{(m-1)} = \{\Theta_{(m-1)}, \mathbf{fl}_{(m-1)}, \eta_{(m-1)}\}$ , where  $\Theta_{(m-1)} = \{\Theta_{(m-1,1)}, \dots, \Theta_{(m-1,\eta)}\}$ ,  $\mathbf{fl}_{(m-1)} = \{\mathbf{fl}_{(m-1,1)}, \dots, \mathbf{fl}_{(m-1,\eta)}\}$ , and  $\eta_{(m-1)} = \eta$ . The operations of swapping and updating  $\mathbf{R}_{(m-1)}$  and  $\mathbf{Q}_{(m-1)}$  are kept in  $\eta_{(m-1)}$ ,  $\gamma_{(m-1)}$ , and  $\Theta_{(m-1,\eta)}$ , where  $\eta_{(m-1)}$  keeps the number of swapping times,  $\gamma_{(m-1)}$  keeps those columns involved in the swaps, and  $\Theta_{(m-1,\eta)}$  keeps the operations of column swaps. From the CLLL (see row (27) in Table 1), we note that  $\mathbf{R}_{(m)}(1 : P(m - 1), P(m - 1) + 1 : Pm)$  is generated by a transformation with  $\Theta_{(m)}$ . Thus, using the information kept in  $\mathcal{B}_{(m-1)}$ , we can generate  $\mathbf{R}_{(m)}(1 : P(m - 1), P(m - 1) + 1 : Pm)$  as shown in rows (9)-(11) of Table 1.

With an updated  $\mathcal{A}_{(m)}$ , one CLLL can be carried out to generate the reduced basis  $\mathbf{G}_{(m)}$ . The calculation of this new basis generation starts with  $\rho = (m - 1)P + 1$ . Hence, the computational complexity of UBLR is evidently reduced as compared to employing one CLLL starting with  $\rho = 2$ . Note that since UBLR and CLLL generate the same LBR  $\mathbf{G}_{(m)}$ , they are expected to provide the same performance.

The UBLR algorithm of the  $m$ th user selection is summarized in Table 1. The inputs of the algorithm of the  $m$ th user selection are  $\{\mathcal{A}_{(m-1)}, \mathcal{B}_{(m-1)}, \mathbf{H}_{(m-1)}, \mathbf{H}_{k(m)}\}$ , while the outputs are  $\{\mathcal{A}_{(m)}, \mathcal{B}_{(m)}\}$ . Note that for the first user selection, with its channel matrix  $\mathbf{H}_{k(1)}$  as the input, instead of using the UBLR, one CLLL is carried out to generate  $\{\mathcal{A}_{(1)}, \mathcal{B}_{(1)}\}$  as the output. Since the outputs of the  $m$ th user selection are regarded as the inputs at the  $(m + 1)$ -th user selection, the algorithm is recursively carried out from  $m = 2$ . The algorithm is terminated if  $m = M$ .

The complexity of CLLL and UBLR algorithms highly depends on the number of column swaps, which is denoted by the output parameter  $\eta$ . In Table 2, the average value of  $\eta$  per iteration is shown when the CLLL-based MMSE-SIC detector is used with the proposed LRG and UBLRG user selection. It is assumed that  $K = 10$  and  $N = 8$  for the two



Average value of $\eta$								
Number of columns in $\mathbf{H}_K$	2	3	4	5	6	7	8	Sum
LRG <sup>1</sup>	0.2909	0.9029	1.8022	3.0633	4.7711	7.2925	12.1228	30.2457
UBLRG <sup>1</sup>	0.2904	0.5851	0.8940	1.2708	1.7653	2.5620	4.7728	12.1404
LRG <sup>2</sup>	0.2926	n/a	1.7977	n/a	4.7663	n/a	12.0856	18.9422
UBLRG <sup>2</sup>	0.2879	n/a	1.4952	n/a	3.0191	n/a	7.3761	12.1783

Note that the superscript <sup>1</sup> denotes the case of  $K = 10, N = 8, (M, P) = (8, 1)$  and the superscript <sup>2</sup> denotes the case of  $K = 10, N = 8, (M, P) = (4, 2)$ , respectively.

**Table 2.** The average value of  $\eta$  in the LRG and UBLRG user selection with the CLLL based MMSE-SIC detector is used.

possible cases of  $(M, P) = (8, 1)$  and  $(M, P) = (4, 2)$ . Based on these results, we can observe that the complexity is significantly reduced if UBLR is employed. We also note that with the LRG, the complexity for the case of  $(M, P) = (8, 1)$  is higher than that of  $(M, P) = (4, 2)$  as expected (a large  $M$  implies a higher complexity). We can also show that the complexity of UBLRG is upper-bounded as  $(K - M + 1)O((MP)^3 N \log(MP)) + \sum_{i=1}^{M-1} O((iP)^3 N \log(iP))$ . Compared to the complexity of LRG which is upper-bounded as  $\sum_{i=1}^M (K - i + 1)O((iP)^3 N \log(iP))$ , the UBLRG scheme has a lower complexity, especially when large  $K$  and  $M$  are considered.

## 6. Diversity Analysis and Numerical Results

In this section, we consider the diversity gain of the combinatorial user selection approaches with various detectors, such as the ML, MMSE, and LR-based SIC detectors. We derive lower bounds on the diversity gain of them. Since the diversity gain analysis of the proposed greedy user selection approach is difficult, we rely on simulations, from which we can show that our proposed LRG/UBLRG user selection approach has a similar diversity gain and comparable performance to the combinatorial one. Throughout this section, we assume that the elements of the channel matrix  $\mathbf{H}_K$  are independent zero-mean CSCG random variables with variance  $\sigma_h^2$ .

### 6.1. Diversity Gain Analysis from Error Probability

Through the following diversity gain analysis, we can see the impact of each MIMO detector on the performance of multiuser systems.

#### 6.1.1. Diversity Gain of Combinatorial User Selection with ML and MMSE Detectors

Using the pairwise error probability (PEP), we can find the diversity order from multiple receive antennas as well as multiple user selection.

**Theorem 6.1.** *The average PEP of the ML detector with the  $M$  selected users under the MDist user selection criterion, denoted by  $P_e^{\text{ml}}$ , is upper-bounded as*

$$P_e^{\text{ml}} \leq c_1 \left( \frac{\|\sigma_h^2 \mathbf{d}\|^2}{N_0} \right)^{-N \lfloor \frac{K}{M} \rfloor} + o \left( \left( \frac{\|\sigma_h^2 \mathbf{d}\|^2}{N_0} \right)^{-N \lfloor \frac{K}{M} \rfloor + 1} \right), \quad (31)$$

where  $c_1 > 0$  is constant, and  $\mathbf{d} = \mathbf{s}_{(1)} - \mathbf{s}_{(2)}$  (here,  $\mathbf{s}_{(i)} \in \mathcal{S}^{MP}$  and  $\mathbf{s}_{(1)} \neq \mathbf{s}_{(2)}$ ).



*Proof.* See Section 8.1. □

This theorem shows that a full receive diversity gain of  $N$  together with a partial multiuser diversity gain of at least  $\lfloor \frac{K}{M} \rfloor$  can be achieved by the ML detector under the MDist user selection criterion. This result is derived under the fact that there are at least  $\lfloor \frac{K}{M} \rfloor$  statistically independent alternative combinations of the composite channel matrix  $\mathbf{H}_{\mathcal{K}}$  for  $M$  users. Hence, this result is a lower bound on the diversity gain. In fact, there are more combinations for  $\mathbf{H}_{\mathcal{K}}$ , which are not independent, that can increase the multiuser diversity gain. By simulations, we will further demonstrate the impact of the combinations of  $M$  selected users that are not independent.

**Theorem 6.2.** *The average PEP of the MMSE detector with the selected  $M$  users under the ME user selection criterion, denoted by  $P_e^{\text{mmse}}$ , is upper-bounded as*

$$P_e^{\text{mmse}} \leq c_2 \left( \frac{\sigma_h^2 \|\mathbf{d}\|^2}{N_0} \right)^{-(N-P+1)\lfloor \frac{K}{M} \rfloor} + o \left( \left( \frac{\sigma_h^2 \|\mathbf{d}\|^2}{N_0} \right)^{-(N-P+1)\lfloor \frac{K}{M} \rfloor + 1} \right), \quad (32)$$

where  $c_2 > 0$  is constant.

*Proof.* See Section 8.2. □

This theorem shows that for the MMSE detector, the ME user selection criterion may not be able to exploit a full receive diversity.

### 6.1.2. Diversity Gain of Combinatorial User Selection with LR-based Detector

**Theorem 6.3.** *The average PEP of the LR-based SIC detector with the selected  $M$  users under the MD user selection criterion, denoted by  $P_e^{\text{lr}}$ , is upper-bounded as*

$$P_e^{\text{lr}} \leq c_3 \left( \frac{\|\sigma_h^2 \mathbf{d}\|^2}{N_0} \right)^{-N\lfloor \frac{K}{M} \rfloor} + o \left( \left( \frac{\|\sigma_h^2 \mathbf{d}\|^2}{N_0} \right)^{-N\lfloor \frac{K}{M} \rfloor + 1} \right), \quad (33)$$

where  $c_3 > 0$  is constant.

*Proof.* See Section 8.3. □

This theorem shows that a full receive diversity gain of  $N$  together with the same partial multiuser diversity gain,  $\lfloor \frac{K}{M} \rfloor$ , as with the ML detector, can be achieved by the LR-based detector under the MD user selection criterion. From these results, we can see that the LR-based detector is as good as the ML detector with respect to the diversity gains.

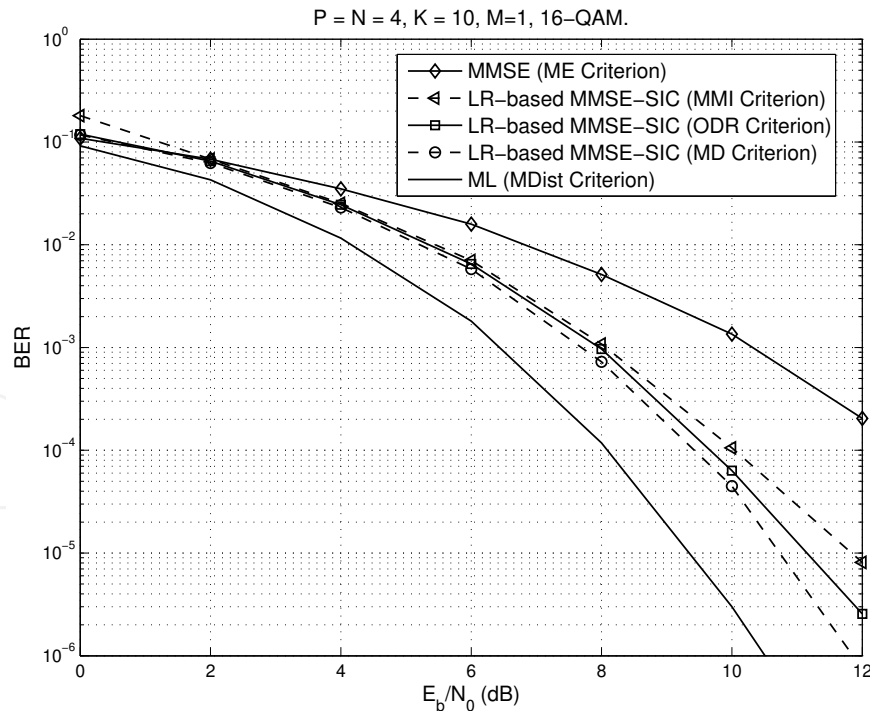
## 6.2. Numerical results

In this subsection, we present simulation results with the MIMO channels of  $\sigma_h^2 = 1$ . The SNR is defined by the energy per bit to the noise power spectral density ratio  $E_b/N_0$ . We used 16 quadrature amplitude modulation (16-QAM) for signaling with Gray mapping. CLLL is carried out for the lattice basis reduction.

### 6.2.1. Single user selection

In order to illustrate the impact of the diversity gain to multiuser MIMO systems, we first present the bit error rate (BER) performance of various multiuser MIMO systems in Fig. 1, where only a single user is selected at one time (i.e.,  $M = 1$ ). Five multiuser MIMO systems are considered with  $P = N = 4$  and  $K = 10$ , namely:

1. MMSE detection under ME criterion: MMSE (ME).
2. ML detection under MDist criterion: ML (MDist).
3. LR-based MMSE-SIC detection under maximize mutual information (MMI) criterion: LR-based MMSE-SIC (MMI).
4. LR-based MMSE-SIC detection under optimal decision region (ODR) criterion: LR-based MMSE-SIC (ODR)
5. LR-based MMSE-SIC detection under MD criterion: LR-based MMSE-SIC (MD)

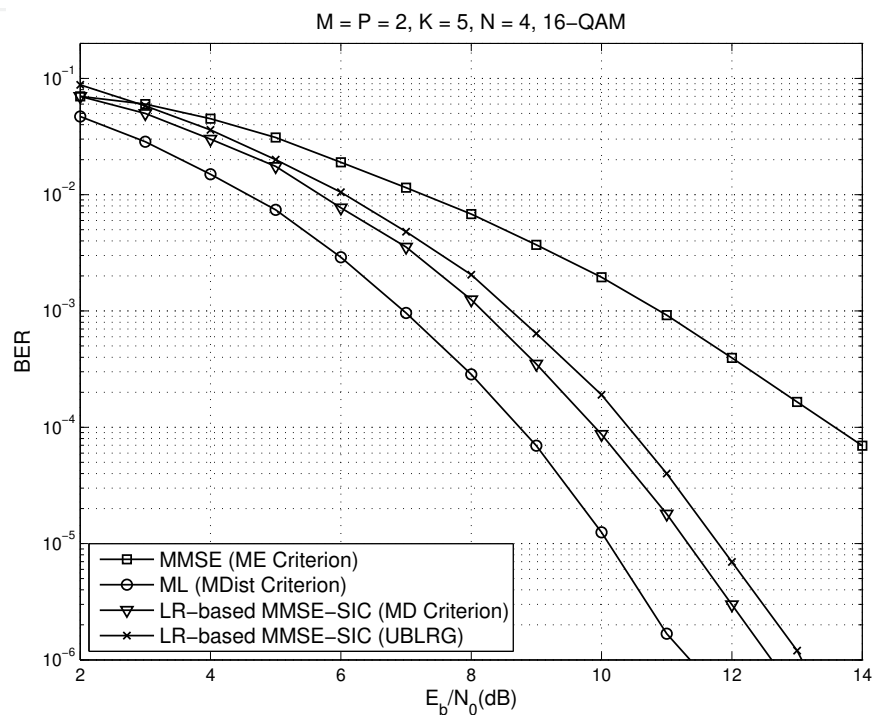


**Figure 1.** BER performance of various multiuser MIMO systems with 16-QAM,  $P = N = 4$ ,  $M = 1$ , and  $K = 10$ .

From Fig. 1, we can observe that the optimal performance is guaranteed by the ML detection under MDist criterion. On the other hand, the conventional MMSE detector with the ME criterion provides poor performance as they cannot fully exploit spatial diversity.

Alternatively, the LR-based SIC detector with the MD criterion can exploit a full diversity as the ML detector with the MDist criterion. It is noteworthy that a full diversity gain cannot be achieved by the LR-based MMSE-SIC detection with the MMI criterion, although the performance can be improved by using the ODR criterion, there is still a BER gap compared to the one with the MD criterion. Overall, it is shown that the best user selection criterion for the LR-based MMSE-SIC detection is the MD criterion.

### 6.2.2. Multiple users selection



**Figure 2.** BER performance of various multiuser MIMO systems with 16-QAM,  $M = P = 2$ ,  $N = 4$ , and  $K = 5$ .

To see the performance of different multiple users selection criteria, the BER results are shown in Fig. 2 for the case of  $M = P = 2$ . We assume that  $K = 5$  and  $N = 4$ . It is shown that, when BER drops from  $10^{-5}$  to  $10^{-6}$ , SNR increases by approximately 1.2 dB. Thus, an estimate of the diversity gain from the simulation becomes  $G \approx 8.3$ , which is greater than the lower bound,  $G_{low} = N \lfloor K/M \rfloor = 8$ , derived from the theoretical analysis in Section 6.1. Moreover, it is shown that the user selection approach with the LR-based detectors has the same diversity gain as in the ML detector, whereas the approach with the MMSE detector has a lower diversity gain. In general, we can show that system of LR-based MMSE-SIC detector with UBLRG can provide a reasonably good performance. Note that, compared with the LR-based MMSE-SIC detector with MD criterion and combinatorial selection, the proposed UBLRG system provides a similar performance; however, as shown in Table 2, by decreasing the number of column swapping, complexity can also be reduced remarkably with more efficient implementations for the proposed UBLRG approach.

## 7. Conclusion

In this chapter, we studied the user selection based on the error probability of an actually employed MIMO detector in multiuser MIMO systems. As the complexity becomes prohibitively high if the user selection is based on exhaustive search (i.e., the combinatorial user selection), we considered a greedy user selection approach to keep the complexity low. We showed that low-complexity suboptimal detectors (i.e., the LR-based MMSE-SIC detector) with the MD criterion for the user selection can fully exploit both multiuser and receive diversity and provide good performance even though their complexity is low, which has been confirmed by both theoretical analysis and simulation results. Moreover, according to the simulation results, it was also shown that the LR-based detection with our proposed greedy user selection approach can achieve a similar diversity gain and have a comparable performance with that based on a combinatorial approach.

## 8. Appendix

### 8.1. Proof of Theorem 6.1

*Proof.* With the selected  $M$  users by the combinatorial user selection approach under the MDist criterion, suppose that we jointly detect  $M$  users' signals with the  $N \times MP$  channel matrix  $\mathbf{H}_{\mathcal{K}}$  using the ML detector. The PEP in detecting  $M$  users' signals has the following upper bound:

$$\Pr(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)}) \leq \operatorname{erfc} \left( \sqrt{\frac{\|\mathbf{H}_{\mathcal{K}} \bar{\mathbf{d}}\|^2}{2N_0}} \right), \quad (34)$$

where

$$\bar{\mathbf{d}} = \arg \min_{\mathbf{d} \in \mathcal{D}, \mathbf{d} \neq \mathbf{0}} \|\mathbf{H}_{\mathcal{K}} \mathbf{d}\|^2, \quad (35)$$

$$\mathcal{D} = \left\{ \mathbf{d} = \mathbf{s} - \mathbf{s}' \mid \mathbf{s} \neq \mathbf{s}' \in \mathcal{S}^{MP} \right\} \subset \mathbb{Z}^{MP} + j\mathbb{Z}^{MP},$$

and  $\operatorname{erfc}(x)$  is the complementary error function of  $x$ , i.e.,  $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-z^2} dz$ .

Let  $\mathcal{V}(\mathbf{H}_{\mathcal{K}})$  denote the length of the shortest non-zero vector of the lattice generated by  $\mathbf{H}_{\mathcal{K}}$ . Then, we have

$$\Pr(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)}) \leq \operatorname{erfc} \left( \sqrt{\frac{\mathcal{V}^2(\mathbf{H}_{\mathcal{K}})}{2N_0}} \right), \quad (36)$$

where

$$\mathcal{V}(\mathbf{H}_{\mathcal{K}}) = \|\mathbf{H}_{\mathcal{K}} \bar{\mathbf{d}}\|. \quad (37)$$

For the case that the MDist criterion is employed, we have

$$\Pr(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)}) \leq \operatorname{erfc} \left( \sqrt{\frac{\max_{\mathcal{K}} \mathcal{V}^2(\mathbf{H}_{\mathcal{K}})}{2N_0}} \right), \quad (38)$$

Note that

$$\max_{\mathcal{K}} \mathcal{V}^2(\mathbf{H}_{\mathcal{K}}) = \max_{\mathcal{K}} \min_{\mathbf{d} \in \mathcal{D}, \mathbf{d} \neq \mathbf{0}} \mathbf{d}^H \mathbf{H}_{\mathcal{K}}^H \mathbf{H}_{\mathcal{K}} \mathbf{d}, \quad (39)$$

Let  $\mathbf{w}_{\mathcal{K}} = \mathbf{H}_{\mathcal{K}} \mathbf{d}$ . Note that  $\mathbf{w}_{\mathcal{K}}$  is a zero-mean CSCG random vector and

$$E[\mathbf{w}_{\mathcal{K}} \mathbf{w}_{\mathcal{K}}^H] = \sigma_h^2 \|\mathbf{d}\|^2 \mathbf{I}. \quad (40)$$

We can show that  $X_{\mathcal{K}} = \|\mathbf{w}_{\mathcal{K}}\|^2$  is a chi-square random variable with  $2N$  degrees of freedom and its pdf is

$$f_X(x_{\mathcal{K}}) = \frac{1}{(\sigma_h^2 \|\mathbf{d}\|^2)^N (N-1)!} x_{\mathcal{K}}^{N-1} e^{-x_{\mathcal{K}}/(\sigma_h^2 \|\mathbf{d}\|^2)}. \quad (41)$$

The cumulative distribution function (cdf) is

$$F_X(x_{\mathcal{K}}) = 1 - e^{-x_{\mathcal{K}}/(\sigma_h^2 \|\mathbf{d}\|^2)} \sum_{q=0}^{N-1} \frac{(x_{\mathcal{K}}/(\sigma_h^2 \|\mathbf{d}\|^2))^q}{q!}. \quad (42)$$

To obtain an upper bound on the error probability, we note that the number of alternative combinations of the channel matrices, which are statistically independent with each other, for selecting  $\mathbf{H}_{\mathcal{K}}$  with the MDist selection is at least  $\lfloor \frac{K}{M} \rfloor$ . Let  $\mathbf{H}_{\mathcal{K}_1}, \mathbf{H}_{\mathcal{K}_2}, \dots, \mathbf{H}_{\mathcal{K}_{\lfloor \frac{K}{M} \rfloor}}$  represent such  $\lfloor \frac{K}{M} \rfloor$  independent alternative combinations of the channel vectors. Then, there are at least  $\lfloor \frac{K}{M} \rfloor$  of  $\mathbf{w}_{\mathcal{K}}$ , i.e.,  $\mathbf{w}_{\mathcal{K}_1}, \mathbf{w}_{\mathcal{K}_2}, \dots, \mathbf{w}_{\mathcal{K}_{\lfloor \frac{K}{M} \rfloor}}$ , which are independent. Let  $V = \max\{X_1, X_2, \dots, X_{\lfloor \frac{K}{M} \rfloor}\}$ , where  $X_m = \|\mathbf{w}_{\mathcal{K}_m}\|^2$ . Using order statistics, the pdf of  $V$  is given by

$$f_V(v) = K F_X^{\lfloor \frac{K}{M} \rfloor - 1}(v) f_X(v) = c'_1 v^{N \lfloor \frac{K}{M} \rfloor - 1} + o(v^{N \lfloor \frac{K}{M} \rfloor - 1 + \epsilon}), \quad (43)$$

where  $c'_1 > 0$  is a constant, and  $\epsilon > 0$ . Thus, we have

$$\begin{aligned} P_e^{\text{ml}} &\leq \sum_{\mathbf{d} \in \mathcal{D}, \mathbf{d} \neq \mathbf{0}} E_V \left[ \operatorname{erfc} \left( \sqrt{\frac{\max_{\mathcal{K}} \mathbf{d}^H \mathbf{H}_{\mathcal{K}}^H \mathbf{H}_{\mathcal{K}} \mathbf{d}}{2N_0}} \right) \right] \\ &= c_1 \left( \frac{\|\sigma_h^2 \mathbf{d}\|^2}{N_0} \right)^{-N \lfloor \frac{K}{M} \rfloor} + o \left( \left( \frac{\|\sigma_h^2 \mathbf{d}\|^2}{N_0} \right)^{-N \lfloor \frac{K}{M} \rfloor + 1} \right), \end{aligned} \quad (44)$$

where  $c_1 > 0$  is a constant. This completes the proof.  $\square$

## 8.2. Proof of Theorem 6.2

*Proof.* It can be shown that under the ME criterion, for a given  $\mathbf{H}_{\mathcal{K}}$ , an upper bound on the error probability in detecting  $M$  users' signals is expressed as

$$\begin{aligned}
 P_e^{\text{mmse}} &\leq \text{erfc} \left( \sqrt{\frac{\max_{\mathcal{K}} \lambda_{\min}(\mathbf{H}_{\mathcal{K}}^H \mathbf{H}_{\mathcal{K}}) \|\mathbf{d}\|^2}{2N_0}} \right) \\
 &= \text{erfc} \left( \sqrt{\frac{\sigma_h^2 \|\mathbf{d}\|^2 \max_{\mathcal{K}} \tilde{X}_{\mathcal{K}}}{2N_0}} \right) \\
 &= \text{erfc} \left( \sqrt{\frac{\sigma_h^2 \|\mathbf{d}\|^2 V}{2N_0}} \right), \tag{45}
 \end{aligned}$$

where  $\tilde{X}_{\mathcal{K}} = \lambda_{\min}(\mathbf{H}_{\mathcal{K}}^H \mathbf{H}_{\mathcal{K}}) / \sigma_h^2$  and  $V = \max_{\mathcal{K}} \tilde{X}_{\mathcal{K}}$ .

Using the pdf of  $V$  (with the same derivation for the ML case in the last subsection), it can be deduced that

$$\begin{aligned}
 P_e^{\text{mmse}} &= E_{\mathbf{H}_{\mathcal{K}}} [\Pr(\mathbf{s}_{(1)} \rightarrow \mathbf{s}_{(2)})] \\
 &\leq E_V \left[ \text{erfc} \left( \sqrt{\frac{\sigma_h^2 \|\mathbf{d}\|^2 V}{2N_0}} \right) \right]. \tag{46}
 \end{aligned}$$

For independent alternative combinations of the channel matrices  $\mathbf{H}_{\mathcal{K}_1}, \mathbf{H}_{\mathcal{K}_2}, \dots, \mathbf{H}_{\mathcal{K}_{\lfloor \frac{K}{M} \rfloor}}$ , similar to the proof of Theorem 5.1, we can obtain that

$$\begin{aligned}
 P_e^{\text{mmse}} &\leq E_V \left[ \text{erfc} \left( \sqrt{\frac{\sigma_h^2 \|\mathbf{d}\|^2 V}{2N_0}} \right) \right] \\
 &\leq \int_0^{+\infty} \text{erfc} \left( \sqrt{\frac{\sigma_h^2 \|\mathbf{d}\|^2 v}{2N_0}} \right) f_V(v) dv \\
 &= c_2 \left( \frac{\sigma_h^2 \|\mathbf{d}\|^2}{N_0} \right)^{-(N-P+1) \lfloor \frac{K}{M} \rfloor} + o \left( \left( \frac{\sigma_h^2 \|\mathbf{d}\|^2}{N_0} \right)^{-(N-P+1) \lfloor \frac{K}{M} \rfloor + 1} \right), \tag{47}
 \end{aligned}$$

where  $c_2 > 0$  is constant. This completes the proof.  $\square$

### 8.3. Proof of Theorem 6.3

*Proof.* In the LR algorithm, we transform the given channel matrix, e.g.,  $\mathbf{H}$ , into a new basis, e.g., denoted by  $\mathbf{G}$ . Here, we have  $\mathcal{L}(\mathbf{G}) = \mathcal{L}(\mathbf{H}) \iff \mathbf{G} = \mathbf{HT}$ , where  $\mathbf{T}$  is an integer unimodular matrix and  $\mathcal{L}(\mathbf{A})$  denotes the lattice generated by  $\mathbf{A}$ . Then,  $\mathbf{G}$  is called LLL-reduced with parameter  $\delta$  if  $\mathbf{G}$  is QR factorized as  $\mathbf{G} = \mathbf{QR}$  where  $\mathbf{Q}$  is unitary,  $\mathbf{R}$  is upper triangular, and the elements of  $\mathbf{R}$  satisfies (29) and (30) with  $m = M$ . We rewrite (30) as

$$\delta |r_{\rho,\rho}|^2 \leq |r_{\rho,\rho+1}|^2 + |r_{\rho+1,\rho+1}|^2, \quad \rho = 1, 2, \dots, MP - 1. \quad (48)$$

Then, we can obtain the following inequalities:

$$|r_{\rho+1,\rho+1}|^2 \geq \beta^{-1} |r_{\rho,\rho}|^2, \quad (49)$$

where  $\beta = (\delta - \frac{1}{4})^{-1} > \frac{4}{3}$ , and

$$\min_{\rho} |r_{\rho,\rho}|^2 \geq \beta^{-MP+1} |r_{1,1}|^2. \quad (50)$$

Since  $\mathbf{G} = \mathbf{QR}$ , we have  $|r_{1,1}|^2 = \|\mathbf{g}_1\|^2$  and

$$\|\mathbf{g}_1\|^2 \geq \min_{\mathbf{d} \in \mathcal{D}, \mathbf{d} \neq 0} \|\mathbf{H}\mathbf{d}\|^2 = \mathcal{V}^2(\mathbf{H}). \quad (51)$$

Thus, we have

$$\min_{\rho} |r_{\rho,\rho}|^2 \geq \beta^{-MP+1} \mathcal{V}^2(\mathbf{H}). \quad (52)$$

In the proposed user selection for selecting  $M$  users with the LR-based SIC detectors, (52) becomes

$$\min_{\rho} |r_{\rho,\rho}|^2 \geq \beta^{-MP+1} \mathcal{V}^2(\mathbf{H}_{\mathcal{K}}), \quad (53)$$

where  $\mathcal{K}$  is the index set of the selected users.

Note that the LR-based SIC detection is considered. Let  $n_{\rho}$  denote the  $\rho$ th element of  $\tilde{\mathbf{n}}$ . Then, the LR-based SIC detection does not have error across all the layers if we have  $\frac{|n_{\rho}|}{|r_{\rho,\rho}|} < \frac{1}{2}$  or  $|n_{\rho}|^2 < \frac{|r_{\rho,\rho}|^2}{4}$  for all  $\rho$ . Thus, the error probability of the LR-based SIC detector can be estimated by

$$\Pr(\text{error}) \simeq \exp\left(-\min_q \frac{|r_{\rho,\rho}|^2}{4N_0}\right). \quad (54)$$

Note that the approximation in above becomes accurate as  $N_0 \rightarrow 0$  (or high SNR).



Substituting (53) into (54), we have

$$\begin{aligned} \Pr(\text{error}) &\leq \exp\left(-\beta^{-MP+1}\mathcal{S}^2(\mathbf{H}_{\mathcal{K}})\right) \\ &\leq \sum_{\mathbf{d}\in\mathcal{D},\mathbf{d}\neq\mathbf{0}} \exp\left(-\beta^{-MP+1}\frac{\max_{\mathcal{K}}\mathbf{d}^H\mathbf{H}_{\mathcal{K}}^H\mathbf{H}_{\mathcal{K}}\mathbf{d}}{2N_0}\right). \end{aligned} \quad (55)$$

Then, with the same approach used in the proof of Theorem 5.1, we can show that the upper bound on the average PEP is

$$P_e^{\text{lr}} \leq c_3 \left(\frac{\|\sigma_h^2\mathbf{d}\|^2}{N_0}\right)^{-N\lfloor\frac{K}{M}\rfloor} + o\left(\left(\frac{\|\sigma_h^2\mathbf{d}\|^2}{N_0}\right)^{-N\lfloor\frac{K}{M}\rfloor+1}\right), \quad (56)$$

where  $c_3 > 0$  is constant. This completes the proof.  $\square$

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