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Chapter 7

Numerical Simulations of Post-Critical Behaviour of Thin-Walled Load-Bearing Structures Applied in Aviation

Tomasz Kopecki

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1. Introduction

The design work on aircraft constructions, with applicable standards and requirements imposed by regulations applicable to aircraft design taken into account, represents a discipline significantly different than other fields of modern engineering. In fact, as opposed to rules commonly applicable to design of technical structures, in view of the need to limit the mass, in the case of airframe structures there is a necessity to allow phenomena involving loss of stability with respect to some of their components under the in-flight conditions.

From the historical point of view, the issue of the loss of stability was a factor significantly slowing down the progress in aviation at early stage of its development. In aspiration to ensure safety, a large group of designers adhered to the lattice structure concepts for many years. The first attempts to develop some more advanced solutions were based on the use of corrugated sheet metal as the skin material for wings and fuselages. Such solution was adopted in numerous constructions manufactured on a mass scale, e.g. Ford Trimotor or Junkers 52 (Fig. 1).

With increasing availability of more and more reliable and powerful aircraft engines and the related improvement of aircraft performance parameters, it became necessary to use smooth skin materials constituting integral components of semi-monocoque and monocoque structures. On the other hand, in striving after development of optimum constructions with the mass criterion met at the same time, it became impossible to use the sheet metal with thickness allowing to achieve critical loads with values exceeding the allowable loads.

Similarly as for other types of constructions, the principal rule involved preventing bar systems, such as stringers, frame components, or spar flanges, from buckling. In the case of the loss of stability, such elements of the structure were considered damaged.

Figure 1. Airplanes with the skin made of corrugated sheet metal: Ford Trimotor (left) and Junkers Ju-52 (right)

At the same time it has been found that the skin stability loss is not dangerous for the whole of the structure provided its character is local, i.e. it occurs within the area of skin segments limited by components of the skeleton, such as frames and stringers, and represents an elastic phenomenon.

In such situation, it has become a standard that some local buckling of airframe skin is admitted in the in-flight conditions. In the current state of the art, the rule applies mainly to isotropic materials, e.g. metals. It should be emphasized that in case when the post-buckling deformation field encompasses also components of the framing, such as bars constituting frame components or stringers, the skin buckling is considered global, and the structure is assumed to be destroyed. It can be therefore concluded that a loss of stability is of a local type when it encompasses skin segments limited with components of the framing.

Further experience collected in operation of airframes constructed with the use of the above-mentioned standard revealed another issue connected with limited operating durability of such structures. In fact, cyclic nature of loads which a plane is subjected to in the course of flight induces occurrence of fatigue phenomena which, when undiagnosed or underestimated, may lead to destruction of the structure. Examples include such aviation accidents as e.g. disasters of De Havilland Comet airplanes in the years 1953–54 or the accident occurring in the course of flight of Aloha Airlines' Boeing 737 when a fatigue-induced gap developed in the skin resulted in explosive decompression of the plane's fuselage and breaking off a large fragment of the fuselage skin (Fig. 2).

Nature and intensity of fatigue-induced changes in a structure is related to the stress distribution which, assuming admissibility of skin stability loss, means that it is necessary to carry out detailed analyses of post-buckling deformation states.

The tool that become used commonly for this purpose is the nonlinear numerical analysis based on the finite element method (FEM), allowing to represent actual deformations of thin-walled structures and the related stress distributions. However, in so far as application of FEM in linear problems became a routine in the engineering practice, and results being obtained with the use of commercial software packages are, on the whole, correct and reliable, the

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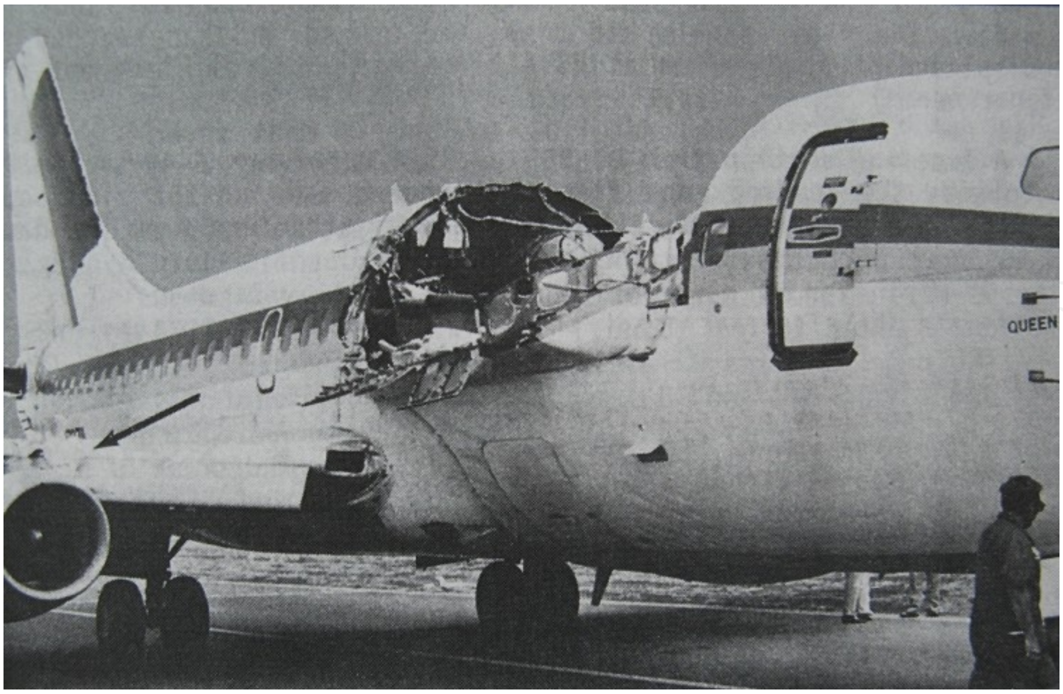

Figure 2. Damaged Aloha Airlines' Boeing 737

nonlinear analysis still causes numerous problems and requires application of additional tools to verify the results.

The essence of FEM-based nonlinear numerical analysis comes down to determination of a relationship between a set of parameters determining the state of the structure, known as state parameters, and a set of control parameters related to the load. The latter can be, in general, related to and expressed by means of a single control parameter. On the other hand, the number of state parameters corresponds to the total number of degrees of freedom of the analyzed system. Such relationship is known as the equilibrium path and for a system with an arbitrary number of degrees of freedom can be interpreted as a hypersurface in the state hyperspace where the dimension of the hyperspace corresponds to the number of degrees of freedom [1-4]. The equilibrium path fulfils the matrix equation of residual forces which, for a single control parameter, has the following form:

$$\mathbf{r}\left(\mathbf{u},\lambda\right) = \mathbf{0}, \tag{1}$$

where $\mathbf{u}$ is the state vector containing components of displacements of nodes of the structure corresponding to its current geometrical configuration, λ is the control parameter representing a function of the load, and $\mathbf{r}$ is the residual vector containing non-balanced force components related to the current system deformation state.

Procedures employed in modern numerical software packages, apart from the prognostic phase allowing to determine the next point on the equilibrium path, comprise also a correction phase offering the possibility to compensate divergences between the actual equilibrium path and the solution determined in the prognostic phase, or the so-called "drift error". The correction phase consists in the use of an additional equation to be met by the system, known as the increment control equation or the equation of constraints,

$$c\left(\Delta \mathbf{u}_n,\ \Delta\lambda_n\right) = 0, \tag{2}$$

where increments $\Delta\mathbf{u}_n = \mathbf{u}_{n+1} - \mathbf{u}_n$ and $\Delta\lambda_n = \lambda_{n+1} - \lambda_n$ correspond to transition from state n to state $n + 1$.

In view of the large number of degrees of freedom and state parameters related to them, deformation processes are represented in practice by means of a relationship between a control parameter related to the load and a selected geometrical quantity linked to deformation of the system. The relationship is called the representative equilibrium path [5-9].

As was already mentioned above, results of FEM-based nonlinear numerical analyses require verification. Relying unquestioningly on such results alone can lead to significant errors in design processes through adopting incorrect solutions as a base for construction design assumptions. The problem consisting in arriving at incorrect deformation patterns as a result of numerical calculations is a consequence of the fact that numerical procedures employed in commercial software packets contain a large number of algorithms the course of which depends on choice of certain control parameters. These in turn follow from the applied boundary conditions, selection of prognostic procedures, correction strategies, and a number of other factors.

In view of practical impossibility to obtain appropriate solutions for complex thin-walled structures in a purely analytical way, the basic tool that can be used for verification of results nonlinear numerical analyses is the experiment, by its nature representing an undertaking relatively expensive and frequently difficult to execute.

In case of systems characterized with high degree of complexity or having geometrical singularities of any kind (e.g. cut-outs), execution of an appropriate experiments is absolutely necessary. It should be however emphasized that semi-monocoque aircraft structures include, in many cases, some typical components with characteristic, repeatable geometrical features. In such cases, it seems to be purposeful to create a base of standard solutions, containing result of experiments aimed at determination of deformation patterns of the analyzed structure for a given range of post-buckling loads justified by actual in-flight conditions. Such data could constitute a base sufficient to verify results of nonlinear numerical analyses.

2. Objective and the point of the research

Thin-walled aircraft load-carrying structures, in view of requirements applicable to them, are typically characterized with significant sophistication of the applied solutions. However, despite their advanced degree of complexity, they still have a number of characteristic features following from assumptions on which their operation is based. Thus, the loss of stability of thin-walled shell segments used in such structures is a result, in general, of a distribution of tangential stresses interpreted as a field of tensions. It can be therefore stated that post-buckling deformation patterns of a skin segments limited by components of the framing depend on factors decisive for stress distributions, i.e. proportions between dimensions of skin segments (rectangular in general), curvature radii related to these dimensions, and the load intensity [10].

Occurrence of post-buckling deformations corresponding to rapid changes in combinations of state parameters, known as bifurcation, in the case of the load only slightly exceeding the critical value, depends in great measure on geometrical imperfections and, to some extent, can exhibit random nature [11, 12]. In most cases, however, the ultimate pattern of post-buckling deformations corresponding to the maximum load is the same in each of load cycles. It is therefore possible to distinct the so-called nature of post-buckling deformations in case of structure designs characterized with specific, fixed geometrical features [13,14]. With knowledge of this nature, i.e. availability of data concerning post-buckling deformation patterns for a sufficiently broad spectrum of variants of the structure, it seems to be possible to use them as a tool for verification of results of nonlinear numerical analyses without necessity to carry out additional experiments.

In the present study, an attempt was made to determine the nature of post-buckling deformation of a characteristic fragment of the typical semi-monocoque aircraft structure by means of carrying out a series of relevant model experiments and confronting the results with the outcome of nonlinear numerical analyses performed with the use of commercial software package.

The subject of the research was a closed, semi-monocoque thin-walled cylindrical shell structure which corresponded to a fragment of an aircraft fuselage tail section (Fig. 3).

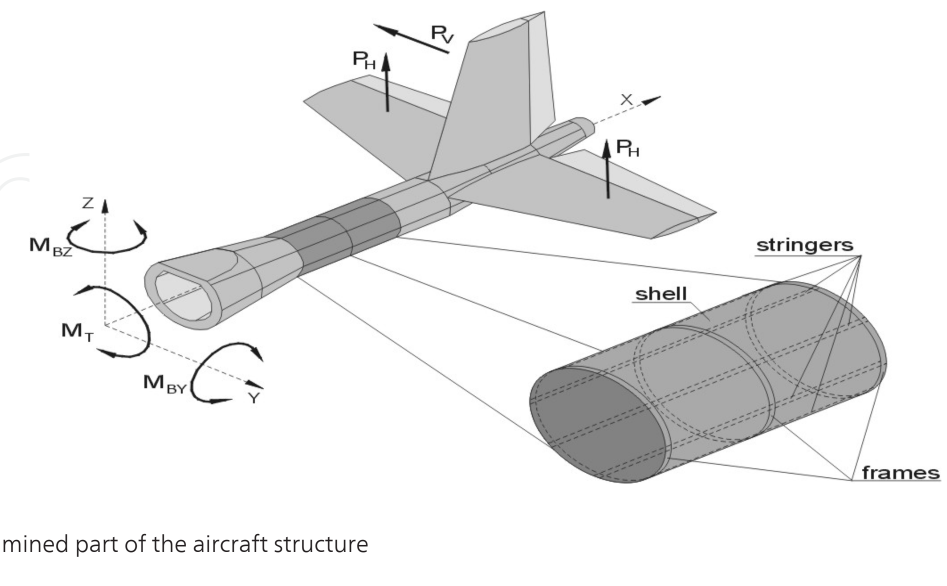

Figure 3. Examined part of the aircraft structure

In the in-flight conditions, the structure can be subjected to bending and twisting, as a result of aerodynamic forces exerted on tail control surfaces. The structure components responsible

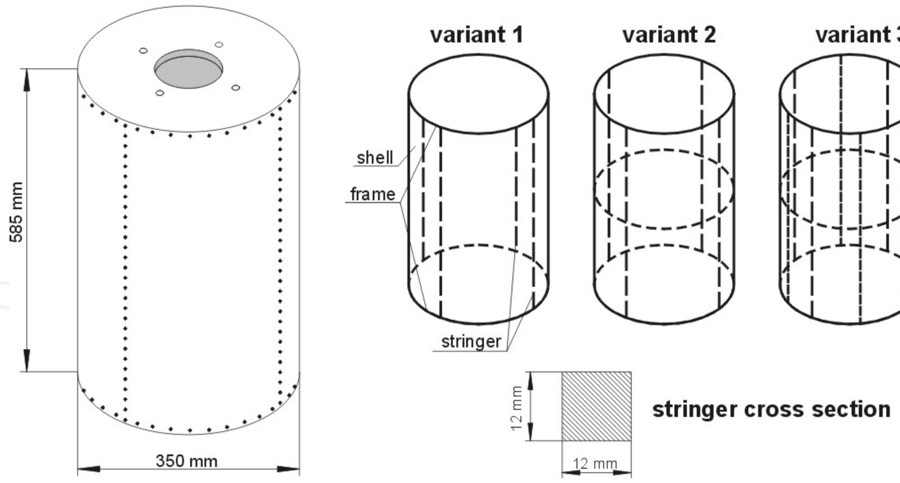

Figure 4. Geometry of the examined structure

for transfer of bending loads are the stringers, cross-sections of which are selected based on the rod stability conditions, with the safety factor provided by aircraft construction regulations taken into account. However, appropriate torsional strength of the structure and its required torsional rigidity must be ensured by the skin in which a distribution of tangential stresses is developed creating conditions favorable for the loss of stability. The distributions, as was already mentioned, depend on a number of factors of geometrical nature, related to the number of frames and stringers determining size and shape of skin segments.

To determine characteristic features of deformation of the tested structure type and examine in detail the nature of the involved phenomena, it is necessary to analyze different geometrical variants. In order to be able to use the obtained results as a universal tool supporting the design process, it would be advisable to examine as broad spectrum of such variants as possible. This study presents only a few examples of such analyses, while the fundamental objective of the work consisted in development of a methodology for creation of sets of results obtained from appropriate model experiments and their numerical representations. Comparison of representative equilibrium paths of the examined systems and convergence of deformation patterns was aimed at determination of recommendations applicable to modeling structures of that type and carrying out nonlinear FEM analyses.

3. Experimental research

The subject of the research were three variants of a thin-walled cylindrical structure, geometrical details of which are presented in Fig. 4. The first variant was a design solution representing a limiting case with particularly small number of framing components. Employing, in the next variant, an additional frame situated half length of the skin segment, and further increasing the number of stringers, corresponded to modifications possible to be employed in practice, as a result of which the size and geometrical relationships characterizing skin segments change significantly.

In all the cases it has been assumed that cross-sections of stringers applied in similar structures have geometrical characteristics preventing them from buckling in conditions of actual operating loads. For this reasons, their dimensions were intentionally exaggerated in model experiments.

All experimental models were made of polycarbonate for which the following material constants have been determined: $E = 3000$ MPa, $v = 0.36$. Selection of the material was dictated by its isotropic properties and low Young modulus which allowed to limit the applied loads to relatively low values.

The models were subjected to constrained torsion with the use of experimental set-up allowing to apply loads gravitationally (Fig. 5). As the representative equilibrium path, the relationship between the total angle of torsion of the structure and the torsional moment was selected. In view of the lack of possibility to register instantaneous changes of the load related to bifurcation changes of combinations of the parameters of state occurring in the structure, the presented equilibrium paths were determined for steady-state conditions as a result of which they have "smooth" courses.

As expected, in the case of the first variant of the examined structure, occurrence of post-buckling deformation had a violent nature. Magnitude of post-buckling deformations and the resulting significant value of the total angle of torsion make application of similar solution in actual aircraft impracticable. Despite the fact that the loss of stability had a local nature, deformations occurring in this case would mean loss of rigidity of the fuselage.

Figure 5. The experimental set-up

The deformation pattern as such was characterized with occurrence of folds observed in all four skin segments (Fig. 6). In the course of experiment, Atos optical scanner of GOM Optical Measuring Techniques brand was used to register the geometry of the deformed skin.

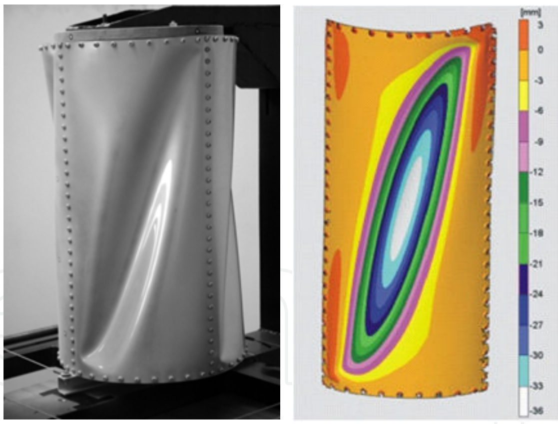

Figure 6. Advanced post-buckling deformation of the examined structure (left) and distribution of contour lines reflecting magnitude of deformations obtained by means of the projection moiré technique (right) — variant 1

In connection with a violent development of deformation, the representative equilibrium path contains a characteristic horizontal segment corresponding to a large change of state parameter combinations at virtually fixed load value (Fig. 9).

The next stage consisted in examination of the second variant of the structure in which an additional frame was employed (Fig. 4, variant 2). The change in proportions of basic skin segment dimensions resulted in occurrence of a post-buckling deformation pattern significantly different than this observed in the first case (Fig. 7).

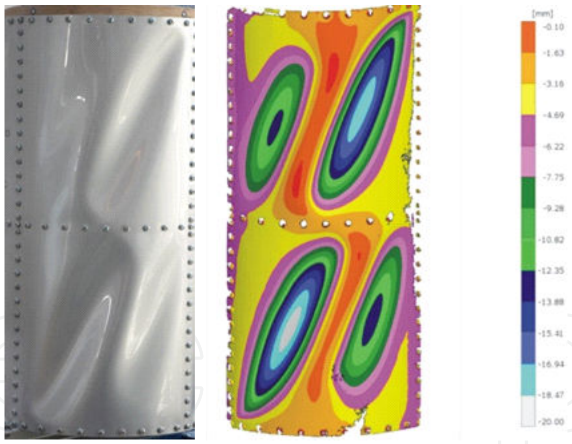

Figure 7. Advanced post-buckling deformation of the examined structure (left) and distribution of contour lines reflecting magnitude of deformations obtained by means of the projection moiré technique (right) — variant 2

Occurrence of a double fold resulted in more gentle course of the phenomenon, which manifested itself in absence of any large jump on the equilibrium path (Fig. 9). An increase of the load critical value and torsional rigidity of the system with respect to the first variant was also observed.

The third variant of the structure was a modification of the second variant reinforced with four evenly distributed additional stringers (Fig. 4, variant 3). As comparison of representative

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equilibrium paths proves, the change of skin segment sizes and proportions as well as changed ratio of the segment dimensions and the curvature radius have brought the effect in the form of further increase of torsional rigidity of the examined structure with simultaneous small reduction of the critical load [10,12].

The pattern of post-buckling deformations has significantly changed, taking the form of single shallow folds (Fig. 8). The course of the phenomenon as such was more gentle in this case than in the variants examined earlier.

Therefore, in this case the skin stability loss resulted in development of small geometrical defects of the fuselage inducing a local drag coefficient increase; however, higher rigidity guarantees that basic aerodynamic properties of the aircraft are maintained.

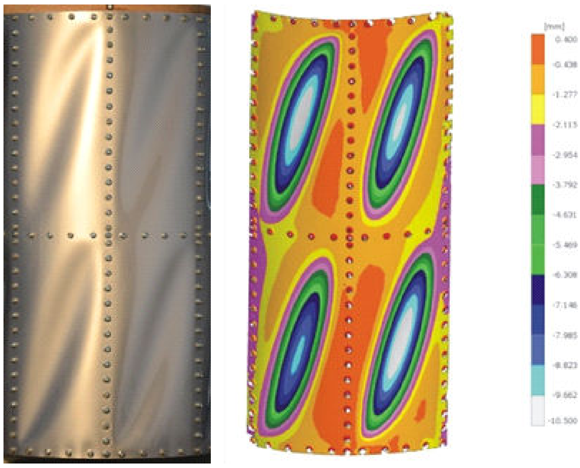

Figure 8. Advanced post-buckling deformation of the examined structure (left) and distribution of contour lines reflecting magnitude of deformations obtained by means of the projection moiré technique (right) — variant 3

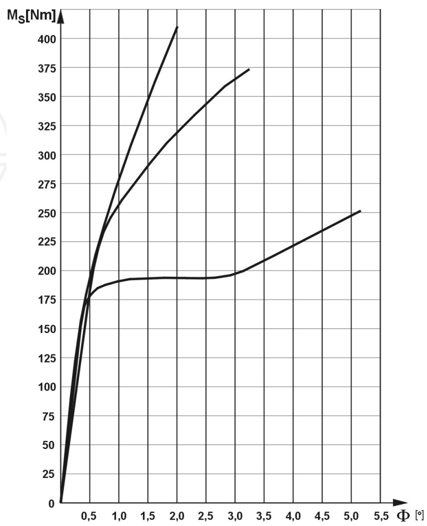

Figure 9. Comparison of representative equilibrium paths determined in the course of experimental research

4. FEM-based numerical analysis

Examining the nature of post-buckling deformations occurring in thin-walled structures with the intent to use the obtained results as a tool useful in aircraft design processes, together with carrying out appropriate experiments, it seems to be purposeful to develop recommendations concerning methods of numerical modeling of considered structures and selection of most effective numerical methods. It should be emphasized that the practice of developing dedicated software based on the finite elements method by entities dealing with aircraft structures design is a relatively rare phenomenon. As a rule, different types of routines are used available on the commercial software market.

Analyses discussed in the present study were carried out on the grounds of MSC MARC program. In all cases, the mounting of the model was reproduced by locking all degrees of freedom of selected points on the upper frame (corresponding to location of bolt joints in the experimental model) and a pair of forces was applied at appropriate points of the lower frame (corresponding to locations where the ties were attached). To represent the skin, four-node shell elements of the thin-shell type were used, with six degrees of freedom at each of the nodes and bilinear shape functions. The frames have been modeled with the use of thick-shell elements with similar properties. In case of stringers, according to software authors' recommendations, beam-type elements were employed, based on the Euler-Bernoulli model [14].

The first variant of the examined structure, in view of the observed post-buckling deformation scale and violent course of the related phenomena, turned out to be very troublesome from the point of view of FEM-based nonlinear numerical simulation. In fact, lack of ability to represent symmetry or antisymmetry of post-buckling deformation states is a characteristic feature of algorithms employed in majority of commercial software packages. In absence of geometrical imperfections of the structure, the state parameter combination change occurred only in one segment of the structure. Errors of that type follow, in general, from unreliability of algorithm used to select an appropriate equilibrium path variant after reaching a bifurcation point [15].

In order to initiate deformation patterns corresponding to actual ones, additional loads in the form of forces with small values normal to the skin applied at central points of skin segments have been introduced to the numerical model (Fig. 10). However, despite the obtained repeatability of deformation in individual segments, faulty results were obtained for a number of consecutive numerical models or solutions in the full load range were impossible to obtain.

A significant improvement of effectiveness of the analysis process and quality of the obtained results was achieved by changing the concept used to model the stringers. In successive versions of numerical models, thick-walled shell elements were used to represent these components of the structure. From among numerical models available in the software package, after a series of tests, a combination of the prognostic secant method and the strain correction method was adopted [16]. The deformation distribution obtained numerically was found satisfactory from the point of view of both qualitative and quantitative similarity to deformation patterns obtained experimentally. Also representative equilibrium paths (Fig. 12) were

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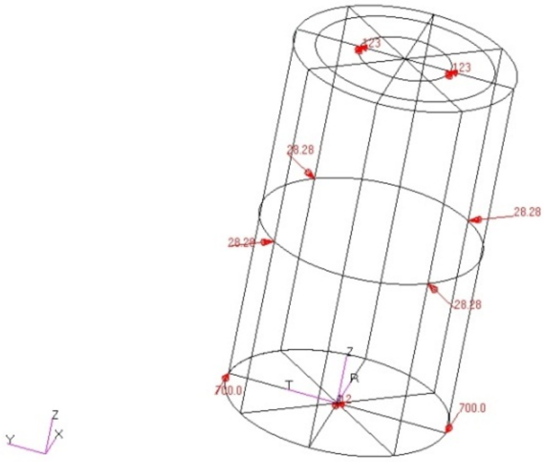

Figure 10. Geometrical model of the structure developed in MSC Patran environment with boundary conditions and load

recognized satisfactorily convergent with experimental characteristics. In line with the rule of uniqueness of solutions, according to which one and only one distribution of the reduced stress corresponds to each deformation state, the obtained reduced stress distribution can be therefore also considered reliable (Fig. 11).

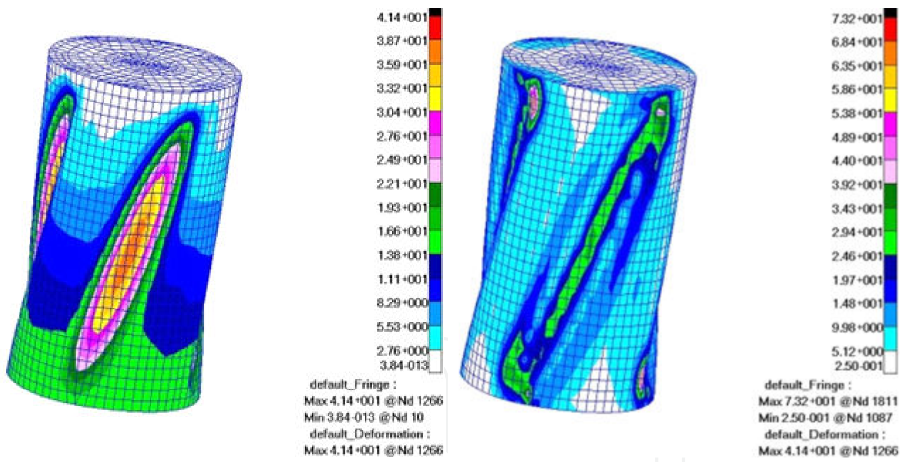

Figure 11. Displacement distribution (left) and the reduced stress according to Huber-Mises hypothesis (right) for 100% of the maximum load (stringers modeled by means of bilinear thick-walled shell elements)

When numerical models for the remaining variants of the structure were developed, the same concept as for application of constraints, loads, and additional forces initiating post-buckling deformation was adopted as in the first variant (Fig. 13).

Application of numerical methods identical to those used previously allowed to obtain post-buckling deformation distributions and the corresponding reduced stress distributions satisfactorily consistent with results of experiments (Figs. 14 and 15).

Plots of representative equilibrium paths for variants 2 and 3 (Figs. 16, 17) are characterized with significantly better conformity with experimental characteristics then in the case of variant 1. In the pre-buckling range, the consistence is almost perfect. On the other hand, for maximum loads, the error of representation of the total angle of torsion does not exceed 10%. It allows to conclude that the adopted modeling method and selection of numerical procedures turned out to be satisfactory.

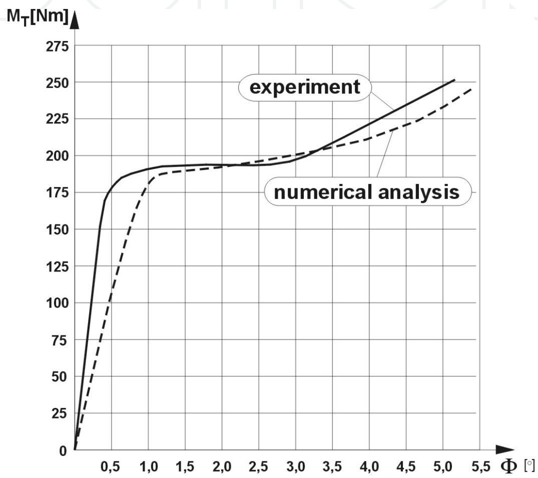

Figure 12. Comparison of representative equilibrium paths — variant 1

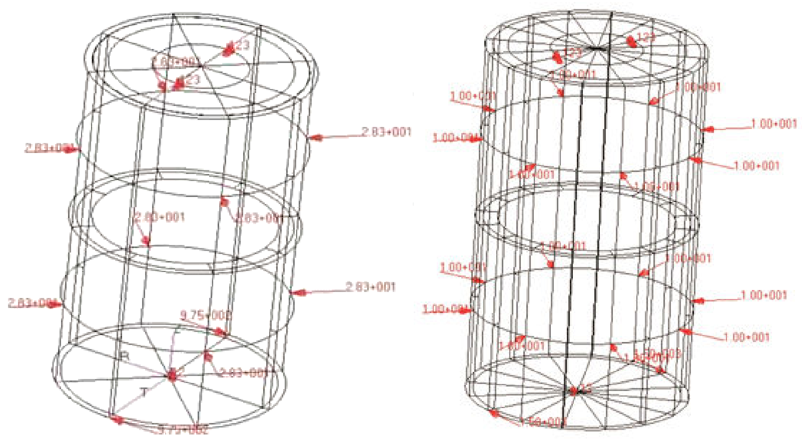

Figure 13. Geometrical models of variant 2 (left) and variant 3 (right) of the examined structure developed in MSC Patran environment, with boundary conditions and loads

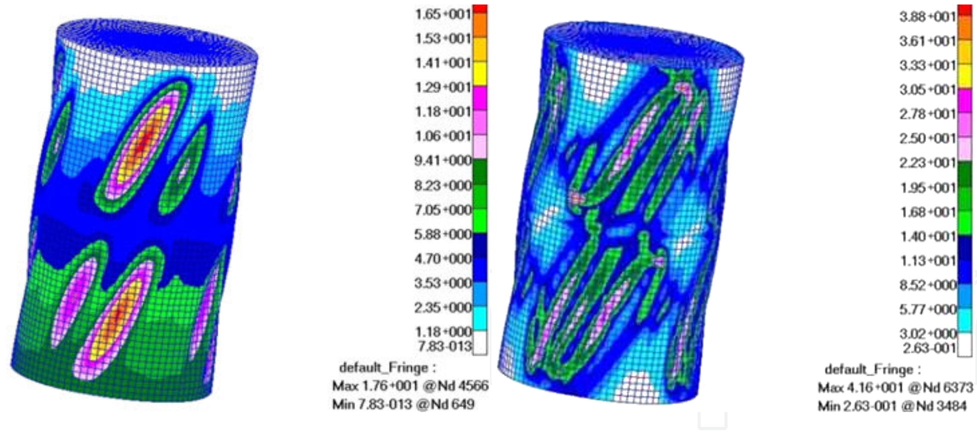

Figure 14. Distribution of the displacement (left) and the reduced stress according to Huber-Mises hypothesis (right) for 100% of the maximum load — variant 2

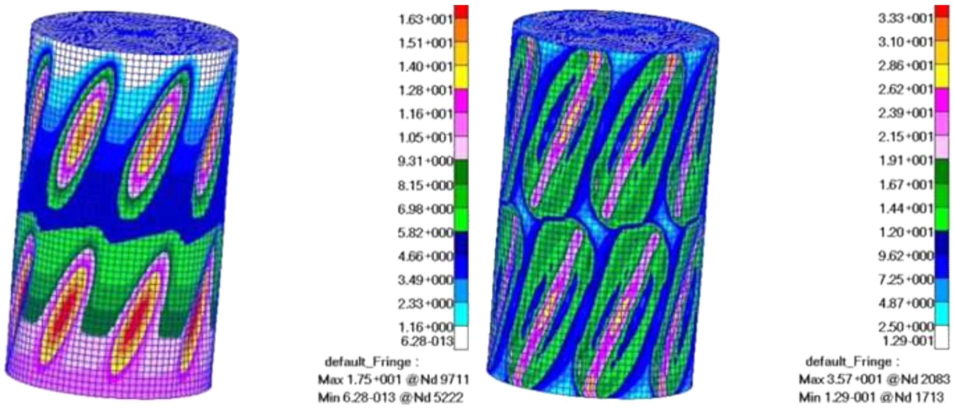

Figure 15. Distribution of the displacement (left) and the reduced stress according to Huber-Mises hypothesis (right) for 100% of the maximum load — variant 3

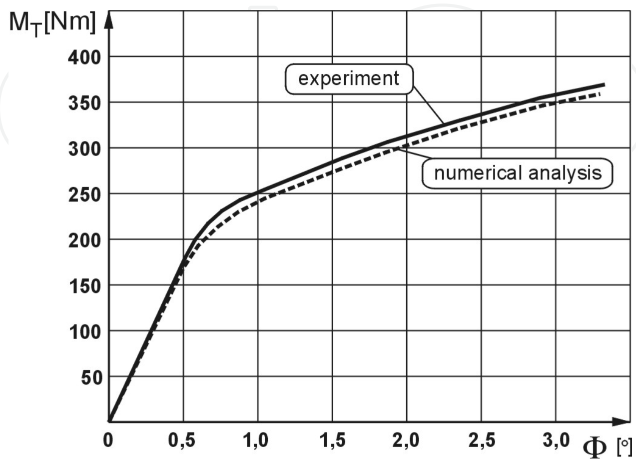

Figure 16. Comparison of representative equilibrium paths — variant 2

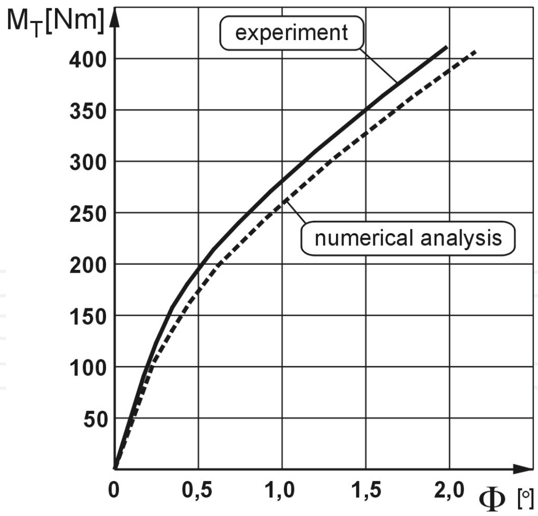

Figure 17. Comparison of representative equilibrium paths — variant 3

5. Summary and conclusions

As it was emphasized in the introduction, the research results presented in this study represent a fragment of the cycle of experiments that should be executed in order to test the whole of the physical phenomena involved in the loss of stability of the examined structure. It can be stated on the grounds of the executed experiments that construction solutions of structures of the type analyzed in this study that comprise too small quantity of framing components, are characterized with deformations far too large to be used in actual aircraft constructions. Considering the three presented variants of the thin-walled cylindrical structure, it seems that the last of them could be used in practical applications.

The fundamental observation that can be made on the grounds of relatively small number of the cases examined here is an increase of torsional rigidity of the structure with increasing number of components of the framing. The increase is caused partly by rigidity of stringers alone, however another reason consists in the change of relationship between the skin segment surface areas and their linear dimensions on one hand and the skin curvature radius on the other. Increasing the number of frames and stringers results in a decrease of average size of skin segments which, at fixed curvature radius value, is the cause of relative „flattening" of skin components. Reduction of the value following from the above-mentioned relationship limits, in a natural way, the depth of folds developed as a result of the loss of stability, and therefore also the scale of deformation. The deformation pattern, and thus also the number and relative position of the folds occurring in individual skin segments, depends also on the ratio of their linear dimensions which is decisive for the nature of the field of tensions developing in the segments [10]. To be able to call a post-buckling deformation research program the completed task, it seems to be necessary to perform a series of experiments aimed at determination of detailed relationships between ratios of geometrical parameters character-

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izing skin segments on one hand and location of folds and the related deformations on the other which in turn determine magnitude of the structure's total angle of torsion. Realization of such research program would require application of the above-described experimental procedure to consecutive versions of the model with fixed curvature radius and different cylinder lengths, and then to another series of models with a fixed length and different diameters. This would allow to determine characteristic combinations of geometrical parameters which are connected to fundamental changes in post-buckling deformation patterns resulting in increase of torsional rigidity of the structure.

As was already noted earlier, results of experiments allow to conclude that, in general, the structure rigidity increases with increasing number of components of the structure framing. It should be however borne in mind that in the case of aircraft structures, there is an absolute necessity to strive after minimization of the mass which limits the possibility to increase the number of frames and stringers. It seems therefore to be possible to determine a limiting number of framing components above which further increase of the weight is no more justified by measurable improvement of strength and rigidity of the structure.

With a sufficiently broad range of test results being available, it would be possible to use them as a base of standards for verification of results of nonlinear numerical analyses, as the nature of post-buckling deformations, with geometrical proportions and rigidity relationships between elements of the structure maintained, is not subject to any major changes when other isotropic materials are used or other load values are applied. This was confirmed by numerical tests performed with the use of models presented here.

The main conclusion following from the presented numerical calculation cases is the necessity to strive to reduce the size of the task and avoid any numerical singularities which, in the case of nonlinear analysis, may result from using different finite element types in the model.

Despite difficulties related frequently to carrying out nonlinear numerical analyses of FEM models of thin-walled structures subjected to advanced deformation states, commercial FEM programs represent a tool allowing for effective determination of stress distributions in such states. However, one should always bear in mind the absolute necessity to verify results obtained this way, either by making use of the above-discussed base of standard solutions, or by performing an appropriate experiment.

Author details

Tomasz Kopecki*

Address all correspondence to: t_kopecki@poczta.wp.pl

* Faculty of Mechanical Engineering and Aeronautics, Rzeszów University of Technology, Rzeszów, Poland

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