Chapter 22

Quantum Dating Market

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Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/53842

1. Introduction

Quantum algorithms have proven to be faster than the fastest known classical algorithms. Clearly, such a superiority means counting on a real quantum computer. Although this essential constraint elimination is in development process, many people is working on that and interesting advances are being made [1–3]. Meanwhile, new algorithms and applications of the existing ones are current research topics [4, 5]. One of the main goals of quantum computing is the application of quantum techniques to classical troubleshooting: the Shor algorithm [6], for example, is a purely quantum-mechanical algorithm which comes to solve the classical factoring problem, also the contribution of Lov Grover [7, 8] to speed up the search for items in an N-item database is very important. Both mathematical finds are the cornerstones of quantum computation, so, considerable amount of work on diverse subjects make use of them. Other algorithms which has been very important for quantum computing progress are Simon's and Deutsch-Jozsa's. Through the quantum games, Meyer in [9] and Eisert in [10], among other, showed that quantum techniques are generalizations of classical probability theory, allowing effects which are impossible in a classical setting. These and many other examples, show that there is no contradiction in using quantum techniques to describe non-quantum mechanical problems and solve hard to solve problems with classical tools. Adding, decision theory and game theory, two examples where probabilities theory is applied, deal with decisions made under uncertain conditions by real humans. Basically, the former considers only one agent and her decisions meanwhile the other considers also the conflicts that two or more players cause to each other through the decisions they take. Due to their inherent complexity this kind of problems results convenient to be analyzed by mean of quantum games models.

Widely observed phenomena of non-commutativity in patterns of behavior exhibited in experiments on human decisions and choices cannot be obtained with classical decision theory [11] but can be adequately described by putting quantum mechanics and decision theory together. Quantum mechanics and decision theory have been recently combined

[11–13] to take into account the indeterminacy of preferences that are determined only when the action takes place. An agent is described by a state that is a superposition of potential preferences to be projected onto one of the possible behaviors at the time of the interaction. In addition to the main goal of modeling uncertainty of preferences that is not due to lack of information, this formalism seems to be adequate to describe widely observed phenomena of non-commutativity in patterns of behavior.

Within this framework, we study the dating market decision problem that takes into account progressive mutual learning [14, 15]. This problem is a variation on the Stable Marriage Problem introduced by Gale and Shapley almost four decades ago [16], that has been recently reformulated in a partial information approach [17, 18]. Specifically, perfect information supposition is very far from being a good approximation for the dating market, in which men and women repeatedly go out on dates and learn about each other.

The dating market problem may be included in a more general category of matching problems where the elements of two sets have to be matched by pairs. Matching problems have broad implications in economic and social contexts [19, 20]. As possible applications one could think of job seekers and employers, lodgers and landlords, men and women who want to date, or solitary ciliates *courtship rituals* [21]. In our model players earn an uncertain payoff from being matched with a particular person on the other side of the market in each time period. Players have a list of preferred partners on the other set. Quantum exploration of partners is compared with classical exploration at the dating set. Nevertheless dating is not just finding, but also being accepted by the partner. The preferences of the chosen partner are important in quantum and classic performances.

Recently [22], we introduced a quantum formulation for decision matching problems, specifically for the dating game that takes into account mutual progressive learning. This learning is accomplished by representing women with quantum states whose associated amplitudes must be modified by men's selection strategies, in order to increase a particular state amplitude and to decrease the others, with the final purpose to achieve the best possible choice when the game finishes. Grover quantum search algorithm is used as a playing strategy. Within the same quantum formulation already used in [22], we will concentrate first on the information associated to the dating market problem. Since we deal with mixed strategies, the density matrix formalism is used to describe the system. There exists a strong relationship between game theories, statistical mechanics and information theories. The bonds between these theories are the density operator and entropy. From the density operator we can construct and understand the statistical behavior about our system by using statistical mechanics. The dating problem is analyzed through information theory under a criterion of maximum or minimum entropy. Even though the decisions players make are based on their payoffs, past experiences, believes, etc., we are not interested in that causes but in the consequences of the decision they take, that is, the influence of the strategies they apply on the quantum system stability. In order to identify the conditions of stability we will use the equivalence between maximum entropy states and those states that obey the Collective Welfare Principle that says that a system is stable only if it maximizes the welfare of the collective above the welfare of the individual [23].

Interesting properties merge when entanglement is considered in quantum models of social decision problems [24]. People decisions are usually influenced by other people actions, opinions, or beliefs, to the extent that they may proceed in ways that they would rarely or never do if moved by their own benefit. Love, hate, envy, or a close friendship, which encase a bit of everything, are examples of relationships between people that may correlate their decisions. So, as driven by a no local force, people may make an inconvenient choice in the heat of a competence. In order to formulate in a mathematical way this sort of problem we remodel the quantum dating between men and women with the inclusion of quantum entanglement between men decision states.

The chapter organization is as follows: First of all, to ease game theory unfamiliar readers comprehension a brief introduction to game basics is presented. In the course of the next sections the quantum dating game is particularly studied. In section 3 the Grover quantum search algorithm as a playing strategy is analyzed. In section 5, the system stability is under study. Finally, section 6 explores entangled strategies performance. At the end of each section the results and the consequent section discussions are set. The chapter ends with a final conclusion.

2. Quantum games

Game theory [25] is a collection of models (games) designed to study competing agents (players) decisions in some conflict situation. It tries to understand the birth and development of conflicting or cooperative behaviors among a group of individuals who behave rationally and strategically according to their personal interests. Although the theory was conceived in order to analyze and solve social and economy problems, existing applications go beyond [26]. Furthermore, the models reach not only individuals but also governments conflicts, institutions trades or smart machines (phones, computers) access management.

Before starting to explain quantum games basics, the classic games notation is presented. The game can be set in strategic (or normal) form or in extensive form, in any of them it has three elements: a set of players $i \in \mathcal{J}$ which is taken to be a finite set $0, ..., N-1$, the set of pure strategies $S_i = \{s_0, s_1, ..., s_{N-1}\}, \quad i = 0, ..., N-1$ which is the set of all strategies available to the player, and the payoffs function $u_i(s_0, s_1, ..., s_{N-1}), \quad i = 0, ..., N-1$, where $s_i \in S_i$. In the strategic form, the game can be denoted by $G(N, S, u)$, where $S = S_0 \times S_1 \times ... \times S_{N-1}$ and $u = u_0 \times u_1 \times ... \times u_{N-1}$. Extensive form representation is useful when it is wanted to include not only who makes the move but also when the move is made. Players apply pure strategies when they are certain of what they want, but such condition is not always possible, so mixed strategies must be considered. A mixed strategy is a probability distribution over S which corresponds to how frequently each move is chosen.

As an example, we can mention the well-known *Prisoners Dilemma (PD)* : Two suspected of committing a crime are caught by the police. As there is insufficient evidence to condemn them, the police place the suspects into separate rooms to convince them to confess. If one of the prisoners confesses, and help the police to condene his partner, he gains his freedom and the other prisoner must serve of 10 years. But if both confess, they must serve a sentence of 3 years. In other case, if both refuse to confess, they both will be convicted of a lesser charge and will have to serve a sentence of only one year in prison. In summary, they can choose between two possible strategies “Confess” (C) or “Not Confess” (N). However, observe that the luck of each player depends both on his election as that of the other. As consequence, confessing is a dominant strategy because regardless the other player decision the one who chooses it avoid the worst conviction. The prisoners know that if neither confesses they must serve a minimum sentence. However, as no one knows the other strategy to do not confess is very risky, specially because camaraderie is not a common quality between criminals. It is very common to represent in a bimatrix the possible strategies combinations with their respective reward. The corresponding bimatrix for the prisoners game is 1.

$\mathbf{S_1 \setminus S_2}$	**C**	**N**
C	3,3	0,10
N	10,0	1,1

Table 1. Prisoners Dilemma: C ≡ confess; N ≡ do not confess. The number on the left is for the years the prisoner S_1 prisoner must serve.

Quantum game theory is a classic game theory generalization. That is, quantum game strategies and outcomes include the classical as particularities, but also quantum features let the application of new strategies which leads to solutions classically imposible. The N players quantum game si denoted by $G(N, \mathcal{H}, \rho, S(\mathcal{H}), u)$, where $\mathcal{H}$ is the Hilbert space of the physical system and $\rho \in S(\mathcal{H})$ is the system initial condition, being $S(\mathcal{H})$ the associated space state. In quantum games, players strategies are represented by unitary operators, which in quantum mechanics are also known as evolution operators related to the system's Hamiltonian [27]. If we call U_i the operator corresponding to player i strategy, the N-players strategies operator results $U = U_0 \otimes U_1 ... U_i \otimes ... \otimes U_{N-1}$. Starting from the initial pure state $|\Psi_0\rangle$ of the system, players apply their strategies U in order to modify it according to their preferences, that is modifying the probability amplitudes associated with each base state. As a consequence, evolution from the initial system state to some state $|\Psi_1\rangle$ is given by $|\Psi_1\rangle = U|\Psi_0\rangle$. Quantum games provide new ways to cooperate, to eliminate dilemmas, and as a consequence new equilibriums arise. As can be seen in [10], for example, the dilemma is avoided in the quantum Prisoner's game. That is, the system equilibrium is not longer (C,C) to be (N,N).

3. Quantum search strategy

In the classic dating market game [28, 29], men choose women simultaneously from N options, looking for those women who would have some "property" they want. Unlike the traditional game, in the quantum version of the dating game, players get the chance to use quantum techniques, for example they can explore their possibilities using a quantum search algorithm. Grover algorithm capitalizes quantum states superposition characteristic to find some "marked" state from a group of possible solutions in considerably less time than a classical algorithm can do [8]. That state space must be capable of being translatable, say to a graph G where to find some particular state which has a searched feature or distinctive mark, throughout the execution of the algorithm. By "distinctive mark" we mean problems whose algorithmic solution are inspired by physical processes. Furthermore it is possible to guarantee that the searched node is marked by a minimum (maximum) value of a physical property included in the algorithm.

Let agents be coded as Hilbert space base states. As a result, men are able to choose from N_w women set $W = \{|0\rangle, |1\rangle, ..., |N_w - 1\rangle\}$. Table 1 displays four women states in the first column and some feature that makes them unique in the second column which we will code with a letter for simplicity.

If a player is looking for a woman with a feature "d", the table must be searched on its second column and when the desired "d" is found, look at the first column where the corresponding

http://dx.doi.org/10.5772/53842

woman	feature
$\|0\rangle$	a
$\|1\rangle$	b
$\|2\rangle$	c
$\|3\rangle$	d

Table 2. Sample woman database. Left column contains women states and right column displays a letter representing some feature or a feature set that characterizes each woman on the left.

chosen woman state is: $|3\rangle$ in this example. The procedure is very simple if the table has just a few rows, but when the database gets bigger, the table in the best case would have to be entered $N_w/2$ times [30]. Under this framework we propose to use Grover algorithm in order to achieve man's decision in less time. Without losing generality let $N_w = 2^n$ being n the qubits needed to code N_w women. Quantum states transformation are made by applying Hilbert space operators U to them, following $\Psi_1 = U_1\Psi_0$ is a new system state starting from Ψ_0. As a consequence any quantum algorithm can be thought as a set of suitable linear transformations. Grover algorithm starts with n qubits in $|0\rangle$, resulting $\psi_{ini} = |00..00\rangle \equiv |0\rangle^{\otimes n}$ the system initial state, where $\otimes$ symbol denotes Kronecker tensor product. Initially, the woman identified by state $|0\rangle$ is chosen with probability one. The next step is to create superposition states and like many other quantum algorithms Grover uses Hadamard transform to do this task since it maps n qubits initialized with $|0\rangle$ to a superposition of all n orthogonal states in the $|0\rangle$, $|1\rangle$,.. $|n-1\rangle$ basis with equal weight, $\psi_1 = H\psi_{ini} = \frac{1}{\sqrt{N_w}} \sum_{i=0}^{N_w-1} |i\rangle$. As an example, when $N_w = 4$, the state results $\psi_1 = H|00\rangle = \frac{1}{2}\sum_{i=0}^{3} |i\rangle = \frac{|00\rangle+|01\rangle+|10\rangle+|11\rangle}{2}$. One-qubit Hadamard transform matrix representation is (1), and n-qubits extension is $H^{\otimes n}$, see [27],

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{1}$$

Another quantum search algorithms characteristic, is the “Oracle", which is basically a black box capable of marking the problem solution. We call U_f the operator which implement the oracle

$$U_f(|w\rangle|q\rangle) = |w\rangle|q \oplus f(w)\rangle, \tag{2}$$

where $f(w)$ is the oracle function which takes the value 1 if w correspond to the searched woman, $f(w) = 1$, and if it is not the case it takes the value 0, $f(w) = 0$. The value of $f(w)$ on a superposition of every possible input w may be obtained [27]. The algorithm sets the target qubit $|q\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. As a result, the corresponding mathematical expression is:

$$|w\rangle(\frac{|0\rangle - |1\rangle}{\sqrt{2}}) \longmapsto^{U_f} (-1)^{f(w)}|w\rangle(\frac{|0\rangle - |1\rangle}{\sqrt{2}}) \tag{3}$$

Observe that the second register is in an eigenstate, so we can ignore it, considering only the effect on the first register.

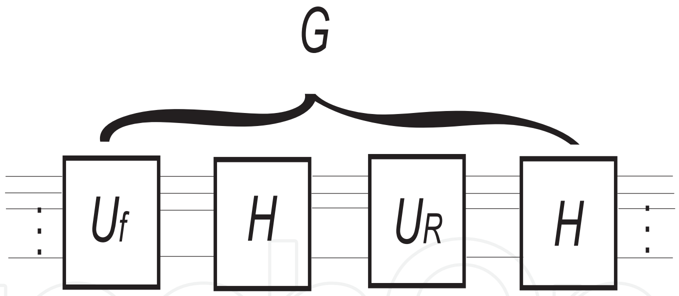

Figure 1. Grover Iteration

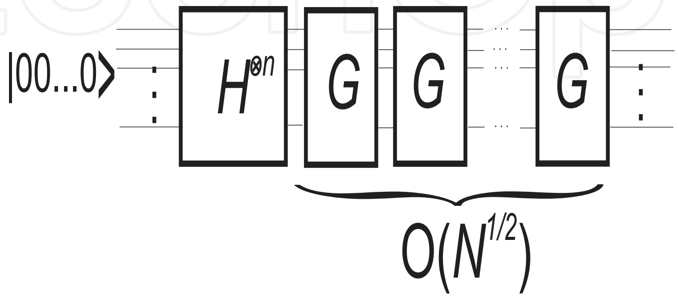

Figure 2. Grover Quantum searching algorithm

$$|w\rangle \longmapsto^{U_f} (-1)^{f(w)}|w\rangle \tag{4}$$

Consequently, if $f(w) = 1$ a phase shift is produced, otherwise nothing happens. As we already stated our algorithm is based on the classical Gale-Shapley (GS) algorithm which assigns the role of proposers to the elements of one set, the men say, and of judges to the elements of the other.

Actually, for a more symmetric formulation of the algorithm where both sets are, at the same time, proposers and judges, it would be necessary another oracle which evaluates women features matching by means of another function $g(x)$ [31], but we will not go into that. As far as we are concerned up to now the Oracle is a device capable of recognizing and “mark" a woman who has some special feature, said hair color, money, good manners, etc. Oracle operator U_f makes one of two central operations comprising of a whole operation named Grover iterate G (Fig.1), and a rotation operator U_R, or conditional phase shift operator represented by equation (5).

U_R and U_f, together with Hadamard transformations represented by H blocks (1), in the order depicted by (Fig. 1), make the initial state vector asymptotically going to reach the solution state vector amplitudes. The symbol I in U_R equation is the identity operator.

$$U_R = 2|0\rangle\langle 0| - I \tag{5}$$

Furthermore, after applying Grover iterate, G, $O(\sqrt{N_w})$ times, the man finds the woman he is looking for. In Figure 1 Grover iterate is shown and Grover quantum algorithm scheme is depicted in 2.

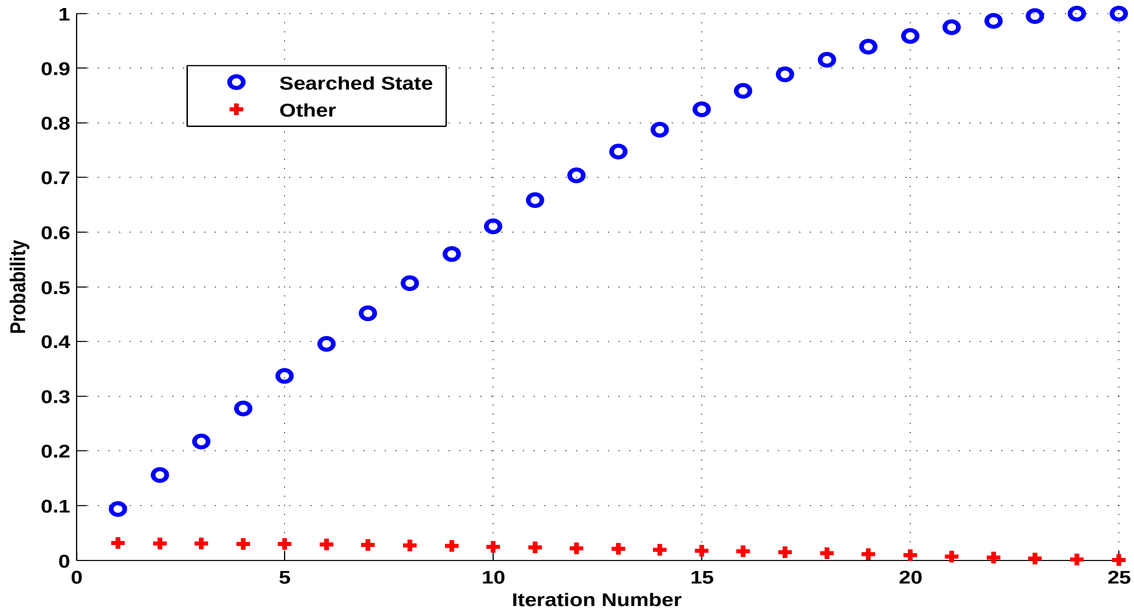

Figure 3. Evolution of the probability to find the chosen woman and the probability to find other woman as a function of the iteration number with Grover's algorithm.

As the number of iterations the algorithm makes depends on the size of the options set, this must be known at the beginning of simulations. Every operator has its matrix representation to be used in simulations. We suppose the player chooses a woman who has some specific particularity that would distinguish her from any other of the group, so we construct matrix U_f and other matrixes for that purpose. The evolution of the squared amplitude with the iteration number is shown in Figure 3. The searched state amplitude is initially the same for all possible states $|i>$ in the Ψ_1 expression. The fast increasing of the probability to find the preferred state on each iteration contrasts with the decreasing of the probability to find every other state. The example displayed is for $N_w = 1024$ women and as the can be seen in Figure 3, the number of iterations needed to get certainty to find the preferred woman are 25. Classically, a statistical algorithm would need approximately $N_w = 1024$ iterations.

Thus when a given man who wants to date a N_w size set selected woman, he must set his own U_f operator out, according to his preferences, and then let the algorithm do the job. The case of N_m men may be obtained generalizing the single man case: every one of them must follow the same steps. Nevertheless, achieving top choice is hard because of competition from other players and your dream partner may not share your feelings. If all players play quantum, the time to find woman is not an issue and the N stable solutions will be the same as for the classic formulation [32].

4. Quantum vs classic

To compare the quantum approach efficiency with the classical one we will consider some players playing quantum and others playing classic. Let us follow the evolution of agents representative from each group, Q and C respectively.

Q, that plays quantum can keep his state as a linear combination of all the prospective results when unitary transforms such as the described above for Grover's algorithm are applied, provided no measurement producing collapse to any of them is done. On the other hand, the only way C has to search such a database is to test the elements sequentially against the condition until the target is found. For a database of size N_w, this brute force search requires an average of $O(N_w/2)$ comparisons [7].

Two different games where both men want to date with the same woman are presented: In the first one player Q gives player C the chance to play first and both have only one attempt per turn, which means only one question to the oracle. The second game, in order that Q plays handicapped, is set out in the way that C can play $N_w/2$ times while Q only once, and player C plays first again. For the last case we analyzed two alternatives for the classic player: in the first one he plays without memory of his previous result and therefore, in every try he has $1/N_w$ probability to find the chosen woman to date, the other alternative permits the classic player to discard previous unfavorable outcomes at any try in order to avoid choosing them again and diminish the selection universe.

The player who invites the chosen woman first has more chances to succeed, as well as that who asks the same woman more times. Nevertheless the woman has the last word, and therefore the dating success for each player depends on that woman preferences. So, let us define P_c^i as the probability that woman i accepts dating the classic player C and P_q^i as the probability that she accepts the quantum player Q proposal. In order to compare performances, we consider $T = 1000$ playing times on turns and count the dating success times, then calculate the mean relative difference between Q and C success total number as $D/T = \frac{Qsuccess - Csuccess}{T}$, for different woman acceptation probabilities.

Initially, both players begin with the system in the initial state $\psi_1 = \frac{1}{\sqrt{N_w}} \sum_{i=0}^{N_w-1} |i\rangle$, therefore the probability to select any woman is the same for both, $p(w_i) = 1/N$. In the next step the Oracle marks one of the prospective women state according men preferences.

The results are highly dependent on the women set size N_w because, as mentioned above, Grover algorithm needs $O(\sqrt{(}N_w))$ steps to find the quantum player's chosen partner while the classic player must use $O(N_w)$ for the same task. In the case of only one woman and one man, for example, classic and quantum will not have any advantage on searching and the dating success difference for the first game will depend only on that woman preferences, that is, if $P_c > P_q$ then $D/T < 0$ and the quantum player will do better when $P_q > P_c$. Similar chances for both players is not usual in most quantum games, such as, for example the coin flip game introduced by Meyer [9] where the quantum player always beats the classic player in a *"mano a mano"* game. For a two women set Q uses only one step, but C needs two steps to find the right partner. In this case Q does better when $P_q > P_c/4$. Winning conditions improve for the quantum player for increasing N_w, but not in a monotonous way, because the number of steps used by Grover algorithm in Q search is an integer that increases in discrete steps.

In order to facilitate comprehension the set size in the simulations results shown is $N_w = 8$, that is the biggest N_w (taken as 2^n) in which Q uses only one step in Grover algorithm.

Under the first game conditions both players have only one attempt by turn. Since C cannot modify state ψ_1 amplitudes, he has $1/8$ chance to be right. On the other hand player Q, using Grover algorithm as his strategy, can modify states amplitudes in order to increase

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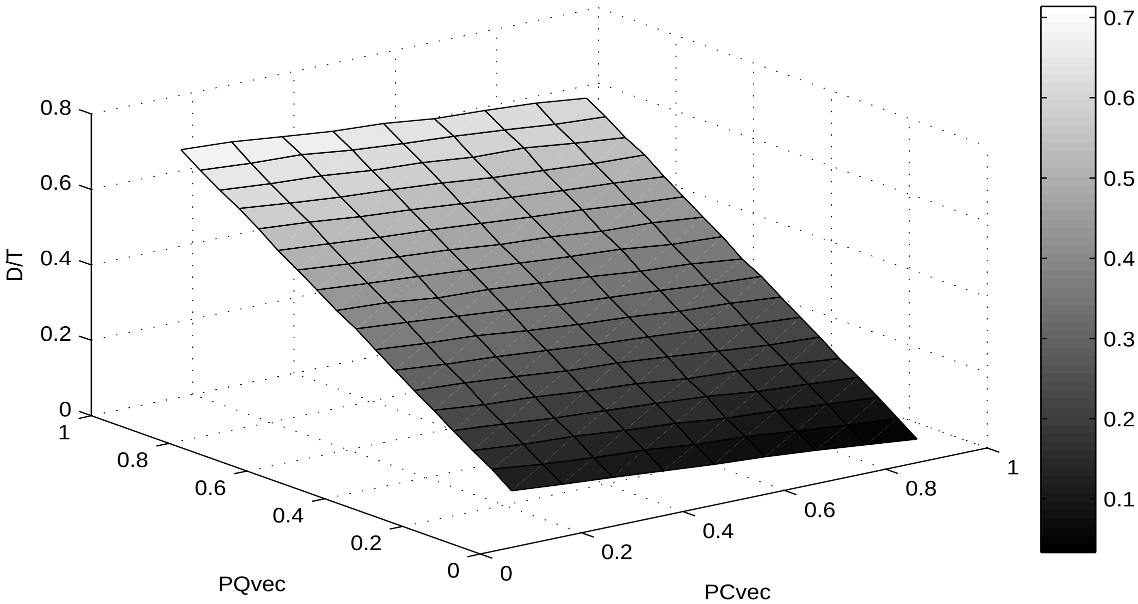

Figure 4. First game: One attempt for both players. Mean relative difference between Q and C success total number as $D/T = \frac{Qsuccess-Csuccess}{T}$, for different woman acceptation probabilities P_c^i and P_q^i. Q outperforms C in all shown cases. The small region where C prevails is not shown.

his chances to win, reaching 0.78 as the probability to find his preferred woman in only one iteration. Figure 4 shows that situation outcomes for different P_c^i and P_q^i combinations. The vertical axis depicts D/T values as a function of P_c^i and P_q^i respectively. D/T is positive for all P_c^i and P_q^i values used in the simulation, which means that even at extremes where $P_c^i >> P_q^i$, the quantum player performs better. However there is a very small region where $P_c^i \approx 1$ and $P_q^i \approx 0$ not shown in the figure that corresponds to a prevailing C.

Under the second game conditions player C have $\frac{N_w}{2} = 4$ attempts before Q plays. After each C attempt the system is forced to collapse to one base state, so a third party, that could be the oracle, arrange the states again and mark the solution. As we explained above, to mark a state means to change its phase but nothing happens to the state amplitude, consequently, for the classic player C, the probability that state results the one the Oracle have signaled is, marked or not, $1/N_w = 1/8$, even though, due to his "insistence", he tries $\frac{N_w}{2} = 4$ times, his dating success chances increase considerably with respect to the first case. Figure 5 shows the corresponding results, where it is possible to see that classic player C begins to outperform Q when $P_c^i >> P_q^i$, that is, when woman has a marked preference for player C.

Player C probability to find the chosen woman can increase to $\frac{1}{2}$ when using a classical algorithm like "Brute-Force algorithm". As shown in figure 5, when C has $\frac{N_w}{2} = 4$ tries while Q has only one, C's odds of success in dating increases, and there are zones on the graph where $D/T < 0$. This implies that player C outperforms player Q. Nevertheless, to achieve that, the chosen woman preferences must be considerably greater for the classic player, that is $P_c^i > 2P_q^i$.

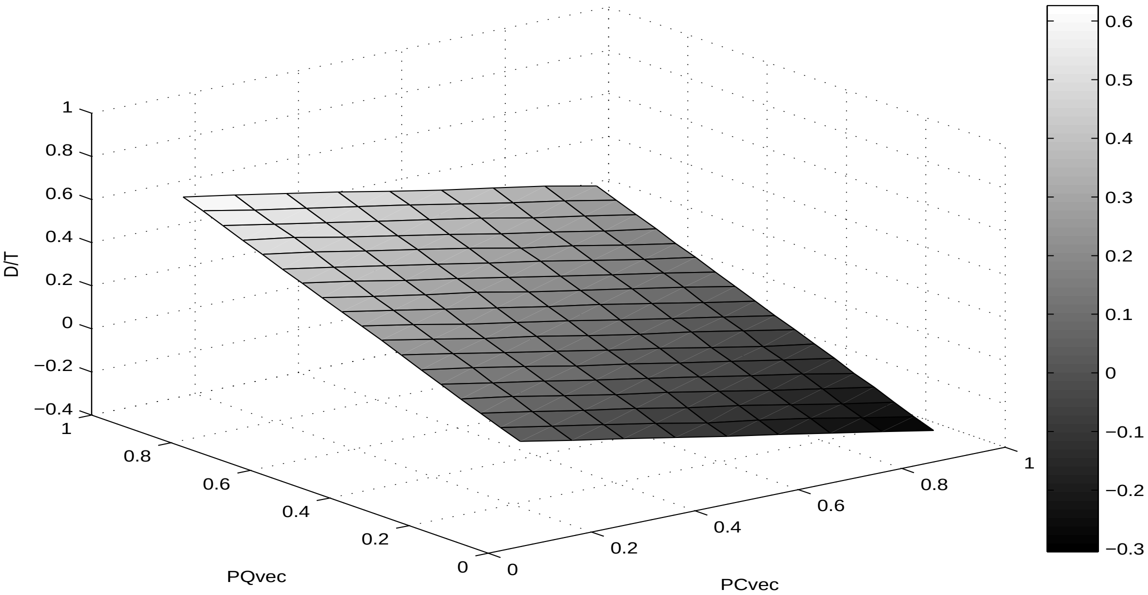

Figure 5. Second game: Classic player C has four tries while Q has only one. Mean relative difference between Q and C success total number as $D/T = \frac{Qsuccess - Csuccess}{T}$, for different woman acceptation probabilities P_c^i and P_q^i. C outperforms Q when $P_c^i >> P_q^i$

4.1. Section discussion

In this section we have introduced a quantum formulation for decision matching problems, specifically for the dating game. In that framework women are represented with quantum states whose associated amplitudes must be modified by men's selection strategies, in order to increase a particular state amplitude and to decrease the others, with the final purpose to achieve the best possible choice when the game finishes. This is a highly time consuming task that takes a $O(N)$ runtime for a classical probabilistic algorithm, being N the women database size. Grover quantum search algorithm is used as a playing strategy that takes the man $O(\sqrt{N})$ runtime to find his chosen partner. As a consequence, if every man uses quantum strategy, no one does better than the others, and stability is quickly obtained.

The performances of quantum vs. classic players depend on the number of players N. In a "one on one" game there is no advantage from any of them and the woman preferences rule. Similar chances for quantum and classic players in "one on one" situation is not usual in most quantum games. Winning conditions improve for the quantum player for increasing N and the same number of attempts, but not in a monotonous way. The comparison between quantum and classic performances shows that for the same numbers of attempts, the quantum approach outperforms the classical approach. If the game is set in order that the classic player has $\frac{N}{2}$ opportunities and the quantum player only one, the former player begins to have an advantage over the quantum one when his probability to be accepted by the chosen woman is much higher than the probability for the quantum player.

5. Stability of couples

There is a group of N_m men and N_w women playing the game. Be $S_i = \{|0\rangle, |1\rangle, ..., |N_w - 1\rangle\}$ the states in a Hilbert space of man i decisions, where $\{0, 1, ..., N_w - 1\}$ are indexes in decimal notation identifying all the women he may choose. As a result each man has been assigned $log_2(N_w)$ qubits in order to identify each woman. Generally, the state vector of one man decisions will be in quantum superposition of the base states, $\Psi_i = \sum_{j=0}^{N_w-1} \alpha_j |j\rangle$, where $|\alpha_j|^2$ is the probability that man i selects woman j when system state is Ψ_i so must satisfied the normalization condition $\sum_{j=0}^{N_w-1} |\alpha_j|^2 = 1$. If there is no correlation between players, the state space of all men decision system is represented through $S_M = S_0 \otimes S_1 \otimes ... \otimes S_{M-2} \otimes S_{M-1}$, where $\otimes$ is the Kronecker product. Note that the S_M extends to any possible combination of men elections. On the other side there are the women who receive men proposals and must decide whether to accept or not one of them. With greater or lesser probability they will receive the all men's proposals, so following the same argument used with the men, be $\Psi_j = \sum_{i=0}^{M-1} \alpha_i |i\rangle$ the woman j acceptation state and be $S_W = S_0 \otimes S_1 \otimes ... \otimes S_{N_w-2} \otimes S_{N_w-1}$ the women acceptances space state. Finally, to close the circle, we define the couples possible states which must include so all possible men's elections as all possible women's acceptances. Accordingly, state space of the couples emerge from the Kronecker product of the men and women spaces,i.e. $S_C = S_M \otimes S_W$.

5.1. Strategies

In quantum games, players strategies are represented by unitary operators, which in quantum mechanics are also known as evolution operators related to the system's Hamiltonian [27]. If we call U_i the operator corresponding to player i strategy, the N-players strategies operator results $U = U_0 \otimes U_1 ... U_i \otimes ... \otimes U_{N-1}$. Starting from the initial pure state $|\Psi_0\rangle$ of the system, players apply their strategies U in order to modify it according to their preferences, that is modifying the probability amplitudes associated with each base state. As a consequence, evolution from the initial system state to some state $|\Psi_1\rangle$ is given by $|\Psi_1\rangle = U|\Psi_0\rangle$. Note that, following the reasoning of the preceding paragraph, when Ψ_0 is the initial state and Ψ_1 is the final state of the couples system, U arises from men and women strategies U_M and U_W respectively through $U = U_M \otimes U_W$. That is, U_M is applied by men to the qubits that identify the women states, meanwhile the women action on the qubits that identify men states is given by U_W.

5.2. Density matrix and system entropy

Often, as in life, players are not completely sure about which strategy to apply, that is, by the way of example, the case where someone chooses between the strategy U_a with probability p_a and U_b with probability $p_b = 1 - p_a$, that situation is referred in a mixed strategies game. Despite the complete system can be represented by its state vector, when it comes to mixed states the density matrix is more suitable. It was introduced by von Neumann to describe a mixed ensemble in which each member has assigned a probability of being in a determined state. The density operator, as it is also commonly called, represents the statistical mixture of all pure states and is defined by the equation

$$\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|, \tag{6}$$

where the coefficients p_i are non-negative and add up to one. From the density operator we can construct and understand the statistical behavior about our system by using statistical mechanics and a criterion of maximum or minimum entropy. Continuing the example, if it is supposed that the system starts in the pure state $\rho_0 = |\Psi_0\rangle\langle\Psi_0|$, after players mixed actions density matrix evolution is

$$\rho_1 = p_a U_a \rho_0 U_a^\dagger + p_b U_b \rho_0 U_b^\dagger. \tag{7}$$

Entropy is the central concept of information theories, [33]. The quantum analogue of entropy was introduced in quantum mechanics by von Neumann,[34] and it is defined by the formula

$$S(\rho) = -Tr\{\rho \log_2 \rho\}. \tag{8}$$

5.3. $N = 2$ Model

In order to set up the notation let us look at the following example of two men and two women that interact for T times periods. Let define $\Psi_0^i = \alpha|0\rangle + \beta|1\rangle$ as the initial decision state of men i which is a linear superposition of the two possible options he has, they are woman 0 or 1. Without losing generalization consider $\alpha = 1$ and $\beta = 0$ which is consistent with thinking that they both have preference for the most popular, the most beautiful, the richest, or any superficial feature that most of the time makes men desire a woman at first glance. Consequently, the men's initial state vector is $\Psi_0^M = \Psi_0^0 \bigotimes \Psi_0^1 = |00\rangle$, where the first qubit represent man's 0 choice and the second is man's 1 choice. As we explain above, the initial quantum pure state is not stable, so during the game the state will change to the general form $\Psi_a^M = \sum_{i=0,j=0}^{1} \alpha_{ij}|ij\rangle$ with probability p_a and $\Psi_b^M = \sum_{i=0,j=0}^{1} \beta_{ij}|ij\rangle$ with probability p_b. As women have the last decision, they must evaluate men proposals and decide to accept one of them or reject all. We consider, just for the example that woman 0 chooses man 0 with p_{0m} and man 1 with probability $1 - p_{0m}$, similar condition for woman 1 but in this case being p_{1m} the probability to choose man 0. That condition doesn't affect system stability but depending on the probabilities values does affect the maximum and minimum of the couple system's entropy. Equation 9 shows the women density matrix which has no off diagonal elements.

$$\rho_{w0} = p_{0m}p_{1m}|00\rangle\langle 00| + p_{0m}(1-p_{1m})|01\rangle\langle 01| + (1-p_{0m})p_{1m}|10\rangle\langle 10| + (1-p_{0m})(1-p_{1m})|11\rangle\langle 11|. \tag{9}$$

The direct product of all possible men proposals with all possible women decisions generates a possible partners state vector which in decimal notation is $\Psi_0^P = \sum_{i=0}^{15} |i\rangle$. Index i is a four qubits number, the first two qubits represent men 0 and 1 choices respectively and the

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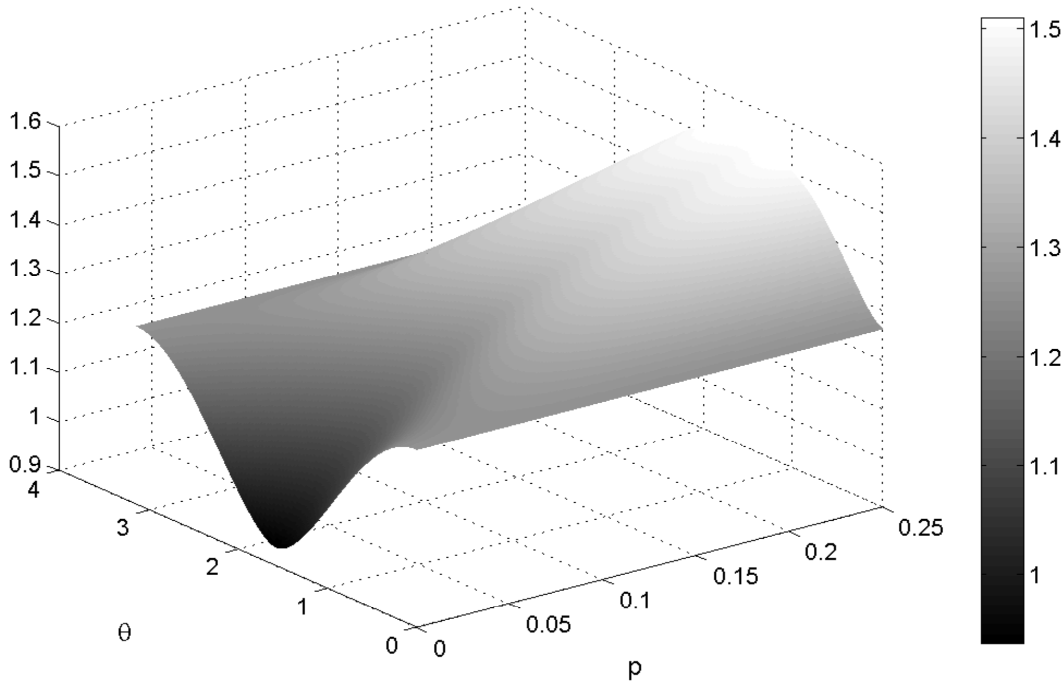

Figure 6. Quantum entropy corresponding to the situation where player 0 varies the probability p to apply strategy U_0^0

other two are the two women possible selections, then 16 are the possible couples states. For example, the state $|0101\rangle$ corresponds to the case that man 0 chooses woman 0 and she accepts him and the same occurs with man 1 and woman 1. Note that not all states corresponds to possible dates, some of them are considering the cases where there are no date, or the ones where only one couple is formed, the state $|0001\rangle$ is an example of the last case where the man man 0 chooses woman 0 and she accepts but on the other hand man 1 also chooses woman but she doesn't and woman 1 does not receive any proposition. As the game progress, probability amplitudes associated with mismatches must decrease, that because it is considered that people prefer to be coupled.

Single players moves or strategies are associated with unitary operators $U_i(\theta)$, with $0 \leq \theta \leq \pi$, applied on each one of their qubits, that in the general case where players have 2^n options, each pure strategy U is composed by n different $U_i(\theta_k)$, being k each state qubit. The general formula of U_i is 10, that are rotation operators, as explained in [27] any qubit operation can be decomposed as a product of rotations. In this work we consider $\gamma = 0$, therefore in what follows $U(\theta, 0)$ is always replaced by the simplest notation $U(\theta)$.

$$U(\theta,\gamma) = \begin{pmatrix} e^{i\gamma} \cdot cos(\theta/2) & sin(\theta/2) \\ -sin(\theta/2) & e^{-i\gamma} \cdot cos(\theta/2) \end{pmatrix} \tag{10}$$

Let p_0 be the probability of player 0 to apply strategy U_0^0 and $1 - p_0$ the probability to apply strategy U_1^0 to the initial state Ψ_0^i, while U_0^1 and U_1^1 are the strategies the man 1 applies with probability p_1 and $1 - p_1$ respectively. The strategies operators used in the examples are defined below, equations 11 and 12 are applied by man 0. Both of them transform the initial

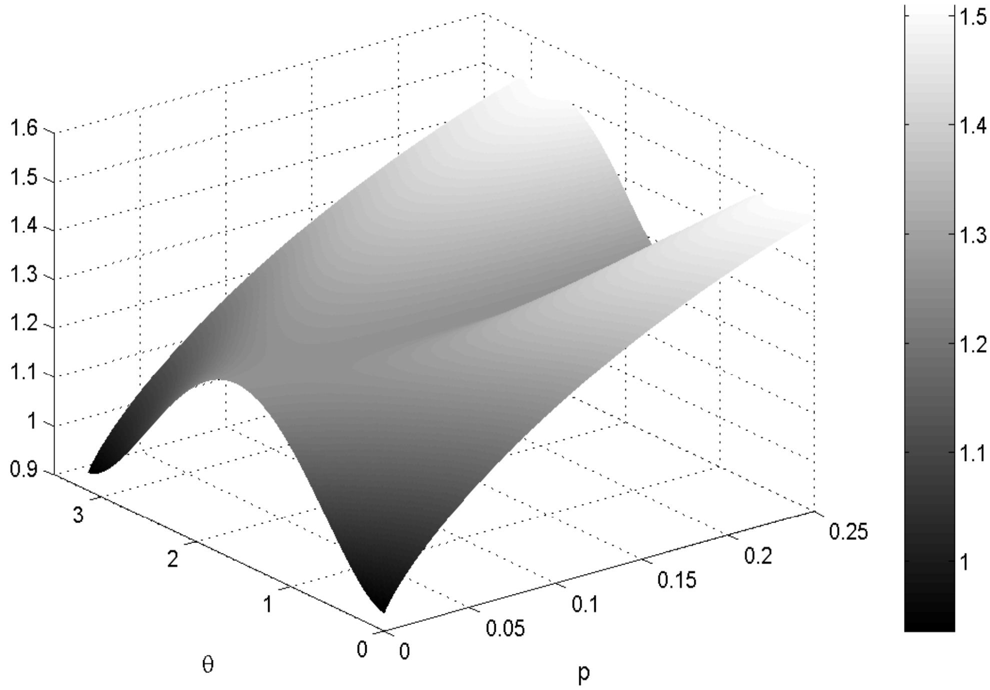

Figure 7. Quantum entropy corresponding to the situation where player 1 varies the probability p to apply strategy U_0^1

state $|0\rangle$ into states that are linear superpositions of 0 and 1, representing states with different probabilities of choosing one woman or the other. On the other hand, strategies applied by man 1 are presented in 13 and 14.

Figures 6 and 7 show two situations where the system entropy varies considerably as a function of the strategies the players use. Figure 2, for example, shows the case where the man 1 applies his strategies with fixed probability, just varying the angle θ while the other man (0) varies both strategies angle and the probability p. In all the cases we present here, in order to simplify the outcomes display, women density matrix doesn't change as explained above.

$$U_0^0 = U(\theta) \quad (11)$$
$$U_1^0 = U(\theta)U(\pi) \quad (12)$$

$$U_0^1 = U(\theta) \quad (13)$$
$$U_1^1 = U(-\theta) \quad (14)$$

For example, if both men choose the same woman with probability one, this is represented in Fig. 1 with $p = 0$. This situation is completely unstable because it is impossible for the woman to choose both of them at the same time (we assume). This correspond to minimum entropy as can be easily seen in Fig. 1. Depending on the strategies applied by men, the whole system entropy, that is the couple system entropy changes reaching maximum and minimum limits. As p increases, the mixing of the strategies also increases producing an increase in entropy that indicates a tendency to stability. The mixing of the strategies means that the men proposals are less focused on one woman. Fig. 2 shows the case where men's role change, that is man 0 fixes his strategies probabilities while 1 varies his own. Although for fixed θ angle, as expected, the minimum entropy points are located where player is applying a pure strategy ($p = 0$), for $\theta = \pi/2$ the entropy value does not change regardless of the value of p, because U_0^0 and U_1^0 are equivalent and therefore player 1 is applying a pure strategy. A result not shown in the figures is that entropy maxima increase when women preferences are the same for every men.

In this way, maxima and minima entropy points may be used to find stable states. Nevertheless, these stable states may not correspond to equilibria states of the game, because the players utilities has not been considered. In order to find Nash equilibria states, these utilities must be considered. This is beyond this chapter goals.

5.4. Section discussion

As a continuation of the analysis of a quantum formulation for the dating game that takes into account mutual progressive learning by representing women with quantum states whose associated amplitudes must be modified by men's selection strategies. we concentrate on the information associated to the problem. Since we deal with mixed strategies, the density matrix formalism is used to describe the system. Even though the decisions players make are based on their payoffs, past experiences, believes, etc., we are not interested in that causes but in the consequences of the decision they take, that is, the influence of the strategies they apply on the quantum system stability by means of the equivalence between maximum entropy states and those states that obey the Collective Welfare Principle that says that a system is stable only if it maximizes the welfare of the collective above the welfare of the individual. Maxima and minima entropy points are used to find characteristic strategies that lead to stable and unstable states. Nevertheless, in order to find Nash equilibria states, the players utilities must be considered.

Maxima and minima entropy do not depend only on the strategies of men but also on women preferences, reaching the highest value when they have no preferences, that is when they choose every man with equal probability. On the other hand, minimum entropy correspond to men betting all chips to a single woman, without giving a chance to other woman.

6. Entangled strategies

The quantum dating market problem has been formulated as a two-sided bandit model [28], where in one side there are the men who must choose one "item" from the other side, which unlike the one side bandit, is composed by women able to reject the invitations.

The quantum formulation, which was presented in previous section, proceeds by assigning one basis of a Hilbert state space to each woman. As a consequence, if N_w is number of women playing, $S_i = \{|0\rangle, |1\rangle, ..., |N_w - 1\rangle\}$ are the states in the Hilbert space representing a man i decisions, therefore every man needs at least $log_2(N_w)$ qubits to identify each woman.

The state of man i decisions is a quantum superposition of the base states, $\Psi_i = \sum_{j=0}^{N_w-1} \alpha_j|j\rangle$, where $|\alpha_j|^2$ is the probability that man i selects woman j when system state is Ψ_i and $|\cdot\rangle$ is known as Dirac's notation. The normalization condition is $\sum_{j=0}^{N_w-1} |\alpha_j|^2 = 1$. On the other side of the market, women receive men proposals and must decide which is the best, accept it and reject the others. Thus, $\Psi_j = \sum_{i=0}^{N_m-1} \alpha_i|i\rangle$ is woman j acceptation state. Finally, combining proposals and acceptances the couples space which is the Kronecker product of all men's and all women's spaces is defined, i.e. $S_C = S_M \otimes S_W$.

Men decision states are separable when there is no connection among players, that is, for instance, no man has any emotional bond with some other that could condition their actions, thus all men decision state, ψ_M, is defined as $\psi_M = \bigotimes_{i=1}^{M-1} \psi_i$. The same reasoning corresponds to women states. On the other hand, if there is some relationship between two or more men, their actions are non-locally correlated, that is, their decisions are far from being independent. John Stuart Bell shown in 1966 that systems in entangled states exhibit correlations beyond those explainable by local "hidden" properties, or in other words, a non-local connection appears when two quantum particles are entangled, [35]. Therefore, we will study the case with correlation between agents by means of quantum entanglement, in other words, how harmful or beneficial can be for players knowing each other in advance.

As we mention in the previous section, players strategies are represented by unitary operators in quantum games. Starting the system in some state $|\Psi_0\rangle$ at time t_0, players apply their strategies U in order to modify it according to their preferences, that is modifying the probability amplitudes associated with each base state. Thus, evolution from the initial system state to some state $|\Psi_1\rangle$ in time t_1 is given by $|\Psi_1\rangle = U|\Psi_0\rangle$. The strategy operator U arises from men and women preferences operators U_M and U_W respectively through $U = U_M \otimes U_W$, where U_M is applied by men to the qubits that identify the women states, meanwhile the women action on the qubits that identify men states is given by U_W.

In order to understand the problem we analyze here a simple example of two men and two women. Single players moves or strategies are associated with 2×2 unitary rotation operators $U_i(\theta, \gamma)$ applied on each one of their qubits (15), where $0 \leq \theta \leq \pi$ and $0 \leq \gamma \leq \pi/2$. Men choices are coded by states $|w_0\rangle = |0\rangle$ and $|w_1\rangle = |1\rangle$, meanwhile women must decide between men $|m_0\rangle = |0\rangle$ and $|m_1\rangle = |1\rangle$. Since any qubit operation can be decomposed as a product of rotations, strategies combinations and possible outcomes are infinite. As a consequence, focusing on men relationship, we study three relevant cases. We suppose, as a measure of satisfaction, that men receive some payoff p_{w_i} if accepted by woman w_i, so for the example we have considered that $p_{w0} = 2$ and $p_{w1} = 5$.

$$U(\theta, \gamma) = \begin{pmatrix} e^{i\gamma} cos(\theta/2) & sin(\theta/2) \\ -sin(\theta/2) & e^{-i\gamma} cos(\theta/2) \end{pmatrix} \tag{15}$$

6.1. Results

For the first case, let us consider $\psi_0 = \frac{\sqrt{2}}{2}(|01\rangle + |10\rangle)$ as the initial state of men decisions system, where the left qubit of ψ_0 is representing man 0 election while the right one represents man 1 choice. As men states are entangled, it is not possible to uncouple their single actions. Therefore judging on the probability amplitudes, there is 50% probability that man 0 chooses woman 0 while man 1 chooses woman 1 and the other 50% for the other case. Since there is no way that men choose the same woman it is a state of mutual cooperation. Women acceptation state is initialized to $\psi_w = 0.5(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$, implying that there is initially 25% chance that they choose the same man. In order to analyze the effect of woman behavior on men payoffs, for this and the following two cases, we consider that men decision state ψ_0 is invariant, while women strategies and their acceptation state ψ_w change. Figure 8 shows the payoff for man 0 as a function of women strategies which are set as $U_i = (t \cdot \pi, t \cdot \pi/2)$ for $t \in [-1, 1]$ and $i = 0, 1$. Different strategies imply changes on women preferences, so some change in $U_i(\theta, \gamma)$ implies that woman i acceptation probability distribution is modified. Following [9], equation 16 represents man 0 payoff, where P_{00} and P_{01} are his chances to be accepted for a date with woman 0 and 1 respectively.

$$\$_{m0} = 2 \cdot P_{00} + 5 \cdot P_{01} \tag{16}$$

In the second example we introduce competition between men. The initial men state is given by $\psi_0 = \frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)$. Figure 9 depicts again the resulting payoffs for man 0 as a function of women strategies. Finally a third case is considered where men decision state is $\psi_m = 0.5(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$. In this case men make independent choices choosing one of four possible options with equal probability. Figure 10 show the resulting payoffs as a function of woman strategies.

As the figures show, the different scenarios present significant differences on payoff topology and maximum payoff values.

The cooperative situation presents the highest payoff compared with the competitive and the independent ones as shown in figure 8. Figures 9 and 10 show that a better payoff may be obtained in the competitive setup compared to the independent one, but on the other hand, also a much lower payoff for other women strategies may be available. The independent decision scenario is thus characterized by lowest maxima and less variation on payoffs.

6.2. Section discussion

We have considered the dating market decision problem under the quantum mechanics point of view with the addition of entanglement between players states. Women and men are represented with quantum states and strategies are represented by means of unitary operator on a complex Hilbert space. Men final payoff, considering payoff as a measure of satisfaction, depends on the woman he is paired with. If men decision states are entangled, their actions are non-locally correlated modeling competition or cooperation scenarios. Three examples are shown in order to illustrate the more usual scenarios. In two of them the men strategies are correlated in a cooperative and a competitive way respectively. In the other example men strategies are independent. Although cooperative and competitive strategies can drive to higher payoffs, changing of women preferences on those scenarios can lead to very low payoffs. The independent decision scenario is characterized by less variation on payoffs.

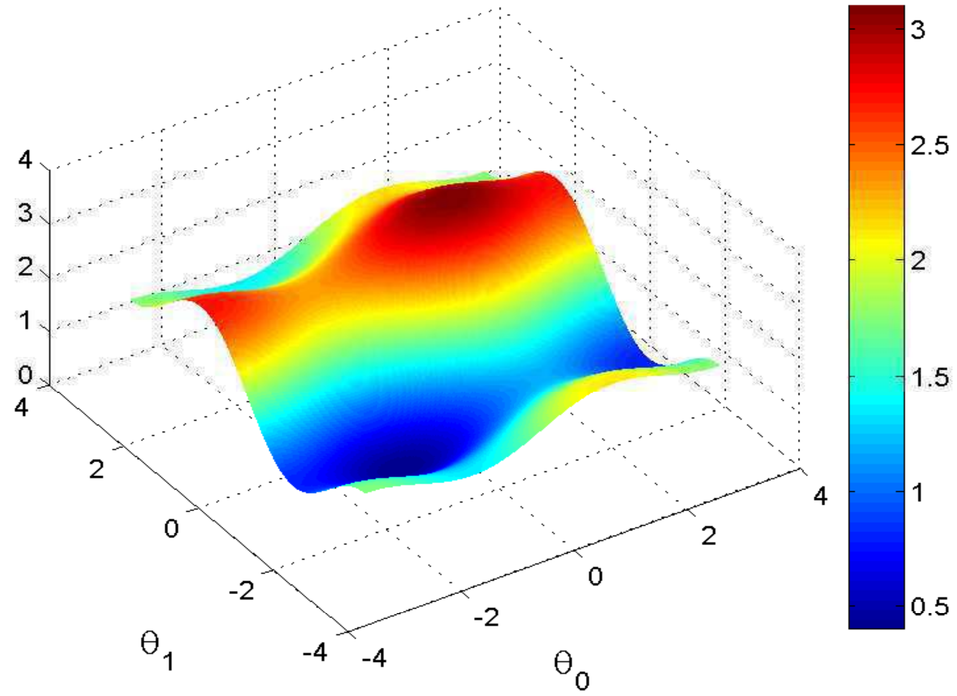

Figure 8. Payoff for man 0 if men never choose the same woman, as function of women acceptation strategies. For the example γ varies as $\theta/2$.

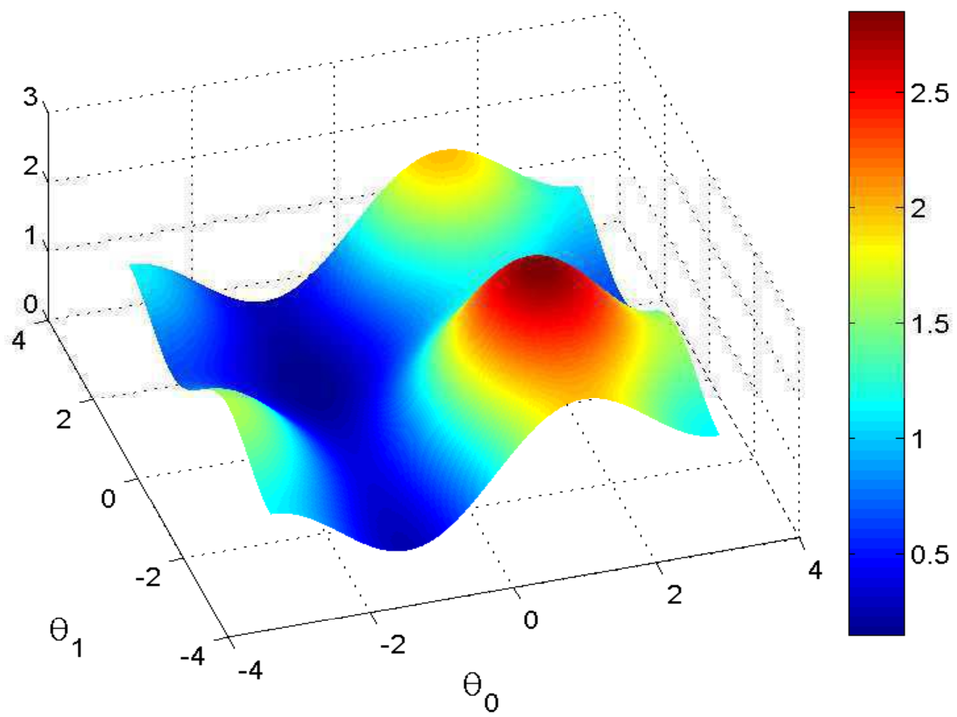

Figure 9. Payoff for man 0 if men always choose the same woman, as function of women acceptation strategies. For the example γ varies as $\theta/2$.

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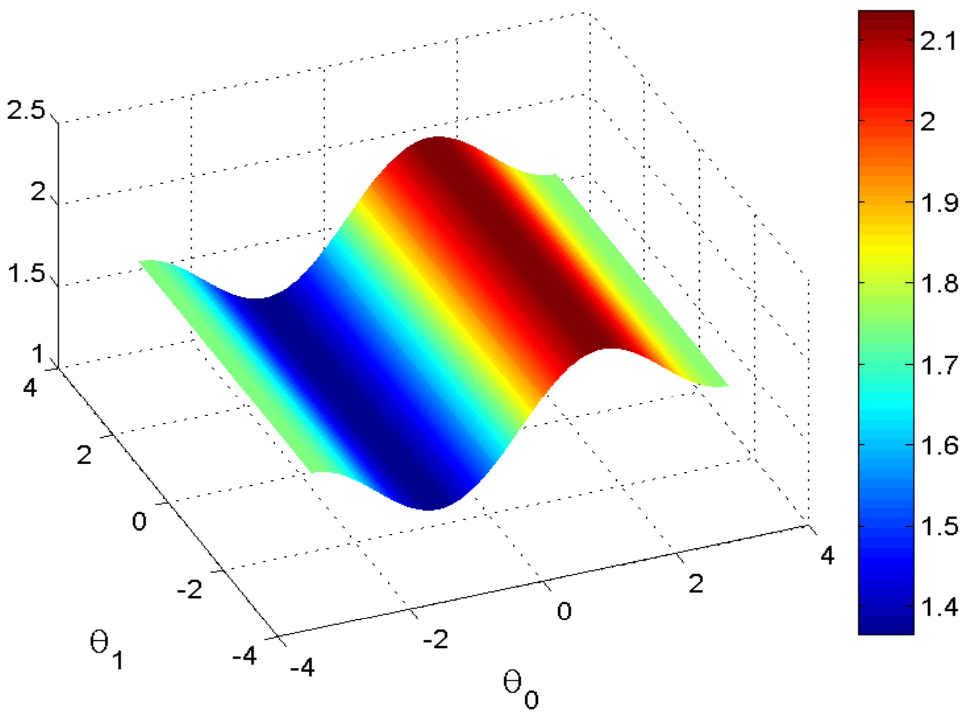

Figure 10. Payoff for man 0 if men choose without restrictions, states are not entangled, as function of women acceptation strategies. For the example γ varies as $\theta/2$.

7. Final remarks

The dating market problem may be included in a more general category of matching problems where the elements of two sets have to be matched by pairs. Matching problems have broad implications not only in economic and social contexts but in other very different research fields such as communications engineering or molecular biology, for example. The main goal of this chapter is to introduce and analyze a quantum formulation for the dating market game, whose nearest classical antecedent is the Stable Marriage Problem. Players strategies are represented by unitary operators, which in quantum mechanics are also known as evolution operators related to the Hamiltonian of the system. Significant outcomes arise when classic players play against quantum ones. For instance, when a quantum player uses Grover search algorithm as her strategy, her winning probabilities grow with increasing number of players, but none leads in a "one on one" game. Besides, from stability point of view, maxima and minima entropy points are used to find characteristic strategies that lead to unstable and stable states, resulting the highest entropy values when women have no preferences, that is, when they choose every man with equal probability. On the other hand, minimum entropy correspond to men betting all chips to a single woman, without giving a chance to other woman. Finally, to model relationships between people that may correlate their decisions, our model consider the situation when men decision states are entangled and their actions are non-locally correlated modeling competition or cooperation scenarios. One of the main outcomes is for example that, although cooperative and competitive strategies can drive to higher payoffs, changing of women preferences on those scenarios can lead to very low payoffs.

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