Chapter 2

Comparisons of Lateral Transshipment with Emergency Order Policies

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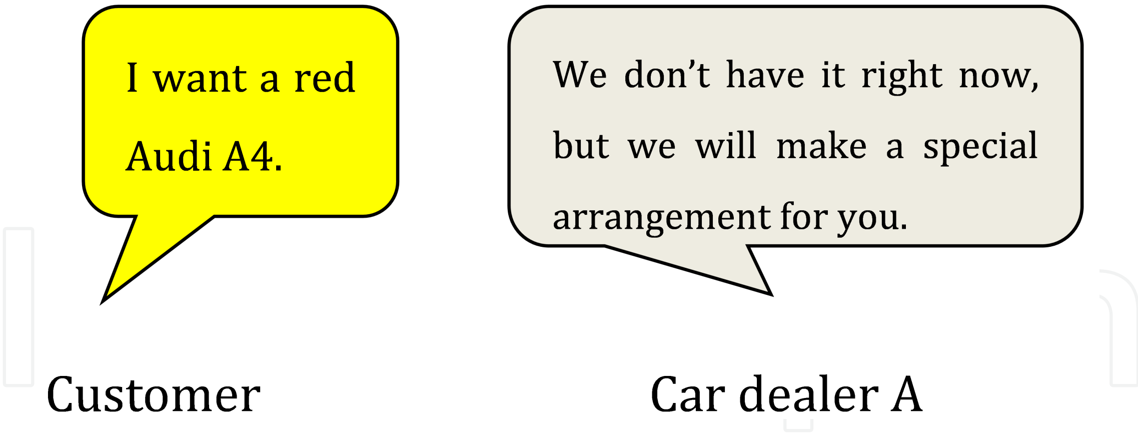
1. Introduction

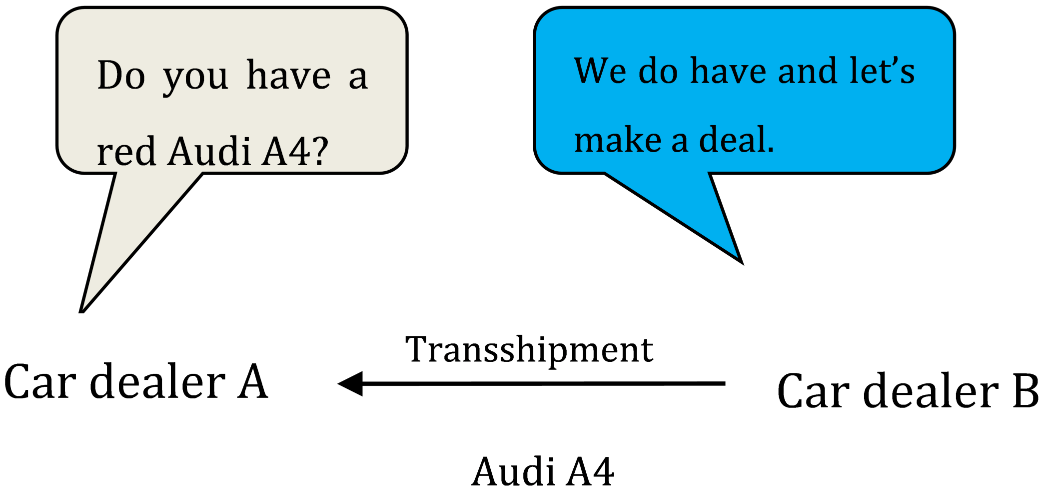
The retail industry has been puzzled by stock-outs for a long time. According to a study report from Supply Chain Digest January 20, 2009, averagely, "more than 1 in every 5 consumers (21.2%) coming into the door of Consumer Electronics retailers leaves without buying at least one product they intended to purchase due to out-of-stocks". For example, Office Max has an out-of-stock rate of 30.6% and is losing $1.96 for every customer coming through their doors due to this reason.

If stock-out occurs, retailers often put emergency orders to meet customer's extra demand. For example, it is very common that oversee employees work over time to fulfill additional orders. On the other hand, transshipment is also a practical business solution to this problem. In the United States, it is commonly observed that if a customer goes to a car dealership and wants a certain type of car, and if the desired car (such as red color) is not in stock, the car dealership will arrange transshipment with another car dealer somewhere in the country with the exact car that the customer wants.

Though transshipment and emergency order problems have been addressed in many perspectives, it is quite rare that two policies are investigated at the same time in a comparative framework, especially with customer requesting behavior and customer switching behavior absorbed. In our research, customer requesting behavior describes that customers who don't acquire their desired products may submit requests to the retailer to ask for being satisfied by emergency orders or transshipments. Meanwhile, customer switching presence refers that some unmet customers may directly switch to another store to search the possibilities of shopping instead of requesting.



Scheme 1.



Scheme 2.

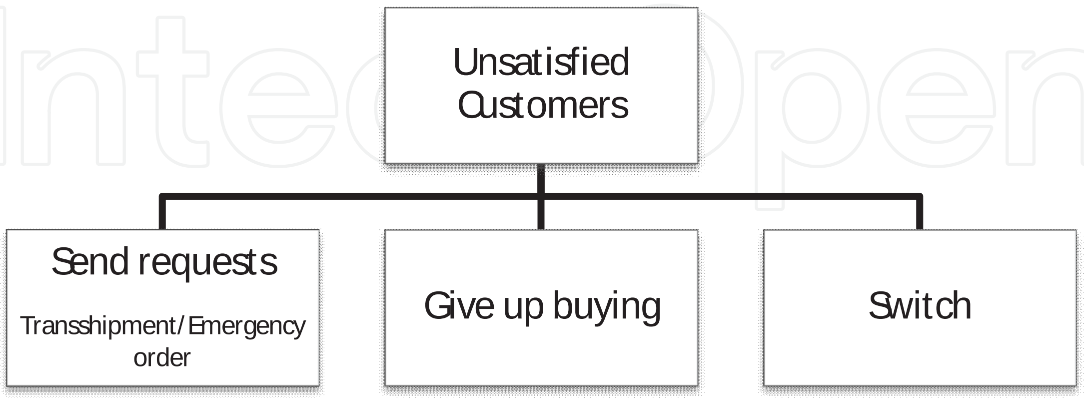


Figure 1. Unsatisfied Customers' Behaviors

In this paper, we study the transshipment and emergency order policies in the presence of "customer requesting" and "customer switching" behaviors, for two retailers under centralized control in a symmetric market system. As obtaining the correct stock balance gives the firm a competitive advantage, we first examine retailers' replenishment decision since in our model, customer demand randomly distributes before selling season. Considering customer requesting and switching factors, we are interested in how those initial inventory decisions should be adjusted correspondingly under two different polices(e.g., transshipment & emergency order). Through numerical experiments, we illustrate that retailer under transshipment policy usually needs to reserve more stock.

Secondly, we contrast the total supply chain's profits in our new model under two policies and aim to find convenient policy-choosing criteria. Under emergency order scenario, any switching customer satisfied by the surplus definitely improves the overall system's profit since the revenue is generated without any additional cost. In the meantime, firm using transshipment as the primary practice to solve out-of-stock issue can also benefit significantly from customer switching behavior by saving transshipment cost. Therefore, there is no straight forward conclusion for retailers regarding profit. We identify that with the same initial replenishment stock, in a symmetric scenario, retailer gains more if emergency cost is less than transshipment charge.

2. Related literature

Emergency order and transshipment, as effective solutions for increasing the multi-echelon supply chain performances have been given attention tremendously.

On one side, a number of models in the literature address models in which there is an option to place new emergency orders if shortage happens. The emergency order often has negligible lead-time, but the unit price is much more expensive. Daniel (1963) studies the optimality of periodic review order-up-to inventory policies when lead time is either 0 or 1 period. Later, Moinzadeh and Nahimas (1988) use a continuous review paradigm to develop a general heuristic policy. The emergency ordering procedure is triggered once on-hand inventory reaches a certain level. Under periodic review inventory system, Chiang and Gutierrez (1998) provide the optimal control policies at each review time point. Other studies can be found in Jain et al. (2010), Lawson and Porteus (2000), and Gaukler et al. (2009).

Although transshipment problem is analyzed in many different perspectives, we only review the research works which are closely related to our paper, where transshipments are conducted after customer demand is realized. Krishnan and Rao (1965) may be the first to explore a single-period two-location problem and its N location extension. Robinson (1990) considers a multi-period, multi-location problem where products are relocated among different locations. Under the assumption of zero transshipment and replenishment lead times, Robinson (1990) derives the optimal ordering policy and finds analytical solutions for the two-location case. Later, Herer and Rashit (1999) consider fixed joint replenishment costs in the similar model. Hu et al. (2008) study multiple period setting but focus on two-location transshipment. Most recently, Olsson (2010) claims that a unidirectional lateral transshipment policy is reasonable if the locations have very different backorder or lost sales costs.

Most early studies assume that there exists a centralized inventory planer coordinating the optimal inventory and transshipment. However, Rudi et al. (2001) initially considers a two retailer decentralized one-period system and proves the uniqueness of Nash equilibrium in order quantities. Hu et al. (2007) discuss the existence of the coordinating transshipment prices. Huang and Sosic (2010) study a repeated inventory sharing game with N retailers and profit of transshipment is distributed among retailers by dual allocation. Other recent transshipment studies can be found in Yu et al. (2011) and Tiacci (2011).

As far as we know, studies mentioned so far all assume that unfilled demand of one retailer never turns to the other retailer. However, it is quite usual that consumers may simply leave and go for shopping at another retailer. In the existing literature, only few studies address the lateral transshipment problem and emergency ordering policy by explicitly incorporating such consumer switching behavior. Lippman and McCardle (1997) implement a rule to split initial and excess demand among competing firms in a competitive newsboy model. Anupindi and Bassok (1999) explore a one manufacturer and N-Retailer system, where a deterministic customer switching rate is assumed, and illustrate that the manufacturer may prefer a decentralized system when market search is intense. Other papers which explore customer switching behavior can be found in Jiang and Anupindi (2010) and Zhao and Atkins (2009). Although demand spills between firms are considered in those studies, transshipment issue and emergency order policy are never studied.

3. Model

In this study, we consider a centralized newsboy model consisting of two local retailers i, j facing their stochastic demands D_i, D_j independently. Before customer comes, the central controller decides the replenishment inventory levels Q_i, Q_j, for retailer i, j. Then, the random demands D_i, D_j are realized and retailers use products on hand to accommodate their own customers respectively. As the local demand is very unpredictable, it is not surprising that retailers may not fulfill all the orders solely by their local inventories. Thereafter, in our model, we anticipate that a fraction of unsatisfied customers are willing to request retailer's transshipment/emergency order arrangement. Other unmet customers may immediately head for other stores and see if they can get their desired products or give up purchasing completely.

Supposing retailer i runs out of products and retailer j's inventory is adequate enough, we assume that $\lambda_i(D_i - Q_i)$ customers see whether transshipment or emergency order can be arranged for them, and remain at retailer i unless their requests are finally rejected, where we refer to the constant fraction parameter λ_i as "customer-requesting rate". Among the rest unmet demand $(1 - \lambda_i)(D_i - Q_i)$, the proportion of customers moving to retailer j instead of leaving directly is A_i. Though customer switching behavior may be influenced by a number of factors, such as distance between stores, availability of substitutable products, or access to

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inventory information, etc, we still can expect that consumer populations from same areas have relatively stable switching rate. Due to this reason, it is appropriate to consider a fixed portion of unsatisfied customers will be triggered to switch by out-of stock issue. At the end of the period, if switching customers still cannot get satisfied, they eventually leave without buying.

Under emergency order setting, we use q_i, q_j to represent the emergency orders placed by retailer i, j respectively. Stick to the same assumption $D_i - Q_i > 0$, $Q_j - D_j > 0$, retailer j's surplus inventory is $Q_j - D_j > 0$, $q_j = \left| \min(Q_j - D_j - A_i(1 - \lambda_i)(D_i - Q_i), 0) \right|$ and $q_i = \lambda_i (D_i - Q_i)$. It is clear that retailer j places emergency orders only if retailer j cannot utilize its surplus to meet all switching customers. However, if the surplus at market retailer j is far beyond the number of switching demand, left stocks at retailer j may cause certain level of overall inefficiency since those products aren't used at all.

Different from emergency order policy, transshipment policy can be regarded as an internal way to enhance supply chain efficiency because no external resource is available in one period. When stock-out happens to both retailers, no transshipment will be conducted. While retailer j has surplus $Q_j - D_j > 0$, $q_{ji} = \min\left(Q_j - D_j - A_i(1 - \lambda_i)(D_i - Q_i), \lambda_i(D_i - Q_i)\right)^+$ is relocated from retailer j to retailer i, in responding to those customers' requests. Clearly, it is not necessary all customers who stay at local retailer i have to be satisfied by transshipment since partial extra products at retailer j are prepared for switching customers because of saving transportation cost. The extreme case occurs when the quantity of switching customers is large enough to meet all left products at retailer j, where retailers end up with no transshipment.

At this moment, we have briefly introduced our research model where retailers can choose one of two alternatives to handle demand uncertainty. With the help of transshipment, firm definitely can take advantage of customer switching behavior by saving shipping cost. Unfortunately, transshipment never meets all customers once out-of-stock takes place since no new merchandises are brought in.

Emergency order policy is also not a perfect substitute because possible waste may be incurred as mentioned early. In the following, our research first considers two policies separately, addressing on some critical operations management decisions, for example replenishment decision in a more realistic model. Furthermore, we pay attention to comparison of two policies and suggest how to decide the optimal policy under different parameter assumptions.

In our research, the regular unit inventory cost is c_n and retailers receive revenue $r > c_n$ for each unit sold locally as well as to switching customers. For each unit of inventory transshipped from retailer i to retailer j, a transshipment expense $t_{ij} < r$ is incurred. Manufacturer charges any emergency order $c_e > c_n$. We summarize the parameters used in our general model below.

Summary of notations

$c_n =$ unit regular product cost;

$c_e =$ unit emergency order cost;

$r =$ unit retail price;

$t_{ij} =$ unit transshipment cost from retailer i to retailer j;

$D_i =$ local demand at retailer i, $cdf = G_i(D_i)$ and $pdf = g_i(D_i)$;

$\lambda_i =$ requesting rate of retailer $i's$ customer;

$A_i =$ switching rate of retailer $i's$ customer;

4. Inventory policies under emergency order Policy

In order to optimize the total profit, the central controller has to plan on replenishment inventory level by minimizing the cost of stocks while trying to make sure that there are enough materials to meet customer demand. Firstly, this research displays an emergency order quantity schedule, and then investigates the consequences of customer switching and requesting behaviors. Finally, we extensively discuss the properties of the optimal replenishment decisions.

4.1. Emergency order schedule

When local demands are perceived and satisfied by retailers' products on hand, the central planning firm needs to arrange emergency orders when it's necessary. Without loss of generality, we present the emergency order schedule in Table 1 and Figure 1.

Event	Description	q_j	q_i
$Event_1$	$Q_i \geq D_i,\ Q_j \geq D_j$	0	0
$Event_2$	$D_i \geq Q_i,\ Q_j \geq D_j$ $Q_j - D_j \geq A_i(1-\lambda_i)(D_i - Q_i)$	0	$\lambda_i(D_i - Q_i)$
$Event_3$	$D_i \geq Q_i,\ Q_j \geq D_j$ $Q_j - D_j \leq A_i(1-\lambda_i)(D_i - Q_i)$	$A_i(1-\lambda_i)(D_i - Q_i)$ $-(Q_j - D_j)$	$\lambda_i(D_i - Q_i)$
$Event_4$	$Q_i \geq D_i,\ D_j \geq Q_j$ $Q_i - D_i \geq A_j(1-\lambda_j)(D_j - Q_j)$	$\lambda_j(D_j - Q_j)$	0
$Event_5$	$Q_i \geq D_i,\ D_j \geq Q_j$ $Q_i - D_i \leq A_j(1-\lambda_j)(D_j - Q_j)$	$\lambda_j(D_j - Q_j)$	$A_j(1-\lambda_j)(D_j - Q_j)$ $-(Q_i - D_i)$
$Event_6$	$Q_i \leq D_i,\ Q_j \leq D_j$	$A_j(1-\lambda_j)(D_j - Q_j)$ $+\lambda_j(D_j - Q_j)$	$A_i(1-\lambda_i)(D_i - Q_i)$ $+\lambda_i(D_i - Q_i)$

Table 1. Emergency Order Schedule

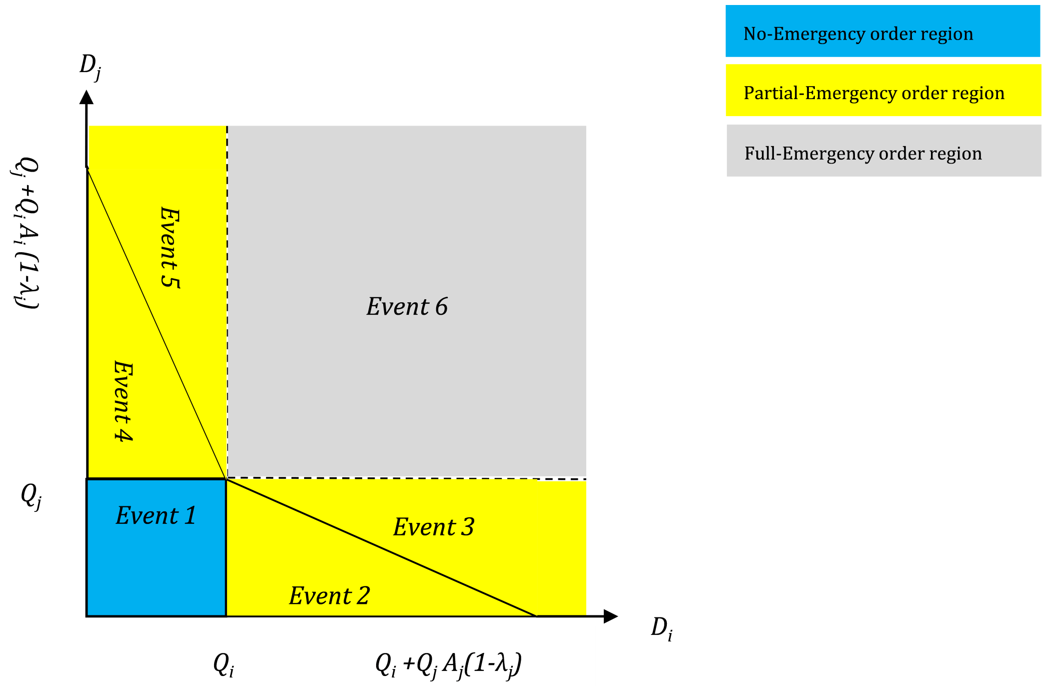

Figure 2. Emergency Order Structure

In Table 1, emergency order is not needed when $Event_1$ take places, since both retailers have surpluses. In Figure 1, this is labeled as a "No-Emergency order region". In $Event_4$ and $Event_5$, $\lambda_j(D_j - Q_j)$ unsatisfied customers wait for emergency orders provided by local retailer's arrangement and $(1 - \lambda_j)(D_j - Q_j)A_j$ customers will go for shopping. It is not hard to find that $Event_2$ and $Event_3$ are the counterparts of $Event_4$ and $Event_5$, e.g., $D_j<Q_j$, $D_i>Q_i$, which can be analyzed similarly by exchanging subscripts i with j.

In events $Event_2$ and $Event_4$, it is suggesting that retailers with extra products don't place any emergency order since extra stocks cover all switching customers. On the other hand, in $Event_3$ and $Event_5$, leftovers are not sufficient enough to serve all switching orders, which require retailers send emergency requests. Although events mentioned above except $Event_1$ are not exactly same, in general, all unmet customers are compensated by products mixed of emergency orders and surplus products. Because of this, we refer these regions as "Partial-Emergency order region" in Figure 1. Nonetheless, since in $Event_6$, emergency orders become the only available resource to solve out-of-stock problem, we label this region as a "Full-emergency order region". Since we are particularly interested in the effects of customer switching and requesting behaviors, we explain their impacts in Proposition 1.

Proposition 1 The amount of emergency orders $q_i + q_j$ is non-decreasing in customer requesting rate λ_i, λ_j, and customer switching rate A_i, A_j.

Recall that in emergency order problems without customer switching and requesting, emergency order decision and retailer's inventory surplus level are isolated from each other. The total amount of emergency orders is simply the sum of all unsatisfied demands from every single shortage retailer. However, when our model adopts customer requesting and customer switching rates, this rule doesn't hold anymore. First, not all unsatisfied customers are willing to stay with the local retailer and wait for emergency orders. Additionally, for non-requesting customers, only a fraction of them will look for products. In the meantime, if two retailers have the opposite positions on inventory level, depending on the left stock amount, switching customers may be partially or completely absorbed by the surplus. Therefore, we conclude that in our model, the total amount of emergency orders never surpasses the number of unmet customers. After we explore the total emergency order amount, we then establish the findings of customer switching and requesting rates in the following. In fact, the sensitivities on rates are quite intuitive. As more customers choose to stay and request for emergency order arrangement, it is natural that more emergency orders need to be added. On the other hand, any increment in customer switching rate also avoids losing customers and more needs are satisfied overall.

Next, we emphasize on addressing the question: How does customers' preference on requesting or switching affect the profit measurement? Intuitively, both rates measure the extent of demand pooling between retailers. But, the profit performance depends on many factors, such as the retail price. Hence, we have,

Proposition 2 Under emergency order setting, the total profit increases in customer switching rate. When $rA_j - r + c_e < 0$, then the total expected profit increases in λ_j.

Compared with requesting rate, customer switching behavior's impact on system's profit is relatively obvious. As analyzed before, the higher proportion of customer switching rate, the fewer customers leave with disappointment since switching customers eventually get fully satisfied by the surplus or emergency order, which helps the supply chain to achieve a better financial performance. At this moment, the retail price does not play a decisive role since switching customers only come from those who prefer not to wait for emergency order.

However, we cannot simply extend customer switching rate sensitivity conclusion to customer requesting rate. We still follow our original assumption that retailer i has surplus and retailer j is short of products. As customer requesting rate rises, more income is generated because of more waiting customers at retailer j. But, at the same time, fewer customers are expected to switch, which results in less revenue from retailer i. Particularly, when retailer i can use its surplus to meet all switching customers, this loss is more significant. Therefore, it is not straightforward that the gain from more waiting customers at retailer j make can make up the loss from fewer switching customers at retailer i without considering the retail price and emergency order cost. Analytically, if one unit of extra inventory at retailer i is sold at price r, the opportunity cost can be calculated as $rA_j - r + c_e$ because one unit of inventory may also be sold to any waiting customer at price r, emergency cost at retailer j is c_e, and expected revenue from retailer i is rA_j. Hence, only the benefit of waiting customers dominates the benefit of switching customers, it is worthwhile to encourage unsatisfied customers to stay.

4.2. Characteristics of optimal replenishment decisions

The replenishment decision needs to be made before demand realization. For any inventory replenishment levels Q_i, Q_j, the total expected profit is given by:

$$E\pi^E = -c_n(Q_i + Q_j) + \int_0^{Q_i}\int_0^{Q_j}\pi_1^E g_i(D_i)g_j(D_j)dD_i dD_j + \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}\pi_6^E g_i(D_i)g_j(D_j)dD_i dD_j \tag{1}$$

$$+\int_0^{Q_i}\int_{Q_j}^{\frac{Q_i-D_i}{(1-\lambda_j)A_j}+Q_j}\pi_2^E g_i(D_i)g_j(D_j)dD_i dD_j + \int_0^{Q_i}\int_{\frac{Q_i-D_i}{(1-\lambda_j)A_j}+Q_j}^{\infty}\pi_3^E g_i(D_i)g_j(D_j)dD_i dD_j \tag{2}$$

$$+\int_0^{Q_j}\int_0^{\frac{Q_j-D_j}{(1-\lambda_i)A_i}+Q_i}\pi_4^E g_i(D_i)g_j(D_j)dD_i dD_j + \int_0^{Q_j}\int_{\frac{Q_j-D_j}{(1-\lambda_i)A_i}+Q_i}^{\infty}\pi_5^E g_i(D_i)g_j(D_j)dD_i dD_j \tag{3}$$

Note: $\pi_i^E,\ i=1,\ \dots 6$ is the net income under different conditions.

π_1^E	$rD_i + rD_j$
π_2^E	$r(D_i + (1-\lambda_j)A_j(D_j - Q_j)) + r(Q_j + \lambda_j(D_j - Q_j)) - c_e\lambda_j(D_j - Q_j)$
π_3^E	$r(D_i + (1-\lambda_j)A_j(D_j - Q_j)) + r(Q_j + \lambda_j(D_j - Q_j))$ $-c_e((1-\lambda_j)A_j(D_j - Q_j) - (Q_i - D_i) + \lambda_j(D_j - Q_j))$
π_4^E	$r(D_j + (1-\lambda_i)A_i(D_i - Q_i)) + r(Q_i + \lambda_i(D_i - Q_i)) - c_e\lambda_i(D_i - Q_i)$
π_5^E	$r(D_j + (1-\lambda_i)A_i(D_i - Q_i)) + r(Q_i + \lambda_i(D_i - Q_i))$ $-c_e((1-\lambda_i)A_i(D_i - Q_i) - (Q_j - D_j) + \lambda_i(D_i - Q_i))$
π_6^E	$rQ_i + rQ_j + (r - c_e)(\lambda_i(D_i - Q_i) + (1-\lambda_j)A_j(D_j - Q_j))$ $+(r - c_e)(\lambda_j(D_j - Q_j) + (1-\lambda_i)A_i(D_i - Q_i))$

Table 2. The Retailers' Net Income Structure under Emergency Order Policy

The first two items in (1) are inventory replenishment costs of regular orders; the third one is the expected revenue generated from local demands when both retailers have extra inventories, and the forth one is the expected income if out of stock problem happens. (2) is the expected income when retailer i has surplus and retailer j runs out of stocks. (3) is the counterpart of (2), where firms' inventory positions are opposite.

Proposition 3 For a given Q_j, denote the optimal Q_i that maximizes the total profit by $Q_i^*(Q_j)$ is unique and decreases in Q_j.

It is not shocking that Q_i^* is declining because extra inventory at retailer j reduces the firm's stock-out risk when retailer i underestimates the level of demand for its products and increases its overstocking risk when retailer i has surplus, thus less inventory is needed. This is different from traditional emergency order problems, where one unit of increment at retailer j has no effect on retailer $i's$ replenishment level. The difference is mainly driven by customer switching behavior.

Furthermore, we extend our research by identifying the intersection of $Q_i^*(Q_j)$ and $Q_j^*(Q_i)$ and explore the existence of the optimal pair (Q_i^E, Q_j^E). Because we introduce customer switching factor and customer requesting issue in our model, its concavity in (Q_i, Q_j) is not as obvious as before.

Theorem 1 The expected total profit $E\pi^E$ is jointly concave in (Q_i, Q_j). Therefore, there exists a unique pair of replenishment levels (Q_i^E, Q_j^E) that maximizes the total expected profit.

Although it is not easy to predict the retailer's replenishment decision's general trend, we can establish some sensitivity conclusions based on the following results.

Proposition 4 For a given Q_j, denote the optimal Q_i that maximizes the total profit by $Q_i^*(Q_j)$ increases in A_j and decreases in λ_j.

The sensitivity on replenishment level is quite intuitive. Besides this, Proposition 4 also describes the relationship between two retailers' replenishment levels when the system deviates from symmetric parameter setting, which provides some managerial insights for retailers.

5. Inventory policies under transshipment policy

We now turn to study the case where a central coordination scheme controls the inventory and transshipment is implemented when it is necessary.

5.1. Transshipment schedule

Similarly, we first check the transshipment decisions after demands at both retailers are observed. Note that transshipment happens only if two retailers have different stock inventory statuses, i.e., one runs out of products and the other one hold too much stock. Below, adopting the framework from Table1, we exhibit the complete transshipment schedule in Table 3

From Table 2, in $Event_1$ and $Event_8$, there is no transshipment since retailer i and retailer j are overstocking their products. In $Event_8$, customer switching behavior doesn't matter since no unsatisfied customer will be served. This is totally different from the same case when retailer utilizes emergency order policy, where all switching demands are finally fulfilled by some

Event	Description	q_{ij}	q_{ji}
$Event_1$	$Q_i \geq D_i,\ Q_j \geq D_j$	0	0
$Event_2$	$Q_i \geq D_i,\ D_j \geq Q_j$ $Q_i - D_i \geq (A_j(1-\lambda_j)$ $+\lambda_j)(D_j - Q_j)$	$\lambda_j(D_j - Q_j)$	0
$Event_3$	$Q_i \geq D_i,\ D_j \geq Q_j$ $Q_i - D_i \geq A_j(1-\lambda_j)(D_j - Q_j)$ $Q_i - D_i \leq (A_j(1-\lambda_j)$ $+\lambda_j)(D_j - Q_j)$	$Q_i - D_i - A_j(1-\lambda_j)$ $(D_j - Q_j)$	0
$Event_4$	$Q_i \geq D_i,\ D_j \geq Q_j$ $Q_i - D_i \leq A_j(1-\lambda_j)(D_j - Q_j)$	0	0
$Event_5$	$D_i \geq Q_i,\ Q_j \geq D_j$ $Q_j - D_j \geq (A_i(1-\lambda_i)+\lambda_i)$ $(D_i - Q_i)$	$\lambda_i(D_i - Q_i)$	0
$Event_6$	$D_i \geq Q_i,\ Q_j \geq D_j$ $Q_j - D_j \geq A_i(1-\lambda_i)(D_i - Q_i)$ $Q_j - D_j \leq (A_i(1-\lambda_i)+\lambda_i)$ $(D_i - Q_i)$	0	$Q_j - D_j - A_i(1-\lambda_i)$ $(D_i - Q_i)$
$Event_7$	$D_i \geq Q_i,\ Q_j \geq D_j$ $Q_j - D_j \leq A_i(1-\lambda_i)(D_i - Q_i)$	0	0
$Event_8$	$Q_i \leq D_i,\ Q_j \leq D_j$	0	0

Table 3. Transshipment Schedule

external emergency orders. Due to this reason, we call emergency order policy "External Policy" versus transshipment policy "Internal Policy".

If we take a look at the transshipment table carefully, there are two other events i.e., $Event_4$ and $Event_7$ ending up with no transshipment since the extras at the surplus retailers are less than the switching demands and there is no reason to arrange any product relocation. Such transshipment pattern is primarily induced by customer switching behavior and is completely unlike classic research. It can also be deduced from Table 2 that in $Event_5$ and $Event_6$, the optimal transshipment quantity is $q_{ji} = \min\left(Q_j - D_j - A_i(1-\lambda_i)(D_i - Q_i),\ \lambda_i(D_i - Q_i)\right)^+,\ \ x^+ = \max\left(x,\ 0\right)$, where $D_j < Q_j$ and $D_i > Q_i$, which are counterparts of $Event_2$ and $Event_3$. The logic behind the schedule is that if the surplus is more than the sum of switching and requesting customers, it is always optimal to transship as much as necessary. In contrast, when the surplus inventory is not enough to match all potential shortfalls, it is better to meet switching customers first, and then transport leftovers.

In the same vein, we also examine how customer switching and requesting behaviors impact the transshipment quantity. The following proposition summarizes some straightforward effects.

Proposition 5 The transshipment amount q_{ij} is increasing in shipping-request rate λ_j and non-increasing in customer switching rate A_j.

Although transshipment policy is quite different from emergency order policy, their structures appear great similarity. Then we have,

Proposition 6 The total profit is increasing in customer switching rate. When $rA_j - r + t_{ij} > 0$, then total expected profit is decreasing in λ_j.

The major difference between Proposition 2 and Proposition 6 is that c_e in Proposition 2 is replaced by $t_{ij.}$ However, we still can develop the similar marginal analysis for this deviation. For each unit inventory relocated from retailer i to retailer j, it is priced at transportation charge t_{ij} and yields revenue $r_.$ Meanwhile, averagely we can regard rA_j as the return brought in by any single switching customer. When $rA_j - r + t_{ij} < 0$, the whole system loses money if customers prefer waiting instead of switching.

5.2. Characteristics of optimal replenishment decisions under transshipment policy

We then move to study the replenishment decisions under transshipment setting. Under central coordination, we choose Q_i, Q_j to maximize the total expected profit which is given by:

$$E\pi^T = -c_n(Q_i + Q_j) + \int_0^{Q_i}\int_0^{Q_j}\pi_1^T g_i(D_i)g_j(D_j)dD_i dD_j + \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}\pi_8^T g_i(D_i)g_j(D_j)dD_i dD_j \tag{4}$$

$$+\int_0^{Q_i}\int_{Q_j}^{\frac{Q_i-D_i}{(1-\lambda_j)A_j+\lambda_j}+Q_j}\pi_2^T g_i(D_i)g_j(D_j)dD_i dD_j + \int_0^{Q_i}\int_{\frac{Q_i-D_i}{(1-\lambda_j)A_j+\lambda_j}+Q_j}^{\frac{Q_i-D_i}{(1-\lambda_j)A_j}+Q_j}\pi_3^T g_i(D_i)g_j(D_j)dD_i dD_j \tag{5}$$

$$+\int_0^{Q_i}\int_{\frac{Q_i-D_i}{(1-\lambda_j)A_j}+Q_j}^{\infty}\pi_4^T g_i(D_i)g_j(D_j)dD_i dD_j + \int_0^{Q_j}\int_0^{\frac{Q_j-D_j}{(1-\lambda_i)A_i+\lambda_i}+Q_i}\pi_5^T g_i(D_i)g_j(D_j)dD_i dD_j \tag{6}$$

$$+\int_0^{Q_j}\int_{\frac{Q_j-D_j}{(1-\lambda_i)A_i+\lambda_i}+Q_i}^{\frac{Q_j-D_j}{(1-\lambda_i)A_i}+Q_i}\pi_6^T g_i(D_i)g_j(D_j)dD_i dD_j + \int_0^{Q_j}\int_{\frac{Q_j-D_j}{(1-\lambda_i)A_i}+Q_i}^{\infty}\pi_7^T g_i(D_i)g_j(D_j)dD_i dD_j \tag{7}$$

Note: π_i^T, $i=1, \ldots 8$ is the net income under different conditions.

π_1^T	$rD_i + rD_j$
π_2^T	$r(D_i + (1-\lambda_j)A_j(D_j - Q_j)) + r(Q_j + \lambda_j(D_j - Q_j)) - t_{ij}\lambda_j(D_j - Q_j)$
π_3^T	$r(D_i + (1-\lambda_j)A_j(D_j - Q_j)) + r(Q_j + (Q_i - D_i) - (1-\lambda_j)A_j(D_j - Q_j))$ $-t_{ij}((Q_i - D_i) - (1-\lambda_j)A_j(D_j - Q_j))$
π_4^T	$rQ_i + rQ_j$
π_5^E	$r(D_j + (1-\lambda_i)A_i(D_i - Q_i)) + r(Q_i + \lambda_i(D_i - Q_i)) - c_{ji}\lambda_i(D_i - Q_i)$
π_6^T	$r(D_j + (1-\lambda_i)A_i(D_i - Q_i)) + r(Q_i + (Q_j - D_j) - (1-\lambda_i)A_i(D_i - Q_i))$ $-t_{ij}((Q_j - D_j) - (1-\lambda_i)A_i(D_i - Q_i))$
π_7^T	$rQ_i + rQ_j$
π_8^T	$rQ_i + rQ_j$

Table 4. The Retailers' Net Income Structure under Transshipment Policy

Proposition 7 For a given Q_j, denote the optimal Q_i that maximizes the total profit by $Q_i^*(Q_j)$ decreases in λ_j and increases in A_j if the demand density function g_i is an increasing function.

Though transshipment and emergency order policy are not identical, the optimal replenishment level possesses some common properties. The essential point is that these decisions are Newsvendor-type inventory model decisions. In the aspect of optimal inventory levels, we can show

Theorem 2 The expected total profit $\pi^T = (Q_i, Q_j)$ is jointly concave in (Q_i, Q_j). Therefore, there exists a unique pair of replenishment levels (Q_i^T, Q_j^T) that maximizes the expected total profit.

Theorem 1 and Theorem 2 exhibit that if two retailers have a common supplier, the optimal replenishment levels definitely can be achieved despite retailers prefer transshipment or emergency order policy. Since good inventory management will lower costs, improve efficiency, Theorem 1 and 2 also provide some operational insights for management.

We have analytically studied customer requesting and customer switching phenomena's impacts on the firm's operations under emergency order policy or transshipment respectively. Although choosing emergency order policy or transshipment is determined by firm's own preference, it is still worthwhile if we can provide some straightforward comparisons between two polices. Then we have,

Proposition 8 When two retailers are symmetric, if emergency order cost is less or equal to transshipment cost, i.e., $c_e < t_{ij}, t_{ji}$, emergency order policy always dominates transshipment policy in profitability with the same initial replenishment inventories.

The conclusion above gives us a simple, practical guideline when two markets are in a similar position. In order to make business more profitable, retailer can easily make the decision if transshipment cost requires more.

6. Computational analysis

In the analytical study part, we have proved that customer switching and requesting factors influence retailers' decisions substantially. In this part, we numerically evaluate these effects and especially interested in comparing the operations index such as replenishment level, profitability of two polices. Because our study focuses on a symmetric scenario, we drop the subscripts "i, j", which are used to distinguish retailers. In our experiments, customer demand is normally distributed with mean 100 and standard deviation 50. Before proceeding to our model's results, we first demonstrate the traditional model's numerical outcomes.

Then, by adjusting the values of λ, A respectively, we investigate the trends of the optimal replenishment levels under different polices in our new framework. Next, we fix the initial inventory amount and examine how profit varies corresponding to changes of requesting rate and customer switching rate. Before moving to the details, we list all notations used in this section below:

A = Customer switching rate

c_e = Emergency order cost

λ = Customer requesting rate

t = Transshipment cost

Q_T = Optimal replenishment level under transshipment

Q_E = Optimal replenishment level under emergency order

PT = Total profit under transshipment

PE = Total profit under emergency order

IPC =Inventory level changes from initial level

6.1. Observation 1

In the classical centralized model, unsatisfied customers at each retailer only wait for transshipment or emergency order and never switch or give up purchasing. We let the unit cost c_n=20, retail price r=40 and list the result in the following table.

Obviously, retailers using emergency order policy require less replenishment inventories and generate more profits. Since holding more stocks increase costs for businesses such as increases warehouse space needed, money spent in stocks could have been allocated etc, it sounds like

c_e	Q_E	PE	Q_T	PT	t
22	39.1	3951.9	113.5	2957.6	2
24	55.3	3837.3	113	2940.9	4
26	66	3744.6	112.5	2923.9	6
28	74.1	3666.2	112	2906.7	8
30	80.5	3598.1	111.4	2889.2	10
32	85.9	3537.9	110.8	2871.4	12
34	90.6	3484	110.2	2853.3	14
36	94.6	3435.2	109.7	2835	16
38	98.2	3390.6	109.1	2816.3	18
40	101.4	3349.6	108.5	2797.3	20

Table 5. Traditional Models' Numerical Results

that retailers should exert emergency order policy as often as possible. However, since the above model's assumption is far from the realistic business environment, we are more concerned about whether the supply chain still behaves in the similar way. In the following, we display our experimental outcome in our modified model.

6.2. Observation 2

We still fix the normal unit cost $c_n = 20$, retail price $r = 40$ and let $A = 0.2$. The emergency cost c_e is priced at 30 and transshipment price t is marketed at 30 and 2 respectively, which can exhibit two extreme cases: (1) no disparity in cost (2) large gap between two charges.

λ	Q_E	PE	$Q_{T\ t=30}$	$PT_{\ t=30}$	$IPC_{\ t=30}$	$Q_{T\ t=2}$	$PT_{\ t=2}$	$IPC_{\ t=2}$
0.1	96.2	2780.5	105.5	2722.9		120.6	2621.4	
0.2	94.8	2780	104.9	2726.5	0.64%	118.2	2692.1	1.99%
0.3	93.4	2777.5	104.2	2728.4	1.22%	115.7	2758	4.06%
0.4	91.9	2772.6	103.7	2728.6	1.75%	113.3	2817.5	6.05%
0.5	90.4	2765	103.2	2727.3	2.23%	110.9	2869.5	8.04%
0.6	88.7	2753.9	102.7	2724.5	2.66%	108.7	2913.7	9.87%
0.7	86.9	2738.9	102.3	2720.5	3.04%	106.6	2950	11.61%
0.8	84.9	2719.9	102	2715.3	3.36%	104.7	2979	13.18%

Table 6. $A = 0.2$

A	Q_E	PE	$Q_{T\ t=30}$	$PT_{t=30}$	$IPC_{t=30}$	$Q_{T\ t=2}$	$PT_{t=2}$	$IPC_{t=2}$
0.1	94.8	2704	105.7	2672.4		119.7	2631.8	
0.2	93.5	2780	104.9	2726.5	0.76%	118.2	2692.1	1.25%
0.3	92.3	2853.3	104.3	2775.5	1.32%	116.8	2746.9	2.42%
0.4	91.2	2923.6	103.9	2819.6	1.70%	115.6	2796.2	3.43%
0.5	90.2	2991	103.5	2859.1	2.08%	114.6	2840	4.26%
0.6	89.2	3055.6	103.5	2894.4	2.08%	113.8	2878.8	4.93%
0.7	88.3	3177.6	103.5	2954.2	2.08%	113.1	2913.1	5.51%
0.8	88.3	3177.2	103.6	2954.2	1.99%	112.6	2943.4	5.93%

Table 7. $\lambda = 0.2$

From Table 6 and Table 7, the inventory level under transshipment always dominates the one under emergency policy despite deviation between transshipment expense and emergency order cost is zero or huge. This discovery displays the same pattern as the traditional model. Intuitively, this justifies our statement mentioned before, where we call transshipment an internal method and emergency order policy an external method. Since retailer cannot get help from outside resource, it is not surprising that retailers conservatively hold more stock to alleviate out-of-stock problem.

But it is also notable that emergency order policy doesn't prevail transshipment policy anymore in profitability under our customer behavior absorbed inventory model framework, for example, $\lambda = 0.5$, $t = 2$, $A = 0.2$, $c_e = 30$, $PT = 2869.5$, $PE = 2727.3$. It advises retailers that they still can be more profitable without placing emergency orders, especially when transshipment cost is relatively low.

Though Table 6 exhibits the impact of customer requesting behavior on replenishment level is trivial when transshipment cost is expensive, this effect becomes more prominent as transshipment price declines. When more customers send requests, we anticipate that more unmet customers are willing to stay. As transshipment price is low, the marginal benefit is considerable if one transshipped unit is sold regarding the corresponding loss of over-stocking, which encourages firm to retain inventory more aggressively. On the other hand, as transshipment expense reaches a certain level, the marginal benefit is not that significant and firm reduces its pace of increasing stock reserving level.

In Table 7, although we conduct the sensitivity analysis on customer switching rate, the replenishment level change under transshipment displays a similar pattern. As customer switching scale rises, more customers are going for shopping instead of waiting for transshipment. At this moment, transshipment price doesn't play a critical role. Due to this reason, it is reasonable that replenishment inventory level change rate doesn't vary rapidly.

6.3. Observation 3

In the previous numerical experiments, we focus on the optimal replenishment inventory levels which heavily depend on parameter settings. Nonetheless, in practice, retailers may simply order a certain number of products, since it is easily handled by employees. Because of this, in the experiments below, we lock the order and contrast the returns between two policies.

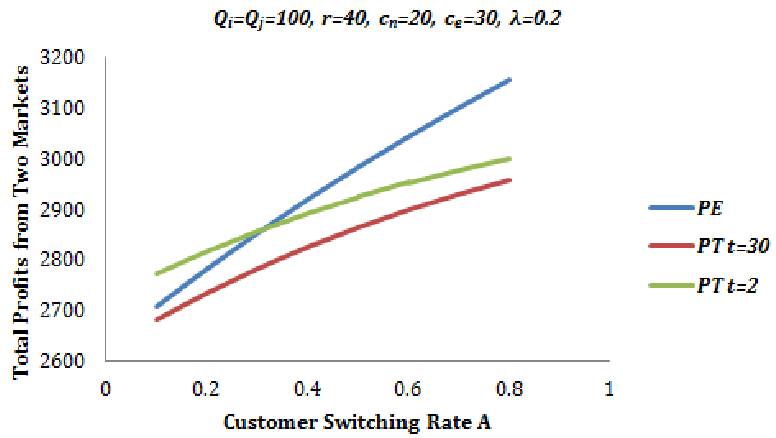

Figure 3. Retailers' Profits V.S Customer Switching Rate

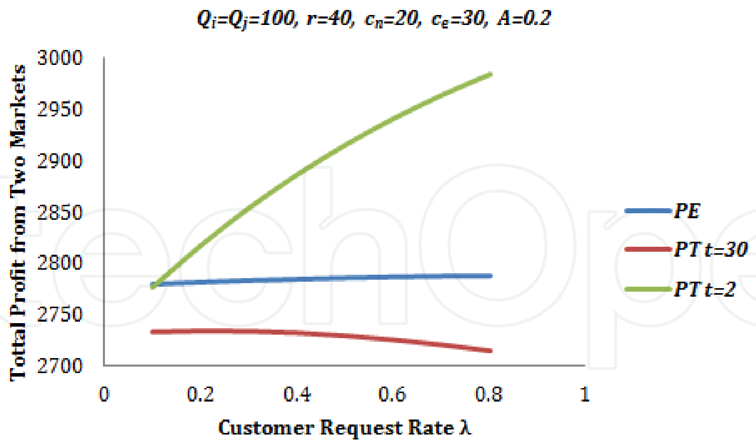

Figure 4. Retailers' Profits V.S Customer Requesting Rate

First of all, from Figure 3 and Figure 4, it is easily indentified that when transshipment cost decreases, the system's profit does increase dramatically. Therefore, firm may take advantage of this property and put effort on reducing transshipment expense since it does not need to consider customer's consumption behavior.

Secondly, we exhibit that as transshipment price is exactly same as emergency cost, emergency order policy completely outperforms transshipment method, which confirms the statement from Proposition 8. This result may be used as one of criteria for firm to choose the appropriate policy if retailers choose to order in a simple manner.

Lastly, when it costs firm high price to conduct transshipment, customer switching behavior and customer requesting rate may affect the total profit in the opposite direction. For example, in Figure 3, when transshipment cost is 30, it demonstrates an increasing trend as customer switching rate increases. In Figure 4, this trend reverses when customer requesting rate rises. The implication behind this is that high transshipment price may take a bite of the total profit if more customers are willing to stay and want for relocated products. Under this condition, retailer may facilitate customers to switch instead of waiting. On the other hand, when transshipment cost is relatively low, customers are more welcome to stay since it enhances the total profit considerably.

7. Summary and future research

This paper utilizes a centralized model to investigate how customers' switching and requesting behaviors affect retailers' operations decisions under emergency order and transshipment policies respectively. We first prove that the optimal replenishment stocking levels exist under two policies. Then, we explore how customer switching and shipping impact transshipment amount and emergency quantity. Furthermore, we numerically compare the system's performance in profitability under two policies, with /without the optimal replenishment level and provide some practical policy choosing criteria for retailers.

Since in our research we consider a centralized one-period two-location model, there are a lot of possible extensions for the future research. At the first step, we may expand our research to a decentralized setting, where retailer maximizes its own profit instead of overall performance. It's also worthwhile to extend our one-period two-location to N-period and N-location model among retailers.

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