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# Estimation and Active Damping of Unbalance Forces in Jeffcott-Like Rotor-Bearing Systems

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Additional information is available at the end of the chapter

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## 1. Introduction

Nowadays, rotordynamics is a technological field in which research is very active because in spite of basic phenomena have been widely studied, there are many aspects that still need theoretical and practical work in order to construct and analyze models to represent with more precision the dynamic behavior of real machines. Dynamic studies in rotordynamics usually are preformed by numerical simulations using the mathematical models reported in literature. The mathematical description of rotating systems allows the possibility to predict their dynamic behavior and to use this information for the design of control algorithms in order to preserve the desired stability and dynamic performance. The accuracy of a model is determined by comparing its response and the response of the real system to the same input signal [19]. Rotor systems are subjected, in an intrinsic way, to endogenous disturbances, centrifugal forces by the inevitable unbalance phenomenon. Magnitude of these unbalance forces depends on the rotor mass, angular speed and distance between geometric center and center mass of rotor [10, 11, 29]. This last parameter is known as eccentricity and represents one of the most difficult parameter to measure or to estimate in a rotor system and consequently, it is an important aspect for the accuracy model.

The recent trends in rotordynamic systems are moving to higher speeds, higher powers, lighter and more compact machinery, which has resulted in machines operating above one or more critical speeds and increasing the vibration problems [5, 31]. In literature, the unbalance phenomenon has been widely reported as the main source of undesired vibration in rotating machinery [5, 10, 11, 29]. An unacceptable level of vibration can cause failure in the bearings, high levels of noise, wearing in the mechanical components and eventually, catastrophic failures in machines [10, 29], hence, control algorithms are needed to reduce the unbalance effects and to take vibrations amplitudes to acceptable values for a safe machine operation.

In literature, some different approaches to solve the problem of the model accuracy have been proposed. An estimation procedure to identify the distributed eccentricity along the shaft of a Jeffcott-like rotor was presented by Yang and Lin (2002). The eccentricity distributions of the shaft are assumed as polynomial of certain grade and the disk eccentricity is considered as lumped. Measurements in different locations along the shaft and to different rotational speeds are needed. By some numerical simulations, they found the method efficiency depends on the number of sensors available and on the number of operating speeds. Some other authors have proposed the problem solution in a lumped parameters approach. Maslen et al. (2002) developed a method to analytically adjust the models of rotor systems to make them consistent with experimental data under the assumption that the predominant uncertainties in the models occur at discrete points, from effects like seal coefficients or foundation interactions. The purpose of the method is to modify the engineering model such that the output of the model matches the experimental data in frequency domain. They show some examples to identify lumped stiffness at the supports and seal coefficients, but not the associated unbalance parameters and the results are presented through numerical simulations. De Queiroz (2009) presented a relatively simple feedback method to identify the unknown unbalance parameters of a Jeffcott rotor based on a dynamic robust control technique, in which the disturbance forces are estimated and then, from these forces, the magnitude and phase of the unbalance are obtained. This strategy is proved by numerical simulations and the rotational speed of the machine has to satisfy the persistency of excitation condition in order to guarantee the convergence of the method. Using curve fitting techniques and optimization procedures based on least-squares methods, Mahfoud et al. (2009) proposed a method to identify the matrices of a rotordynamic model expressed in state variables, measuring the full state vector (displacement, speed and acceleration) in three steps. The impulse response for a null rotational speed is used to identify the speed non-dependent matrix, the control matrix is identified using the steady-state response and the dependent dynamic matrix is calculated from the permanent time response of the system at an operational speed. Finally, the external forces can be found proposing an inverse problem from the model with the three matrices previously determined. Recently, some results in this issue have been published in specialized literature, Sudhakar and Sekhar (2011) estimated the unbalance faults in a Jeffcott-like rotor system with fault identification approach, obtaining good results in both numerical and experimental ways, showing the need of new methods and techniques to solve the unbalance forces estimation problem.

The developments in the fields of electronics, computing and control systems have changed the approach to reduce the level of vibration amplitudes in mechanical systems. In the traditional approach, changes in stiffness or damping system parameters cause changes in the dynamic system behavior. Nowadays, control systems can adapt dynamically these stiffness or damping parameters depending on the requirements or apply force directly to reduce the vibration effects. This trend is increasingly applied in rotating machinery and other fields of structural mechanics [11]. For control purposes, many passive, semi-active and active devices have been proposed [31]. Active Magnetic Bearings (AMB) have found an important field of application in rotor systems because the advantages over other devices. The absence of contact between an AMB and rotor avoids wearing and the need of lubrication, in addition, AMB dynamics is relatively easy to control [21]. For this, many researchers have reported results about showing the viability for AMB application in vibration control of rotating machinery since the 1990's [16, 26] until recent days [1, 13, 20, 28]. Piezoelectric actuators are other devices with an increasing application in rotor systems, they represent an alternative

because their main characteristics: very precise movement, compact, high force, low energy consumption, quick response time, no electromagnetic interference. Some researchers have presented numerical and experimental results in the active control of unbalance response in rotors using piezoelectric actuators showing that it is possible to control vibration amplitudes by these devices [15, 17, 22]. Finally, another alternative to vibration control in rotor systems are the semi-active devices. A semi-active vibration control system replaces the actuators to apply directly force for devices which can change the stiffness and/or damping system parameters. Due to their low energy consumption, their application in theoretical and practical issues in mechanical systems tends to increase in the last years. Magnetorheological dampers represent the most used semi-active device in rotor systems [3, 9, 12].

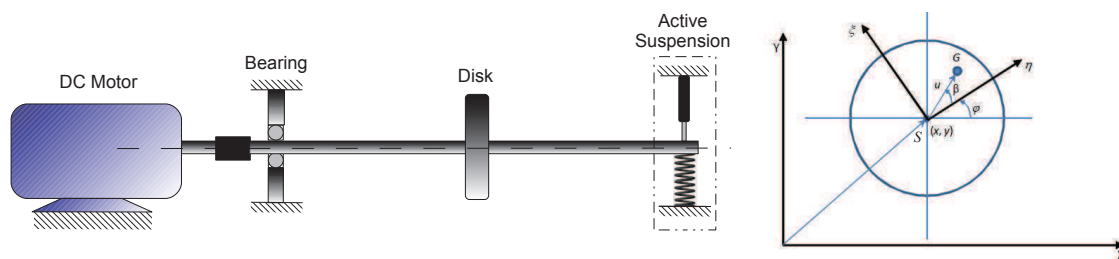
Generally, a control scheme to vibration attenuation is designed using a system model, so that it is very important that the model represents to the real system behavior with good accuracy. As we mentioned above, in the models used to describe the dynamic behavior of rotor systems, the amount and location of unbalance are some of the most difficult parameters to be measured and, therefore, estimation techniques are needed to establish these and other parameters to get the required accuracy in the model. Observers (or estimators) can be designed, from measurements of the input and of the response of the system to provide an approximation of system states or disturbances that can not be directly measured [14]. Observers can be considered as subsystems that combine sensed signals with other knowledge of the control system to produce estimated signals and offer important advantages: they can remove sensors, which reduces cost and improves reliability, and improve the quality of signals that come from the sensors, allowing performance enhancement. However, observers have disadvantages: they can be complicated to implement and they expend computational resources. Also, because observers form software control loops, they can become unstable under certain conditions. Observers can also provide observed disturbance signals, which can be used to improve disturbance response. In spite of observers add complexity to the system and require computational resources, an observer applied with skill can bring substantial performance benefits and do so, in many cases, while reducing cost or increasing reliability [6].

This chapter deals with the active control problem of unbalance-induced synchronous vibrations in variable-speed Jeffcott-like non-isotropic rotor-bearing systems using only measurements of the radial displacement close to the disk. In this study, the rotor-bearing system is supported by a conventional bearing at its left end and by an active control device at the right one, which is used to provide the control forces. A robust and efficient active unbalance control scheme based on on-line compensation of rotor unbalance-induced perturbation force signals is proposed to suppress the undesirable vibrations affecting the rotor-bearing system dynamics. The methodology presented by Sira-Ramirez et al. (2008) is applied to design a Luenberger linear state observer to estimate the unbalance force signals and velocities of the coordinates of the rotor center, which are required to implement the proposed control scheme. The designed state observer is called the Generalized Proportional Integral (GPI) observer because its design approach is the dual counterpart of the so-called GPI controller [7]. A state-space based extended linear mathematical model is developed to locally describe the dynamics of the perturbed rotor-bearing system for design purposes of the disturbance observer. The modelling approach of disturbance signals through a family of Taylor time-polynomials of fourth degree described in [23] is used to locally reconstruct such unknown signals. A similar approach to reconstruct disturbance signals based also

on its Taylor time-polynomial expansion has been previously proposed by Sira-Ramirez et al. (2007) using the on-line algebraic parameter identification methodology described in [8]. Additionally, a Proportional-Integral (PI) control law is designed to perform robust tracking tasks of smooth rotor speed reference profiles described by Bézier interpolation polynomials. Simulation results are provided to show the efficient and robust performance of the active vibration control scheme, estimation of the unbalance forces and rotor speed controller for the tracking of a speed reference profile that takes the rotor system from a rest initial speed to an operation speed above its first critical speeds.

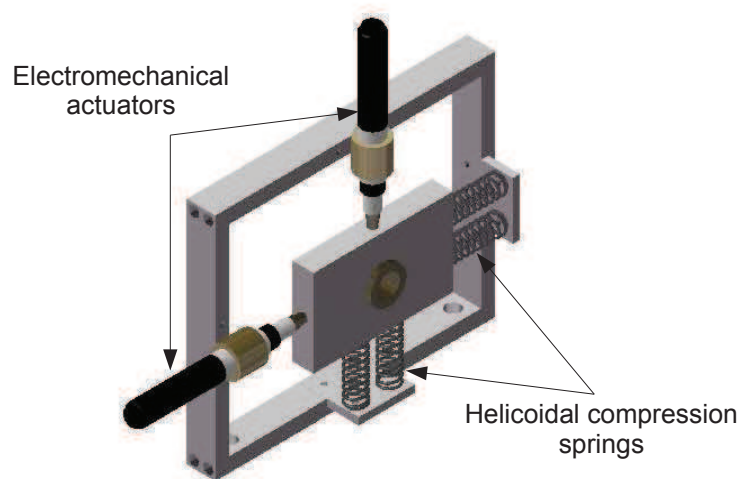
## 2. Rotor system model

The rotor system in a Jeffcott configuration is shown in Fig. 1. The rotor is supported by a conventional bearing at its left end and by an active suspension at the right one.



**Figure 1.** Jeffcott like rotor system with active suspension.

In this study, the active suspension presented by Arias-Montiel and Silva-Navarro (2010b) is considered. This control device is based on two linear electromechanical actuators and helicoidal compression springs. Electromechanical actuators provide the control forces in two perpendicular directions in order to compensate actively the unbalance effects and to get reductions in vibration amplitudes. The active suspension is depicted in Fig. 2.



**Figure 2.** Active suspension with linear actuators.

The Jeffcott like rotor system consists of a disk with mass  $m$  mounted at the mid span of a flexible shaft. In Fig. 1,  $x$  and  $y$  denote the orthogonal coordinates of rotor geometric center,  $u$  is the distance between the gravity center  $G$  and the geometric center  $S$ , which is known as rotor eccentricity. Moreover,  $k_x$ ,  $k_y$ ,  $c_x$  and  $c_y$  are the shaft stiffness and viscous damping

coefficients in  $x$  and  $y$  directions, respectively,  $\varphi$  and  $\omega$  are the angular displacement and velocity, respectively,  $J$  is the polar moment of inertia of the rotor,  $c_\varphi$  is the rotational viscous damping coefficient and  $\tau$  is the control torque to smoothly regulate the rotor speed through an traditional PID driver. Considering  $u_x$  and  $u_y$  as the radial control forces provided by the active suspension used to compensate the unbalance effects on the rotor in each movement plane and  $\tau$  as the control torque provided by the motor, the rotor system model can be obtained by Euler-Lagrange formulation. Defining the coordinates of rotor gravity center  $x_G$  and  $y_G$  as

$$\begin{aligned} x_G &= x + u \cos(\varphi + \beta) \\ y_G &= y + u \sin(\varphi + \beta) \end{aligned} \quad (1)$$

and their time derivatives

$$\begin{aligned} \dot{x}_G &= \dot{x} - u\dot{\varphi}(\sin\varphi + \beta) \\ \dot{y}_G &= \dot{y} + u\dot{\varphi}(\cos\varphi + \beta) \end{aligned} \quad (2)$$

We can obtain the system kinetic energy as

$$T = \frac{1}{2}m\dot{x}_G^2 + \frac{1}{2}m\dot{y}_G^2 + \frac{1}{2}J\dot{\varphi}^2 \quad (3)$$

and the potential energy as

$$V = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 \quad (4)$$

So, the system Lagrangian is given by

$$L = T - V = \frac{1}{2}m\dot{x}_G^2 + \frac{1}{2}m\dot{y}_G^2 + \frac{1}{2}J\dot{\varphi}^2 - \frac{1}{2}k_x x^2 - \frac{1}{2}k_y y^2 \quad (5)$$

and proposing the dissipation function of Rayleigh of the form

$$D = \frac{1}{2}c_x \dot{x}^2 + \frac{1}{2}c_y \dot{y}^2 + \frac{1}{2}c_\varphi \dot{\varphi}^2 \quad (6)$$

Then, considering  $\dot{\varphi} = \omega$  the dynamics equations for the system can be achieved from

$$\begin{aligned}
\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{x}} \right] - \frac{\partial L}{\partial x} + \frac{\partial D}{\partial \dot{x}} &= u_x \\
\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{y}} \right] - \frac{\partial L}{\partial y} + \frac{\partial D}{\partial \dot{y}} &= u_y \\
\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{\omega}} \right] - \frac{\partial L}{\partial \phi} + \frac{\partial D}{\partial \dot{\omega}} &= \tau
\end{aligned} \tag{7}$$

From equations (7), one obtains

$$\begin{aligned}
m\ddot{x} + c_x\dot{x} + k_x x - m\omega^2 u \cos(\varphi + \beta) - m\dot{\omega} u \sin(\varphi + \beta) &= u_x \\
m\ddot{y} + c_y\dot{y} + k_y y - m\omega^2 u \sin(\varphi + \beta) + m\dot{\omega} u \cos(\varphi + \beta) &= u_y \\
(J + mu^2)\dot{\omega} + c_\varphi\omega - m\dot{x}u \sin(\varphi + \beta) + m\dot{y}u \cos(\varphi + \beta) &= \tau
\end{aligned} \tag{8}$$

and rewriting these last equations

$$\begin{aligned}
m\ddot{x} + c_x\dot{x} + k_x x &= u_x + \xi_x \\
m\ddot{y} + c_y\dot{y} + k_y y &= u_y + \xi_y \\
J_e\dot{\omega} + c_\varphi\omega &= \tau + \xi_w \\
\dot{\phi} &= \omega
\end{aligned} \tag{9}$$

with

$$\begin{aligned}
\xi_x &= mu \left[ \dot{\omega} \sin(\varphi + \beta) + \omega^2 \cos(\varphi + \beta) \right] \\
\xi_y &= mu \left[ -\dot{\omega} \cos(\varphi + \beta) + \omega^2 \sin(\varphi + \beta) \right] \\
\xi_w &= mu \left[ \dot{x} \sin(\varphi + \beta) - \dot{y} \cos(\varphi + \beta) \right] \\
J_e &= J + mu^2
\end{aligned} \tag{10}$$

In the above,  $\xi_x$ ,  $\xi_y$  and  $\xi_w$  are the centrifugal forces and perturbation torque, respectively, induced by the rotor unbalance.

Defining the state space variables as  $z_1 = x$ ,  $z_2 = \dot{x}$ ,  $z_3 = y$ ,  $z_4 = \dot{y}$ ,  $z_5 = \varphi$ ,  $z_6 = \dot{\phi}$ , the generalized forces as  $u_1 = u_x$ ,  $u_2 = u_y$  and  $u_3 = \tau$  and the total unbalance amplitude  $y_u$  as system output, one obtains the following state-space description of system (8):



$$\begin{aligned}
 \dot{z}_1 &= z_2 \\
 \dot{z}_2 &= \frac{1}{\Delta} f_1 + \frac{1}{\Delta} g_1 \\
 \dot{z}_3 &= z_4 \\
 \dot{z}_4 &= \frac{1}{\Delta} f_2 + \frac{1}{\Delta} g_2 \\
 \dot{z}_5 &= z_6 \\
 \dot{z}_6 &= \frac{1}{\Delta} f_3 + \frac{1}{\Delta} g_3 \\
 y_u &= \sqrt{z_1^2 + z_3^2}
 \end{aligned} \tag{11}$$

where

$$\begin{aligned}
 f_1 &= -k_x \left[ \frac{1}{m} - \frac{u^2}{J_e} \cos^2(z_5 + \beta) \right] z_1 - c_x \left[ \frac{1}{m} - \frac{u^2}{J_e} \cos^2(z_5 + \beta) \right] z_2 \\
 &+ \frac{u^2 k_y}{J_e} \cos(z_5 + \beta) \sin(z_5 + \beta) z_3 \\
 &+ \frac{c_y u^2}{J_e} \cos(z_5 + \beta) \sin(z_5 + \beta) z_4 - \frac{c_\varphi u}{J_e} z_6 \sin(z_5 + \beta) \\
 &+ \left[ mu \left( \frac{1}{m} - \frac{u^2}{J_e} \cos^2(z_5 + \beta) \right) \cos(z_5 + \beta) - \frac{mu^3}{J_e} \cos(z_5 + \beta) \sin^2(z_5 + \beta) \right] z_6^2
 \end{aligned}$$

$$g_1 = \left[ \frac{1}{m} - \frac{u^2}{J_e} \cos^2(z_5 + \beta) \right] u_x - \frac{u^2}{J_e} \cos(z_5 + \beta) \sin^2(z_5 + \beta) u_y + \frac{u}{J_e} \sin(z_5 + \beta) \tau$$

$$\begin{aligned}
 f_2 &= \frac{u^2 k_x}{J_e} \cos(z_5 + \beta) \sin(z_5 + \beta) z_1 + \frac{c_x u^2}{J_e} \cos(z_5 + \beta) \sin(z_5 + \beta) z_2 \\
 &- k_y \left[ \frac{1}{m} - \frac{u^2}{J_e} \sin^2(z_5 + \beta) \right] z_3 - c_y \left[ \frac{1}{m} - \frac{u^2}{J_e} \sin^2(z_5 + \beta) \right] z_4 \\
 &+ \frac{c_\varphi u}{J_e} z_6 \cos(z_5 + \beta) + mu \left[ \frac{1}{m} - \frac{u^2}{J_e} \sin^2(z_5 + \beta) \right] \sin(z_5 + \beta) z_6^2 \\
 &- \frac{mu^3}{J_e} \cos^2(z_5 + \beta) \sin(z_5 + \beta) z_6^2
 \end{aligned}$$



$$g_2 = -\frac{u^2}{J_e} \cos(z_5 + \beta) \sin(z_5 + \beta) u_x + \left[ \frac{1}{m} - \frac{u^2}{J_e} \sin^2(z_5 + \beta) \right] u_y - \frac{u}{J_e} \cos(z_5 + \beta) \tau$$

$$f_3 = -\frac{uk_x}{J_e} \sin(z_5 + \beta) z_1 - \frac{c_x u}{J_e} \sin(z_5 + \beta) z_2 + \frac{uk_y}{J_e} \cos(z_5 + \beta) z_3 + \frac{c_y u}{J_e} \cos(z_5 + \beta) z_4 - \frac{c_\varphi}{J_e} z_6$$

$$g_3 = \frac{u}{J_e} \sin(z_5 + \beta) u_x - \frac{u}{J_e} \cos(z_5 + \beta) u_y + \frac{\tau}{J_e}$$

$$\Delta = \frac{1}{J_e} (J_e - mu^2)$$

### 3. Active unbalance control

For the design of the active unbalance control scheme proposed in this chapter, consider the nonlinear ordinary differential equations that describe the dynamics of the rotor center, where only the position coordinates are available for measurement

$$\begin{aligned} m\ddot{x} + c_x \dot{x} + k_x x &= u_x + \xi_x \\ m\ddot{y} + c_y \dot{y} + k_y y &= u_y + \xi_y \end{aligned} \quad (12)$$

where

$$\begin{aligned} \xi_x &= mu \left[ \dot{\omega} \sin(\varphi + \beta) + \omega^2 \cos(\varphi + \beta) \right] \\ \xi_y &= mu \left[ -\dot{\omega} \cos(\varphi + \beta) + \omega^2 \sin(\varphi + \beta) \right] \end{aligned} \quad (13)$$

In our design approach, the unbalance forces  $\xi_x$  and  $\xi_y$  will be considered as unknown disturbance signals.

For the active unbalance suppression, Proportional-Derivative (PD) controllers with compensation of the rotor unbalance-induced disturbance signals are proposed

$$\begin{aligned} u_x &= -\alpha_{1,x}\hat{x} - \alpha_{0,x}x - \hat{\zeta}_x(t) \\ u_y &= -\alpha_{1,y}\hat{y} - \alpha_{0,y}y - \hat{\zeta}_y(t) \end{aligned} \quad (14)$$

where  $\hat{\zeta}_x(t)$  and  $\hat{\zeta}_y(t)$  are estimated perturbation signals of the actual time-varying unbalance forces  $\zeta_x(t)$  and  $\zeta_y(t)$ , respectively, and  $\hat{x}$  and  $\hat{y}$  are estimates of the velocities of the rotor center in  $x$  and  $y$  directions, respectively.

In this chapter, an on-line estimation approach based on Luenberger linear estate observers is proposed to estimate the disturbance and velocity signals, using measurements of the rotor center coordinates  $(x, y)$ , which could be obtained employing proximity sensors or accelerometers in practical applications. A state-space based extended linear mathematical model will be developed to locally describe the dynamics of the perturbed rotor-bearing system for design purposes of the disturbance observer. A family of Taylor time polynomials of fourth degree will be used to locally describe the unknown disturbance signals.

The use of the controllers (14) in the rotor-bearing system (12) yields the following closed-loop dynamics for the rotor center coordinates

$$\begin{aligned} \ddot{x} + \frac{1}{m} (c_x + \alpha_{1,x}) \dot{x} + \frac{1}{m} (k_x + \alpha_{0,x}) x &= 0 \\ \ddot{y} + \frac{1}{m} (c_y + \alpha_{1,y}) \dot{y} + \frac{1}{m} (k_y + \alpha_{0,y}) y &= 0 \end{aligned} \quad (15)$$

whose characteristic polynomials are given by

$$\begin{aligned} p_x(s) &= s^2 + \frac{1}{m} (c_x + \alpha_{1,x}) s + \frac{1}{m} (k_x + \alpha_{0,x}) \\ p_y(s) &= s^2 + \frac{1}{m} (c_y + \alpha_{1,y}) s + \frac{1}{m} (k_y + \alpha_{0,y}) \end{aligned} \quad (16)$$

Therefore, by selecting the controller gains  $\alpha_{i,j}$ ,  $i = 0, 1$ ,  $j = x, y$ , so that the characteristic polynomials (16) be Hurwitz, one can guarantee that the dynamics of the rotor center coordinates be globally asymptotically stable, i.e.,  $\lim_{t \rightarrow \infty} x(t) = 0$  and  $\lim_{t \rightarrow \infty} y(t) = 0$ . It can be observed that a consequence of the unbalance cancellation is that the rotor unbalance-induced perturbation torque signal  $\zeta_\omega$  affecting additively the rotor velocity dynamics is also suppressed, i.e.,  $\lim_{t \rightarrow \infty} \zeta_\omega(t) = 0$ .

For the closed-loop dynamics of the coordinates of the rotor center, the following reference system is proposed

$$\begin{aligned}\ddot{x} + 2\zeta_x\omega_{nx}\dot{x} + \omega_{nx}^2x &= 0 \\ \ddot{y} + 2\zeta_y\omega_{ny}\dot{y} + \omega_{ny}^2y &= 0\end{aligned}\quad (17)$$

where  $\zeta_i, \omega_{ni} > 0, i = x, y$ , are the viscous damping ratios and natural frequencies for the rotor center dynamics. Then, the gains of the controllers (14) are calculated as

$$\begin{aligned}\alpha_{1,x} &= 2m\zeta_x\omega_{nx} - c_x \\ \alpha_{1,y} &= 2m\zeta_y\omega_{ny} - c_y \\ \alpha_{0,x} &= m\omega_{nx}^2 - k_x \\ \alpha_{0,y} &= m\omega_{ny}^2 - k_y\end{aligned}$$

Otherwise, a Proportional-Integral (PI) control law is proposed for tracking tasks of an angular speed profile  $\omega^*(t)$  specified for the rotor system

$$\begin{aligned}\tau &= J_e v + c_\varphi \omega \\ v &= \dot{\omega}^*(t) - \alpha_{1,\omega} [\omega - \omega^*(t)] - \alpha_{0,\omega} \int_0^t [\omega - \omega^*(t)] dt\end{aligned}\quad (18)$$

By replacing the control law (18) into the rotor-bearing system (9), it is obtained the homogenous differential equation that describes the dynamics of the angular speed tracking error  $e_\omega = \omega - \omega^*(t)$ , under the assumption that the unbalance was canceled by the action of the PD control forces (14),

$$\ddot{e}_\omega + \alpha_{1,\omega}\dot{e}_\omega + \alpha_{0,\omega}e_\omega = 0\quad (19)$$

Then, the asymptotic convergence of the tracking error  $e_\omega$  to zero can be achieved selecting the design parameters  $\alpha_{0,\omega}$  and  $\alpha_{1,\omega}$  such as the characteristic polynomial associated to tracking error dynamics in closed loop (19) given by

$$p_\omega(s) = s^2 + \alpha_{1,\omega}s + \alpha_{0,\omega}\quad (20)$$

be a Hurwitz polynomial. In this case, the asymptotic tracking of the specified angular speed profile can be verified, i.e.,

$$\lim_{t \rightarrow \infty} e_{\omega}(t) = 0 \Rightarrow \lim_{t \rightarrow \infty} \omega(t) = \omega^*(t)$$

In this chapter, the following Hurwitz polynomial is proposed for the closed-loop rotor angular speed dynamics

$$p_{\omega d}(s) = s^2 + 2\zeta_r \omega_{nr} s + \omega_{nr}^2 \quad (21)$$

where  $\zeta_r$  and  $\omega_{nr} > 0$  are the viscous damping ratio and natural frequency for rotor angular speed dynamics. Then, the gains of the controller (18) are calculated as

$$\begin{aligned} \alpha_{0,\omega} &= \omega_{nr}^2 \\ \alpha_{1,\omega} &= 2\zeta_r \omega_{nr} \end{aligned}$$

#### 4. Asymptotic estimation of unbalance forces

In the design process of the disturbance observer, it is assumed that the perturbation force signals  $\zeta_x$  and  $\zeta_y$  can be locally approximated by a family of fourth degree Taylor time-polynomials [23]:

$$\zeta_i(t) = \sum_{j=0}^4 p_{j,i} t^j, \quad i = x, y \quad (22)$$

where the coefficients  $p_{j,i}$  are completely unknown.

The perturbation signals can then be locally described by the following state-space based linear mathematical model:

$$\begin{aligned} \dot{\zeta}_{1,i} &= \zeta_{2,i} \\ \dot{\zeta}_{2,i} &= \zeta_{3,i} \\ \dot{\zeta}_{3,i} &= \zeta_{4,i} \\ \dot{\zeta}_{4,i} &= \zeta_{5,i} \\ \dot{\zeta}_{5,i} &= 0 \end{aligned} \quad (23)$$

where  $\tilde{\zeta}_{1,i} = \tilde{\zeta}_i$ ,  $\tilde{\zeta}_{2,i} = \dot{\tilde{\zeta}}_i$ ,  $\tilde{\zeta}_{3,i} = \ddot{\tilde{\zeta}}_i$ ,  $\tilde{\zeta}_{4,i} = \tilde{\zeta}_i^{(3)}$ ,  $\tilde{\zeta}_{5,i} = \tilde{\zeta}_i^{(4)}$ ,  $i = x, y$ .

Therefore, an extended state space model for the perturbed rotor center dynamics is given by

$$\begin{aligned}
 \dot{\eta}_{1,i} &= \eta_{2,i} \\
 \dot{\eta}_{2,i} &= -\frac{k_i}{m}\eta_{1,i} - \frac{c_i}{m}\eta_{2,i} + \frac{1}{m}u_i + \frac{1}{m}\tilde{\zeta}_{1,i} \\
 \dot{\tilde{\zeta}}_{1,i} &= \tilde{\zeta}_{2,i} \\
 \dot{\tilde{\zeta}}_{2,i} &= \tilde{\zeta}_{3,i} \\
 \dot{\tilde{\zeta}}_{3,i} &= \tilde{\zeta}_{4,i} \\
 \dot{\tilde{\zeta}}_{4,i} &= \tilde{\zeta}_{5,i} \\
 \dot{\tilde{\zeta}}_{5,i} &= 0
 \end{aligned} \tag{24}$$

where  $\eta_{1,i} = i$ ,  $\eta_{2,i} = \dot{\eta}_{1,i}$ ,  $i = x, y$ .

From system (24), the following Luenberger linear state observer is proposed to estimate the disturbance and rotor center velocity signals

$$\begin{aligned}
 \hat{\eta}_{1,i} &= \hat{\eta}_{2,i} + \beta_{6,i}(\eta_{1,i} - \hat{\eta}_{1,i}) \\
 \hat{\eta}_{2,i} &= -\frac{k_i}{m}\hat{\eta}_{1,i} - \frac{c_i}{m}\hat{\eta}_{2,i} + \frac{1}{m}u_i + \frac{1}{m}\hat{\zeta}_{1,i} + \beta_{5,i}(\eta_{1,i} - \hat{\eta}_{1,i}) \\
 \hat{\tilde{\zeta}}_{1,i} &= \hat{\tilde{\zeta}}_{2,i} + \beta_{4,i}(\eta_{1,i} - \hat{\eta}_{1,i}) \\
 \hat{\tilde{\zeta}}_{2,i} &= \hat{\tilde{\zeta}}_{3,i} + \beta_{3,i}(\eta_{1,i} - \hat{\eta}_{1,i}) \\
 \hat{\tilde{\zeta}}_{3,i} &= \hat{\tilde{\zeta}}_{4,i} + \beta_{2,i}(\eta_{1,i} - \hat{\eta}_{1,i}) \\
 \hat{\tilde{\zeta}}_{4,i} &= \hat{\tilde{\zeta}}_{5,i} + \beta_{1,i}(\eta_{1,i} - \hat{\eta}_{1,i}) \\
 \hat{\tilde{\zeta}}_{5,i} &= \beta_{0,i}(\eta_{1,i} - \hat{\eta}_{1,i})
 \end{aligned} \tag{25}$$

The dynamics of the estimation errors,  $e_{1,i} = \eta_{1,i} - \hat{\eta}_{1,i}$ ,  $e_{2,i} = \eta_{2,i} - \hat{\eta}_{2,i}$ ,  $e_{p_k,i} = \tilde{\zeta}_{k,i} - \hat{\tilde{\zeta}}_{k,i}$ ,  $k = 1, 2, \dots, 5$ ,  $i = x, y$ , are then given by

$$\begin{aligned}
 \dot{e}_{1,i} &= -\beta_{6,i}e_{1,i} + e_{2,i} \\
 \dot{e}_{2,i} &= -\beta_{5,i}e_{1,i} - \frac{k_i}{m}e_{1,i} - \frac{c_i}{m}e_{2,i} + \frac{1}{m}e_{p_1,i} \\
 \dot{e}_{p_1,i} &= -\beta_{4,i}e_{1,i} + e_{p_2,i} \\
 \dot{e}_{p_2,i} &= -\beta_{3,i}e_{1,i} + e_{p_3,i} \\
 \dot{e}_{p_3,i} &= -\beta_{2,i}e_{1,i} + e_{p_4,i} \\
 \dot{e}_{p_4,i} &= -\beta_{1,i}e_{1,i} + e_{p_5,i} \\
 \dot{e}_{p_5,i} &= -\beta_{0,i}e_{1,i}
 \end{aligned} \tag{26}$$

Thus, the characteristic polynomials of the dynamics of the observation errors (26) are

$$p_{o,i}(s) = s^7 + \left(\beta_{6,i} + \frac{c_i}{m}\right)s^6 + \left(\beta_{5,i} + \frac{k_i}{m} + \frac{c_i}{m}\beta_{6,i}\right)s^5 + \frac{1}{m}\beta_{4,i}s^4 + \frac{1}{m}\beta_{3,i}s^3 + \frac{1}{m}\beta_{2,i}s^2 + \frac{1}{m}\beta_{1,i}s + \frac{1}{m}\beta_{0,i} \quad (27)$$

which are completely independents of any coefficients  $p_{j,i}$  of the Taylor polynomial expansions of disturbance signals  $\zeta_i(t)$ .

The design parameter for the state observer (25) are selected so that the characteristic polynomials (27) be Hurwitz polynomials. Particularly, these polynomials are proposed of the form

$$p_{o,i}(s) = (s + p_{o,i}) \left(s^2 + 2\zeta_{o,i}\omega_{o,i}s + \omega_{o,i}^2\right)^3, \quad i = x, y. \quad (28)$$

with  $p_{o,i}, \zeta_{o,i}, \omega_{o,i} > 0$ .

Equating term by term the coefficients of both polynomials (28) and (27), one obtains that

$$\begin{aligned} \beta_{0,i} &= m\omega_{o,i}^6 p_{o,i} \\ \beta_{1,i} &= m \left(\omega_{o,i}^6 + 6\zeta_{o,i}p_{o,i}\omega_{o,i}^5\right) \\ \beta_{2,i} &= m \left(6\omega_{o,i}^5\zeta_{o,i} + 12p_{o,i}\omega_{o,i}^4\zeta_{o,i}^2 + 3p_{o,i}\omega_{o,i}^4\right) \\ \beta_{3,i} &= m \left(12\omega_{o,i}^4\zeta_{o,i}^2 + 3\omega_{o,i}^4 + 8p_{o,i}\omega_{o,i}^3\zeta_{o,i}^3 + 12p_{o,i}\omega_{o,i}^3\zeta_{o,i}\right) \\ \beta_{4,i} &= m \left(8\omega_{o,i}^3\zeta_{o,i}^3 + 12\omega_{o,i}^3\zeta_{o,i} + 12p_{o,i}\omega_{o,i}^2\zeta_{o,i}^2 + 3p_{o,i}\omega_{o,i}^2\right) \\ \beta_{5,i} &= 12\omega_{o,i}^2\zeta_{o,i}^2 + 3\omega_{o,i}^2 + 6p_{o,i}\omega_{o,i}\zeta_{o,i} - \frac{c_i}{m}\beta_{6,i} - \frac{k_i}{m} \\ \beta_{6,i} &= p_{o,i} + 6\omega_{o,i}\zeta_{o,i} - \frac{c_i}{m} \end{aligned}$$

## 5. Simulation results

In order to verify the dynamic behavior of the rotor speed controller, active unbalance control scheme and estimation of the unbalance forces, some numerical simulations were carried out using the numerical parameters shown in Table 1.

The performance of the rotor speed controller (18) was evaluated for the tracking of the smooth speed reference profile  $\omega^*(t)$  shown in Fig. 3, which allows to take the rotor from

$m = 3.85 \text{ kg}$	$c_y = 14 \text{ N s/m}$	$u = 222 \text{ }\mu\text{m}$
$k_x = 1.9276 \times 10^5 \text{ N/m}$	$d = 0.020 \text{ m}$	$\beta = \frac{\pi}{4} \text{ rad}$
$c_x = 12 \text{ N s/m}$	$r_{\text{disk}} = 0.076 \text{ m}$	$c_\varphi = 1.5 \times 10^{-3} \text{ Nm s/rad}$
$k_y = 2.0507 \times 10^5 \text{ N/m}$	$l = 0.7293 \text{ m}$	

**Table 1.** Rotor System Parameters.

an initial speed  $\bar{\omega}_1$  for  $t \leq T_1$  to the desired final operation speed  $\bar{\omega}_2$  for  $t \geq T_2$ . In general, the unbalance response has more interest when the rotor is running above its first critical speeds  $\omega_{cr1x} = \sqrt{k_x/m} = 223.76 \text{ rad/s} = 2136.8 \text{ rpm}$  and  $\omega_{cr1y} = \sqrt{k_y/m} = 230.79 \text{ rad/s} = 2203.9 \text{ rpm}$ .

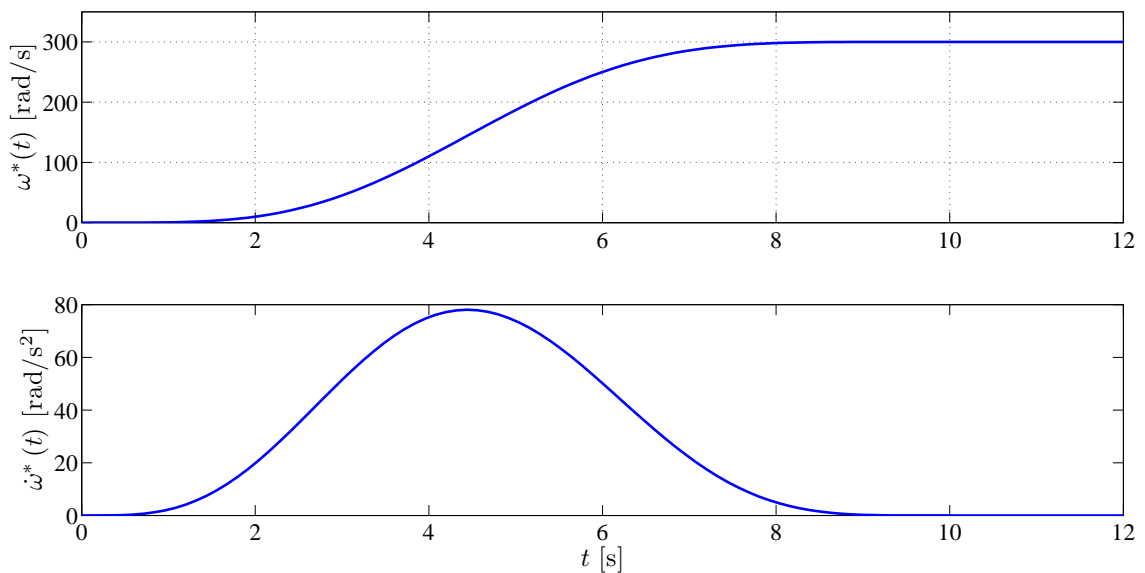
The speed profile specified for the rotor system is described by

$$\omega^*(t) = \begin{cases} \bar{\omega}_1 & \text{for } 0 \leq t < T_1 \\ \bar{\omega}_1 + (\bar{\omega}_2 - \bar{\omega}_1) \psi(t, T_1, T_2) & \text{for } T_1 \leq t \leq T_2 \\ \bar{\omega}_2 & \text{for } t > T_2 \end{cases} \quad (29)$$

where  $\bar{\omega}_1 = 0 \text{ rad/s}$ ,  $\bar{\omega}_2 = 300 \text{ rad/s} = 2864.8 \text{ rpm}$ ,  $T_1 = 0 \text{ s}$ ,  $T_2 = 10 \text{ s}$  and  $\psi(t, T_1, T_2)$  is a Bézier polynomial defined as

$$\psi(t) = \left( \frac{t - T_1}{T_2 - T_1} \right)^5 \left[ r_1 - r_2 \left( \frac{t - T_1}{T_2 - T_1} \right) + r_3 \left( \frac{t - T_1}{T_2 - T_1} \right)^2 - \dots - r_6 \left( \frac{t - T_1}{T_2 - T_1} \right)^5 \right]$$

with constants  $r_1 = 252$ ,  $r_2 = 1050$ ,  $r_3 = 1800$ ,  $r_4 = 1575$ ,  $r_5 = 700$  and  $r_6 = 126$ .

**Figure 3.** Smooth reference profiles for the rotor speed and acceleration,  $\omega^*(t)$  and  $\dot{\omega}^*(t)$ .

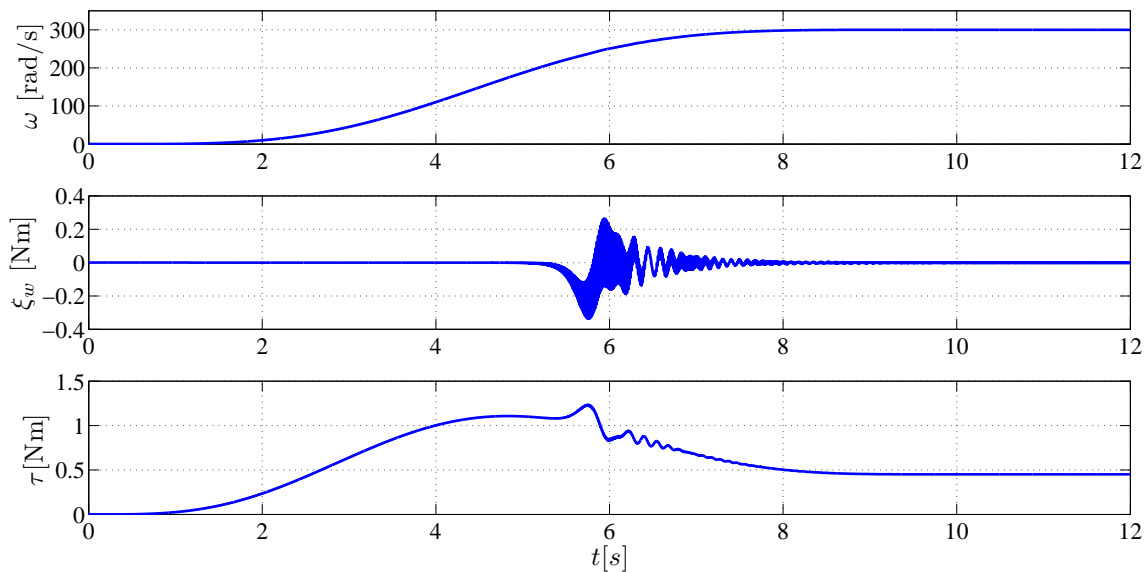


In Fig. 4 the robust and efficient performance of the PI speed controller (18) is shown. Here, the active unbalance control scheme (14) is not performed, i.e.,  $u_x = u_y \equiv 0$ . Therefore, some irregularities of the control torque action can be observed when the rotor passes through its first critical speeds. Fortunately, the presented speed controller results quite robust against the bounded torque perturbation input signal  $\xi_w$  induced by the rotor unbalance.

It is important to note that the control gains were selected to get a closed-loop rotor speed dynamics having a Hurwitz characteristic polynomial:

$$P_\omega(s) = s^2 + 2\zeta_r\omega_{nr}s + \omega_{nr}^2$$

with  $\omega_{nr} = 15$  rad/s and  $\zeta_r = 0.7071$ .



**Figure 4.** Rotor system response using local PI speed controller without active unbalance control.

Fig. 5 depicts the open-loop rotor unbalance response while the rotor is taken from the rest initial speed ( $\bar{\omega}_1 = 0$  rad/s) to the operating speed ( $\bar{\omega}_2 = 300$  rad/s) above its first critical speeds by using the PI rotor speed controller (18). The presence of high vibration amplitude levels (above 9 mm) at the resonant peaks can be observed. Note in Fig. 6 that the centrifugal forces induced by the rotor unbalance are quite significant. Thus, the active rotor balancing controllers (14) should be actively compensate those perturbation forces in real time.

On the other hand, Figs. 7-9 depict the closed-loop rotor-bearing system response by using simultaneously the disturbance observer-based active unbalance control scheme (14), PI rotor speed controller (18) and disturbance observer (25). One can see in Fig. 7 the robust performance of the PI speed controller (18), achieving an effective tracking of the smooth speed reference profile (29). Since the rotor unbalance-induced torque perturbation input signal  $\xi_w$  is canceled by the active balancing controllers (14), a smooth curve of the control torque is accomplished, eliminating the irregularities presented in the control torque response without active unbalance control (4).

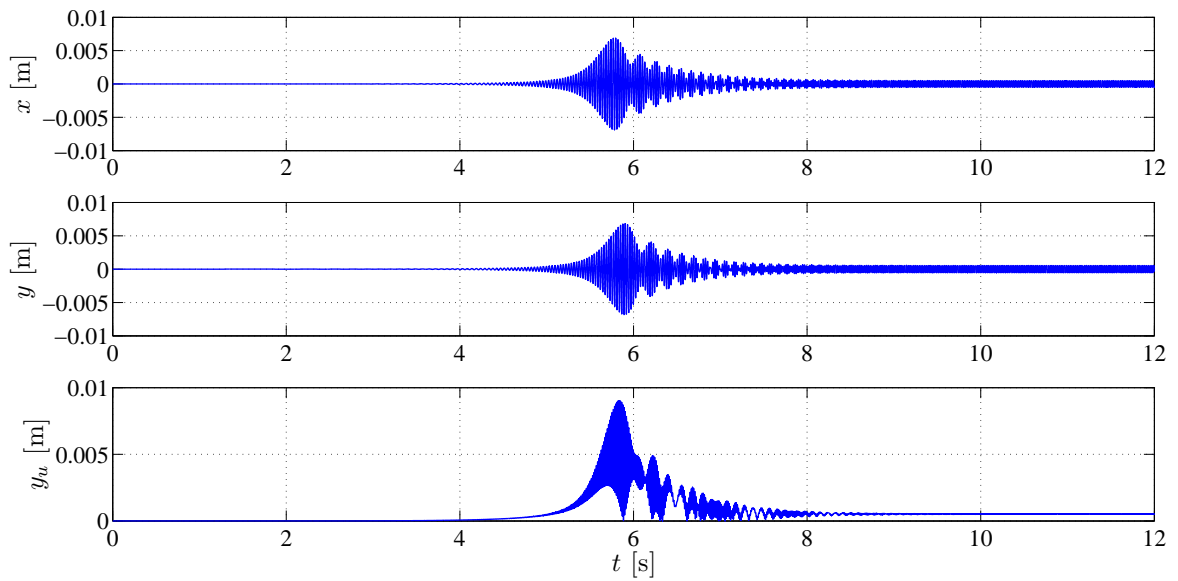


Figure 5. Open-loop rotor unbalance response with local PI rotor speed controller.

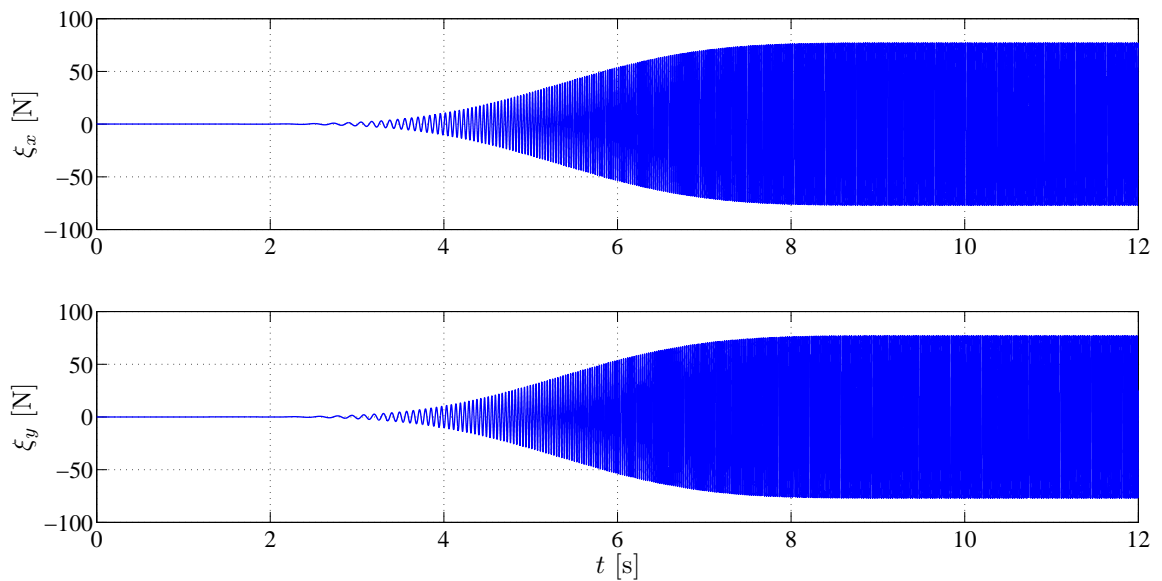


Figure 6. Rotor unbalance forces without active unbalance control.

The closed-loop rotor unbalance response is described in Fig. 8. The active unbalance suppression can be clearly noted. In this case, the gains of the active unbalance controllers were selected to get a closed-loop system dynamics having the Hurwitz characteristic polynomials:

$$P_x(s) = s^2 + 2\zeta_x\omega_{nx}s + \omega_{nx}^2$$

$$P_y(s) = s^2 + 2\zeta_y\omega_{ny}s + \omega_{ny}^2$$

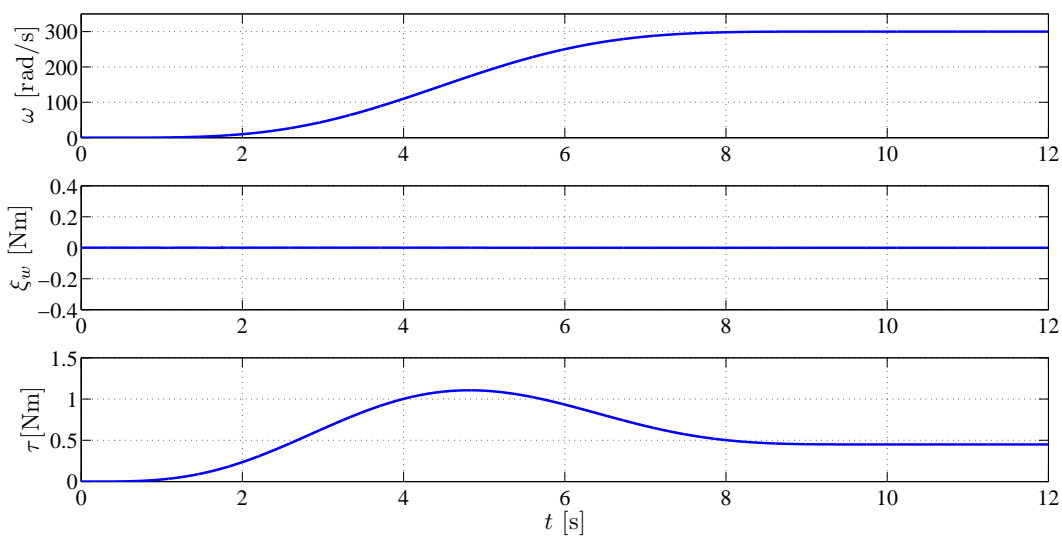
with  $\omega_{nx} = \omega_{ny} = 10$  rad/s and  $\zeta_x = \zeta_y = 0.7071$ .

Fig. (9) describes the active vibration control scheme response, which applies the active compensation of the estimated unbalance force signals  $\hat{\xi}_x$  and  $\hat{\xi}_y$  shown in Fig. (10).

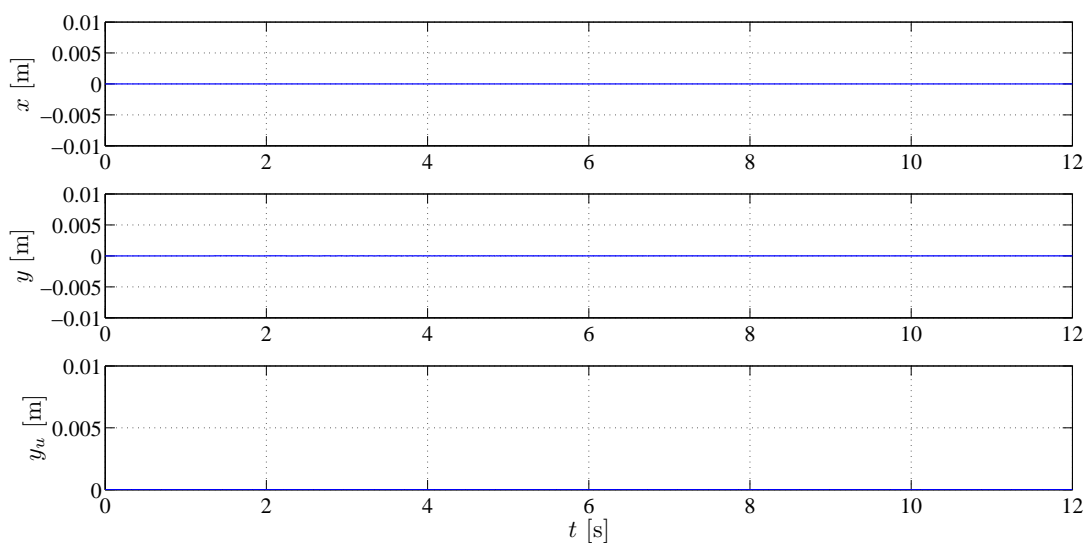
The characteristic polynomials, assigned to the observation error dynamics, must be faster than the rotordynamics and, therefore, are specified as

$$P_{o,i}(s) = (s + p_{o,i}) (s^2 + 2\zeta_{o,i}\omega_{o,i}s + \omega_{o,i}^2)^3, \quad i = x, y.$$

with desired parameters  $p_{o,i} = \omega_{o,i} = 1200$  rad/s and  $\zeta_{o,i} = 100$ .



**Figure 7.** Rotor system response using local PI speed controller with active unbalance control.



**Figure 8.** Closed-loop rotor unbalance response with local PI rotor speed controller.

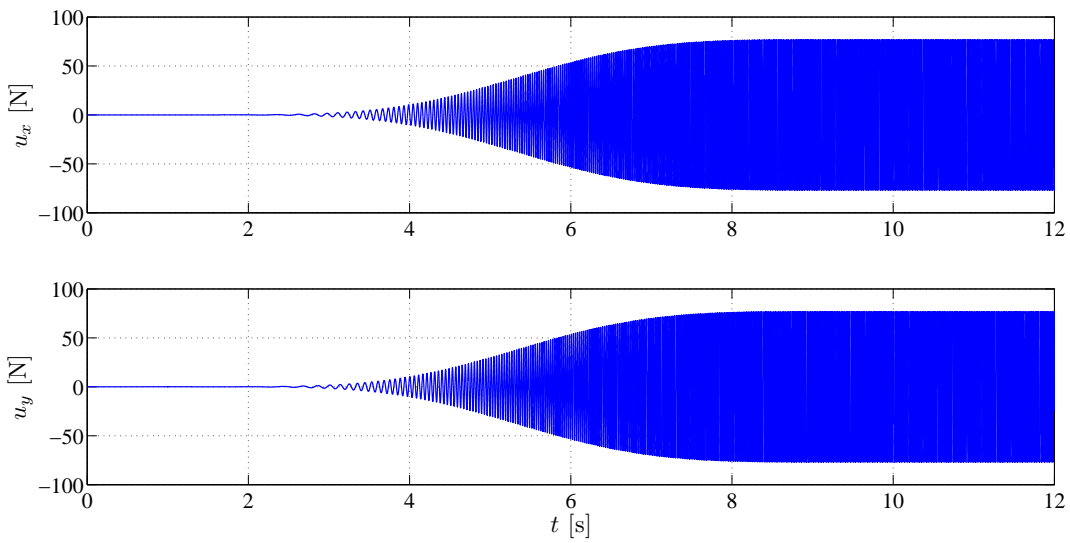


Figure 9. Response of active unbalance Controllers.

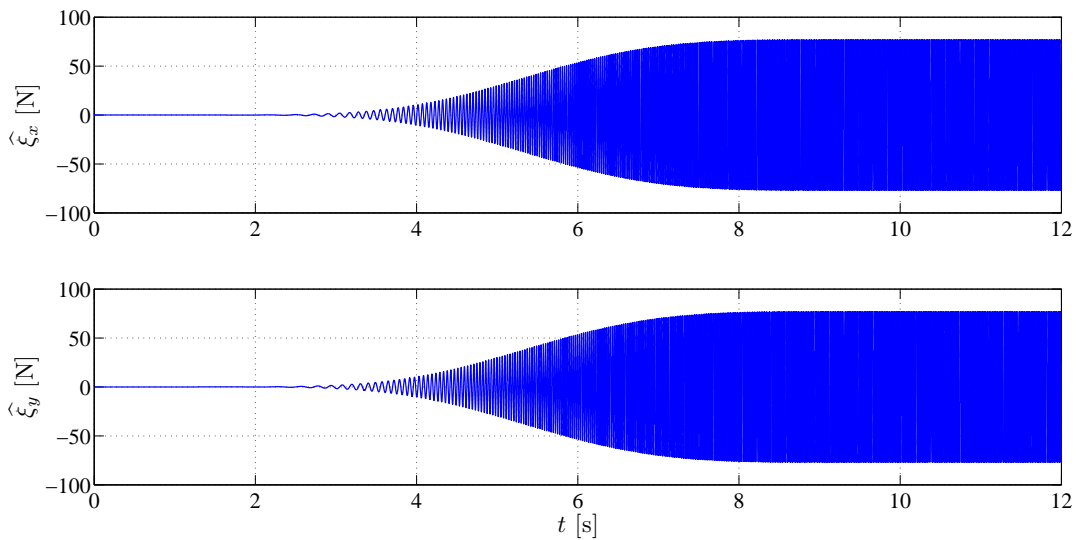


Figure 10. Estimates of the closed-loop rotor unbalance force signals using disturbance observer (25).

### 6. Conclusions

In this chapter, we have proposed a PD-like active vibration control scheme for robust and efficient suppression of unbalance-induced synchronous vibrations in variable-speed Jeffcott-like non-isotropic rotor-bearing systems of three degrees of freedom using only measurements of the radial displacement close to the disk. In this study, we have considered the application of an active suspension device, which is based on two linear electromechanical actuators and helicoidal compression springs, to provide the control forces required for on-line balance of the rotor system. The presented control approach is mainly based on the compensation of bounded perturbation force signals induced by the rotor unbalance, and the specification of the desired closed-loop rotor-bearing system dynamics (viscous damping ratios and natural frequencies). A robust and fast estimation scheme of the

perturbation force signals and velocities of the rotor center coordinates based on Luenberger linear state observers has also been proposed. In the state observer design process, the perturbation force signals were locally approximated by a family of Taylor time-polynomials of fourth degree. Therefore, each perturbation signal was locally described by a state space-based linear mathematical model of fifth order. Then, an extended linear mathematical model was obtained to locally describe the dynamics of the perturbed rotor system to be used in the design of the disturbance and state observer. In addition, a PI rotor speed controller was proposed to perform robust tracking tasks of smooth rotor speed reference profiles described by Bézier interpolation polynomials. Simulations results show the robust and efficient performance of the active vibration control scheme and rotor speed controller proposed in this chapter, as well as the fast and effective estimation of the perturbation force signals, when the rotor system is taken from a rest initial speed to an operation speed above its first critical velocities. The proposed methodology can be applied for more complex and realistic rotor-bearing systems (e.g., more disks, turbines, shaft geometries), finite element models, monitoring and fault diagnosis quite common in industrial rotating machinery.

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## References

- [1] Arias-Montiel, M. & Silva-Navarro, G. (2010). Active Unbalance Control in a Two Disks Rotor System Using Lateral Force Actuators, *Proceeding of 7th International Conference on Electrical Engineering, Computing Science and Automatic Control*, pp. 440-445, Tuxtla Gutierrez, México.
- [2] Arias-Montiel, M. & Silva-Navarro, G. (2010b). Finite Element Modelling and Unbalance Compensation for an Asymmetrical Rotor-Bearing System with Two Disks, *In: New Trends in Electrical Engineering, Automatic Control, Computing and Communication Sciences*, Edited by C.A. Coello-Coello, Alex Pozniak, José A. Moreno-Cadenas and Vadim Azhmyakov, pp. 127-141, Logos Verlag Berlin GmbH, Germany.
- [3] Cabrera-Amado, M. & Silva-Navarro, G. (2010). Semiactive Control for the Unbalance Compensation in a Rotor-Bearing System, *In: New Trends in Electrical Engineering, Automatic Control, Computing and Communication Sciences*, Edited by C.A. Coello-Coello, Alex Pozniak, José A. Moreno-Cadenas and Vadim Azhmyakov, pp. 143-158, Logos Verlag Berlin GmbH, Germany.

- [4] De Queiroz, M. S. (2009). An Active Identification Method of Rotor Unbalance Parameters, *Journal of Vibration and Control*, Vol. 15, pp. 1365-1374.
- [5] De Silva, C. W. (2007). *Vibration Damping, Control and Design*, CRC Press, USA.
- [6] Ellis, G. (2002). *Observers in Control Systems*, Academic Press, USA.
- [7] Fliess, M., Marquez, R., Delaleau, E. & Sira-Ramirez, H. (2002). Correcteurs Proportionnels-Integraux Généralisés, *ESAIM Control, Optimisation and Calculus of Variations*, Vol. 7, No. 2, pp. 23-41.
- [8] Fliess, M. & Sira-Ramirez, H. (2003). An algebraic framework for linear identification, *ESAIM: Control, Optimization and Calculus of Variations*, Vol. 9, pp. 151-168.
- [9] Forte, P., Paterno, M. & Rustighi, E. (2004). A Magnetorheological Fluid Damper for Rotor Applications, *International Journal of Rotating Machinery*, Vol. 10, No. 3, pp. 175-182.
- [10] Friswell, M. I., Penny, J. E. T., Garvey, S. D. & Lees, A. W. (2010). *Dynamics of Rotating Machines*, Cambridge University Press, USA.
- [11] Genta, G. (2009). *Vibration Dynamics and Control*, Springer, Berlin, Germany.
- [12] Guldbakke, J. M. & Hesselbach, J. (2006). Development of Bearings and a Damper Based on Magnetically Controllable Fluids, *Journal of Physics: Condensed Matter*, Vol. 18, pp. S2959-S2972.
- [13] Gosiewski, Z. & Mystkowsky, A. (2008). Robust Control of Active Magnetic Suspension: Analytical and Experimental Results, *Mechanical Systems and Signal Processing*, Vol. 22, No. 6, pp. 1297-1303.
- [14] Inman, D. J. (2006). *Vibration with Control*, John Wiley & Sons, England.
- [15] Jung-Ho, P., Young-Bog, H., So-Nam, Y. & Hu-Seung, L. (2010). Development of a Hybrid Bearing Using Permanent Magnets and Piezoelectric Actuators, *Journal of Korean Physical Society*, Vol. 57, No. 4, pp. 907-912.
- [16] Knospe, C. R., Hope, R. W., Fedigan, S. J. & Williams, R. D. (1995). Experiments in the Control of Unbalance Response Using Magnetic Bearings, *Mechatronics*, Vol. 5, No. 4, pp. 385-400.
- [17] Lihua, Y., Yanhua, S. & Lie, Y. (2012). Active Control of Unbalance Response of Rotor Systems Supported by Tilting-Pad Gas Bearings, *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, Vol. 226, No. 2, pp. 87-98.
- [18] Mahfoud J., Der Hagopian J., Levecque N. & Steffen Jr. V. (2009), Experimental model to control and monitor rotating machines, *Mechanism and Machine Theory*, Vol. 44, pp. 761-771 .

- [19] Maslen, E. H., Vázquez, J. A. & Sortore, C. K. (2002). Reconciliation of Rotordynamic Models with Experimental Data, *Journal of Engineering for Gas Turbines and Power*, Vol. 124, pp. 351-356.
- [20] Pi-Cheng, T., Mong-Tao, T, Kuan-Yu, C., Yi-Hua, F. & Fu-Chu, C. (2011). Design of Model-Based Unbalance Compensator with Fuzzy Gain Tuning Mechanism for an Active Magnetic Bearing System, *Expert Systems with Applications*, Vol. 38, No. 10, pp. 12861-12868.
- [21] Schweitzer, G. & Maslen, E. H. (Eds) (2010). *Magnetic Bearings - Theory, Design and Application to Rotating Machinery*, Springer, Germany.
- [22] Simoes, R. C., Steffen Jr., V., Der Hagopian, J. & Mahfoud, J. (2007). Modal Active Vibration Control of a Rotor Using Piezoelectric Stack Actuators, *Journal of Vibration and Control*, Vol. 13, No. 1, pp. 45-64.
- [23] Sira-Ramirez, H., Beltran-Carbajal, F. & Blanco-Ortega, A. (2008). A Generalized Proportional Integral Output Feedback Controller for the Robust Perturbation Rejection in a Mechanical System, *e-STA*, Vol. 5, No. 4, pp. 24-32.
- [24] Sira-Ramirez, H., Feliu-Batlle, V., Beltran-Carbajal, F. & Blanco-Ortega, A. (2008). Sigma-Delta modulation sliding mode observers for linear systems subject to locally unstable inputs, *16th Mediterranean Conference on Control and Automation*, pp. 344-349, Ajaccio, France, June 25-27.
- [25] Sira-Ramirez, H., Silva-Navarro, G. & Beltran-Carbajal, F. (2007). On the GPI Balancing Control of an Uncertain Jeffcott Rotor Model, *Proceeding of 4th International Conference on Electrical and Electronics Engineering (ICEEE 2007)*, pp. 306-309, Mexico city, Mexico.
- [26] Sivrioglu, S. & Nonami, K. (1998). Sliding Mode Control with Time Varying Hyperplane for AMB Systems, *IEEE/ASME Transactions on Mechatronics*, Vol. 3, No. 1, pp. 51-59.
- [27] Sudhakar, G. N. D. S. & Sekhar, A. S. (2011). Identification of Unbalance in a Rotor Bearing System, *Journal of Sound and Vibration*, Vol. 330, pp. 2299-2313.
- [28] Tammi, K. (2009). Active Control of Rotor Vibrations by Two Feedforward Control Algorithms, *Journal of Dynamic Systems, Measurement, and Control*, Vol. 131, pp. 1-10.
- [29] Vance, J. M., *Rotordynamics of Turbomachinery* (1988). John Wiley & Sons, USA.
- [30] Yang, T. & Lin, C. (2002). Estimation of Distributed Unbalance of Rotors, *Journal of Engineering for Gas Turbines and Power*, Vol. 124, pp. 976-983.
- [31] Zhou, S. & Shi, J. (2001). Active Balancing and Vibration Control of Rotating Machinery: A Survey, *The Shock and Vibration Digest*, Vol. 33, No. 4, pp. 361-371.



