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# The Behavior in Stationary Regime of an Induction Motor Powered by Static Frequency Converters

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Additional information is available at the end of the chapter

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## 1. Introduction

Generally, the electric induction motors are designed for supply conditions from energy sources in which the supply voltage is a sinusoidal wave. The parameters and the functional sizes of the electric motors are guaranteed by designers only for it. If the electric motor is powered through an inverter, due to the presence in the input voltage waveform of superior time harmonics, both its parameters and its functional characteristic sizes will be more or less different from those in the case of the sinusoidal supply. The presence of these harmonics will result in the appearance of a deforming regime in the machine, generally with adverse effects in its operation. Under loading and speed conditions similar to those in the case of the sinusoidal supply, it is registered an amplification of the losses of the machine, of the electric power absorbed and thus a reduction in efficiency. There is also a greater heating of the machine and an electromagnetic torque that at a given load is not invariable, but pulsating, in rapport with the average value corresponding to the load. The occurrence of the deforming regime in the machine is inevitable, because any inverter produces voltages or printed currents containing, in addition to the fundamental harmonic, superior time harmonics of odd order. The deforming regime in the electric machine is unfortunately reflected in the supply power grid that powers the inverter. Generalizing, the output voltage harmonics are grouped into families centered on frequencies:

$$f_j = Jm_f f_c = Jm_f f_1 \quad (J = 1, 2, 3, \dots), \quad (1)$$

and the various harmonic frequencies in a family are:

$$f_{(v)} = f_j \pm k f_c = (Jm_f \pm k) f_c = (Jm_f \pm k) f_1, \quad (2)$$

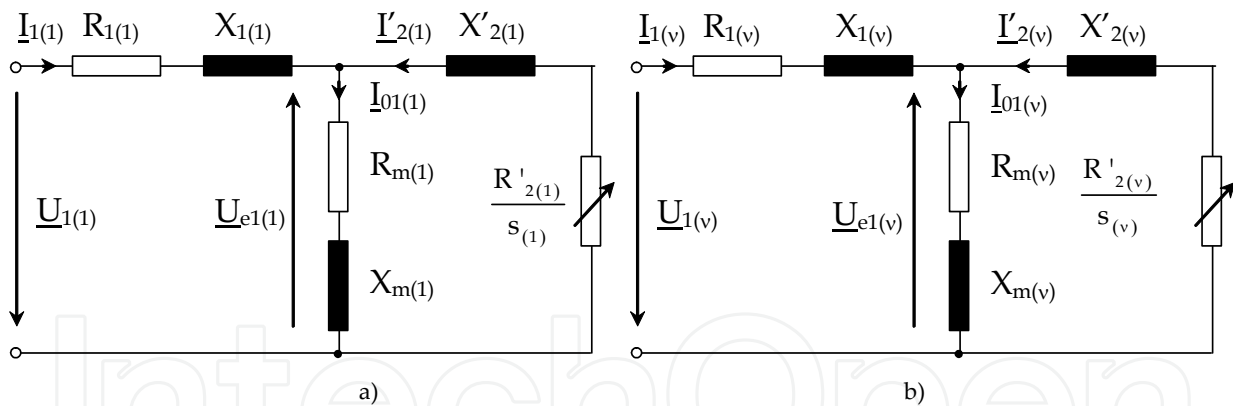
with

$$v = Jm_f \pm k \quad (3)$$

In the above relations,  $m_f$  represents the frequency modulation factor,  $f_1$  is the fundamental's frequency and  $f_c$  is the frequency of the control modulating signal. Whereas the harmonic spectrum contains only  $v$  order odd harmonics, in order that  $(Jm_f \pm k)$  is odd, an odd  $J$  determines an even  $k$  and vice versa. The present chapter aims to analyze the behavior of the induction motor when it is supplied through an inverter. The purpose of this study is to develop the theory of three-phase induction machine with a squirrel cage, under the conditions of the non-sinusoidal supply regime to serve as a starting point in improving the methodology of its constructive-technological design as advantageous economically as possible.

## 2. The mathematical model of the three-phase induction motor in the case of non-sinusoidal supply

In the literature there are known various mathematical models associated to induction machines fed by static frequency and voltage converters. The majority of these models are based on the association between an induction machine and an equivalent scheme corresponding to the fundamental and a lot of schemes corresponding to the various  $v$  frequencies, corresponding to the Fourier series decomposition of the motor input voltage - see Fig. 1 (Murphy & Turnbull, 1988). In this model the skin effect is not considered.



**Figure 1.** Equivalent scheme of the machine supplied through frequency converter: a) for the case of fundamental; b) for the  $v$  order harmonics (positive or negative sequence).

For the equivalent scheme in Fig.1.a, corresponding to the fundamental, the electrical parameters are defined as:

$$\begin{aligned} R_{1(1)} &= R_1 = R_{1n}; & X_{1(1)} &= X_1 = aX_{1n}; \\ R'_{2(1)} &= R'_2 = R'_{2n}; & X'_{2(1)} &= X'_2 = aX'_{2n}; \\ R_{m(1)} &= R_m = a^2R_{mn}; & X_{m(1)} &= X_m = aX_{mn}; \end{aligned} \quad (4)$$

$$\frac{R'_{2(1)}}{s_{(1)}} = \frac{R'_2}{s} = \frac{a}{c} R'_{2n}$$

In relations (4),  $R_{1n}$ ,  $X_{1n}$ ,  $R'_{2n}$ ,  $X'_{2n}$ ,  $R_{mn}$ ,  $X_{mn}$  represents the values of the parameters  $R_1$ ,  $X_1$ ,  $R'_2$ ,  $X'_2$ ,  $R_m$  and  $X_m$  in nominal operating conditions (fed from a sinusoidal power supply, rated voltage frequency and load) and

$$a = \frac{f_1}{f_{1n}} = \frac{\omega_1}{\omega_{1n}} = \frac{n_1}{n_{1n}}; \quad c = \frac{n_1 - n}{n_{1n}} = \frac{n_1 - n}{n_1} \cdot \frac{n_1}{n_{1n}} = s \cdot a \quad (5)$$

In the relations (5),  $f_1$  and  $f_{1n}$  are random frequencies of the rotating magnetic field, and the nominal frequency of the rotating magnetic field respectively. For  $v$  order harmonics, the scheme from Fig. 1.b is applicable. The slip  $s_{(v)}$ , corresponding to the  $v$  order harmonic is:

$$s_{(v)} = \frac{vn_1 \mp n}{vn_1} = 1 \mp \frac{n}{vn_1} = 1 \mp \frac{1}{v} \pm \frac{c}{a} \frac{1}{v}, \quad (6)$$

where sign (-) (from the first equality) corresponds to the wave that rotates within the sense of the main wave and the sign (+) in the opposite one. For the case studied in this chapter - that of small and medium power machines - the resistances  $R_{1(v)}$  and reactances  $X_{1(v)}$  values are not practically affected by the skin effect. In this case we can write:

$$R_{1(v)} = R_{1(1)} = R_1 = R_{1n}, \quad (7)$$

$$X_{1(v)} = \omega_{1(v)} \cdot L_{1\sigma(v)} = v\omega_1 L_{1\sigma(v)}, \quad (8)$$

where  $L_{1\sigma(v)}$  is the stator dispersion inductance corresponding to the  $v$  order harmonic. If it is agreed that the machine cores are linear media (the machine is unsaturated), it results that the inductance can be considered constant, independently of the load (current) and flux, one can say that:

$$L_{1\sigma(v)} = L_{1\sigma(1)} = L_{1\sigma} \quad (9)$$

By replacing the inductance  $L_{1\sigma(v)}$  expression from relation (9) in relation (8), we obtain:

$$X_{1(v)} = v\omega_1 L_{1\sigma} = vX_1 = vaX_{1n} \quad (10)$$

For the rotor resistance and rotor leakage reactance, corresponding to the  $v$  order harmonic, both reduced to the stator the following expressions were established:

$$R'_{2(v)} = R'_{2(1)} = R'_2 = R'_{2n}, \quad (11)$$

$$X'_{2(v)} = v \cdot X'_2 = v \cdot a \cdot X'_{2n} \quad (12)$$

The magnetization resistance corresponding to the  $\nu$  order harmonic,  $R_{m\nu}$ , is given by the relation:

$$R_{m(\nu)} = k_{k''} \cdot \nu^2 \cdot a^2 \cdot R_{mn} \quad (13)$$

$k_{k''}$  is a coefficient dependent on iron losses and on the magnetic field variation. The magnetization reluctance corresponding to the magnetic field produced by the  $\nu$  order harmonic is:

$$X_{m(\nu)} = k_{k'} \cdot \nu \cdot a \cdot X_{mn} \quad (14)$$

Further the author intends to establish a single mathematical model associated to induction motors, supplied by static voltage and frequency converter, which consists of a single equivalent scheme and which describes the machine operation, according to the presence in the input power voltage of higher time harmonics. For this, the following simplifying assumptions are taken into account:

- the permeability of the magnetic core is considered infinitely large comparing to the air permeability and the magnetic field lines are straight perpendicular to the slot axis;
- both the ferromagnetic core and rotor cage (bar + short circuit rings) are homogeneous and isotropic media;
- the marginal effects are neglected, the slot is considered very long on the axial direction. The electromagnetic fields are considered, in this case plane-parallel;
- the skin effect is taken into account in the calculations only in bars that are in the transverse magnetic field of the slot. For the bar portions outside the slot and in short circuit rings, current density is considered as constant throughout the cross section of the bar;
- the passing from the constant density zone into the variable density zone occurs abruptly;
- in the real electric machines the skin effect is often influenced by the degree of saturation but the simultaneous coverage of both phenomena in a mathematical relationships, easily to be applied in practice is very difficult, even precarious. Therefore, the simplifying assumption of neglecting the effects of saturation is allowed as valid in establishing the relationships for equivalent parameters;
- the local variation of the magnetic induction and of current density is considered sinusoidal in time, both for the fundamental and for each  $\nu$  harmonic;
- one should take into account only the fundamental space harmonic of the EMF.

Under these conditions of non-sinusoidal supply, the asynchronous motor may be associated to an equivalent scheme, corresponding to all harmonics. The scheme operates in the fundamental frequency  $f_{1(1)}$  and it is represented in Fig. 2. According to this scheme, it can be formally considered that the motors, in the case of supplying through the power frequency converter (the corresponding parameters and the dimensions of this situation are marked with index "CSF") behave as if they were fed in sinusoidal regime at fundamental's frequency,  $f_{1(1)}$  with the following voltages system:

$$u_A = \sqrt{2} \cdot U_{1(\text{CSF})} \cdot \sin \omega_1 t; u_B = \sqrt{2} \cdot U_{1(\text{CSF})} \cdot \sin \left( \omega_1 t - \frac{2\pi}{3} \right); u_C = \sqrt{2} \cdot U_{1(\text{CSF})} \cdot \sin \left( \omega_1 t + \frac{2\pi}{3} \right), \quad (15)$$

where,

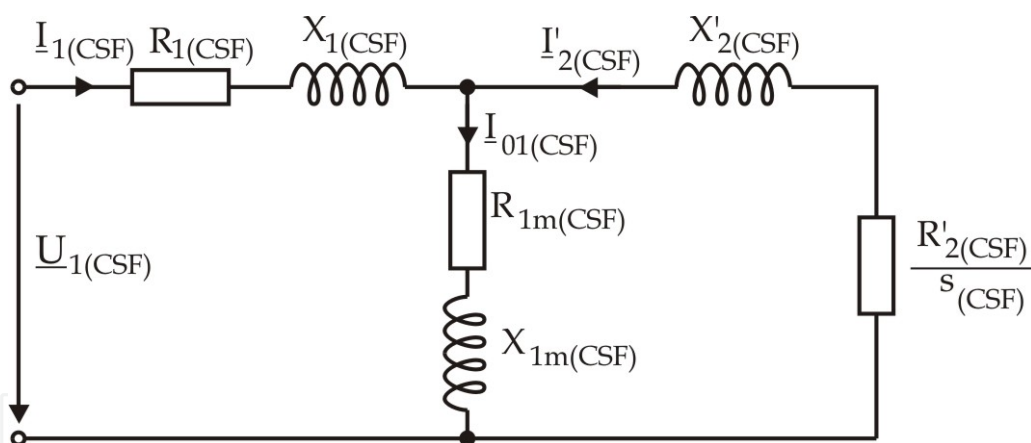
$$U_{1(\text{CSF})} = \sqrt{U_{1(1)}^2 + \sum_{v \neq 1} U_{1(v)}^2} \quad (16)$$

$U_{1(v)}$  is the phase voltage supply corresponding to the  $v$  order harmonic. Corresponding to the system supply voltages, the current system which go through the stator phases is as follows:

$$\begin{cases} i_A = \sqrt{2} \cdot I_{1(\text{CSF})} \cdot \sin \left( \omega_1 t - \varphi_{1(\text{CSF})} \right) \\ i_B = \sqrt{2} \cdot I_{1(\text{CSF})} \cdot \sin \left( \omega_1 t - \varphi_{1(\text{CSF})} - \frac{2\pi}{3} \right), \\ i_C = \sqrt{2} \cdot I_{1(\text{CSF})} \cdot \sin \left( \omega_1 t - \varphi_{1(\text{CSF})} - \frac{4\pi}{3} \right) \end{cases}, \quad (17)$$

where  $I_{1(\text{CSF})}$  is given by:

$$I_{1(\text{CSF})} = \sqrt{I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2} \quad (18)$$



**Figure 2.** The equivalent scheme of the asynchronous motor powered by a static frequency converter.

Power factor in the deforming regime is defined as the ratio between the active power and the apparent power, as follows:

$$\Delta_{(\text{CSF})} = \frac{P_{1(\text{CSF})}}{S_{1(\text{CSF})}} = \frac{P_{1(\text{CSF})}}{U_{1(\text{CSF})} I_{1(\text{CSF})}} \quad (19)$$

If we consider the non-sinusoidal regime, the active power absorbed by the machine  $P_{1(\text{CSF})}$  is defined, as in the sinusoidal regime, as the average in a period of the instantaneous power. The following expression is obtained:

$$P_{1(\text{CSF})} = \frac{1}{T} \int_0^T p \cdot dt = \sum_{v=1} U_{1(v)} I_{1(v)} \cos \varphi_{1(v)} = U_1 I_1 \cos \varphi_1 + \sum_{v \neq 1} U_{1(v)} I_{1(v)} \cos \varphi_{1(v)} \quad (20)$$

Therefore, the active power absorbed by the motor when it is supplied through a power static converter is equal to the sum of the active powers, corresponding to each harmonic (the principle of superposition effects is found). In relation (20),  $\cos \varphi_{1(v)}$  is the power factor corresponding to the  $v$  order harmonic having the expression:

$$\cos \varphi_{1(v)} = \frac{R_{1(v)} + \frac{R'_{2(v)}}{s_{(v)}}}{\sqrt{\left(R_{1(v)} + \frac{R'_{2(v)}}{s_{(v)}}\right)^2 + \left(X_{1(v)} + X'_{2(v)}\right)^2}} \quad (21)$$

The apparent power can be defined in the non-sinusoidal regime also as the product of the rated values of the applied voltage and current:

$$S_{1(\text{CSF})} = U_{1(\text{CSF})} \cdot I_{1(\text{CSF})}, \quad (22)$$

Taken into account the relations (20), (21) and (22), the relation (19) becomes:

$$\Delta_{(\text{CSF})} = \frac{U_1 I_1 \cos \varphi_1 + \sum_{v \neq 1} U_{1(v)} I_{1(v)} \cos \varphi_{1(v)}}{\sqrt{U_{1(1)}^2 + \sum_{v \neq 1} U_{1(v)}^2} \cdot \sqrt{I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2}} \quad (23)$$

Because  $\Delta_{(\text{CSF})} \leq 1$ , formally (the phase angle has meaning only in harmonic values) an angle  $\varphi_{1(\text{CSF})}$  can be associated to the power factor  $\Delta_{(\text{CSF})}$ , as:  $\cos \varphi_{1(\text{CSF})} = \Delta_{(\text{CSF})}$ . With this, the relation (23) can be written:

$$\cos \varphi_{1(\text{CSF})} = \frac{\cos \varphi_1 + \sum_{v \neq 1} \frac{U_{1(v)}}{U_{1(1)}} \frac{I_{1(v)}}{I_1} \cos \varphi_{1(v)}}{\sqrt{1 + \sum_{v \neq 1} \left(\frac{U_{1(v)}}{U_{1(1)}}\right)^2} \cdot \sqrt{1 + \sum_{v \neq 1} \left(\frac{I_{1(v)}}{I_{1(1)}}\right)^2}} \quad (24)$$

If one takes into account the relation (Murphy&Turnbull, 1988):

$$\frac{I_{1(v)}}{I_{1(1)}} = \frac{1}{v} \cdot \frac{1}{f_{1r} \cdot x_{sc}^*} \cdot \frac{U_{1(v)}}{U_{1(1)}}, \quad (25)$$

where  $x_{sc}^*$  is the reported short-circuit impedance, measured at the frequency  $f_1 = f_{1n}$ , relation (24) becomes:

$$\cos \varphi_{1(\text{CSF})} = \frac{\cos \varphi_1 + \sum_{v \neq 1} \frac{1}{v} \cdot \frac{1}{f_{1r} \cdot x_{sc}^*} \cdot \left( \frac{U_{1(v)}}{U_{1(1)}} \right)^2 \cos \varphi_{1(v)}}{\sqrt{\left[ 1 + \sum_{v \neq 1} \left( \frac{U_{1(v)}}{U_{1(1)}} \right)^2 \right] \cdot \left[ 1 + \sum_{v \neq 1} \left( \frac{1}{v} \cdot \frac{1}{f_{1r} \cdot x_{sc}^*} \cdot \frac{U_{1(v)}}{U_{1(1)}} \right)^2 \right]}} \quad (26)$$

### 3. The determination of the equivalent parameters of the stator winding

The equivalent parameters of the scheme have been calculated at the fundamental's frequency, under the presence of all harmonics in the supply voltage. Under these conditions, we note by  $p_{\text{Cu1(CSF)}}$  the losses that occur in the stator winding when the motor is supplied through a power frequency converter. These losses are in fact covered by some active power absorbed by the machine from the network, through the converter,  $P_{1(\text{CSF})}$ . According to the principle of the superposition effects, it can be considered:

$$P_{\text{Cu1(CSF)}} = P_{\text{Cu1(1)}} + \sum_{v \neq 1} P_{\text{Cu1(v)}} = 3R_{1(1)} I_{1(1)}^2 + 3 \sum_{v \neq 1} R_{1(v)} I_{1(v)}^2 \quad (27)$$

Further, the stator winding resistance corresponding to the fundamental,  $R_{1(1)}$  and stator winding resistances corresponding to the all higher time harmonics  $R_{1(v)}$ , are replaced by a single equivalent resistance  $R_{1(\text{CSF})}$ , corresponding to all harmonics, including the fundamental. The equalization is achieved under the condition that in this resistance the same loss  $p_{\text{Cu1(CSF)}}$  occurs, given by relation (27), as if considering the "v" resistances  $R_{1(v)}$ , each of them crossed by the current  $I_{1(v)}$ . This equivalent resistance,  $R_{1(\text{CSF})}$ , determined at the fundamental's frequency, is traversed by the current  $I_{1(\text{CSF})}$ , with the expression given by (18). Therefore:

$$P_{\text{Cu1(CSF)}} = 3R_{1(\text{CSF})} \cdot I_{1(\text{CSF})}^2 = 3R_{1(\text{CSF})} \left( I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) \quad (28)$$

Making the relations (27) and (28) equal, it results:

$$3R_{1(\text{CSF})} \left( I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) = 3R_{1(1)} \left( I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) = 3R_{1(v)} \left( I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right), \quad (29)$$

from which:

$$R_{1(\text{CSF})} = R_{1(1)} = R_1. \quad (30)$$

Applying the principle of the superposition effects to the reactive power absorbed by the stator winding  $Q_{\text{Cu1(CSF)}}$ , the following expression is obtained:

$$Q_{\text{Cu1(CSF)}} = Q_{\text{Cu1(1)}} + \sum_{v \neq 1} Q_{\text{Cu1(v)}} = 3 \cdot X_{1(1)} I_{1(1)}^2 + 3 \sum_{v \neq 1} X_{1(v)} I_{1(v)}^2 \quad (31)$$



As in the previous case, the stator winding reactance corresponding to the fundamental,  $X_{1(1)}$  (determined at the fundamental's frequency  $f_{1(1)}$ ) and the stator winding reactances, corresponding to all higher time harmonics  $X_{1(v)}$  (determined at frequencies  $f_{1(v)}=v \cdot f_1$  where  $J_{m \pm k}$ ) are replaced by an equivalent reactance,  $X_{1(\text{CSF})}$ , determined at fundamental's frequency. This equivalent reactance, traversed by the current  $I_{1(\text{CSF})}$ , conveys the same reactive power,  $Q_{\text{Cu1}(\text{CSF})}$  as in the case of considering "v" reactances  $X_{1(v)}$ , (each of them determined at  $f_{1(v)}$  frequency and traversed by the current  $I_{1(v)}$ ). Following the equalization, the following expression can be written:

$$Q_{\text{Cu1}(\text{CSF})} = 3X_{1(\text{CSF})}I_{1(\text{CSF})}^2 = 3X_{1(\text{CSF})} \left( I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) \quad (32)$$

Making the relations (31) and (32) equal, it results:

$$X_{1(\text{CSF})} \left( I_{1(1)}^2 + \sum_{v \neq 1} I_{1(v)}^2 \right) = X_{1(1)}I_{1(1)}^2 + \sum_{v \neq 1} vX_{1(1)}I_{1(v)}^2 = X_1 \left( I_{1(1)}^2 + \sum_{v \neq 1} vI_{1(v)}^2 \right) \quad (33)$$

One can notice the following:

$$k_{X1} = \frac{X_{1(\text{CSF})}}{X_1}$$

the factor that highlights the changes that the reactants of the stator phase value suffer in the case of a machine supplied through a power frequency converter, compared to sinusoidal supply, both calculated at the fundamental's frequency. From relations (25) and (33) it follows:

$$k_{X1} = \frac{X_{1(\text{CSF})}}{X_1} = \frac{1 + \sum_{v \neq 1} v \left( \frac{1}{f_{1r} X_{sc}^*} \right)^2 \cdot \frac{1}{v^2} \left( \frac{U_{1(v)}}{U_{1(1)}} \right)^2}{1 + \sum_{v \neq 1} \left( \frac{1}{f_{1r} X_{sc}^*} \right)^2 \cdot \frac{1}{v^2} \left( \frac{U_{1(v)}}{U_{1(1)}} \right)^2} = \frac{1 + \sum_{v \neq 1} \frac{1}{v} \left( \frac{1}{f_{1r} X_{sc}^*} \right)^2 \left( \frac{U_{1(v)}}{U_{1(1)}} \right)^2}{1 + \sum_{v \neq 1} \frac{1}{v^2} \left( \frac{1}{f_{1r} X_{sc}^*} \right)^2 \left( \frac{U_{1(v)}}{U_{1(1)}} \right)^2} \quad (34)$$

where:

$$X_{sc}^* = \frac{X_{sc}}{Z_{(1)}}$$

- is the short circuit impedance reported, corresponding to the frequency  $f_1=f_{1n}$  and  $f_{1r}$  is the reported frequency. One can notice that:  $k_{X1} > 1$ . With the equivalent resistance given by (30) and the equivalent reactance resulting from the relationship (34) we can now write the relation for the equivalent impedance of the stator winding,  $\underline{Z}_{1(\text{CSF})}$  covering all frequency harmonics and including the fundamental:

$$\underline{Z}_{1(\text{CSF})} = R_{1(\text{CSF})} + jX_{1(\text{CSF})} = R_{1(\text{CSF})} + jk_{X1}X_1 \quad (35)$$

#### 4. Determining the equivalent global change parameters for the power rotor fed by the static frequency converter

Further, it is considered a winding with multiple cages whose bars (in number of "c") are placed in the same notch of any form, electrically separated from each other (see Fig. 3). These bars are connected at the front by short-circuiting rings (one ring may correspond to several bars notch). This "generalized" approach, pure theoretically in fact, has the advantage that by its applying the relations of the two equivalent factors  $k_{r(CSF)}$  and  $k_{x(CSF)}$ , valid for any notch type and multiple cages, are obtained. The rotor notch shown in Fig. 3 is the height  $h_c$  and it is divided into "n" layers (strips), each strip having a height  $h_s = h_c/n$ . The number of layers "n" is chosen so that the current density of each band should be considered constant throughout the height  $h_s$  (and therefore not manifesting the skin effect in the strip). The notch bars are numbered from 1 to c, from the bottom of the notch. The lower layer of each bar is identified by the index "i" and the top layer by the index "s". Thus, for a bar with index  $\delta$  characterized by a specific resistance  $\rho_\delta$  and an absolute magnetic permeability  $\mu_\delta$ , the lower layer is noted with  $N_{\delta i}$  and the extremely high layer with  $N_{\delta s}$ . The current that flows through the bar  $\delta$  is noted with  $i_{c\delta}$  ( $I_{c\delta}$  - rated value). The length of the bar, over which the skin effect occurs, is L. For the beginning, let us consider only the presence of the fundamental in the power supply, which corresponds to the supply pulsation,  $\omega_{1(1)} = \omega_1 = 2\pi f_1$ . In this case:

$$k_{r\delta(1)} = \frac{R_{\delta(1)}}{R_{\delta}} = \frac{1}{I_{c\delta(1)}^2} \cdot \sum_{\varepsilon=N_{\delta i}}^{N_{\delta s}} \frac{I_{\varepsilon(1)}^2}{b_{\varepsilon}} \cdot \sum_{\varepsilon=N_{\delta i}}^{N_{\delta s}} b_{\varepsilon} \quad (36)$$

$$k_{x\delta(1)} = \frac{L_{\delta n\sigma(1)}}{L_{\delta n\sigma}} = \frac{\left| \text{Re} \left[ \Psi_{\delta n\sigma(1)} \right] \right| \left( \sum_{\varepsilon=N_{\delta i}}^{N_{\delta s}} b_{\varepsilon} \right)^2}{\sqrt{2} \mu_\delta L h_s I_{c\delta(1)} \cdot \sum_{\lambda=N_{\delta i}}^{N_{\delta s}} \frac{1}{b_\lambda} \left[ \left( \sum_{\varepsilon=N_{\delta i}}^{\lambda-1} b_{\varepsilon} \right) \left( \sum_{\varepsilon=N_{\delta i}}^{\lambda} b_{\varepsilon} \right) + \frac{b_\lambda^2}{3} \right]} \quad (37)$$

where  $b_\lambda$  and  $b_\varepsilon$  are the width of  $\lambda$  and  $\varepsilon$  order strips and  $\Psi_{\delta n\sigma(1)}$  is the  $\delta$  bar flux corresponding to the fundamental of the own magnetic field, assuming that for the  $\lambda$  order strip, the magnetic linkage corresponds to a constant repartition of the fundamental current density on the strip.

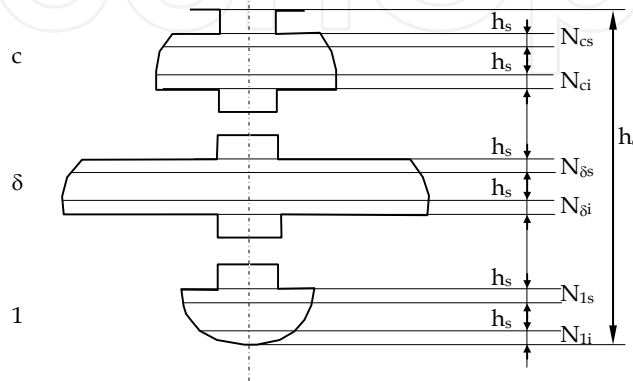


Figure 3. Notch generalized for multiple cages.

If in the motor power supply one considers only the  $v$  order harmonic which corresponds to the supply pulsation  $\omega_{1(v)}=v\omega_1$ , the relations (36) and (37) remain valid with the following considerations: index "1" is replaced by index "v" and the rotor phenomena are with the pulsation  $\omega_{2(v)}$  given by the relation:

$$\omega_{2(v)} = s_{(v)} \cdot \omega_{1(v)} = \left( 1 \mp \frac{1}{v} \pm \frac{s}{v} \right) \cdot v \cdot \omega_1 , \quad (38)$$

Subsequently we shall consider the real case, where in the  $\delta$  bar both the fundamental and  $v$  order time harmonics are present. For this, the equivalent d.c. global factor of the  $\delta$  bar resistance modification is calculated with the relation:

$$k_{r\delta(CSF)} = \frac{P_{\delta(CSF)}^-}{P_{\delta(CSF)}^-} = \frac{R_{\delta(CSF)}^-}{R_{\delta(CSF)}^-} , \quad (39)$$

where  $p_{\delta(CSF)}^-$  represents the total a.c. losses in  $\delta$  bar (considering the appropriate skin effect for all harmonics) and  $p_{\delta(CSF)}^-$  represents the bar  $\delta$  total losses, without considering the repression phenomenon. The a.c. total losses in the  $\delta$  bar are obtained by applying the effects superposition principle by adding all the  $\delta$  bar a.c. losses caused by each  $v$  order time, including the fundamental. Therefore one can obtain:

$$P_{\delta(CSF)}^- = P_{\delta(1)}^- + \sum_{v \neq 1} P_{\delta(v)}^- , \quad (40)$$

The a.c. loss in  $\delta$  bar, corresponding to the fundamental,  $p_{\delta(1)}^-$ , is calculated with the following relation:

$$P_{\delta(1)}^- = I_{c\delta(1)}^2 \cdot k_{r\delta(1)} \cdot R_{\delta}^- \quad (41)$$

In the same way, the expression of the  $\delta$  bar a.c. losses produced by some  $v$  order time harmonic is obtained:

$$P_{\delta(v)}^- = I_{c\delta(v)}^2 \cdot R_{\delta(v)}^- = I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \cdot R_{\delta}^- \quad (42)$$

By replacing the relations (41) and (42) in relation (40), it results:

$$P_{\delta(CSF)}^- = I_{c\delta(1)}^2 \cdot k_{r\delta(1)} \cdot R_{\delta}^- + \sum_{v \neq 1} I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \cdot R_{\delta}^- = R_{\delta}^- \left( I_{c\delta(1)}^2 \cdot k_{r\delta(1)} + \sum_{v \neq 1} I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \right) . \quad (43)$$

The  $\delta$  bar losses without considering the repression phenomenon in the bar are calculated using the following relationship:

$$P_{\delta(CSF)}^- = I_{c\delta(CSF)}^2 \cdot R_{\delta}^- , \quad (44)$$

where:

$$I_{c\delta(CSF)} = \sqrt{I_{c\delta(1)}^2 + \sum_{v \neq 1} I_{c\delta(v)}^2} \quad (45)$$

is the rated value of the current which runs through the  $\delta$  bar, in the case of a motor supplied by a frequency converter. By replacing the relation (45) in relation (44):

$$P_{\delta(CSF)} = R_{\delta} \cdot \left( I_{c\delta(1)}^2 + \sum_{v \neq 1} I_{c\delta(v)}^2 \right) \quad (46)$$

By replacing the relations (43) and (46) in (39) one obtains the expression for the global equivalent factor of the a.c. increasing resistance in the bar  $\delta$ ,  $k_{r\delta(CSF)}$ , in case of the presence of all harmonics in the motor power:

$$k_{r\delta(CSF)} = \frac{P_{\delta(CSF)} \sim}{P_{\delta(CSF)} -} = \frac{R_{\delta} \cdot \left( I_{c\delta(1)}^2 \cdot k_{r\delta(1)} + \sum_{v \neq 1} I_{c\delta(v)}^2 \cdot k_{r\delta(v)} \right)}{R_{\delta} \cdot \left( I_{c\delta(1)}^2 + \sum_{v \neq 1} I_{c\delta(v)}^2 \right)} = \frac{k_{r\delta(1)} + \sum_{v \neq 1} k_{r\delta(v)} \left( \frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2}{1 + \sum_{v \neq 1} \left( \frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2} \quad (47)$$

The global equivalent change of a.c.  $\delta$  bar inductance modification has the expression:

$$k_{x\delta(CSF)} = \frac{q_{\delta(CSF)} \sim}{q_{\delta(CSF)} -} \quad (48)$$

where  $q_{\delta(CSF)} \sim$  is the a.c. total reactive power, in the  $\delta$  bar, and  $q_{\delta(CSF)} -$  is the total reactive power for a uniform current distribution  $\delta$  in the bar. Applying the superposition in the case of a.c. total reactive power, the following relationship is obtained:

$$q_{\delta(CSF)} \sim = q_{\delta(1)} \sim + \sum_{v \neq 1} q_{\delta(v)} \sim \quad (49)$$

A.c. reactive power corresponding to the fundamental is calculated using the following relation:

$$q_{\delta(1)} \sim = \omega_1 \cdot k_{x\delta(1)} \cdot L_{\delta\sigma} \cdot I_{c\delta(1)}^2 \quad (50)$$

In the same way, the expression of the a.c. reactive power in the  $\delta$  bar corresponding to the  $v$  order harmonic is obtained:

$$q_{\delta(v)} \sim = \omega_{1(v)} L_{\delta\sigma(v)} \cdot I_{c\delta(v)}^2 = v \cdot \omega_1 \cdot k_{x\delta(v)} \cdot L_{\delta\sigma} \cdot I_{c\delta(v)}^2 \quad (51)$$

By replacing the relations (50) and (51) in the relation (28), the expression for calculating the total a.c. reactive power in the  $\delta$  bar is obtained:

$$\begin{aligned}
q_{\delta(\text{CSF})-} &= \omega_1 \cdot k_{x\delta(1)} \cdot L_{\delta n\sigma-} \cdot I_{c\delta(1)}^2 + \left( k_{x\delta(1)} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot k_{x\delta(v)} \cdot I_{c\delta(v)}^2 \right) = \\
&= \omega_1 \cdot L_{\delta n\sigma-} \cdot \left( k_{x\delta(1)} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot k_{x\delta(v)} \cdot I_{c\delta(v)}^2 \right)
\end{aligned} \quad (52)$$

The total reactive power for an uniform current repartition in the  $\delta$  bar, in the case of a motor supplied through a frequency converter, is calculated by the relation:

$$q_{\delta(\text{CSF})-} = q_{\delta(1)-} + \sum_{v \neq 1} q_{\delta(v)-}, \quad (53)$$

where  $q_{\delta(1)-}$  is the reactive power corresponding to the fundamental, in case of an uniform current distribution  $I_{c\delta(1)}$  in the  $\delta$  bar, while  $q_{\delta(v)-}$  is the reactive power corresponding to the  $v$  harmonic in case of a uniform current distribution  $I_{c\delta(v)}$  in the  $\delta$  bar:

$$q_{\delta(1)-} = \omega_{1(1)} L_{\delta n\sigma-} \cdot I_{c\delta(1)}^2 = \omega_1 \cdot L_{\delta n\sigma-} \cdot I_{c\delta(1)}^2. \quad (54)$$

Similarly, for the reactive power corresponding to the  $v$  harmonic, in the case of an uniform current  $I_{c\delta(v)}$  repartition in the  $\delta$  bar, the following relation is obtained:

$$q_{\delta(v)-} = \omega_{1(v)} \cdot L_{\delta n\sigma-} \cdot I_{c\delta(v)}^2 = v \cdot \omega_1 \cdot L_{\delta n\sigma-} \cdot I_{c\delta(v)}^2 \quad (55)$$

By replacing the relations (54) and (55) in relation (53), the expression for the total reactive power for a uniform current distribution in the  $\delta$  bar becomes:

$$q_{\delta(\text{CSF})-} = \omega_1 \cdot L_{\delta n\sigma-} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot \omega_1 \cdot L_{\delta n\sigma-} \cdot I_{c\delta(v)}^2 = \omega_1 \cdot L_{\delta n\sigma-} \cdot \left( I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot I_{c\delta(v)}^2 \right) \quad (56)$$

By replacing the relations (52) and (56) in relation (48), the expression for the global equivalent factor of the a.c. modifying inductance is obtained:

$$k_{x\delta(\text{CSF})} = \frac{q_{\delta(\text{CSF})-}}{q_{\delta(\text{CSF})-}} = \frac{\omega_1 L_{\delta n\sigma-} \cdot \left( k_{x\delta(1)} \cdot I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot k_{x\delta(v)} \cdot I_{c\delta(v)}^2 \right)}{\omega_1 L_{\delta n\sigma-} \cdot \left( I_{c\delta(1)}^2 + \sum_{v \neq 1} v \cdot I_{c\delta(v)}^2 \right)} = \frac{k_{x\delta(1)} + \sum_{v \neq 1} \left[ v \cdot \left( \frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2 \cdot k_{x\delta(v)} \right]}{1 + \sum_{v \neq 1} \left[ v \cdot \left( \frac{I_{c\delta(v)}}{I_{c\delta(1)}} \right)^2 \right]} \quad (57)$$

## 5. Determining the equivalent parameters of the winding rotor, considering the skin effect

The rotor winding's parameters are affected by the skin effect, at the start of the motor and also at the nominal operating regime. For establishing the relations that define these parameters, considering the skin effect, the expression of the rotor phase impedance

reduced to the stator is used. For this, the rotor with multiple bars is replaced by a rotor with a single bar on the pole pitch. Initially only the fundamental present in the power supply of the motor is considered. The rotor impedance reduced to the stator has the equation:

$$\underline{Z}'_{2(1)} = \frac{R'_{2(1)}}{s_{(1)}} + jX'_{2(1)} \quad (58)$$

Knowing that the induced EMF by the fundamental component of the main magnetic field from the machine in the pole pitch bars is:

$$\underline{U}_{e(1)} = \underline{I}'_{2(1)} \cdot \underline{Z}'_{2(1)} \quad (59)$$

where, for the general case of multiple cages is valid the relation:

$$\underline{I}'_{2(1)} = \sum_{\delta=1}^c \underline{I}'_{c\delta(1)} = \frac{\underline{U}_{e(1)}}{\underline{\Delta}_{(1)}} \sum_{\delta=1}^c \underline{\Delta}_{\delta(1)} \quad (60)$$

In the relation (60), the number of the cages and respectively the rotor bars/ pole pitch is equal to "c". In the case of motors with the power up to 45 [kW], c=1 (simple cage or high bars) or c=2 (double cage).  $\underline{\Delta}_{(1)}$  is the determinant corresponding to the equation system:

$$\underline{U}_{e(1)} = \sum_{\varepsilon=1}^c \underline{R}_{\delta\varepsilon(1)} \cdot \underline{I}_{c\varepsilon(1)} \quad , \quad \varepsilon=1, 2, \dots, c \quad (61)$$

having the expression:

$$\underline{\Delta}_{(1)} = \begin{vmatrix} \underline{R}_{11(1)} & \dots & \underline{R}_{1n(1)} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \underline{R}_{n1(1)} & \dots & \underline{R}_{nn(1)} \end{vmatrix} \quad (62)$$

$\underline{\Delta}_{\delta(1)}$  is the determinant corresponding to the fundamental obtained from  $\underline{\Delta}_{(1)}$ , where column  $\delta$  is replaced by a column of 1:

$$\underline{\Delta}_{\delta(1)} = \begin{vmatrix} \underline{R}_{11(1)} & \dots & \underline{R}_{1, \delta-1(1)} & 1 & \underline{R}_{1, \delta+1(1)} & \dots & \underline{R}_{1n(1)} \\ \cdot & & \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot & & \cdot \\ \underline{R}_{n1(1)} & \dots & \underline{R}_{n, \delta-1(1)} & 1 & \underline{R}_{n, \delta+1(1)} & \dots & \underline{R}_{nn(1)} \end{vmatrix} \quad (63)$$

Because in the first phase the steady-state regime is under focus, the phenomenon in the rotor corresponding to the fundamental has the pulsation  $\omega_{2(1)}=s\omega_1$ , where s is the motor slip for the sinusoidal power supply in the steady-state regime. If the relation (63) is introduced

in (60), the expression of the equivalent impedance of the rotor phase reduced to the stator, corresponding to the fundamental valid when considering the skin effect is obtained:

$$\underline{Z}'_{2(1)} = \frac{\underline{\Delta}_{(1)}}{\sum_{\delta=1}^c \underline{\Delta}_{\delta(1)}} \quad (64)$$

Thus, the expressions for the rotor phase resistance and inductance reduced to the stator, corresponding to the fundamental, both affected by the skin effect can be written.

$$\frac{R'_{2(1)}}{s_{(1)}} = \Re \left[ \underline{Z}'_{2(1)} \right], \quad (65)$$

$$X'_{2(1)} = \Im \left[ \underline{Z}'_{2(1)} \right] \quad (66)$$

By considering in the motor power supply the  $v$  harmonic only, similar expressions are obtained for the corresponding rotor parameters. Thus:

$$\underline{Z}'_{2(v)} = \frac{\underline{\Delta}_{(v)}}{\sum_{\delta=1}^c \underline{\Delta}_{\delta(v)}}, \quad (67)$$

$$\frac{R'_{2(v)}}{s_{(v)}} = \Re \left[ \underline{Z}'_{2(v)} \right], \quad (68)$$

$$X'_{2(v)} = \Im \left[ \underline{Z}'_{2(v)} \right] \quad (69)$$

Further on we consider the real case of an electric induction machine fed by a frequency converter. For the beginning, the case of simple cage respectively high bars induction motors will be analyzed. Thus, a rotor phase resistance corresponding to the fundamental,  $R'_{2(1)}$ , and rotor phase resistance corresponding to higher order harmonics  $R'_{2(v)}$  are replaced by an equivalent resistance  $R'_{2(\text{CSF})}$ , which dissipates the same part of active power as in the case of “ $v$ ” resistances. This equivalent resistance is defined at the fundamental’s frequency and it is traversed by the  $I'_{2(\text{CSF})}$  current:

$$I'_{2(\text{CSF})} = \sqrt{I'_{2(1)2} + \sum_{v \neq 1} I'_{2(v)2}} \quad (70)$$

For the rotor phase equivalent resistance reduced to the stator, corresponding to all harmonics, defined at the fundamental’s frequency, one can write:

$$R'_{2(\text{CSF})} = k_{r(\text{CSF})} \cdot R'_{2c} + R'_{2i}, \quad (71)$$

where:  $R'_{2c}$  is the resistance, considered at the fundamental's frequency of a part from the rotor phase winding from notches and reported to the stator,  $R'_{2i}$  is the resistance of a part of the rotor winding, neglecting skin effect reported to the stator,  $k_{r(CSF)}$  is the global modification factor of the rotor winding resistance, having the expression given by the relation (47). To track the changes that appear on the resistance of the rotor winding when the machine is supplied through a frequency converter, comparing to the case when the machine is fed in the sinusoidal regime, the  $k_{R_2}$  factor is introduced:

$$k_{R_2} = \frac{R'_{2(CSF)}}{R'_2}, \quad (72)$$

where  $R'_2$  is the rotor winding resistance reported to the stator, when the machine is fed in the sinusoidal regime:

$$R'_2 = k_r R'_{2c} + R'_{2i}, \quad (73)$$

where  $k_r$  is the modification factor of the a.c. rotor resistance, in the case of sinusoidal:  $k_r \cong k_{r(1)}$ . It is obtained:

$$k_{R_2} = \frac{k_{r(CSF)} R'_{2c} + R'_{2i}}{k_r R'_{2c} + R'_{2i}} \quad (74)$$

If both the nominator and the denominator of the second member on the relation (74) are divided by  $k_r$  and then by  $R'_{2c}$ , the following expression is obtained:

$$k_{R_2} = \frac{\frac{k_{r(CSF)}}{k_r} + \frac{R'_{2i}}{R'_{2c}} \cdot \frac{1}{k_r}}{1 + \frac{R'_{2i}}{R'_{2c}} \cdot \frac{1}{k_r}} = \frac{k_{kr} + r_2 \cdot \frac{1}{k_r}}{1 + r_2 \cdot \frac{1}{k_r}}, \quad (75)$$

where:

$$r_2 = \frac{R'_{2i}}{R'_{2c}} \cong \text{const.},$$

which is constant for the same motor, at a given fundamental's frequency. For  $c=1$ ,  $k_{kr} > 1$ , it results that  $k_{R_2} > 1$ , which means that  $R'_{2(CSF)} > R'_2$  also. The procedure is similar for the reactance. The rotor phase reactance, corresponding to the fundamental,  $X'_{2(1)}$ , and also the reactance corresponding to the higher harmonics,  $X'_{2(v)}$ , are replaced by an equivalent reactance  $X'_{2(CSF)}$ . As in the case of the rotor resistance, we can write:

$$k_{X_2} = \frac{X'_{2(CSF)}}{X'_2}, \quad (76)$$



where  $X'_{2(\text{CSF})}$  is the equivalent reactance of the rotor phase, reduced to the stator, corresponding to all harmonics, including the fundamental, on the fundamental's frequency:

$$X'_{2(\text{CSF})} = k_{X(\text{CSF})} X'_{2c} + X'_{2i} \quad (77)$$

and  $X'_2$  is the reactance of the rotor phase reduced to the stator which characterizes the machine when it is fed in the sinusoidal regime:

$$X'_2 = k_X X'_{2c} + X'_{2i} \quad (78)$$

In relation (77) and (78), we noted:  $X'_{2c}$  - the reactance of the rotor winding part from the notches, reduced to the stator, in which the skin effect is present,  $X'_{2i}$  - the reactance of the rotor winding phase where the skin effect can be neglected.  $k_{X(\text{CSF})}$  is defined in relation (57), where  $c \cong 1$ . Taking into account the relations (77) and (78), the relation (76) becomes:

$$k_{X'_2} = \frac{k_{X(\text{CSF})} X'_{2c} + X'_{2i}}{k_X X'_{2c} + X'_{2i}} = \frac{\frac{k_{X(\text{CSF})}}{k_X} + \frac{X'_{2i}}{X'_{2c}} \cdot \frac{1}{k_X}}{1 + \frac{X'_{2i}}{X'_{2c}} \cdot \frac{1}{k_X}} = \frac{k_{k_X} + x_2 \frac{1}{k_X}}{1 + x_2 \frac{1}{k_X}} \quad (79)$$

where:

$$x_2 = \frac{X'_{2i}}{X'_{2c}} \quad (80)$$

is a constant for the same motor at a given fundamental's frequency  $k_{k_X} < 1$ , with the consequences  $k_{X'_2} < 1$  and  $X'_{2(\text{CSF})} < X'_2$ . With this, the impedance of a rotor phase reported to the stator in the case of a machine supplied by a power converter, receives the form:

$$\underline{Z}'_{2(\text{CSF})} = \frac{R'_{2(\text{CSF})}}{S_{(\text{CSF})}} + jX'_{2(\text{CSF})} \quad (81)$$

where:

$$S_{(\text{CSF})} = \frac{R'_{2(\text{CSF})} I'_{2(\text{CSF})}}{U_{e1(\text{CSF})}} \quad (82)$$

and:

$$U_{e1(\text{CSF})} = \sqrt{U_{e1(1)}^2 + \sum_{v \neq 1} U_{e1(v)}^2} \quad (83)$$

In the case of double cage induction motors, the rotor parameters are necessary to be determined for both cages. The principle of calculation keeps its validity from the above presented case,

the induction motors with simple cage, respectively cage with high bars, with one remark: in the relations for determining  $k_{r(\text{CSF})}$  respectively  $k_{x(\text{CSF})}$ , it is considered that  $c=2$  (for  $\delta=1$  the working work cage results and for  $\delta=c=2$  the startup cage results). The complex structure of the used algorithm and its component computing relations synthetically presented in the paper, request a very high volume of calculation. Therefore the presence of a computer in solving this problem is absolutely necessary. In the Laboratory of Systems dedicated to control the electrical servomotors from the Polytechnic University of Timișoara the software calculation CALCMOT has been designed. It allows the determination and the analysis of the factors  $k_{r(\text{CSF})}$ ,  $k_{x(\text{CSF})}$  and the parameters of the equivalent winding machine induction in the non-sinusoidal regime. Further on, the expressions of the equivalent parameters for the magnetic circuit will be set (corresponding to all harmonics). Thus, to determine the equivalent resistance of magnetization  $R_{1m(\text{CSF})}$ , we have to take into account that this is determined only by the ferromagnetic stator core losses which are covered directly by the stator power without making the transition through the stereo-mechanical power. By approximating that  $I_{01(\text{CSF})} \approx I_{\mu(\text{CSF})}$ , for  $R_{1m(\text{CSF})}$  it is obtained:

$$R_{1m(\text{CSF})} = \frac{P_{z1(\text{CSF})} + P_{j1(\text{CSF})}}{3I_{\mu(\text{CSF})}^2}, \quad (83)$$

where  $p_{z1(\text{CSF})}$  and  $p_{j1(\text{CSF})}$  are global losses occurring respectively in the stator teeth and in the yoke due to the supplying of the motor through the frequency converter. In determining the total magnetization current  $I_{\mu(\text{CSF})}$ , the principle of the superposition effects is applied:

$$I_{\mu(\text{CSF})} = \sqrt{I_{\mu(1)}^2 + \sum_{v \neq 1} I_{\mu(v)}^2} \quad (84)$$

For the equivalent magnetizing reactance, corresponding to all harmonics, determined at the fundamental's magnetization frequency  $f_{l(1)}$ , we obtain:

$$X_{1m(\text{CSF})} \cong \sqrt{\left(\frac{U_{1(\text{CSF})}}{I_{\mu(\text{CSF})}}\right)^2 - \left(R_{1(\text{CSF})} + R_{1m(\text{CSF})}\right)^2} \quad (85)$$

For the equivalent impedance of the magnetization circuit it can be written:

$$\underline{Z}_{1m(\text{CSF})} = R_{1m(\text{CSF})} + j \cdot X_{1m(\text{CSF})} \quad (86)$$

Given these assumptions and considering that the equivalent parameters were calculated reduced to the fundamental's frequency (in the conditions of a sinusoidal regime), one may formally accept the calculation in complex quantities. Corresponding to the unique scheme shown in Fig. 2, the motor equations are:

$$\underline{U}_{1(\text{CSF})} = \underline{Z}_{1(\text{CSF})} \cdot \underline{I}_{1(\text{CSF})} - \underline{U}_{e1(\text{CSF})};$$

$$\underline{U}'_{e2(\text{CSF})} = \underline{Z}'_{2(\text{CSF})} \cdot \underline{I}'_{2(\text{CSF})} = \underline{U}'_{e1(\text{CSF})};$$

$$\underline{U}'_{e1(\text{CSF})} = -\underline{Z}'_{m(\text{CSF})} \cdot \underline{I}'_{01(\text{CSF})}; \quad (87)$$

$$\underline{I}'_{01(\text{CSF})} = \underline{I}'_{1(\text{CSF})} + \underline{I}'_{2(\text{CSF})}$$

## 6. Experimental validation

The induction machines which have been tested are: MAS 0,37 [kW] x 1500 [rpm] and MAS 1,1 [kW] x 1500 [rpm]. To validate the experimental studies of the theoretical work, tests were made both for the operation of motors supplied by a system of sinusoidal voltages, and for the operation in case of static frequency converter supply. In Tables 1 and 2 are presented theoretical values (obtained by running the calculation program) and the results of measurements, for  $k_{R'2}$  and  $k_{X'2}$ , factors, respectively the calculation errors of, for both motors tested.

Nr.	$f_{1(1)}$ [Hz]	$k_{R'2} = \frac{R'_{2(\text{CSF})}}{R'_2}$ (calculated)	$k_{R'2}$ (measured)	$\varepsilon_{k_{R'2}}$ [%]	$k_{X'2} = \frac{X'_{2(\text{CSF})}}{X'_2}$ (calculated)	$k_{X'2}$ (measured)	$\varepsilon_{k_{X'2}}$ [%]
1.	25	1,048	1,11	5,58	0,863	0,894	3,6
2.	30	1,026	1,077	4,97	0,912	0,857	-6,03
3.	40	1,021	1,061	3,77	0,944	0,884	-6,35
4.	50	1,014	1,075	6,01	0,967	0,897	-7,23
5.	60	1,011	1,079	6,82	0,975	0,914	-6,25

**Table 1.** The theoretical and experimental values of factors  $k_{R'2}$  and  $k_{X'2}$ , respectively the errors of calculation, corresponding to 0.37 [kW] x 1500 [rpm] MAS.

Nr.	$f_{1(1)}$ [Hz]	$k_{R'2} = \frac{R'_{2(\text{CSF})}}{R'_2}$ (calculated)	$k_{R'2}$ (measured)	$\varepsilon_{k_{R'2}}$ [%]	$k_{X'2} = \frac{X'_{2(\text{CSF})}}{X'_2}$ (calculated)	$k_{X'2}$ (measured)	$\varepsilon_{k_{X'2}}$ [%]
1.	20	1,098	1,185	7,92	0,812	0,821	1,108
2.	30	1,041	1,120	7,58	0,886	0,916	3,386
3.	40	1,034	1,106	6,96	0,926	0,891	-3,77
4.	50	1,023	1,089	6,45	0,956	0,863	-9,72
5.	60	1,018	1,082	6,28	0,966	0,871	-9,83

**Table 2.** The theoretical and experimental values of factors  $k_{R'2}$  și  $k_{X'2}$ , respectively the errors of calculation, corresponding to 1.1 [kW] x 1500 [rpm] MAS.

Parameters of the winding machine supplied by the power converter can be calculated with errors less than 10 [%]. The main cause of errors is the assumption of saturation neglect. Even in this case the results can be considered satisfactory, which leads to validate the theoretical study carried out in the paper.

## 7. Theoretical analysis of the magnetic losses

### 7.1. Statoric iron losses

#### 7.1.1. The main stator iron losses

##### A. The main stator teeth losses

In the teeth, the magnetic field is alternant and generates this type of losses. In the case of the direct supplying system the total losses from the stator teeth  $p_{z1}$  are being composed by the magnetic hysteresis losses,  $p_{z1h}$  and the eddy currents losses,  $p_{z1w}$ :

$$p_{z1} = (k_{zh} \cdot \sigma_h \cdot f_1 + k_{zw} \cdot \sigma_w \cdot f_1^2 \cdot \Delta^2) \cdot B_{z1m}^2 \cdot G_{z1}, \quad (88)$$

where:  $\sigma_h$  is a material constant depending on the thickness and the quality of the steel sheet,  $f_1$  is the supplying frequency,  $B_{z1m}$  represents the magnetic induction in the middle of the stator tooth,  $G_{z1}$  represent the weight of the stator teeth,  $\sigma_w$  is a material constant similar to  $\sigma_h$ , depending on the sheet thickness and quality and  $\Delta$  represents the thickness of the sheet.  $k_{zh}$  and  $k_{zw}$  are two factors which have the mission of underlining respectively the hysteresis losses increment and the eddy currents losses increment due to the mechanical modifications of the stator's sheets. In the case of converters-mode supplying system, at the total losses from the stators teeth caused by the fundamental the losses induced by the higher time harmonics must be taken into account. For an exact analytic expression in the following it is proposed an analysis method of the iron losses based upon the equalization of the hysteresis losses with the eddy currents ones. For the start, only the fundamental is considered present in the supplying system. Distinct from the sine-mode supplying system, when in most cases the supplying frequency is  $f_1=f_{1n}=50$  [Hz], is the fact that in the case of the inverter based supplying system the fundamental frequency can take values higher than 50 [Hz]. At very high magnetization frequencies the influence of the skin effect must be taken in consideration. In the following, the minimum value of the magnetization frequency is being determined and for that the skin effect must be considered. The computing relation for the magnetization frequency  $f_1$  is the following:

$$f_1 = \left( \frac{\xi}{\Delta} \right)^2 \cdot \frac{\rho}{\mu\pi}, \quad (89)$$

where  $\xi$  is the refulation factor.

The minimum magnetization frequency  $f_{min}$ , computed with the relation (89), from which the skin effect must be considered is 140[Hz]. Consequently, in the fundamental - wave

supplying mode, at which usually we have  $f_1 \leq 120$  [Hz], the principal losses from the stators teeth, can be written as following:

$$P_{z1(1)} = (k_{zh} \cdot \sigma_h \cdot f_1 + k_{zw} \cdot \sigma_w \cdot f_1^2 \cdot \Delta^2) \cdot B_{z1m(1)}^2 \cdot G_{z1} , \quad (90)$$

where  $B_{z1m(1)}$  represents the magnetic induction from the middle of the tooth,  $B_{z1m(1)} = B_{z1m}$ . In order to be able to apply the principle of over position effects, the machine is being considered as being ideal; therefore we neglect the hysteresis phenomenon. For this, we proposed the equalization of the hysteresis losses with the eddy current losses, an assumption that allows the linearization of the machines' equations. Through this equalization, the real machine – that is practically non-linear and in which the principal losses are made of a sum of two components: the one of eddy currents losses and the one of hysteresis losses - is being replaced with a theoretical linear machine, characterized only by its eddy currents losses. Energetically speaking, the two machines must be equivalent. As a following, if we take  $p_{z1w(1)}^*$  as the eddy currents losses corresponding to the fundamental, which appear in the theoretical model of the machine adopted, than these losses must be equal to the main losses from the stator teeth characteristic to the real machine, losses given through the relation:

$$P_{z1w(1)}^* = P_{z1(1)} \quad (91)$$

We consider these equivalent losses,  $p_{z1w(1)}^*$ , equal to the real losses through the eddy currents corresponding to the fundamental,  $p_{z1w(1)}$ , multiplied with a  $k_{z1e(1)}$  factor. This is an equalization factor of the real losses from the stators teeth with losses resulted only from "p<sub>z1w(1)</sub>" – fundamental-mode supplying state:

$$P_{z1w(1)}^* = k_{z1e(1)} \cdot P_{z1w(1)} \quad (92)$$

We consider that through this equalization factor a covering value of the principal stator teeth losses is obtained. The relation (91) made explicit becomes:

$$(k_{zh} \cdot \sigma_h \cdot f_1 + k_{zw} \cdot \sigma_w \cdot f_1^2 \cdot \Delta^2) \cdot B_{z1m(1)}^2 \cdot G_{z1} = k_{z1e(1)} \cdot k_{zw} \cdot \sigma_w \cdot f_1^2 \cdot \Delta^2 \cdot B_{z1m(1)}^2 \cdot G_{z1} . \quad (93)$$

Because of the fact that the usually used sheets have the thickness  $\Delta=0.5$  [mm]=const, one can consider that:

$$k_{z1e(1)} = 1 + \frac{K_{z\Delta}}{f_1} \quad (94)$$

where we have

$$K_{z\Delta} = K_z / \Delta^2 \text{ with } K_z = \frac{\sigma_h \cdot k_{zh}}{\sigma_w \cdot k_{zw}}$$

In the following part we consider that only the  $v$  order harmonic is present in the supplying wave, characterized by the magnetization frequency  $f_{1(v)}=v \cdot f_1$ . Therefore, the principal losses

in the stator teeth occurring in the real machine corresponding to the  $v$  order time harmonic must be corrected through the two factors  $k_{h(v)}$  and  $k_{w(v)}$ , which are a function of the reaction of the eddy currents:

$$P_{z1(v)} = \left( k_{zh} \cdot k_{h(v)} \cdot \sigma_h \cdot v \cdot f_1 + k_{zw} \cdot k_{w(v)} \cdot \sigma_w \cdot v^2 \cdot f_1^2 \cdot \Delta^2 \right) \cdot B_{z1m(v)}^2 \cdot G_{z1} \quad (95)$$

In the relation (95),  $B_{z1m(v)}$  represents the magnetic induction according to the  $v$  order time harmonic from the middle of the tooth. The factors  $k_{h(v)}$  and  $k_{w(v)}$  have the expressions:

$$k_{h(v)} = \frac{\xi_{(v)}}{2} \cdot \frac{\text{sh}\xi_{(v)} + \sin \xi_{(v)}}{\text{ch}\xi_{(v)} - \cos \xi_{(v)}}; \quad k_{w(v)} = \frac{3}{\xi_{(v)}} \cdot \frac{\text{sh}\xi_{(v)} - \sin \xi_{(v)}}{\text{ch}\xi_{(v)} - \cos \xi_{(v)}}; \quad (96)$$

As in the case of the fundamental-wave supplying case, the real machine is replaced by a theoretical linear machine which has only losses given by the eddy currents. Reasoning as in the case of the fundamental, we obtain:

$$k_{z1e(v)} = 1 + \frac{K_z}{\Delta^2} \cdot \frac{1}{v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}} = 1 + \frac{K_{z\Delta}}{v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}}, \quad (97)$$

$$P_{z1(v)} = P_{z1w(v)}^* = k_{z1e(v)} \cdot P_{z1w(v)} = k_{z1e(v)} \cdot k_{zw} \cdot k_{w(v)} \cdot \sigma_w \cdot v^2 \cdot f_1^2 \cdot \Delta^2 \cdot B_{z1m(v)}^2 \cdot G_{z1} \quad (98)$$

where  $P_{z1w(v)}^*$  are the equivalent losses corresponding to the  $v$  harmonic. If we have  $P_{z1(\text{CSF})}$  for the losses from the stators teeth with the machine supplied by inverters, by applying the principle of over position effects for the theoretical linear model of the machine, it will be written:

$$P_{z1(\text{CSF})} = k_{zw} \cdot \sigma_w \cdot f_1^2 \cdot \Delta^2 \cdot B_{z1m(1)}^2 \cdot G_{z1} \left[ k_{z1e(1)} + \sum_{v \neq 1} k_{z1e(v)} \cdot k_{w(v)} \cdot v^2 \left( \frac{B_{z1m(v)}}{B_{z1m(1)}} \right)^2 \right] \quad (99)$$

In order to analyze the modifications suffered by the main losses in the stators teeth while the motor is supplied by an inverter versus the sine-mode supplying system, we analyze the ratio between the relations (99) and (88). After making the intermediary computations in which the relations (93), (94) and (99) are taken into account we obtain:

$$k_{pz1} = \frac{P_{z1(\text{CSF})}}{P_{z1}} = 1 + \sum_{v \neq 1} \left( \frac{k_{z1e(v)}}{k_{z1e(1)}} \cdot k_{w(v)} \cdot v^2 \cdot k_{Bz1(v,1)}^2 \right), \quad (100)$$

where  $k_{Bz(v,1)} = B_{z1m(v)} / B_{z1m(1)}$ .

## B. The principal losses in the stator yoke

In the case of the direct – mode supplying system of the machine, the principal yoke losses consist of the hysteresis losses,  $p_{j1h}$  and eddy currents losses,  $p_{j1w}$ :

$$p_{j1} = \left( \sigma_h \cdot f_1 \cdot k_{j1h} + \sigma_w \cdot \Delta^2 \cdot f_1^2 \cdot k_{j1w} \right) \cdot B_{j1}^2 \cdot G_{j1} \quad (101)$$

where:  $B_{j1}$  is the magnetic induction in the stator yoke,  $G_{j1}$  represents the weight of the stator yoke,  $k_{j1w} = k_{j1w1} \cdot k_{j1w2}$ , where  $k_{j1w1}$  is a coefficient that corresponds to the non uniform repartition of the magnetic induction in the yoke and  $k_{j1w2}$  is a coefficient that corresponds to the currents closing perpendicular to the sheets, through the places with imperfections in the sheets isolation layer and also in the wholes made in the cutting process. In the case on an inverter supplying system at the total losses from the stator yoke caused by the fundamental, the superior time harmonics losses must be added. In order to apply the principle of over-position effect the method is similar to the one used in the case of the principal losses in the teeth. We equalize energetically the real machine with the linear theoretical one where we consider only the eddy currents losses. As a following, for the fundamental supplying mode, the principal losses in the stator yoke for a real machine,  $p_{j1(1)}$  are:

$$p_{j1(1)} = \left( \sigma_h \cdot f_1 \cdot k_{j1h} + \sigma_w \cdot \Delta^2 \cdot f_1^2 \cdot k_{j1w} \right) \cdot B_{j1(1)}^2 \cdot G_{j1} \quad (102)$$

If we have  $p_{j1w(1)}^*$  as losses in eddy currents, than these must be equalized with the principal losses from the stator yoke described with the relation (102):

$$p_{j1w(1)}^* = p_{j1(1)} \quad (103)$$

These equivalent losses,  $p_{j1w(1)}^*$  are considered equal to the real eddy currents losses  $p_{j1w(1)}$ , multiplied with an equalizing factor of the real yoke losses with “ $p_{j1w(1)}$ ” type losses,  $k_{j1e(1)}$ :

$$p_{j1w(1)}^* = k_{j1e(1)} \cdot p_{j1w(1)} \quad (104)$$

Similarly to point A, as a following of the equalization we obtain the relation:

$$k_{j1e(1)} = 1 + \frac{K_w}{\Delta^2 \cdot f_1} = 1 + \frac{K_{w\Delta}}{f_1}, \quad (105)$$

where we have:

$$K_w = \frac{\sigma_h \cdot k_{j1h}}{\sigma_w \cdot k_{j1w}} \quad \text{and} \quad K_{w\Delta} = \frac{K_w}{\Delta^2}$$

As a following we consider present in the supplying system of the machine only the  $v$  order superior time harmonic. Because of the fact that the magnetization frequency  $f_{1(v)}$  is the fundamental one multiplied with  $v$ , the principal losses from the stator yoke which appear in the fundamental must be adjusted with the two coefficients:  $k_{h(v)}$  and  $k_{w(v)}$ . These factors take into account respectively the skin effect and the eddy currents reaction.

$$p_{j1(v)} = \left( k_{h(v)} \cdot \sigma_h \cdot v \cdot f_1 \cdot k_{j1h} + k_{w(v)} \cdot \sigma_w \cdot \Delta^2 \cdot v^2 \cdot f_1^2 \cdot k_{j1w} \right) \cdot B_{j1(v)}^2 \cdot G_{j1} \quad (106)$$



In the relation (106),  $B_{j1(v)}$  represents the magnetic induction accordingly to the  $v$  order harmonic. Through the energetically equalization realized from the replacement of the real machine with the linear model, we obtain the equalizing factor of the stator yoke losses, with the “ $p_{j1w(v)}$ ” type losses:

$$k_{j1e(v)} = 1 + \frac{K_w}{\Delta^2} \cdot \frac{1}{v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}} = 1 + \frac{K_{w\Delta}}{v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}} \quad (107)$$

In conclusion, the principal losses in the stator yoke, corresponding to the  $v$  order time harmonic can be written by equalizing as:

$$P_{j1(v)} = P_{j1w(v)}^* = k_{j1e(v)} \cdot P_{j1w(v)}, \quad (108)$$

where:

$$P_{j1w(v)} = k_{w(v)} \cdot \sigma_w \cdot \Delta^2 \cdot v^2 \cdot f_1^2 \cdot k_{j1w} \cdot B_{j1(v)}^2 \cdot G_{j1} \quad (109)$$

As a following we have considered the situation of the machine supplied by the fundamental and the superior time harmonics as well. Taking  $p_{j1(CSF)}$  as the global losses occurring in the stator yoke due to the converter supplying mode, by applying the over position effect principle on the theoretical linear model we can write:

$$P_{j1(CSF)} = \sigma_w \cdot f_1^2 \cdot \Delta^2 \cdot k_{j1w} \cdot B_{j1(1)}^2 \cdot G_{j1} \left[ k_{j1e(1)} + \sum_{v \neq 1} k_{j1e(v)} \cdot k_{w(v)} \cdot v^2 \left( \frac{B_{j1(v)}}{B_{j1(1)}} \right)^2 \right] \quad (110)$$

In order to analyze the changes that the principal losses from the stator yoke suffer when the machine is being supplied through an inverter versus the sine-mode supplying case, we divide the relation (110) at (101). After finishing the computations we have:

$$k_{pj1} = \frac{P_{j1(CSF)}}{P_{j1}} = 1 + \sum_{v \neq 1} \left( \frac{k_{j1e(v)} \cdot k_{w(v)} \cdot v^2 \cdot k_{Bj1(v,1)}^2}{k_{j1e(1)}} \right), \quad (111)$$

where:  $k_{Bj1(v,1)} = B_{j1(v)} / B_{j1(1)}$ .

### 7.1.2. The supplementary stator iron losses

#### A. Surface supplementary losses

In the case of a network supplying mode, the magnetic induction distribution curve over the polar step is not very different from a sine-curve. The surface stator losses are given by the expression:

$$P_{\sigma 1} = \frac{1}{2} \cdot l \cdot \pi \cdot D \cdot \frac{\tau_{c1} - b_{41}}{\tau_{c1}} \cdot k_o \cdot (N_{c2} \cdot n)^{1,5} \cdot (\tau_{c2} \cdot \beta_2 \cdot k_{\delta 2} \cdot B_{\delta})^2 \quad (112)$$



In the relation (112) the significance of the sizes is the following:  $D$  is the inner diameter of the stator,  $\tau_{c1}$  is the step of the stator slot and  $\tau_{c2}$  is the step of the rotor slot,  $b_{41}$  is the opening of the stator slot,  $N_{c2}$  is the number of stator slots,  $n$  is the rotation speed,  $\beta_2$  is a factor dependent on the ratio  $b_{42}/\delta$  ( $b_{42}$  is the opening of the rotor slot),  $k_{\delta 2}$  is an air gap factor,  $k_o$  is an adjustment factor which depends on the materials resistivity and its magnetic permeability. In the case of the inverter supplying method, due to the deforming state at the supplementary losses produced by the fundamental, the surface losses produced by the superior time harmonics must be considered. Because of the fact that the surface losses in the polar pieces are treated as the eddy current losses developed in the inductor sheets, we can apply the over position effect principle without any further parallelism. Therefore, the surface supplementary losses in the stator in the case of a machine supplied by inverters can be computed with the relation:

$$P_{\sigma 1(\text{CSF})} = \frac{1}{2} \cdot l \cdot \pi \cdot D \cdot \frac{\tau_{c1} - b_{41}}{\tau_{c1}} \cdot k_o \cdot (N_{c2} \cdot n)^{1,5} \cdot \left( \tau_{c2} \cdot \beta_2 \cdot k_{\delta 2} \cdot B_{\delta(1)} \right)^2 \left[ 1 + \sum_{v \neq 1} \left( \frac{B_{\delta(v)}}{B_{\delta(1)}} \right)^2 \right] \quad (113)$$

Dividing the supplementary losses in the stator surface when having an inverter supplying system for the machine,  $P_{\sigma 1(\text{CSF})}$ , by the supplementary losses in the stator surface when we have the sine-mode supplying system for the machine,  $P_{\sigma 1}$ , and making the intermediary computations we obtain the increment factor of the supplementary stator surface losses in the inverter versus the sine-mode supplying case,  $k_{P_{\sigma 1}}$ , as following:

$$k_{P_{\sigma 1}} = \frac{P_{\sigma 1(\text{CSF})}}{P_{\sigma 1}} = 1 + \sum_{v \neq 1} \left( \frac{B_{\delta(v)}}{B_{\delta(1)}} \right)^2 = 1 + \sum_{v \neq 1} k_{B_{\delta(v,1)}}^2 > 1, \quad (114)$$

where  $k_{B_{\delta(v,1)}} = B_{\delta(v)} / B_{\delta(1)}$ . By analyzing the relation (114) one can notice the fact that the  $k_{P_{\sigma 1}}$  factor tends to 1 because of the fact that the value is practically very low. Consequently, the surface supplementary losses increase due to the inverter supplying system to an extent that is not to be taken into consideration.

### B. The pulsation supplementary losses

In the case of the sine-mode supplying system, the pulsation supplementary losses in the stator, provided that the magnetic field along the polar step is not much different from a sine-wave, has the following expression:

$$P_{P1} = \frac{1}{2} \cdot \sigma_w \cdot k_{wP1} \cdot (\Delta N_{c2} n)^2 \cdot \left( \frac{\gamma_2 \delta k_{\delta}}{2 \tau_{c1}} \right)^2 \cdot G_{z1} \cdot B_{z1m}^2, \quad (115)$$

where  $k_{wP1}$  is an increment coefficient of the stator losses by eddy currents due to processing,  $k_{\delta}$  is the total air gap factor and  $\gamma_2$  is constant for the one and the same machine, depended on the opening of the stator slot and the air gap dimension. In the situation in which the

machine is supplied by inverters, by applying the over position effect principle, the following expression for the supplementary pulsation losses in the stator  $P_{P1(\text{CSF})}$  is obtained:

$$P_{P1(\text{CSF})} = \frac{1}{2} \cdot \sigma_w \cdot k_{wP1} \cdot (\Delta N_{c2} n)^2 \cdot \left( \frac{\gamma_2 \delta k_\delta}{2\tau_{c1}} \right)^2 \cdot G_{z1} \cdot B_{z1m(1)}^2 \left[ 1 + \sum_{v \neq 1} \left( \frac{B_{z1m(v)}}{B_{z1m(1)}} \right)^2 \right] \quad (116)$$

Dividing the pulsation stator losses in the case of the inverter supplying system  $P_{P1(\text{CSF})}$ , by the pulsation stator losses in the case of sine-mode supplying system  $P_{P1}$ , we obtain the increment factor of the supplementary pulsation losses in the inverter versus sine-wave supplying system,  $k_{Pp1}$ :

$$k_{Pp1} = \frac{P_{P1(\text{CSF})}}{P_{P1}} = 1 + \sum_{v \neq 1} \left( \frac{B_{z1m(v)}}{B_{z1m(1)}} \right)^2 = 1 + \sum_{v \neq 1} k_{Bz1(v,1)}^2 > 1 \quad (117)$$

By analyzing the relation (117) we can state that in the case of an inverter supplied machine we have not obtained a significant increment of the pulsation losses in the stator due to the small value of the  $k_{Bz1(v,1)}^2$ .

## 7.2. Rotor iron losses

### 7.2.1. Principal losses in the rotor iron

#### A. The principal losses in the rotor's teeth

Firstly, only one superior time harmonic is considered present in the supplying system of the machine, of an average order  $v$ . The real losses that this harmonic produces in the rotor teeth have the expression:

$$p_{z2(v)} = \left( k_{zh} \cdot k_{h(v)} \cdot \sigma_h \cdot s_{(v)} \cdot v \cdot f_1 + k_{zw} \cdot k_{w(v)} \cdot \sigma_w \cdot s_{(v)}^2 \cdot v^2 \cdot f_1^2 \cdot \Delta^2 \right) B_{z2m(v)}^2 \cdot G_{z2} \quad (118)$$

In the relation (118),  $B_{z2m(v)}$  represents the magnetic induction corresponding to the  $v$  order harmonic from the middle of the rotor tooth. In the theoretical model adopted, these losses given by the relation (118) are produced only by eddy currents:

$$P_{z2(v)} = P_{z2w(v)}^* = k_{z2e(v)} \cdot P_{z2w(v)} \quad (119)$$

where  $k_{z2e(v)}$  is an equalizing factor of the real losses from the rotor teeth, only with the losses of "p<sub>z2w(v)</sub>" type, corresponding to the  $v$  order time harmonic. Developing the relation (119) by using the relation (118), after finishing the intermediary computations we obtain:

$$k_{z2e(v)} = 1 + \frac{K_z}{\Delta^2} \cdot \frac{1}{s_{(v)} \cdot v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}} = 1 + \frac{K_{z\Delta}}{s_{(v)} \cdot v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}} \quad (120)$$

Therefore, the principal losses from the rotor teeth, corresponding to the  $v$  order time harmonic can be written by equalization as it follows:

$$P_{z2(v)} = k_{z2e(v)} \cdot k_{zw} \cdot k_{w(v)} \cdot \sigma_w \cdot s_{(v)}^2 \cdot v^2 \cdot f_1^2 \cdot \Delta^2 \cdot B_{z2m(v)}^2 \cdot G_{z2} \quad (121)$$

In the conditions in which in the supplying system of the machine all the superior time harmonics are present, the principal losses in the rotor teeth can be written as:

$$P_{z2(\text{CSF})} = \sum_{v \neq 1} P_{z2(v)} \quad (122)$$

### B. The principal losses from the rotor's yoke

In the hypotheses in which in the supplying system only the  $v$  order harmonic is present, the real principal losses induced by it in the rotor yoke have the expression:

$$P_{j2(v)} = \left( k_{h(v)} \cdot \sigma_h \cdot s_{(v)} \cdot v \cdot f_1 \cdot k_{j1h} + k_{w(v)} \cdot \sigma_w \cdot s_{(v)}^2 \cdot v^2 \cdot f_1^2 \cdot \Delta^2 \cdot k_{j2w} \right) \cdot B_{j2(v)}^2 \cdot G_{j2} \quad (123)$$

Through the energetic equalization, due to the replacement of the real machine by a theoretical linear model we can obtain the equality:

$$P_{j2(v)} = P_{j2w(v)}^* = k_{j2e(v)} \cdot P_{j2w(v)} \quad (124)$$

Reasoning as in the previous cases, we can determine the equalizing factor of the real losses in the rotor yoke, only with losses of the type “ $p_{j2w(v)}$ ” type as it follows:

$$k_{j2e(v)} = 1 + \frac{K_w}{\Delta^2} \cdot \frac{1}{s_{(v)} \cdot v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}} = 1 + \frac{K_{w\Delta}}{s_{(v)} \cdot v \cdot f_1} \cdot \frac{k_{h(v)}}{k_{w(v)}} \quad (125)$$

Consequently, the principal rotor yoke losses corresponding to the  $v$  order harmonic can be written by equalization in the form:

$$P_{j2(v)} = k_{j2e(v)} \cdot k_{j2w} \cdot k_{w(v)} \cdot \sigma_w \cdot s_{(v)}^2 \cdot v^2 \cdot f_1^2 \cdot \Delta^2 \cdot B_{j2(v)}^2 \cdot G_{j2} \quad (126)$$

Disregarding all these, in the case of the inverter supplying system the total principal losses in the rotor yoke,  $p_{j2(\text{CSF})}$ , are computed with the relation:

$$P_{j2(\text{CSF})} = \sum_{v \neq 1} P_{j2(v)} \quad (127)$$

## 7.2.2. The supplementary losses in the rotor iron

### A. The surface supplementary losses

If the machine is directly supplied from the power supply, the surface supplementary rotor losses are calculated with the relation:

$$P_{\sigma 2} = \frac{1}{2} \cdot p_{\sigma 2} \cdot 1 \cdot \pi \cdot (\Delta - 2\delta) \cdot \frac{\tau_{c2} - b_{42}}{\tau_{c2}}, \quad (128)$$

where the specific rotor surface losses  $p_{\sigma 2}$  have the expression:

$$p_{\sigma 2} = k_o (N_{c1} \cdot n)^{1.5} \cdot (\tau_{c1} \cdot \beta_1 \cdot k_{\delta 1} \cdot B_{\delta})^2 \quad (129)$$

In the relations (128) and (129) we noted by  $b_{42}$  the opening of the rotor slot,  $N_{c1}$  the number of rotor slots,  $\beta_1$  a factor dependent on the  $b_{41}/\delta$  ratio and  $k_{\delta 1}$  the air gap factor. Proceeding similarly we can obtain the expression of the increment factor of the supplementary losses in the rotor surface while the machine is being supplied by inverters versus the sine-mode supplying system,  $k_{P\delta 2}$ :

$$k_{P\sigma 2} = \frac{P_{\sigma 2(\text{CSF})}}{P_{\sigma 2}} = 1 + \sum_{v \neq 1} \left( \frac{B_{\delta(v)}}{B_{\delta(1)}} \right)^2 = 1 + \sum_{v \neq 1} k_{B\delta(v,1)}^2 = k_{P\sigma 1} > 1 \quad (130)$$

## B. The supplementary pulsation losses

The supplementary pulsation rotor losses, in the sine-mode supplying system have the following expression:

$$P_{P2} = \frac{1}{2} \sigma_w \cdot k_{wP2} (\Delta \cdot N_{c1} \cdot n \cdot B_{P2})^2 \cdot G_{z2} \quad (131)$$

$B_{P2}$  represents the pulsation induction in the rotor teeth. Consequently, taking into account the fact that:

$$\frac{B_{z2m(v)}}{B_{z2m(1)}} = \frac{B_{\delta(v)}}{B_{\delta(1)}} = k_{B\delta(v,1)}, \quad (132)$$

we obtain:

$$k_{PP2} = \frac{P_{P2(\text{CSF})}}{P_{P2}} = 1 + \sum_{v \neq 1} k_{B\delta(v,1)}^2 > 1 \quad (133)$$

## 8. Conclusions

This paper aims to study the theoretical behavior of asynchronous three-phase motor in the case of supplying through a power frequency converter. This study has aimed to develop the theory of the asynchronous three-phase motor in non-sinusoidal periodic regime to serve as a starting point in optimizing the design methodology. Given that the asynchronous three-phase motor is fed through a static frequency converter, the machine operation in the presence of higher time harmonics in the supply voltage can be described by a single mathematical model. The model consists of a single equivalent scheme corresponding to all harmonics and it is defined at the fundamental frequency.

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