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# dc Josephson Current Between an Isotropic and a d-Wave or Extended s-Wave Partially Gapped Charge Density Wave Superconductor

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Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/46073>

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## 1. Introduction

The discovery and further development of superconductivity is extremely interesting because of its pragmatic (practical) and purely academic reasons. At the same time, the superconductivity science is very remarkable as an important object for the study in the framework of the history and methodology of science, since all the details are well documented and well-known to the community because of numerous interviews by participants including main heroes of the research and the fierce race for higher critical temperatures of the superconducting transition,  $T_c$ . Moreover, the whole science has well-documented dates, starting from the epoch-making discovery of the superconducting transition by Heike Kamerlingh-Onnes in 1911 [1–7], although minor details of this and, unfortunately, certain subsequent discoveries in the field were obscured [8–11]. As an illustrative example of a senseless dispute on the priority, one can mention the controversy between the recognition of Bardeen-Cooper-Schrieffer (BCS) [12] and Bogoliubov [13] theories.

If one looks beyond superconductivity, it is easy to find quite a number of controversies in different fields of science [14, 15]. Recent attempts [16–18] to contest and discredit the Nobel Committee decision on the discovery of graphene by Andre Geim and Kostya Novoselov [19, 20] are very typical. The reasons of a widespread disagreement concerning various scientific discoveries consist in a continuity of scientific research process and a tense competition between different groups, as happened at liquefying helium and other cryogenic gases [9, 21–24] and was reproduced in the course of studying graphite films [25, 26]. At the same time, the authors and the dates of major discoveries and predictions in the science of superconductivity are indisputable, fortunately to historians and teachers.

Macroscopic manifestations of the superconducting state and diverse properties of the plethora of superconductors are consequences of main fundamental features: (i) zero

resistivity found already by Kamerlingh-Onnes (sometimes the existence of persistent currents discovered by him in 1914 is considered more prominent and mysterious [27]), (ii) expulsion of a weak magnetic field (the Meissner effect [28]), and (iii) the Josephson effects [29–37], i.e. the possibility of dc or ac super-currents in circuits, containing thin insulating or normal-metal interlayers between macroscopic superconducting segments. Of course, the indicated properties are interrelated. For instance, a macroscopic superconducting loop with three Josephson junctions can exhibit a superposition of two states with persistent currents of equal magnitudes and opposite polarity [38].

We note that those findings, reflecting a cooperative behavior of conducting electrons (later interpreted in terms of a quantum-mechanical wave function [12, 39–43]), had to be augmented by the observed isotope dependence of  $T_c$  [44, 45] in order that the first successful semi-microscopic (it is so, because the declared electron-phonon interaction was, in essence, reduced to the phenomenological four-fermion contact one) BCS theory of superconductivity [12] would come into being. Sometimes various ingenious versions of the BCS theory, explicitly taking into account the momentum and energy dependences of interaction matrix elements, as well as the renormalization of relevant normal-state properties by the superconducting reconstruction of the electron spectrum [46–50], are called “the BCS theory”. Nevertheless, such extensions of the initial concept, explicitly related to Ref. [12] and results obtained therein, are inappropriate. This circumstance testifies that one should be extremely accurate with scientific terms, since otherwise it may lead to reprehensible misunderstandings [51].

Whatever be a theory referred to as “the BCS one” or as “the theory of superconductivity” [52], we still lack a true consistent microscopic picture scenario (scenarios?) of superconducting pairing in different various classes of superconductors. As a consequence, all existing superconducting criteria [53–72] are empirical rather than microscopic, although based on various relatively well-developed theoretical considerations. Hence, materials scientists must rely on their intuition to find new promising superconductors [73–78], although bearing also in mind a deep qualitative theoretical reasoning [43, 79–83].

It is no wonder that unusual transport properties of superconductors together with their magnetic-field sensibility led to a number of practically important applications. Namely, features (i) and (ii) indicated above made it possible to manufacture large-scale power cables, fly-wheel energy storage devices, bearings, high field magnets, fault current limiters, superconductor-based transformers, levitated trains, motors and power generators [84–93]. At the same time, the Josephson (weak-coupling) feature (iii) became the basis of small-scale superconducting electronics [88, 94–98], which also uses the emergence of half-integer magnetic flux quantization in circuits with superconducting currents [99, 100]. Smartly designed SQUID devices with several Josephson junctions and a quantized flux serve as sensible detectors of magnetic field and electromagnetic waves, which, in their turn, are utilized in industry, research, and medicine [95–98, 101]. Recently oscillatory effects inherent to superfluid  $^3\text{He}$  [102–104] and  $^4\text{He}$  [103–105], which are similar to the Josephson one, were used to construct superfluid helium quantum interference devices (SHeQUIDs) [106].

High- $T_c$  oxide superconductors found in 1986 [107] and including large families of materials with  $T_c \leq 138$  K [108–112] extended the application domain of superconductivity, because,

first, liquid-nitrogen temperatures were achieved and, second, the predominant  $d_{x^2-y^2}$ -order parameter symmetry (at least in hole-doped oxides) made possible applications in electronics and quantum computation more diverse [37, 113–122].

While studying high- $T_c$  cuprates, superconductivity was shown to compete with charge density waves (CDWs), so that the observed properties in the superconducting state must be modified by CDWs [123–128]. It should concern Josephson currents phenomenon too [129–134], although this topic has not been properly developed so far.

Of course, other superconducting materials found after the discovery of high- $T_c$  oxide materials are also very remarkable, because of their non-trivial electron spectra, so that Josephson currents through junctions involving those materials should possess interesting features. We mean, in particular,  $\text{MgB}_2$  with  $T_c \leq 40$  K [135] and a multiple energy-gap structure [136, 137], as well as Fe-based pnictides and chalcogenides with  $T_c \leq 56$  K and concomitant spin density waves (SDWs) suspected to have deep relations with superconductivity in those materials [78].

In this paper, we present our theoretical studies of dc Josephson currents between conventional superconductors and partially CDW-gapped materials with an emphasis on cuprates, although the gross features of the model can be applied to other CDW superconductors as well. The next Section 2 contains the justification of the approach and the formulation of the problem, whereas numerical results of calculations, as well as the detailed discussion, are presented in Section 3. Section 4 contains some general conclusions concerning dc Josephson currents across junctions involving partially gapped CDW superconductors.

A more involved case of Josephson junctions between two CDW superconductors with various symmetries of superconducting pairing will be treated elsewhere.

## 2. Theoretical approach

### 2.1. $d$ -wave versus $s$ -wave order parameter symmetry

Coherent properties of Fermi liquids in the paired state are revealed by measurements of dc or ac Josephson tunnel currents between two electrodes possessing such properties. The currents depend on the phase difference between superconducting order parameters of the electrodes involved [30, 31, 119]. Manifestations of the coherent pair tunneling are more complex for superconductors with anisotropic order parameters than for those with an isotropic energy gap. In particular, it is true for  $d$ -wave superconductors, where the order parameter changes its sign on the Fermi surface (FS) [119, 138–143]. As was indicated above, high- $T_c$  oxides are usually considered as such materials, where the  $d_{x^2-y^2}$  pairing is usually assumed at least as a dominating one [117, 144–152]. However, conventional  $s$ -wave contributions were also detected in electron tunneling experiments [153–160] and, probably, in nuclear magnetic resonance (NMR) and nuclear quadrupole resonance measurements [161]. Therefore, only a minority of researchers prefer to accept the isotropic  $s$ -wave (or extended  $s$ -wave) nature of superconductivity in cuprates [162–175]. Notwithstanding the existing fundamental controversies, the  $d$ -wave specificity of high- $T_c$  oxide superconductivity has already been used in technical devices [95, 116, 118–120, 122].

## 2.2. Pseudogaps as a manifestation of non-superconducting gapping

In addition to the complex character of superconducting order parameter, cuprates reveal another intricacy of their electron spectrum. Namely, the pseudogap is observed both below and above  $T_c$  [176–180]. Here, various phenomena manifesting themselves in resistive, magnetic, optical, photoemission (ARPES), and tunnel (STM and break-junction) measurements are considered as a consequence of the “pseudogap”-induced depletion in the electron density of states, in analogy to what is observed in quasi-one-dimensional compounds above the mean-field phase-transition temperature [181, 182].

Notwithstanding large theoretical and experimental efforts, the pseudogap nature still remains unknown [126–128, 133, 178, 183–201]. Namely, some researchers associate them with precursor order parameter fluctuations, which might be either of a superconducting or some other competing (CDWs, SDWs, etc.) origin. Another viewpoint consists in relating pseudogaps to those competing orderings, but treating them, on the equal footing with superconductivity, as well-developed states that can be made allowance for in the mean field approximation, fluctuation effects being non-crucial. We believe that the available observations support the latter viewpoint (see, e.g., recent experimental evidences of CDW formation in various cuprates [202–205]). Moreover, although undoped cuprates are antiferromagnetic insulators [206], the CDW seems to be a more suitable candidate responsible for the pseudogap phenomena, which competes with Cooper pairing in doped high- $T_c$  oxide samples [123–127], contrary to what is the most probable for iron-based pnictides and chalcogenides [78, 207]. Nevertheless, the type of order parameter competing with Cooper pairing in cuprates is not known with certainty. For instance, neutron diffraction studies of a number of various high- $T_c$  oxides revealed a nonhomogeneous magnetic ordering (usually associated with SDWs) in the pseudogap state [208, 209].

## 2.3. Superconducting order parameter symmetry scenarios

Bearing in mind all the aforesaid, we present here the following scenarios of dc Josephson tunneling between a non-conventional partially gapped CDW superconductor and an ordinary  $s$ -wave one. The Fermi surface (FS) of the former is considered two-dimensional with a  $d_{x^2-y^2}$ ,  $d_{xy}$ - or extended  $s$ -wave (with a constant order parameter sign) four-lobe symmetry of superconducting order parameter and a CDW-related doping-dependent dielectric order parameter. The CDWs constitute a system with a four-fold symmetry emerging inside the superconducting lobes in their antinodal directions for cuprates (the  $d_{x^2-y^2}$ -geometry of the superconducting order parameter, see Figure 1) or in the nodal directions for another possible configuration allowed by symmetry (the  $d_{xy}$ -geometry of the superconducting order parameter). (Below, for the sake of brevity, when considering the extended  $s$ -wave geometries for the superconducting order parameter, we use the corresponding mnemonic notations  $s_{x^2-y^2}^{\text{ext}}$  and  $s_{xy}^{\text{ext}}$ .) Thus, the CDW order parameter  $\Sigma$  competes with its superconducting counterpart  $\Delta$  over the whole area of their coexistence, which gives rise to an interesting phenomena of temperature- ( $T$ -) reentrant  $\Sigma$  [126–128, 210, 211]. In this paper, the main objective of studies are the angular dependences, which might be observed in the framework of the adopted model. Of course, any admixture of Cooper pairing with a symmetry different from  $d_{x^2-y^2}$ -one [148, 154, 160, 212, 213] may alter the results. Moreover, the superconducting

order parameter symmetry might be doping-dependent [214]. To obtain some insight into such more cumbersome situations, we treat here the pure isotropic  $s$ -wave case as well. Other possibilities for predominantly  $d$ -wave superconductivity coexisting with CDWs lie somewhere between those pure  $s$ - and  $d$ - extremes.

#### 2.4. Formulation of the problem

The dc Josephson critical current through a tunnel junction between two superconductors, whatever their origin, is given by the general equation [30, 35]

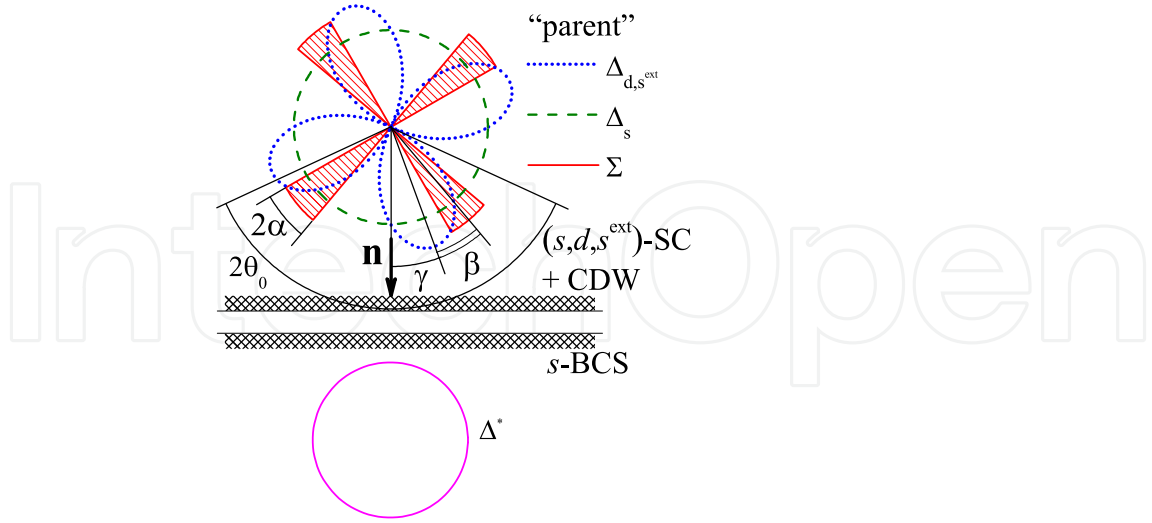
$$I_c(T) = 4eT \sum_{\mathbf{p}\mathbf{q}} \left| \tilde{T}_{\mathbf{p}\mathbf{q}} \right|^2 \sum_{\omega_n} F_{\text{HTSC}}^+(\mathbf{p}; \omega_n) F_{\text{OS}}(\mathbf{q}; -\omega_n), \quad (1)$$

Here,  $\tilde{T}_{\mathbf{p}\mathbf{q}}$  are matrix elements of the tunnel Hamiltonian corresponding to various combinations of FS sections for superconductors taken on different sides of tunnel junction,  $\mathbf{p}$  and  $\mathbf{q}$  are the transferred momenta,  $e > 0$  is the elementary electrical charge,  $F_{\text{HTSC}}^+(\mathbf{p}; \omega_n)$  and  $F_{\text{OS}}(\mathbf{q}; -\omega_n)$  are Gor'kov Green's functions for  $d$ -wave (CDW gapped!) and ordinary  $s$ -wave superconductor, respectively, and the internal summation is carried out over the discrete fermionic "frequencies"  $\omega_n = (2n + 1) \pi T$ ,  $n = 0, \pm 1, \pm 2, \dots$ . The external summation should take into account both the anisotropy of electron spectrum  $\zeta(\mathbf{p})$  in a superconductor in the manner suggested long time ago for all kinds of anisotropic superconductors [215], the directionality of tunneling [216–220], and the concomitant dielectric (CDW) gapping of the nested FS sections [129].

Hereafter, we shall assume that the ordinary superconductor has the isotropic order parameter  $\Delta^*(T)$ . At the same time, the superconducting order parameter of the high- $T_c$  CDW superconductor has the properly rotated (see Figure 1) pure  $d$ -wave form  $\Delta(T) \cos [2(\theta - \gamma)]$ , the angle  $\theta$  being reckoned from the normal  $\mathbf{n}$  to the junction plane and  $\gamma$  is a tilt angle between  $\mathbf{n}$  and the bisectrix of the nearest positive lobe. Note that, for the  $s^{\text{ext}}$ -symmetry, the gap profile is the same as in the  $d$ -case, but the signs of all lobes are identical rather than alternating (for definiteness, let this sign be positive).

The dielectric order parameter  $\Sigma(T)$  corresponds to the checkerboard system of mutually perpendicular CDWs (observed in various high- $T_c$  oxides [221–223]). In the adopted model, it is nonzero inside four sectors, each of the width  $2\alpha$ , with their bisectrices rotated by the angle  $\beta$  with respect to the bisectrices of superconducting order parameter lobes [126–128, 210, 211]. Actually, we shall assume  $\beta$  to be either 0 or  $\pi/4$ . Since the nesting vectors are directed along the  $\mathbf{k}_x$ - and  $\mathbf{k}_y$ -axes in the momentum space [126, 224], the adopted choice corresponds to the choice between  $d_{x^2-y^2}$ - and  $d_{xy}$ -symmetry. Another possible, unidirectional CDW geometry is often observed in cuprates as well [225–227]. It can be treated in a similar way, but we shall not consider it in this work.

Note also that, in agreement with previous studies [216–220, 228], the tunnel matrix elements  $\tilde{T}_{\mathbf{p}\mathbf{q}}$  in Eq. (1) should make allowance for the tunnel directionality (the angle-dependent probability of penetration through the barrier) [140, 229, 230]. We factorize the corresponding directionality coefficient  $w(\theta)$ . The weight factor  $w(\theta)$  effectively disables the FS outside a certain given sector around  $\mathbf{n}$ , thus governing the magnitude and the sign of the Josephson



**Figure 1.** Geometry of the junction between a conventional  $s$ -wave superconductor ( $s$ -BCS) and a  $d$ -,  $s$ -extended ( $s^{\text{ext}}$ ) or  $s$ -superconductor partially gapped by charge density waves (CDWs, induced by dielectric, i.e. electron-hole, pairing). The angle  $\alpha$  denotes the half-width of each of four angular sectors at the Fermi surface, where the CDW gap appears. The gap profiles for the parent CDW insulator ( $\Sigma$ ),  $s$ - ( $\Delta_s$ ),  $d$ - ( $\Delta_d$ ), and  $s$ -extended ( $\Delta_{s^{\text{ext}}}$ ) superconductors, and conventional superconductor ( $\Delta^*$ ) are shown.  $\beta$  is a misorientation angle between the nearest superconducting lobe and CDW-gapped sector,  $\gamma$  is a tilt angle of superconducting lobe with respect to the junction plane determined by the normal  $\mathbf{n}$ ,  $\theta_0$  is a measure of tunneling directionality (see explanations in the text).

tunnel current. Specifically, we used the following model for  $w(\theta)$ :

$$w(\theta) = \exp \left[ - \left( \frac{\tan \theta}{\tan \theta_0} \right)^2 \right], \quad (2)$$

where  $\theta_0$  is an angle describing the effective width of the directionality sector. We emphasize that, for tunneling between two anisotropic superconductors, two different coefficients  $w(\theta)$  associated with  $\mathbf{p}$ - and  $\mathbf{q}$ -distributions in the corresponding electrodes come into effect [216].

In accordance with the previous treatment of partially gapped  $s$ -wave CDW superconductors [123–125, 129, 130, 132, 231–234] and its generalization to their  $d$ -wave counterparts [126–128, 210, 211, 235] and in line with the basic theoretical framework for unconventional superconductors [236, 237], the anomalous Gor'kov Green's functions for high- $T_c$  oxides are assumed to be different for angular sectors with coexisting CDWs and superconductivity ( $d$  sections of the FS) and the “purely superconducting” rest of the FS ( $nd$  sections)

$$F_{\text{HTSC},nd}(\mathbf{p};\omega_n) = \frac{\Delta(T) \cos [2(\theta - \gamma)]}{\omega_n^2 + \Delta^2(T) \cos^2 [2(\theta - \gamma)] + \zeta_{nd}^2(\mathbf{p})}, \quad (3)$$

$$F_{\text{HTSC},d}(\mathbf{p};\omega_n) = \frac{\Delta(T) \cos [2(\theta - \gamma)]}{\omega_n^2 + \Delta^2(T) \cos^2 [2(\theta - \gamma)] + \Sigma^2(T) + \zeta_d^2(\mathbf{p})}. \quad (4)$$

Here, we explicitly took into account a possible angle deviation  $\gamma$  of the  $\Delta$ -lobe direction, which is governed by the crystal lattice geometry, from the normal  $\mathbf{n}$  to the junction plane; the latter is created artificially and, generally speaking, can be not coinciding with a crystal facet. The concomitant rotation of the CDW sectors is made allowance for implicitly. The quasiparticle spectra  $\xi_d(\mathbf{p})$  and  $\xi_{nd}(\mathbf{p})$  correspond to “hot” and “cold” spots of the cuprate FS, respectively (see, e.g., Refs. [176, 238–240]).

Substituting Eqs. (2), (3), and (4) into Eq. (1) and carrying out standard transformations [30, 35], we obtain

$$I_c(T) = \frac{\Delta(0)\Delta^*(0)}{2eR_N} i_c(T), \quad (5)$$

$$i_c(T) = \frac{1}{2\pi} \int_{\theta_d} w(\theta) \cos[2(\theta - \gamma)] P \left[ \Delta^*(T), \sqrt{\Sigma^2 + \Delta^2(T) \cos^2[2(\theta - \gamma)]} \right] d\theta \\ + \frac{1}{2\pi} \int_{\theta_{nd}} w(\theta) \cos[2(\theta - \gamma)] P \left[ \Delta^*(T), |\Delta(T) \cos 2(\theta - \gamma)| \right] d\theta. \quad (6)$$

Here,  $R_N$  is the normal-state resistance of the tunnel junction, determined by  $|\tilde{T}_{\mathbf{p}\mathbf{q}}|^2$  without the factorized multiplier  $w(\theta)$ , the integration is carried out over the CDW-gapped and CDW-free FS sections (the FS-arcs  $\theta_d$  and  $\theta_{nd}$ , respectively, in the two-dimensional problem geometry),  $\Delta^*(T)$  is the order parameter of the ordinary isotropic superconductor, whereas the function  $P(\Delta_1, \Delta_2)$  is given by the expression [129, 215]

$$P(\Delta_1, \Delta_2) = \int_{\min\{\Delta_1, \Delta_2\}}^{\max\{\Delta_1, \Delta_2\}} \frac{dx \tanh \frac{x}{2T}}{\sqrt{(x^2 - \Delta_1^2)(\Delta_2^2 - x^2)}}. \quad (7)$$

Modified Eqs. (3)-(6) turn out valid for the calculation of dc Josephson current through a tunnel junction between an ordinary  $s$ -wave superconductor and a partially gapped CDW superconductor with an extended  $s$ -symmetry of superconducting order parameter [142, 241]. For this purpose, it is enough to substitute the cosine functions in Eqs. (3)-(6) by their absolute values.

At  $w(\theta) \equiv 1$  (the absence of tunnel directionality),  $\Sigma \equiv 0$  (the absence of CDW-gapping), and putting  $\cos 2(\theta - \gamma) \equiv 1$  (actually, it is a substitution of an isotropic  $s$ -superconductor for the  $d$ -wave one), Eq. (6) expectedly reproduces the famous Ambegaokar–Baratoff result for tunneling between  $s$ -wave superconductors [30, 31, 35, 242].

Note that, in Eq. (6), the directionality is made allowance for only by introducing the angular function  $w(\theta)$  reflecting the angle-dependent tunnel-barrier transparency. On the other hand, the tunneling process, in principle, should also take into account the factors  $|\mathbf{v}_{g,nd} \cdot \mathbf{n}|$  and  $|\mathbf{v}_{g,d} \cdot \mathbf{n}|$ , responsible for extra directionality [140, 219, 230], where  $\mathbf{v}_{g,nd} = \nabla \xi_{nd}$  and  $\mathbf{v}_{g,d} = \nabla \xi_d$  are the quasiparticle group velocities for proper FS sections. Those factors can be considered as proportional to a number of electron attempts to penetrate the barrier [139]. They were introduced decades ago in the framework of general problem dealing with tunneling in heterostructures [243–245]. Nevertheless, we omitted here the



group-velocity-dependent multiplier, since it requires that the FS shape should be specified, thus going beyond the applied semi-phenomenological scheme, as well as beyond similar semi-phenomenological approaches of other groups [138, 139, 141, 236, 246]. We shall take the additional directionality factor into account in subsequent publications, still being fully aware of the phenomenological nature of both  $|\mathbf{v}_g \cdot \mathbf{n}|$  and  $w(\theta)$  functions.

It is well known [143] that, in the absence of directionality, the Josephson tunneling between  $d$ - and  $s$ -wave superconductors is weighted-averaged over the FS, with the cosine multiplier in Eq. (6) playing the role of weight function. In this case, the Josephson current has to be strictly equal to zero. However, it was found experimentally that the dc Josephson current between  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  and Pb [155],  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  and Nb [247],  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  and PbIn [248],  $\text{Y}_{1-x}\text{Pr}_x\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$  and Pb [153] differ from zero. Hence, either a subdominant  $s$ -wave component of the superconducting order parameter does exist in cuprate materials, as was discussed above, or the introduction of directionality is inevitable to reconcile any theory dealing with tunneling of quasiparticles from (to) high- $T_c$  oxides and the experiment.

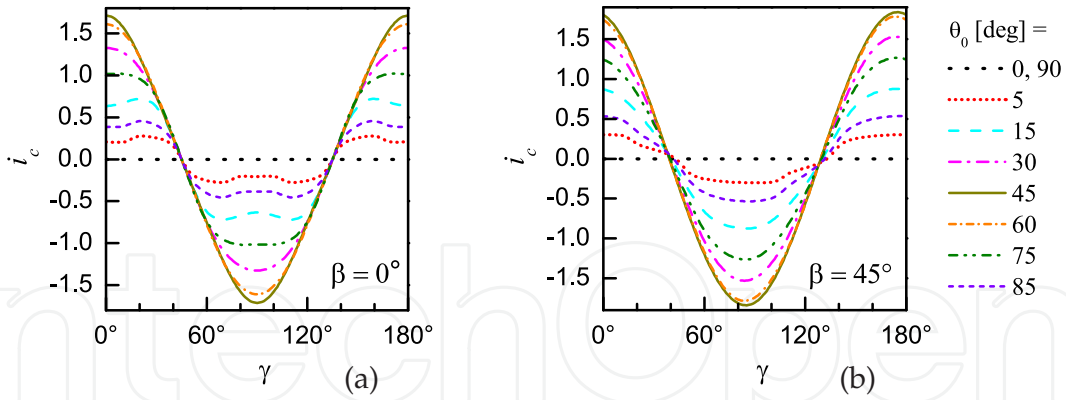
We restrict ourselves mostly to the case  $T = 0$ , when formula (7) is reduced to elliptic functions [30, 249], although some calculations will be performed for  $T \neq 0$  as well. The reason consists in the smallness of  $T_c$  for conventional  $s$ -wave superconductors (in our case, it is Nb, see below) as compared to  $T_c$  of anisotropic  $d$ -wave oxides. Hence, all effects concerning  $T$ -dependent interplay between  $\Delta$  and  $\Sigma$  including possible reentrance of  $\Sigma(T)$  [126–128, 210, 211, 235] become insignificant in the relevant  $T$ -range cut off by the  $s$ -wave-electrode order parameter. On the contrary, in the symmetrical case, when one studies tunneling between different high- $T_c$ -oxide grains,  $T$ -dependences of the Josephson current are expected to be very interesting. This more involved situation will be investigated elsewhere.

### 3. Results and discussion

#### 3.1. Total currents

In what follows, we shall consider in parallel the dc Josephson currents between a more or less conventional (weak-coupling BCS  $s$ -wave) Nb with a zero- $T$  energy gap  $\Delta^*(0) = 1.4$  meV and  $T_c = 9.2$  K [247] and either a  $d_{x^2-y^2}$ - or a  $d_{xy}$ - superconductor ( $\beta = 0$  and  $\pi/4$ , respectively). The latter is also possible from the symmetry viewpoint, but have not yet been found among existing classes of CDW superconductors.

The dependences of the dimensionless current  $i_c(T = 0)$  on the tilt angle  $\gamma$  are shown in Figure 2(a) for  $\alpha = 15^\circ$  and various values of the parameter  $\theta_0$  describing the degree of directionality. Since  $T = 0$ , there is no need to solve the equation set for  $\Sigma(T)$  and  $\Delta(T)$  for partially CDW-gapped  $s$ -wave [233] or  $d$ -wave [128] superconductors self-consistently. Instead, for definiteness, we chose the experimental values  $\Sigma(0) = 36.3$  meV and  $\Delta(0) = 28.3$  meV appropriate to slightly overdoped  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  samples [250] as input parameters. The half-width  $\alpha$  of each of the four CDW sectors was *rather arbitrarily* chosen as  $15^\circ$ . In fact, it is heavily dependent on the doping extent and cannot be unambiguously extracted even from the most precise angle-resolved photoemission spectra (ARPES) [200, 251, 252]. Thus, hereafter we consider the parameter of dielectric FS gapping  $\alpha$  as a *phenomenological* one on the same footing as the tunneling directionality parameter  $\theta_0$ .



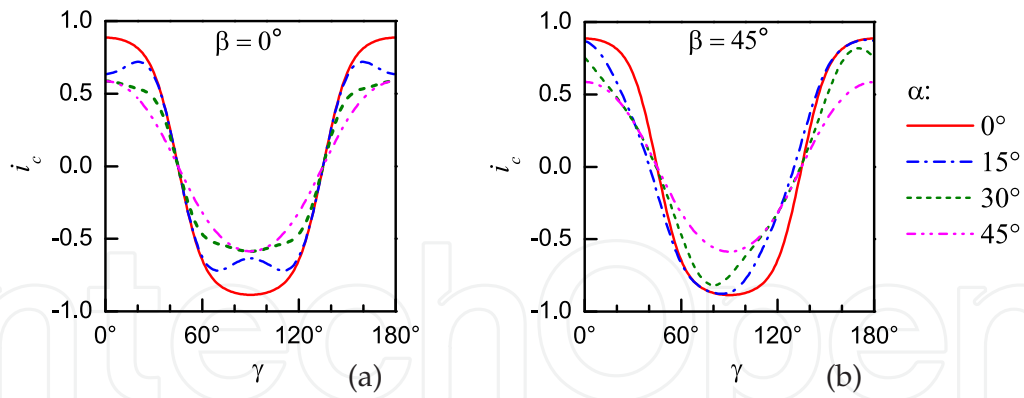
**Figure 2.** (a) Zero-temperature ( $T = 0$ ) dependences on  $\gamma$  of the dimensionless dc Josephson current  $i_c$  for the tunnel junction between an  $s$ -wave superconductor and a CDW  $d_{x^2-y^2}$ -wave one ( $\beta = 0^\circ$ ) for various  $\theta_0$ 's. The specific gap values for electrodes correspond to the experimental data for Nb ( $\Delta^*(T = 0) = 1.4$  meV) and  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  ( $\Sigma(T = 0) = 36.3$  meV and  $\Delta(T = 0) = 28.3$  meV). The calculation parameter  $\alpha = 15^\circ$ . See further explanations in the text. (b) The same as in panel (a), but for a CDW  $d_{xy}$ -superconductor ( $\beta = 45^\circ$ ).

It is evident that, if the sector  $\theta_0$  of effective tunneling equals zero, the Josephson current vanishes. It is also natural that, in the case of  $d$ -wave pairing and the absence of tunneling directionality ( $\theta_0 = 90^\circ$ ), the Josephson current disappears due to the exactly mutually compensating contributions from superconducting order parameter lobes with different signs [119, 138, 143]. Intermediate  $\theta_0$ 's correspond to non-zero Josephson tunnel current of either sign (conventional 0- and  $\pi$ -junctions [120, 122, 253]) except at the tilt angle  $\gamma = 45^\circ$ , when  $i_c = 0$ . In this connection, one should recognize that the energy minimum for non-conventional anisotropic superconductors can occur, in principle, at any value of the order parameter phase [254]. As is seen from Figure 2(a), the existence of CDWs in cuprates ( $\alpha \neq 0$ ,  $\Sigma \neq 0$ ) influences the  $\gamma$ -dependences of  $i_c$ , which become non-monotonic for  $\theta_0$  close to  $\alpha$  demonstrating a peculiar resonance between two junction characteristics. The effect appears owing to the actual  $d_{x^2-y^2}$ - pattern with the coinciding bisectrices of CDW sectors and superconducting lobes ( $\beta = 0^\circ$ ). This circumstance may ensure the finding of CDWs (pseudogaps) by a set of relatively simple transport measurements.

At the same time, for the hypothetical  $d_{xy}$  order parameter symmetry ( $\beta = 45^\circ$ , Figure 2(b)), when hot spots lie in the nodal regions, the dependences  $i_c(\gamma)$  become asymmetrical relative to  $\gamma = 90^\circ$  and remain monotonic as for CDW-free  $d$ -wave superconductors.

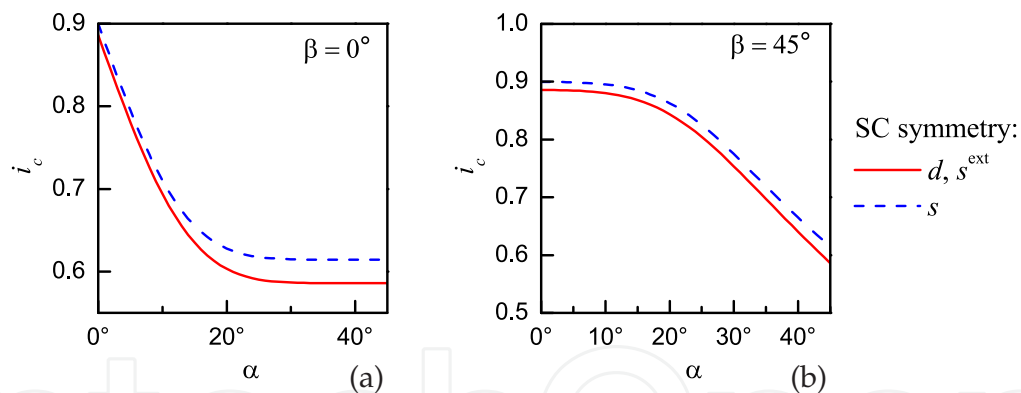
The role of superconducting-lobe and CDW (governed by the crystalline structure) orientation with respect to the junction plane (the angle  $\gamma$ ) is most clearly seen for varying  $\alpha$ , which is shown in Figure 3. The indicated above “resonance” between  $\theta_0$  and  $\alpha$  is readily seen in Figure 3(a). One also sees that the Josephson current amplitude is expectedly reduced with the increasing  $\alpha$ , since CDWs suppress superconductivity [123–127, 255]. For  $\beta = 45^\circ$  (Figure 3(b)), the curves  $i_c(\gamma)$  are non-symmetrical, and their form is distorted by CDWs relative to the case of “pure” superconducting  $d$ -wave electrode.

The dependence of  $i_c$  on the CDW-sector width, i.e. the degree of dielectric FS gapping, is a rapidly dropping one, which is demonstrated in Figures 4(a) (for  $\beta = 0^\circ$ , i.e. for  $d_{x^2-y^2}$  or  $s_{x^2-y^2}^{\text{ext}}$  symmetries) and 4(b) (for  $\beta = 45^\circ$ , i.e. for  $d_{xy}$  or  $s_{xy}^{\text{ext}}$  symmetries) calculated for



**Figure 3.** The same as in Figure 2, but for  $\theta_0 = 15^\circ$  and various  $\alpha$ 's.

$\gamma = 0^\circ$  and  $\theta_0 = 15^\circ$ . Indeed, for cuprates, where the directions of superconducting lobes and CDW sectors coincide, an extending CDW-induced gap reduces the electron density of states available to superconducting pairing until  $\alpha$  becomes equal to  $\theta_0$  (see Figure 4(a)). A further increase of the pseudogapped FS arc has no influence on  $i_c$ , since it falls outside the effective tunneling sector. We note that the  $\alpha$ -dependence of  $i_c$  for cuprates can be, in principle, non-linearly mapped onto the doping dependence of the pseudogap [200, 251, 252]. It is remarkable that, qualitatively, the results are the same for the extended  $s$ -symmetry (denoted as  $s^{\text{ext}}$ ) of the superconducting order parameter and are very similar to those for the assumed  $s$ -wave order parameter (curves marked by  $s$ ).

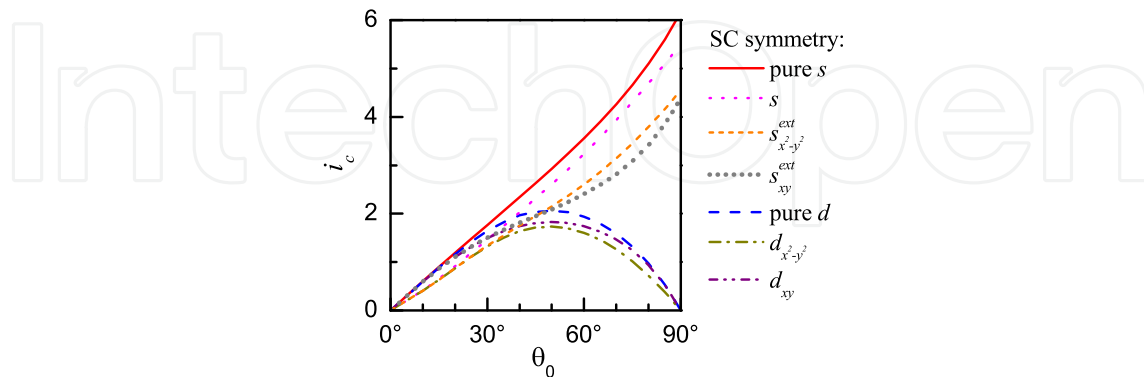


**Figure 4.** Dependences  $i_c(\alpha)$  for  $\gamma = 0^\circ$  and  $\theta_0 = 15^\circ$  for  $d$ -,  $s$ -extended, and  $s$ -symmetries of superconducting order parameter.

At the same time, if the CDW sectors are rotated in the momentum space by  $45^\circ$  with respect to the superconducting lobes and/or the directional-tunneling  $\theta_0$ -cone (see Figure 4(b)), the dependences  $i_c(\alpha)$  are very weak at small  $\alpha$  and become steep for  $\alpha > \theta_0$ . This result is true for the  $d_{xy}$ -, rotated extended  $s$ -, and isotropic  $s$ -symmetries of the superconducting order parameter coexisting with its dielectric counterpart.

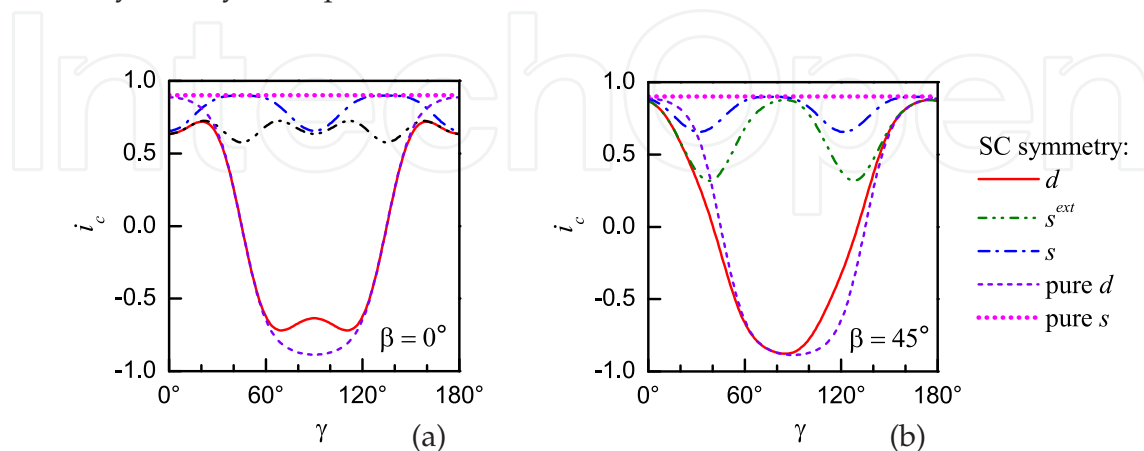
One sees from Figure 4 that, for small  $\theta_0 = 15^\circ$ , the  $d$ - and extended  $s$ -order parameters result in the same  $i_c(\alpha)$ . Of course, it is no longer true for larger  $\theta_0$ , when contributions from different lobes into the total Josephson current start to compensate each other for  $d$ -wave superconductivity, whereas no compensation occurs for the extended  $s$ -wave scenario. To

make sure that this assertion is valid, we calculated the dependences  $i_c(\theta_0)$  for  $\gamma = 0^\circ$ ,  $\alpha = 15^\circ$ , and  $\beta = 0^\circ$  and  $45^\circ$ . The results are presented in Figure 5. Indeed, for  $\theta_0 \geq 30^\circ$ , the curves corresponding to  $d$ -wave and extended  $s$ -wave superconductors come apart, as it has to be. Thus, Josephson currents between isotropic and CDW  $d$ -wave superconductors, similarly to the CDW-free case, are non-zero only because the tunneling is non-isotropic.

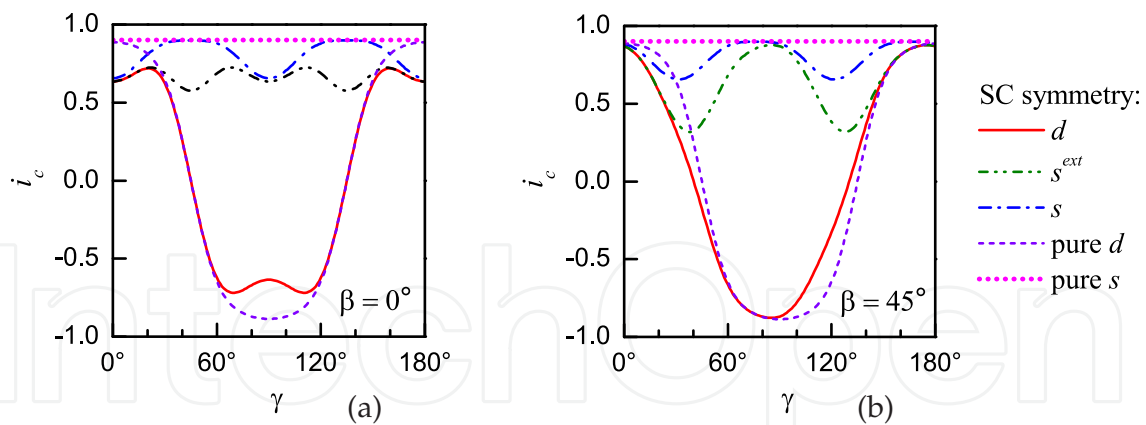


**Figure 5.** Dependences  $i_c(\theta_0)$  for  $\gamma = 0^\circ$ ,  $\beta = 0^\circ$ , and  $\alpha_0 = 15^\circ$  for various symmetries of superconducting order parameter.

It is instructive to compare the tilt-angle- $\gamma$  dependences of the Josephson currents  $i_c$  for possible superconducting order parameter symmetries, which are considered, in particular, for cuprates. The results of calculations are displayed in Figure 7 for  $\alpha = \theta_0 = 15^\circ$ . For an  $s$ -wave CDW-free superconductor,  $i_c(\gamma) = \text{const}$ . The reference curve  $i_c(\gamma)$  for a CDW-free  $d_{x^2-y^2}$ -wave superconductor (Figure 7(a)) is periodic and alternating. CDWs distort both curves. Namely, the CDW  $d_{x^2-y^2}$ -wave superconductor demonstrates a non-monotonic behavior of  $i_c(\gamma)$ , as was indicated above, whereas  $i_c(\gamma)$  for the  $s$ -wave CDW superconductor becomes a periodic dependence of a constant sign. The curve  $i_c(\gamma)$  for the extended  $s$ -wave CDW superconductor has a different form than in the  $s$ -wave case, although being qualitatively similar. The presented data demonstrate that CDWs can significantly alter angle dependences often considered as a smoking gun, when determining the actual order parameter symmetry for cuprates or other like materials.



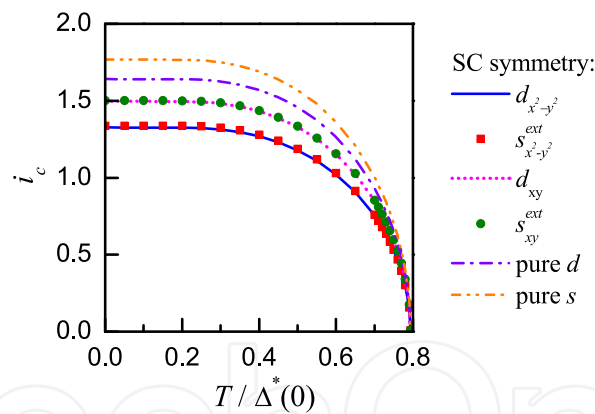
**Figure 6.** The same as in Figure 2, but for  $\theta_0 = 15^\circ$  and various symmetries of superconducting order parameter.



**Figure 7.** The same as in Figure 2, but for  $\theta_0 = 15^\circ$  and various symmetries of superconducting order parameter.

The results for  $\beta = 45^\circ$  (Figure 7(b)) differ quantitatively from their counterparts found for  $\beta = 0^\circ$ , but qualitative conclusions remain the same.

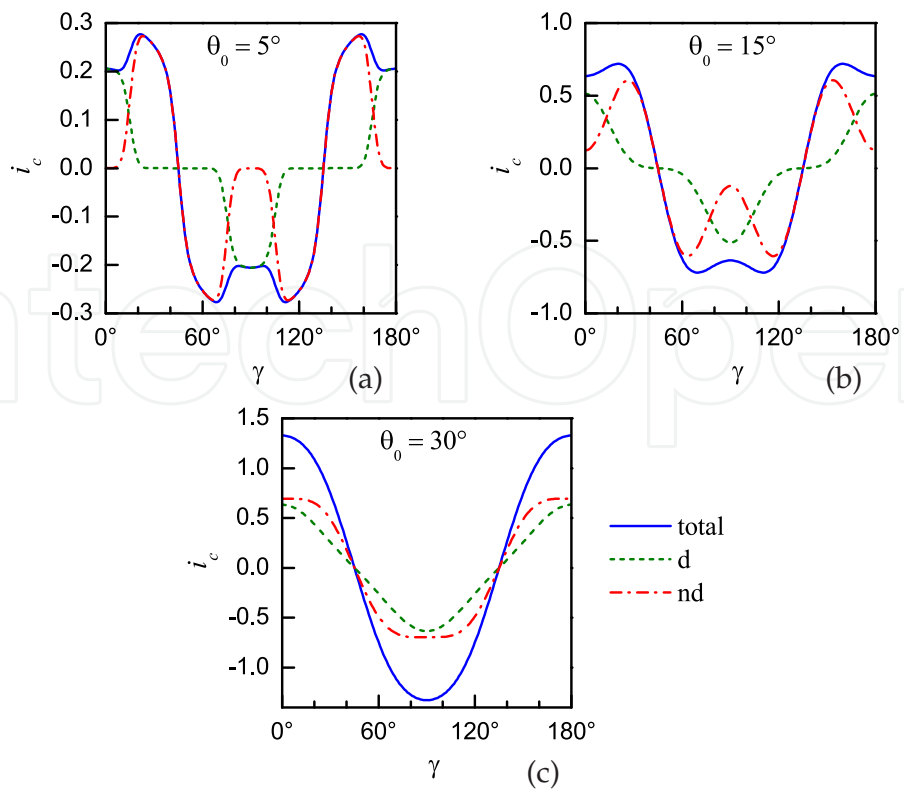
As was indicated above, the temperature behavior of  $i_c$  between ordinary superconductors and cuprates is determined by the order parameter dependence  $\Delta^*(T)$  for the material with much lower  $T_c$ , Nb in our case. This is demonstrated in Figure 8 for  $d$ -, extended  $s$ - and  $s$ -wave CDW high- $T_c$  superconductors. One sees that all curves  $i_c(T)$  are similar, differing only in magnitudes.



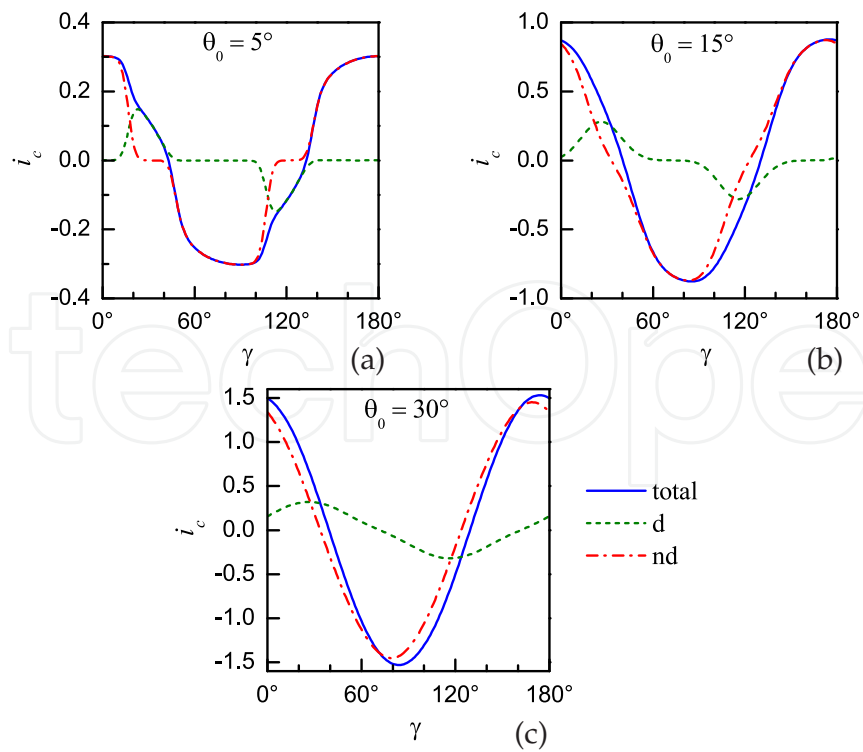
**Figure 8.** Dependences  $i_c(T)$  for  $\gamma = 0^\circ$ ,  $\theta_0 = 30^\circ$ ,  $\alpha_0 = 15^\circ$  and various symmetries of superconducting order parameter.

### 3.2. Analysis of current components

In Figure 9, the dependences  $i_c(\gamma)$  resolved into  $d$  and  $nd$  components are shown for CDW  $d$ -wave superconductors with  $\beta = 0^\circ$ ,  $\alpha = 15^\circ$ , and various  $\theta_0$ 's. Note that the order parameter amplitudes at  $T = 0$  are the same throughout the paper! It comes about that, for a narrow directionality cone  $\theta_0$ , the contribution of the nested ( $d$ ) FS sections has quite a different tilt ( $\gamma$ ) angle behavior as compared to their  $nd$  counterparts. All that gives rise to a non-monotonic pattern seen, e.g., in Figure 2(a).



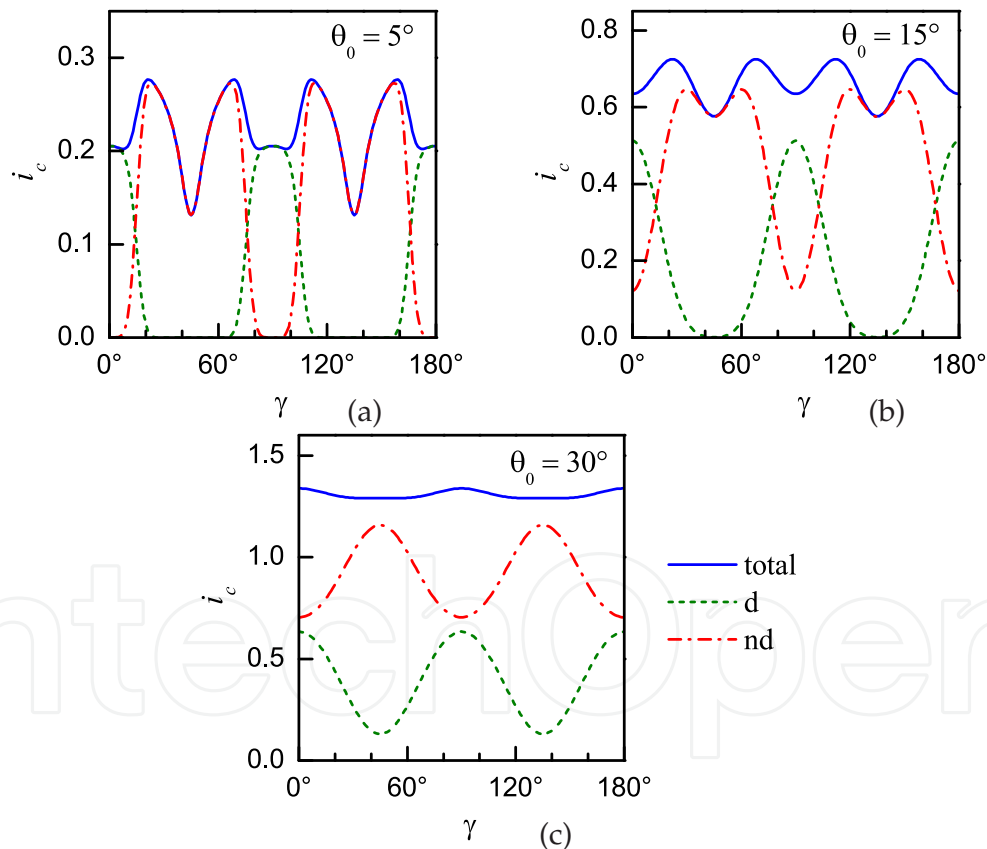
**Figure 9.** Dependences of  $i_c$  and its d and nd components on  $\gamma$  for  $d_{x^2-y^2}$  order parameter symmetry,  $\alpha_0 = 15^\circ$ , and various  $\theta_0$ 's (panel a to c).



**Figure 10.** The same as in Figure 9, but for  $d_{xy}$  order parameter symmetry.

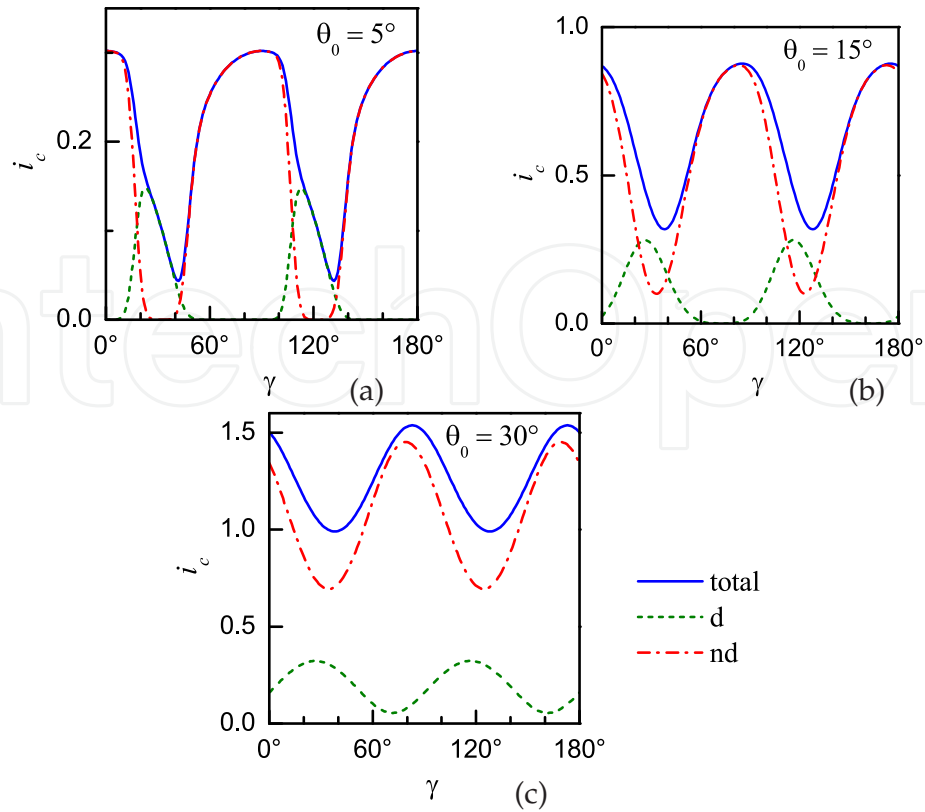
In Figure 10, the same dependences as in Figure 9 are shown, but for  $\beta = 45^\circ$ . One sees that, whatever complex is the  $\gamma$ -angle behavior of d contribution to the overall tunnel currents between a  $d_{xy}$ -superconductor and Nb, the CDW influence is much weaker in governing the dependences  $i_c(\gamma)$ .

It is illustrative to carry out the same analysis in the scenario, when the high- $T_c$  CDW superconductor is assumed to be an extended  $s$ -wave one, i.e. when the sign of superconducting order parameter is the same for all lobes. In the case  $\beta = 0^\circ$ , the corresponding results can be seen in Figure 11, where the  $\gamma$ -dependences of d and nd components of  $i_c$ , as well as the total  $i_c(\gamma)$  dependences, are depicted for the same parameter set as in Figure 9. We see that the d and nd contributions oscillate with the varying  $\gamma$  almost in antiphase, remaining, nevertheless, positive. For large  $\theta_0 = 30^\circ$  (Figure 11(c)), oscillations largely compensate each other making the curve  $i_c(\gamma)$  almost flat, which mimics the behavior appropriate to CDW-free isotropic  $s$ -wave superconductors. However, we emphasize that this, at the first glance, dull result obtained for a relatively wide CDW sector is actually a consequence of a peculiar superposition involving the periodic dependences of d and nd components on  $\gamma$  with rapidly varying amplitudes.



**Figure 11.** The same as in Figure 9, but for  $s_{x^2-y^2}^{\text{ext}}$  order parameter symmetry.

The same plots as in Figure 11 were calculated for  $\beta = 45^\circ$  and depicted, in Figure 12. Here, the directionality angle  $\theta_0$  is the main factor determining the amplitude of  $i_c$ , the role of CDWs being much weaker than in the case  $\beta = 0^\circ$ . It is natural, because now CDW-gapping is concentrated in the nodal regions.



**Figure 12.** The same as in Figure 11, but for  $s_{xy}^{\text{ext}}$  order parameter symmetry.

## 4. Conclusions

The results obtained confirm that the dc Josephson current, probing coherent superconducting properties [30, 31, 33, 37, 119, 256–258], is always suppressed by the electron-hole CDW pairing, which, in agreement with the totality of experimental data, is assumed here to compete with its superconducting electron-electron (Cooper) counterpart [129, 130, 132, 259–262]. We emphasize that, as concerns the quasiparticle current, the results are more ambiguous. In particular, the states on the FS around the nodes of the  $d$ -wave superconducting order parameter are engaged into CDW gapping [126–128, 210, 211, 235, 263], so that the ARPES or tunnel spectroscopy feels the overall energy gaps being larger than their superconducting constituent.

Our examination demonstrates that the emerging CDWs should distort the dependences  $i_c(\gamma)$ , whatever is the symmetry of superconducting order parameter. It is easily seen that, for equal (or almost equal)  $\theta_0$  and  $\alpha$ , CDWs make the  $i_c(\gamma)$  curves non-monotonic and quantitatively different from their CDW-free counterparts. In particular,  $i_c$  values are conspicuously smaller for  $\Sigma \neq 0$ . The required resonance between  $\theta_0$  and  $\alpha$  can be ensured by the proper doping, i.e. a series of samples and respective tunnel junctions should be prepared with attested tilt angles  $\gamma$ , and the Josephson current should be measured for them. Of course, such measurements could be very cumbersome, although they may turn out quite realistic to be performed.



At the same time, when an *s*-wave contribution to the actual order parameter in a cuprate sample is dominant up to the complete disappearance of the *d*-wave component, the  $i_c(\gamma)$  dependences for junctions involving CDW superconductors are no longer constant as in the CDW-free case. This prediction can be verified for CDW superconductors with *a fortiori* *s*-wave order parameters (such materials are quite numerous [123–128]).

In this paper, our approach was purely theoretical. We did not discuss unavoidable experimental difficulties to face with in fabricating Josephson junctions necessary to check the results obtained here. We are fully aware that the emerging problems can be solved on the basis of already accumulated knowledge concerning the nature of grain boundaries in high- $T_c$  oxides [37, 115–119, 122, 264–268]. Note that required junctions can be created at random in an uncontrollable fashion using the break-junction technique [250]. This method allows to comparatively easily detect CDW (pseudogap) influence on the tilt-angle dependences.

To summarize, measurements of the Josephson current between an ordinary superconductor and a *d*-wave or extended *s*-wave one (e.g., a high- $T_c$  oxide) would be useful to detect a possible CDW influence on the electron spectrum of the latter. Similar studies of iron-based superconductors with doping-dependent spin density waves (SDWs) would also be of benefit (see, e.g., recent Reviews [78, 269–275]), since CDW and SDW superconductors have similar, although not identical, properties [123–125].

## Acknowledgements

AIV is grateful to Kasa im. Józefa Mianowskiego, Fundacja na Rzecz Nauki Polskiej, and Fundacja Zygmunta Zaleskiego for the financial support of his visits to Warsaw. The work was partially supported by the Project N 8 of the 2012-2014 Scientific Cooperation Agreement between Poland and Ukraine.

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