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Fuzzy Control of Nonlinear Systems with General Performance Criteria

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1. Introduction

Research on control of non-linear systems over the years has produced many results: control based on linearization, global feedback linearization, non-linear H_∞ control, sliding mode control, variable structure control, state dependent Riccati equation control, etc [5]. This chapter will focus on fuzzy control techniques. Fuzzy control systems have recently shown growing popularity in non-linear system control applications. A fuzzy control system is essentially an effective way to decompose the task of non-linear system control into a group of local linear controls based on a set of design-specific model rules. Fuzzy control also provides a mechanism to blend these local linear control problems all together to achieve overall control of the original non-linear system. In this regard, fuzzy control technique has its unique advantage over other kinds of non-linear control techniques. Latest research on fuzzy control systems design is aimed to improve the optimality and robustness of the controller performance by combining the advantage of modern control theory with the Takagi-Sugeno fuzzy model [7–10, 13, 14].

In this chapter, we address the non-linear state feedback control design of both continuous-time and discrete-time non-linear fuzzy control systems using the Linear Matrix Inequality (LMI) approach. We characterize the solution of the non-linear control problem with the LMI, which provides a sufficient condition for satisfying various performance criteria. A preliminary investigation into the LMI approach to non-linear fuzzy control systems can be found in [7, 8, 13]. The purpose behind this novel approach is to convert a non-linear system control problem into a convex optimization problem which is solved by a LMI at each time. The recent development in convex optimization provides efficient algorithms for solving LMIs. If a solution can be expressed in a LMI form, then there exist optimization algorithms providing efficient global numerical solutions [3]. Therefore if the LMI is feasible, then LMI control technique provides globally stable solutions satisfying the corresponding mixed performance criteria [4, 6, 15–20]. We further propose to employ mixed performance criteria to design the controller guaranteeing the quadratic sub-optimality with inherent stability property in combination with dissipative type of disturbance attenuation.

In the following sections, we first introduce the Takagi-Sugeno fuzzy modelling for non-linear systems in both continuous time and discrete time. We then propose the general performance criteria in section 3. Then, the LMI control solutions are derived to characterize the optimal and robust fuzzy control of continuous time and discrete time non-linear systems, respectively. The inverted pendulum system control is used as an illustrative example to demonstrate the effectiveness and robustness of our proposed approaches.

The following notation is used in this work: $x \in \mathcal{R}^n$ denotes n-dimensional real vector with norm $\|x\| = (x^T x)^{1/2}$ where $(.)^T$ indicates transpose. $A \geq 0$ for a symmetric matrix denotes a positive semi-definite matrix. $\mathcal{L}_2$ and l_2 denotes the space of infinite sequences of finite dimensional random vectors with finite energy, i.e. $\int_0^\infty \|x_t\|^2 < \infty$ in continuous-time, and $\Sigma_{k=0}^\infty \|x_k\|^2 < \infty$ in discrete-time, respectively.

2. Takagi-Sugeno system model

The importance of the Takagi-Sugeno fuzzy system model is that it provides an effective way to decompose a complicated non-linear system into local dynamical relations and express those local dynamics of each fuzzy implication rule by a linear system model. The overall fuzzy non-linear system model is achieved by fuzzy "blending" of the linear system models, so that the overall non-linear control performance is achieved. Both of the continuous-time and the discrete-time system models are summarized below.

2.1. Continuous-time Takagi-Sugeno system model

The ith rule of the Takagi-Sugeno fuzzy model can be expressed by the following forms:

Model Rule i:
If $\varphi_1(t)$ is $M_{i1}, \varphi_2(t)$ is $M_{i2},...$ and $\varphi_p(t)$ is M_{ip},
Then the input-affine continuous-time fuzzy system equation is:

$$\dot{x}(t) = A_i x(t) + B_i u(t) + F_i w(t)$$
$$y(t) = C_i x(t) + D_i u(t) + Z_i w(t)$$
$$i = 1, 2, 3, ..., r \qquad (1)$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the control input vector, $y(t) \in \mathcal{R}^q$ is the performance output vector, $w(t) \in \mathcal{R}^s$ is $\mathcal{L}_2$ type of disturbance, r is the total number of model rules, M_{ij} is the fuzzy set. The coefficient matrices are $A_i \in \mathcal{R}^{n \times n}, B_i \in \mathcal{R}^{n \times m}, F_i \in \mathcal{R}^{n \times s}, C_i \in \mathcal{R}^{q \times n}, D_i \in \mathcal{R}^{q \times m}, Z_i \in \mathcal{R}^{q \times s}$. And $\varphi_1, ..., \varphi_p$ are known premise variables, which can be functions of state variables, external disturbance and time.

It is assumed that the premises are not the function of the input vector $u(t)$, which is needed to avoid the defuzzification process of fuzzy controller. If we use $\varphi(t)$ to denote the vector containing all the individual elements $\varphi_1(t), \varphi_2(t), ..., \varphi_p(t)$, then the overall fuzzy system is

$$\dot{x}(t) = \frac{\sum_{i=1}^r g_i(\varphi(t))[A_i x(t) + B_i u(t) + F_i w(t)]}{\sum_{i=1}^r g_i(\varphi(t))} = \sum_{i=1}^r h_i(\varphi(t))[A_i x(t) + B_i u(t) + F_i w(t)]$$

$$y(t) = \frac{\sum_{i=1}^r g_i(\varphi(t))[C_i x(t) + D_i u(t) + Z_i w(t)]}{\sum_{i=1}^r g_i(\varphi(t))} = \sum_{i=1}^r h_i(\varphi(t))[C_i x(t) + D_i u(t) + Z_i w(t)] \quad (2)$$

where

$$\varphi(t) = [\varphi_1(t), \varphi_2(t), ..., \varphi_p(t)] \tag{3}$$

$$g_i(\varphi(t)) = \prod_{j=1}^{p} M_{ij}(\varphi_j(t)) \tag{4}$$

$$h_i(\varphi(t)) = \frac{g_i(\varphi(t))}{\sum_{i=1}^{r} g_i(\varphi(t))} \tag{5}$$

for all time t. The term $M_{ij}(\varphi_j(t))$ is the grade membership function of $\phi_j(t)$ in M_{ij}.

Since, the following properties hold

$$\sum_{i=1}^{r} g_i(\varphi(t)) > 0$$

$$g_i(\varphi(t)) \geq 0, i = 1, 2, 3, ..., r \tag{6}$$

We have

$$\sum_{i=1}^{r} h_i(\varphi(t)) = 1$$

$$h_i(\varphi(t)) \geq 0, i = 1, 2, 3, ..., r \tag{7}$$

for all time t.

It is assumed that the state feedback is available and the non-linear state feedback control input is given by

$$u(t) = -\sum_{i=1}^{r} h_i(\varphi(t))K_i x(t) \tag{8}$$

Substituting this into the system and performance output equation, we have

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))(A_i - B_i K_j)x(t) + \sum_{i=1}^{r} h_i(\varphi(t))F_i w(t)$$

$$y(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))(C_i - D_i K_j)x(k) + \sum_{i=1}^{r} h_i(\varphi(t))Z_i w(t) \tag{9}$$

Using the notation

$$G_{ij} = A_i - B_i K_j$$
$$H_{ij} = C_i - D_i K_j \tag{10}$$

then the system equation becomes

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))G_{ij}x(t) + \sum_{i=1}^{r} h_i(\varphi(t))F_i w(t)$$

$$y(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))H_{ij}x(t) + \sum_{i=1}^{r} h_i(\varphi(t))Z_i w(t) \tag{11}$$

2.2. Discrete-time Takagi-Sugeno system model

At time step k, the ith rule of the Takagi-Sugeno fuzzy model can be expressed by the following forms:

Model Rule i:
If $\varphi_1(k)$ is $M_{i1}, \varphi_2(k)$ is $M_{i2},...$ and $\varphi_p(k)$ is M_{ip},
Then the input-affine discrete-time fuzzy system equation is:

$$x(k+1) = A_i x(k) + B_i u(k) + F_i w(k)$$
$$y(k) = C_i x(k) + D_i u(k) + Z_i w(k)$$
$$i = 1, 2, 3, ..., r \tag{12}$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $u(k) \in \mathcal{R}^m$ is the control input vector, $y(k) \in \mathcal{R}^q$ is the performance output vector, $w(k) \in \mathcal{R}^s$ is l_2 type of disturbance, r is the total number of model rules, M_{ij} is the fuzzy set. The coefficient matrices are $A_i \in \mathcal{R}^{n \times n}, B_i \in \mathcal{R}^{n \times m}, F_i \in \mathcal{R}^{n \times s}, C_i \in \mathcal{R}^{q \times n}, D_i \in \mathcal{R}^{q \times m}, Z_i \in \mathcal{R}^{q \times s}$. And $\varphi_1, ..., \varphi_p$ are known premise variables which can be functions of state variables, external disturbance and time.

It is assumed that the premises are not the function of the input vector $u(k)$, which is needed to avoid the defuzzification process of fuzzy controller. If we use $\varphi(k)$ to denote the vector containing all the individual elements $\varphi_1(k), \varphi_2(k), ..., \varphi_p(k)$, then the overall fuzzy system is

$$x(k+1) = \frac{\sum_{i=1}^r g_i(\varphi(k)) A_i x(k) + B_i u(k) + F_i w(k)}{\sum_{i=1}^r g_i(\varphi(k))} = \sum_{i=1}^r h_i(\varphi(k)) A_i x(k) + B_i u(k) + F_i w(k)$$

$$y(k) = \frac{\sum_{i=1}^r g_i(\varphi(k)) C_i x(k) + D_i u(k) + Z_i w(k)}{\sum_{i=1}^r g_i(\varphi(k))} = \sum_{i=1}^r h_i(\varphi(k)) C_i x(k) + D_i u(k) + Z_i w(k) \tag{13}$$

where

$$\varphi(k) = [\varphi_1(k), \varphi_2(k), ..., \varphi_p(k)] \tag{14}$$

$$g_i(\varphi(k)) = \prod_{j=1}^p M_{ij}(\varphi_j(k)) \tag{15}$$

$$h_i(\varphi(k)) = \frac{g_i(\varphi(k))}{\sum_{i=1}^r g_i(\varphi(k))} \tag{16}$$

for all k. The term $M_{ij}(\varphi_j(k))$ is the grade membership function of $\phi_j(k)$ in M_{ij}.

Since, the following properties hold

$$\sum_{i=1}^r g_i(\varphi(k)) > 0$$
$$g_i(\varphi(k)) \geq 0, i = 1, 2, 3, ..., r \tag{17}$$

We have

$$\sum_{i=1}^r h_i(\varphi(k)) = 1$$
$$h_i(\varphi(k)) \geq 0, i = 1, 2, 3, ..., r \tag{18}$$

for all k.

It is assumed that the state feedback is available and the non-linear state feedback control input is given by

$$u(k) = -\sum_{i=1}^{r} h_i(\varphi(k))K_i x(k) \tag{19}$$

Substituting this into the system and performance output equation, we have

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))(A_i - B_i K_j)x(k) + \sum_{i=1}^{r} h_i(\varphi(k))F_i w(k)$$

$$y(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))(C_i - D_i K_j)x(k) + \sum_{i=1}^{r} h_i(\varphi(k))Z_i w(k) \tag{20}$$

Using the notation

$$G_{ij} = A_i - B_i K_j$$
$$H_{ij} = C_i - D_i K_j \tag{21}$$

then the system equation becomes

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))G_{ij}x(k) + \sum_{i=1}^{r} h_i(\varphi(k))F_i w(k)$$

$$y(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))H_{ij}x(k) + \sum_{i=1}^{r} h_i(\varphi(k))Z_i w(k) \tag{22}$$

3. General performance criteria

In this section, we propose the general performance criteria for non-linear control design, which yields a mixed Non-Linear Quadratic Regular (NLQR) in combination with $\mathcal{H}_\infty$ or dissipative performance index. The commonly used system performance criteria, including bounded-realness, positive-realness, sector boundedness and quadratic cost criterion, become special cases of the general performance criteria. Both the continuous-time and discrete-time general performance criteria are given below:

3.1. Continuous-time general performance criteria

Consider the quadratic Lyapunov function

$$V(t) = x^T(t)Px(t) > 0 \tag{23}$$

for the following difference inequality

$$\dot{V}(t) + x^T(t)Qx(t) + u^T(t)Ru(t) + \alpha y^T(t)y(t) - \beta y^T(t)w(t) + \gamma w^T(t)w(t) \le 0 \tag{24}$$

with $Q > 0, R > 0$ functions of $x(t)$.

Note that upon integration over time from 0 to T_f, (24) yields

$$V(T_f) + \int_0^{T_f} [(x^T(t)Qx(t) + u^T(t)Ru(t)]dt +$$

$$\int_0^{T_f} [\alpha y^T(t)y(t) - \beta y^T(t)w(t) + \gamma w^T(t)w(t)]dt \leq V(0) \tag{25}$$

By properly specifying the value of weighing matrices Q, R, C_i, D_i, Z_i and α, β, γ, mixed performance criteria can be used in non-linear control design, which yields a mixed Non-linear Quadratic Regulator (NLQR) in combination with dissipative type performance index with disturbance reduction capability. For example, if we take $\alpha = 1, \beta = 0, \gamma < 0$, (25) yields

$$V(T_f) + \int_0^{T_f} [(x^T(t)Qx(t) + u^T(t)Ru(t) + y^T(t)y(t)]dt +$$

$$\leq V(0) - \gamma \int_0^{T_f} [w^T(t)w(t)]dt \tag{26}$$

which is a mixed $NLQR - H_\infty$ Design [16–18].

Other possible performance criteria which can be used in this framework with various design parameters α, β, γ are given in Table.1. Design coefficients α and γ can be maximized or minimized to optimize the controller behavior. It should also be noted that the satisfaction of any of the criteria in Table 1 will also guarantee asymptotic stability of the controlled system.

3.2. Discrete-time general performance criteria

Consider the quadratic Lyapunov function

$$V(k) = x^T(k)Px(k) \tag{27}$$

for the following difference inequality

$$V(k+1) - V(k) + x^T(k)Qx(k) + u^T(k)Ru(k) + \alpha y^T(k)y(k) - \beta y^T(k)w(k) + \gamma w^T(k)w(k) \leq 0 \tag{28}$$

with $Q > 0, R > 0$ functions of $x(k)$.

Note that upon summation over k, (28) yields

$$V(N) + \sum_{k=0}^{N-1} (x^T(k)Qx(k) + u^T(k)Ru(k) + \alpha y^T(k)y(k) - \beta y^T(k)w(k) + \gamma w^T(k)w(k)) \leq V(0) \tag{29}$$

By properly specifying the value of weighing matrices Q, R, C_i, D_i, Z_i and α, β, γ, mixed performance criteria can be used in non-linear control design, which yields a mixed Non-linear Quadratic Regulator (NLQR) in combination with dissipative type performance index with disturbance reduction capability. For example, if we take $\alpha = 1, \beta = 0, \gamma < 0$, (29) yields

$$V(N) + \sum_{k=0}^{N-1} (x^T(k)Qx(k) + u^T(k)Ru(k) + \alpha y^T(k)y(k)) \leq V(0) - \gamma \sum_{k=0}^{N-1} w^T(k)w(k) \tag{30}$$

which is a mixed $NLQR - H_\infty$ Design [16–18]. In (19), γ can be minimized to achieve a smaller $l_2 - l_2$ or H_∞ gain for the closed loop system.

Other possible performance criteria which can be used in this framework with various design parameters α, β, γ are given in Table.1. Design coefficients α and γ can be maximized or minimized to optimize the controller behavior. It should also be noted that the satisfaction of any of the criteria in Table 1 will also guarantee asymptotic stability of the controlled system.

α	β	γ	Performance Criteria
1	0	<0	$NLQR - \mathcal{H}_\infty$ Design
0	1	0	$NLQR - Passivity\ Design$
0	1	>0	$NLQR - Input\ Strict\ Passivity\ Design$
>0	1	0	$NLQR - Output\ Strict\ Passivity\ Design$
>0	1	>0	$NLQR - Very\ Strict\ Passivity$

Table 1. Various performance criteria in a general framework

4. Fuzzy LMI control of continuous time non-linear systems with general performance criteria

The main results of this chapter are summarized in section 4 and section 5. The following theorem provides the fuzzy LMI control to the continuous time non-linear systems with general performance criteria.

Theorem 1 Given the system model and performance output (2) and control input (8), if there exist matrices $S = P^{-1} > 0$ for all $t \geq 0$, such that the following LMI holds:

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\ * & \Lambda_{22} & \Lambda_{23} & 0 & 0 \\ * & * & I & 0 & 0 \\ * & * & * & R^{-1} & 0 \\ * & * & * & * & I \end{bmatrix} \geq 0 \tag{31}$$

where

$$\Lambda_{11} = -\frac{1}{2}[SA_i^T - M_j B_i^T + SA_j^T - M_i^T B_j^T + A_i S - B_i M_j + A_j S - B_j M_i]$$

$$\Lambda_{12} = -\frac{1}{2}(F_i + F_j) + \frac{\beta}{4}[SC_i^T - M_j D_i^T + SC_j^T - M_i^T D_j^T]$$

$$\Lambda_{13} = \frac{1}{2}\alpha^{1/2}[SC_i^T - M_j D_i^T + SC_j^T - M_i^T D_j^T]$$

$$\Lambda_{14} = \frac{1}{2}(M_i^T + M_j^T)$$

$$\Lambda_{15} = SQ^{T/2}$$

$$\Lambda_{22} = -\gamma I + \frac{1}{2}\beta(Z_i + Z_j)^T$$

$$\Lambda_{23} = \frac{1}{2}\alpha^{1/2}[Z_i + Z_j]^T \tag{32}$$

using the notation

$$M_i = K_i P^{-1} = K_i S \tag{33}$$

then inequality (24) is satisfied.

Proof

By applying system model and performance output (2)(11), and state feedback input (8), the performance index inequality (24) becomes

$$[\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))G_{ij}x(t) + \sum_{i=1}^{r} h_i(\varphi(t))F_i w(t)]^T P x(t) +$$

$$x^T(t)P[\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))G_{ij}x(t) + \sum_{i=1}^{r} h_i(\varphi(t))F_i w(t)] +$$

$$x^T(t)Qx(t) + [-\sum_{i=1}^{r} h_i \varphi(t)K_i x(t)]^T R[-\sum_{i=1}^{r} h_i \varphi(t)K_i x(t)]$$

$$\alpha[\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))H_{ij}x(t) + \sum_{i=1}^{r} h_i(\varphi(t))Z_i w(t)]^T$$

$$\times [\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))H_{ij}x(t) + \sum_{i=1}^{r} h_i(\varphi(t))Z_i w(t)]$$

$$-\beta[\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\varphi(t))h_j(\varphi(t))H_{ij}x(t) + \sum_{i=1}^{r} h_i(\varphi(t))Z_i w(t)]^T \times w(t)$$

$$+\gamma w^T(t)w(t) \le 0 \tag{34}$$

Inequality (34) is equivalent to

$$[x^T(t) \ w^T(t)] \times \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ * & \Delta_{22} \end{bmatrix} \times \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \le 0 \tag{35}$$

where

$$\Delta_{11} = (\sum_i \sum_j h_i h_j G_{ij})^T P + P(\sum_i \sum_j h_i h_j G_{ij}) + Q + [\sum_i h_i K_i]^T R[\sum_i h_i K_i] +$$

$$\alpha[\sum_i \sum_j h_i h_j H_{ij}]^T [\sum_i \sum_j h_i h_j H_{ij}]$$

$$\Delta_{12} = P(\sum_i h_i F_i) + \alpha[\sum_i \sum_j h_i h_j H_{ij}]^T [\sum_i h_i Z_i] - \frac{\beta}{2}[\sum_i \sum_j h_i h_j H_{ij}]^T$$

$$\Delta_{22} = \gamma I + \alpha[\sum_i h_i Z_i]^T [\sum_i h_i Z_i] - \beta[\sum_i h_i Z_i]^T \tag{36}$$

Inequality (35) can be rewritten as

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} - \alpha \begin{bmatrix} [\sum_i \sum_j h_i h_j H_{ij}]^T \\ [\sum_i h_i Z_i]^T \end{bmatrix} \times [[\sum_i \sum_j h_i h_j H_{ij}] \ [\sum_i h_i Z_i]] \ge 0 \tag{37}$$

where

$$\Theta_{11} = -(\sum_i \sum_j h_i h_j G_{ij})^T P - P(\sum_i \sum_j h_i h_j G_{ij}) - Q - [\sum_i h_i K_i]^T R[\sum_i h_i K_i]$$

$$\Theta_{12} = -P(\sum_i h_i F_i) + \frac{\beta}{2}[\sum_i \sum_j h_i h_j H_{ij}]^T$$

$$\Theta_{22} = -\gamma I + \beta[\sum_i h_i Z_i]^T \tag{38}$$

By applying Schur complement to inequality (37), we have

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \alpha^{1/2}[\sum_i \sum_j h_i h_j H_{ij}]^T \\ * & \Theta_{22} & \alpha^{1/2}[\sum_i h_i Z_i]^T \\ * & * & I \end{bmatrix} \geq 0 \tag{39}$$

Similarly, inequality (39) can also be written as

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \alpha^{1/2}[\sum_i \sum_j h_i h_j H_{ij}]^T \\ * & \Phi_{22} & \alpha^{1/2}[\sum_i h_i Z_i]^T \\ * & * & I \end{bmatrix} - \begin{bmatrix} [\sum_i h_i K_i]^T \\ 0 \\ 0 \end{bmatrix} R \left[[\sum_i h_i K_i] \ 0 \ 0 \right] \geq 0 \tag{40}$$

where

$$\Phi_{11} = -(\sum_i \sum_j h_i h_j G_{ij})^T P - P(\sum_i \sum_j h_i h_j G_{ij}) - Q$$

$$\Phi_{12} = -P(\sum_i h_i F_i) + \frac{\beta}{2}[\sum_i \sum_j h_i h_j H_{ij}]^T$$

$$\Phi_{22} = -\gamma I + \beta[\sum_i h_i Z_i]^T \tag{41}$$

By applying Schur complement again to (40), we have

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \alpha^{1/2}[\sum_i \sum_j h_i h_j H_{ij}]^T & [\sum_i h_i K_i]^T \\ * & \Phi_{22} & \alpha^{1/2}[\sum_i h_i Z_i]^T & 0 \\ * & * & I & 0 \\ * & * & * & R^{-1} \end{bmatrix} \geq 0 \tag{42}$$

Equivalently, we have

$$\sum_i \sum_j h_i h_j \times \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ * & \Gamma_{22} & \Gamma_{23} & 0 \\ * & * & I & 0 \\ * & * & * & R^{-1} \end{bmatrix} \geq 0 \tag{43}$$

where

$$\Gamma_{11} = -\frac{1}{2}[(A_i - B_iK_j) + (A_j - B_jK_i)]^T P - \frac{1}{2}P[(A_i - B_iK_j) + (A_j - B_jK_i)] - Q$$

$$\Gamma_{12} = -\frac{1}{2}P(F_i + F_j) + \frac{\beta}{4}[(C_i - D_iK_j) + (C_j - D_jK_i)]^T$$

$$\Gamma_{13} = -\frac{1}{2}\alpha^{1/2}[(C_i - D_iK_j) + (C_j - D_jK_i)]^T$$

$$\Gamma_{14} = -\frac{1}{2}(K_i + K_j)^T$$

$$\Gamma_{22} = -\gamma I + \frac{1}{2}\beta(Z_i + Z_j)^T$$

$$\Gamma_{23} = \frac{1}{2}\alpha^{1/2}(Z_i + Z_j)^T \tag{44}$$

Therefore, we have the following LMI

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ * & \Gamma_{22} & \Gamma_{23} & 0 \\ * & * & I & 0 \\ * & * & * & R^{-1} \end{bmatrix} \geq 0 \tag{45}$$

By multiplying both sides of the LMI above by the block diagonal matrix $diag\{S, I, I, I\}$, where $S = P^{-1}$, and using the notation

$$M_i = K_iP^{-1} = K_iS \tag{46}$$

we obtain

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ * & X_{22} & X_{23} & 0 \\ * & * & I & 0 \\ * & * & * & R^{-1} \end{bmatrix} \geq 0 \tag{47}$$

where

$$X_{11} = -\frac{1}{2}[SA_i^T - M_jB_i^T + SA_j^T - M_i^TB_j^T + A_iS - B_iM_j + A_jS - B_jM_i] - SQS$$

$$X_{12} = -\frac{1}{2}(F_i + F_j) + \frac{\beta}{4}[SC_i^T - M_j^TD_i^T + SC_j^T - M_i^TD_j^T]$$

$$X_{13} = \frac{1}{2}\alpha^{1/2}[SC_i^T - M_j^TD_i^T + SC_j^T - M_i^TD_j^T]$$

$$X_{14} = \frac{1}{2}(M_i^T + M_j^T)$$

$$X_{22} = -\gamma I + \frac{1}{2}\beta(Z_i + Z_j)^T$$

$$X_{23} = \frac{1}{2}\alpha^{1/2}(Z_i + Z_j)^T \tag{48}$$

By applying Schur complement again, the final LMI is derived

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\ * & \Lambda_{22} & \Lambda_{23} & 0 & 0 \\ * & * & I & 0 & 0 \\ * & * & * & R^{-1} & 0 \\ * & * & * & * & I \end{bmatrix} \geq 0 \tag{49}$$

where

$$\Lambda_{11} = -\frac{1}{2}[SA_i^T - M_j B_i^T + SA_j^T - M_i^T B_j^T + A_i S - B_i M_j + A_j S - B_j M_i]$$

$$\Lambda_{12} = -\frac{1}{2}(F_i + F_j) + \frac{\beta}{4}[SC_i^T - M_j D_i^T + SC_j^T - M_i^T D_j^T]$$

$$\Lambda_{13} = \frac{1}{2}\alpha^{1/2}[SC_i^T - M_j D_i^T + SC_j^T - M_i^T D_j^T]$$

$$\Lambda_{14} = \frac{1}{2}(M_i^T + M_j^T)$$

$$\Lambda_{15} = SQ^{T/2}$$

$$\Lambda_{22} = -\gamma I + \frac{1}{2}\beta(Z_i + Z_j)^T$$

$$\Lambda_{23} = \frac{1}{2}\alpha^{1/2}[Z_i + Z_j]^T \tag{50}$$

Hence, if the LMI (49) holds, inequality (24) is satisfied. This concludes the proof of the theorem.

Remark 1: For the chosen performance criterion, the LMI (49) need to be solved at each time to find matrices S, M, by using relation (33), we can find the feedback control gain, therefore, the feedback control can be found to satisfy the chosen criterion.

5. Fuzzy LMI control of discrete time non-linear systems with general performance criteria

This section summarizes the main results for fuzzy LMI control of discrete time non-linear systems with general performance criteria:

Theorem 2: Given the closed loop system and performance output (13), and control input (19), if there exist matrices $S = P^{-1} > 0$ for all $k \geq 0$, such that the following LMI holds:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 \\ * & * & S & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 \\ * & * & * & * & R^{-1} & 0 \\ * & * & * & * & * & I \end{bmatrix} \geq 0 \tag{51}$$

where

$$\Xi_{11} = S$$

$$\Xi_{12} = \frac{\beta}{4}(C_i S - D_i Y_j + C_j S - D_j Y_i)^T$$

$$\Xi_{13} = \frac{1}{2}(A_i S - B_i Y_j + A_j S - B_j Y_i)^T$$

$$\Xi_{14} = \frac{1}{2}\alpha^{1/2}(C_i S - D_i Y_j + C_j S - D_j Y_i)^T$$

$$\Xi_{15} = \frac{1}{2}(Y_i + Y_j)^T$$

$$\Xi_{16} = SQ^{T/2}$$

$$\Xi_{22} = -\gamma I + \frac{\beta}{2}(Z_i + Z_j)^T$$

$$\Xi_{23} = \frac{1}{2}\alpha^{1/2}(F_i + F_j)^T$$

$$\Xi_{24} = \frac{1}{2}\alpha^{1/2}(Z_i + Z_j)^T \tag{52}$$

and

$$S(k+1) > S(k) \tag{53}$$

where $S(k) = P^{-1}(k)$, then (28) is satisfied with the feedback control gain being found by

$$K(k) = Y(k)P(k) \tag{54}$$

Proof
The performance index inequality (28) can be explicitly written as

$$[\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))G_{ij}x(k) + \sum_{i=1}^{r} h_i(\varphi(k))F_i w(k)]^T$$

$$\times P \times [\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))G_{ij}x(k) + \sum_{i=1}^{r} h_i(\varphi(k))F_i w(k)]$$

$$-x^T(k)Px(k) + x^T(k)Qx(k) + [-\sum_{i=1}^{r} h_i(\varphi(k))K_i x(k)]^T R[-\sum_{i=1}^{r} h_i(\varphi(k))K_i x(k)] +$$

$$\alpha[\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))H_{ij}x(k) + \sum_{i=1}^{r} h_i(\varphi(k))Z_i w(k)]^T$$

$$\times [\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))H_{ij}x(k) + \sum_{i=1}^{r} h_i(\varphi(k))Z_i w(k)]$$

$$-\beta[\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\varphi(k))h_j(\varphi(k))H_{ij}x(k) + \sum_{i=1}^{r} h_i(\varphi(k))Z_i w(k)]^T \times w(k)$$

$$+\gamma w^T(k)w(k) \leq 0 \tag{55}$$

Equivalently,

$$
\begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix} \begin{bmatrix} -P+Q & 0 \\ 0 & \gamma I \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} +
$$

$$
\begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix} \begin{bmatrix} (\sum_i \sum_j h_i h_j G_{ij}) & (\sum_i h_i F_i) \end{bmatrix}^T \times P \times \begin{bmatrix} (\sum_i \sum_j h_i h_j G_{ij}) & (\sum_i h_i F_i) \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} +
$$

$$
+ x^T(k) [\sum_i h_i K_i]^T R [\sum_i h_i K_i] x(k) +
$$

$$
\alpha \begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix} \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix}^T \times \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} +
$$

$$
- \beta \begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix} \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix}^T w(k) \leq 0
$$

$$(56)$$

which can be written, after collecting terms, as

$$
\begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix} \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ * & \Upsilon_{22} \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} +
$$

$$
\begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix} \begin{bmatrix} (\sum_i \sum_j h_i h_j G_{ij}) & (\sum_i h_i F_i) \end{bmatrix}^T \times P \times \begin{bmatrix} (\sum_i \sum_j h_i h_j G_{ij}) & (\sum_i h_i F_i) \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} +
$$

$$
\alpha \begin{bmatrix} x^T(k) & w^T(k) \end{bmatrix} \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix}^T \times \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} \geq 0
$$

$$(57)$$

where

$$
\Upsilon_{11} = P - Q - [\sum_i h_i K_i]^T R [\sum_i h_i K_i]
$$

$$
\Upsilon_{12} = \frac{\beta}{2} [\sum_i \sum_j h_i h_j H_{ij}]^T
$$

$$
\Upsilon_{22} = -\gamma I + \beta [\sum_i h_i Z_i]^T
$$

$$(58)$$

Equivalently, we have

$$
\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ * & \Upsilon_{22} \end{bmatrix} - \begin{bmatrix} (\sum_i \sum_j h_i h_j G_{ij}) & (\sum_i h_i F_i) \end{bmatrix}^T \times P \times \begin{bmatrix} (\sum_i \sum_j h_i h_j G_{ij}) & (\sum_i h_i F_i) \end{bmatrix} -
$$

$$
\alpha \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix}^T \times \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix} \geq 0
$$

$$(59)$$

By applying Schur complement, we obtain

$$
\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & (\sum_i \sum_j h_i h_j G_{ij}))^T \\ * & \Upsilon_{22} & (\sum_i h_i F_i)^T \\ * & * & P^{-1} \end{bmatrix} - \alpha \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix}^T \times \begin{bmatrix} (\sum_i \sum_j h_i h_j H_{ij}) & (\sum_i h_i Z_i) \end{bmatrix} \geq 0
$$

$$(60)$$

By applying Schur complement again, we obtain

$$
\begin{bmatrix}
\Upsilon_{11} & \Upsilon_{12} & (\sum_i \sum_j h_i h_j G_{ij}))^T & \alpha^{1/2}(\sum_i \sum_j h_i h_j H_{ij})^T \\
* & \Upsilon_{22} & (\sum_i h_i F_i)^T & \alpha^{1/2}(\sum_i h_i Z_i)^T \\
* & * & P^{-1} & 0 \\
* & * & * & I
\end{bmatrix} \geq 0
$$

(61)

Equivalently, the following inequality holds

$$
\begin{bmatrix}
\Psi_{11} & \Psi_{12} & (\sum_i \sum_j h_i h_j G_{ij}))^T & \alpha^{1/2}(\sum_i \sum_j h_i h_j H_{ij})^T \\
* & \Psi_{22} & (\sum_i h_i F_i)^T & \alpha^{1/2}(\sum_i h_i Z_i)^T \\
* & * & P^{-1} & 0 \\
* & * & * & I
\end{bmatrix} -
$$
$$
\begin{bmatrix}
(\sum_i h_i K_i)^T \\
0 \\
0 \\
0
\end{bmatrix} \times R \times \left[(\sum_i h_i K_i)\ 0\ 0\ 0 \right] \geq 0
$$

(62)

where

$$
\Psi_{11} = P - Q
$$

$$
\Psi_{12} = \frac{\beta}{2} [\sum_i \sum_j h_i h_j H_{ij}]^T
$$

$$
\Psi_{22} = -\gamma I + \beta [\sum_i h_i Z_i]^T
$$

(63)

By applying Schur complement one more time, we have

$$
\begin{bmatrix}
\Psi_{11} & \Psi_{12} & (\sum_i \sum_j h_i h_j G_{ij}))^T & \alpha^{1/2}(\sum_i \sum_j h_i h_j H_{ij})^T & (\sum_i h_i K_i)^T \\
* & \Psi_{22} & (\sum_i h_i F_i)^T & \alpha^{1/2}(\sum_i h_i Z_i)^T & 0 \\
* & * & P^{-1} & 0 & 0 \\
* & * & * & I & 0 \\
* & * & * & * & R^{-1}
\end{bmatrix} \geq 0
$$

(64)

By factoring out the $\sum_i \sum_j h_i(\varphi_k) h_j(\varphi_k)$ term, we have

$$
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\
* & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 \\
* & * & P^{-1} & 0 & 0 \\
* & * & * & I & 0 \\
* & * & * & * & R^{-1}
\end{bmatrix} \geq 0
$$

(65)

where

$$\Omega_{11} = P - Q$$

$$\Omega_{12} = \frac{\beta}{4}[H_{ji} + H_{ij}]^T$$

$$\Omega_{13} = \frac{1}{2}(G_{ji} + G_{ij}))^T$$

$$\Omega_{14} = \frac{1}{2}\alpha^{1/2}(H_{ij} + H_{ji})^T$$

$$\Omega_{15} = \frac{1}{2}(K_i + K_j)^T$$

$$\Omega_{22} = -\gamma I + \frac{\beta}{2}(Z_i + Z_j)^T$$

$$\Omega_{23} = \frac{1}{2}(F_i + F_j)^T$$

$$\Omega_{24} = \frac{1}{2}\alpha^{1/2}(Z_i + Z_j)^T$$

(66)

By pre-multiplying and post-multiplying the matrix with the block diagonal matrix $diag(S, I, I, I, I)$, where $S = P^{-1}$, and applying Schur complement again, the following LMI result is obtained

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 \\ * & * & S & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 \\ * & * & * & * & R^{-1} & 0 \\ * & * & * & * & * & I \end{bmatrix} \geq 0$$

(67)

where

$$\Xi_{11} = S$$

$$\Xi_{12} = \frac{\beta}{4}(C_iS - D_iY_j + C_jS - D_jY_i)^T$$

$$\Xi_{13} = \frac{1}{2}(A_iS - B_iY_j + A_jS - B_jY_i)^T$$

$$\Xi_{14} = \frac{1}{2}\alpha^{1/2}(C_iS - D_iY_j + C_jS - D_jY_i)^T$$

$$\Xi_{15} = \frac{1}{2}(Y_i + Y_j)^T$$

$$\Xi_{16} = SQ^{T/2}$$

$$\Xi_{22} = -\gamma I + \frac{\beta}{2}(Z_i + Z_j)^T$$

$$\Xi_{23} = \frac{1}{2}\alpha^{1/2}(F_i + F_j)^T$$

$$\Xi_{24} = \frac{1}{2}\alpha^{1/2}(Z_i + Z_j)^T$$

(68)

where $S(k) = P^{-1}(k)$, then (28) is satisfied with the feedback control gain being found by

$$K(k) = Y(k)P(k) \tag{69}$$

6. Application to the inverted pendulum system

The inverted pendulum on a cart problem is a benchmark control problem used widely to test control algorithms. A pendulum beam attached at one end can rotate freely in the vertical 2-dimensional plane. The angle of the beam with respect to the vertical direction is denoted at angle θ. The external force u is desired to set angle of the beam θ (x_1) and angular velocity $\dot{\theta}$ (x_2) to zero while satisfying the mixed performance criteria. A model of the inverted pendulum on a cart problem is given by [1, 9]:

$$\dot{x}_1 = x_2 + \epsilon_1 w$$
$$\dot{x}_2 = \frac{gsin(x_1) - amLx_2^2 sin(2x_1)/2 - acos(x_1)u}{4L/3 - amLcos^2(x_1)} + \epsilon_2 w \tag{70}$$

where x_1 is the angle of the pendulum from vertical direction, x_2 is the angular velocity of the pendulum, g is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, L is the length of the center of mass (the entire length of the pendulum beam equals $2L$), u is the external force, control input to the system, w is the $\mathcal{L}_2$ type of disturbance, $a = \frac{1}{m+M}$ is a constant, and $\epsilon_1.\epsilon_2$ is the weighing coefficients of disturbance.

Due to the system non-linearity, we approximate the system using the following two-rule fuzzy model:

continuous-time fuzzy model

Rule 1: If $|x_1(t)|$ is close to zero,
Then $\dot{x}(t) = A_1 x(t) + B_1 u(t) + F_1 w(t)$

Rule 2: If $|x_1(t)|$ is close to $\pi/2$,
Then $\dot{x}(t) = A_2 x(t) + B_2 u(t) + F_2 w(t)$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4L/3 - amL} & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4L/3 - amL} \end{bmatrix} \quad F_1 = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4L/3 - amL\delta^2)} & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ -\frac{a\delta}{4L/3 - amL\delta^2} \end{bmatrix} \quad F_1 = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \quad with \; \delta = cos(80^o) \tag{71}$$

discrete-time fuzzy model

Rule 1: If $|x_1(k)|$ is close to zero,
Then $x(k+1) = \mathcal{A}_1 x(k) + \mathcal{B}_1 u(k) + \mathcal{F}_1 w(k)$

Rule 2: If $|x_1(k)|$ is close to $\pi/2$,
Then $x(k+1) = \mathcal{A}_2 x(k) + \mathcal{B}_2 u(k) + \mathcal{F}_2 w(k)$

where

$$\mathcal{A}_1 = \begin{bmatrix} 1 & T \\ \frac{gT}{4L/3-amL} & 1 \end{bmatrix} \mathcal{B}_1 = \begin{bmatrix} 0 \\ -\frac{aT}{4L/3-amL} \end{bmatrix} \mathcal{F}_1 = \begin{bmatrix} \epsilon_1 T \\ \epsilon_2 T \end{bmatrix}$$

$$\mathcal{A}_2 = \begin{bmatrix} 1 & T \\ \frac{2gT}{\pi(4L/3-amL\delta^2)} & 1 \end{bmatrix} \mathcal{B}_2 = \begin{bmatrix} 0 \\ -\frac{a\delta T}{4L/3-amL\delta^2} \end{bmatrix} \mathcal{F}_2 = \begin{bmatrix} \epsilon_1 T \\ \epsilon_2 T \end{bmatrix}$$

$$\text{with } \delta = \cos(80^o), \text{Sampling time } T = 0.001 \tag{72}$$

The following values are used in our simulation:

$$M = 8kg, m = 2kg, L = 0.5m, g = 9.8m/s^2, \epsilon_1 = 1, \epsilon_2 = 0$$

and the initial condition of $x_1(0) = \pi/6, x_2(0) = -\pi/6$. The membership function of Rule 1 and Rule 2 is shown below in Fig.1.
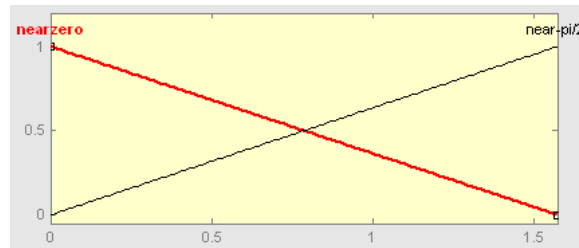

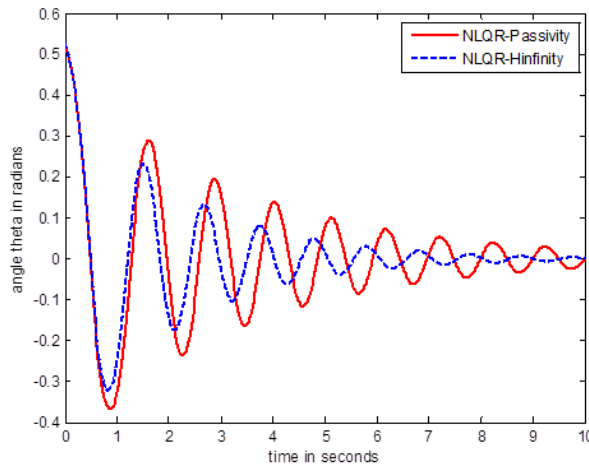**Figure 1.** Membership functions of Rule 1 and Rule 2.

Figure 2. Angle trajectory of the inverted pendulum.

The feedback control gain can be found from (31)(51) by solving the LMI at each time. The following design parameters are chosen to satisfy:

Mixed $NLQR - \mathcal{H}_\infty$ criteria:

$$C = [1 \quad 1], D = [1], Q = diag[1001], R = 1, \alpha = 1, \beta = 0, \gamma = -5$$

Mixed $NLQR - passivity$ criteria:

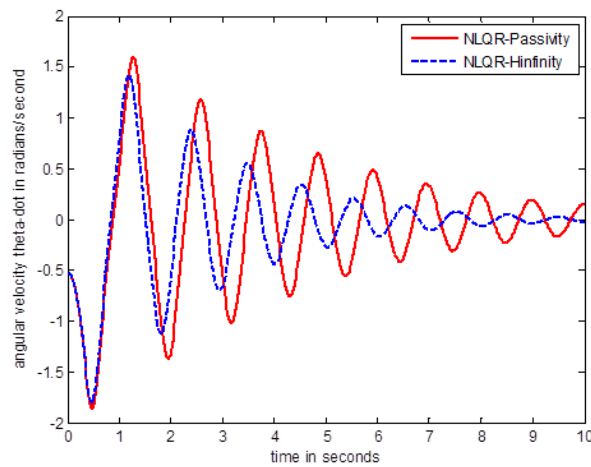$$C = [1 \quad 1], D = [1], Q = diag[1001], R = 1, \alpha = 1, \beta = 5, \gamma = 0$$

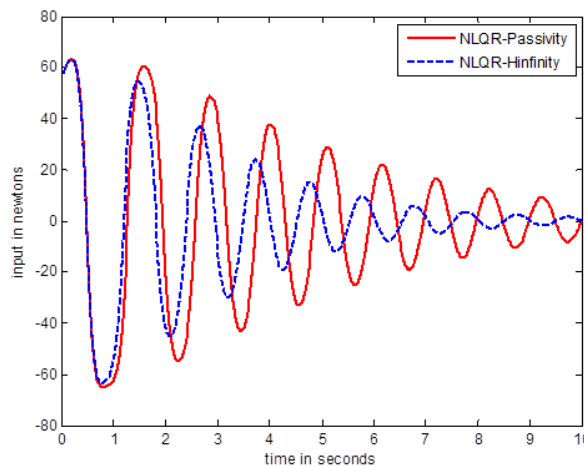**Figure 3.** Angular velocity trajectory of the inverted pendulum.

Figure 4. Control input applied to the inverted pendulum.

The mixed criteria control performance results are shown in the Figs.2-4. From these figures, we find that the novel fuzzy LMI control has satisfactory performance. The mixed $NLQR - H_\infty$ criteria control has a smaller overshoot and a faster response than the one with passivity property. The new technique controls the inverted pendulum very well under the effect of finite energy disturbance. It should also be noted that the LMI fuzzy control with mixed performance criteria satisfies global asymptotic stability.

7. Summary

This chapter presents a novel fuzzy control approach for both of continuous time and discrete time non-linear systems based on the LMI solutions. The Takagi-Sugeno fuzzy model is applied to decompose the non-linear system. Multiple performance criteria are used to design the controller and the relative weighting matrices of these criteria can be achieved by choosing different coefficient matrices. The optimal control can be obtained by solving LMI at each time. The inverted pendulum is used as an example to demonstrate its effectiveness. The

simulation studies show that the proposed method provides a satisfactory alternative to the existing non-linear control approaches.

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