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# Delay-Dependent Generalized H$_2$ Control for Discrete-Time Fuzzy Systems with Infinite-Distributed Delays

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## 1. Introduction

In recent years, there has been significant interest in the study of stability analysis and controller synthesis for Takagi-Sugeno(T-S) fuzzy systems, which has been used to approximate certain complex nonlinear systems [1]. Hence it is important to study their stability analysis and controller synthesis. A rich body of literature has appeared on the stability analysis and synthesis problems for T-S fuzzy systems [2-6]. However, these results rely on the existence of a common quadratic Lyapunov function (CQLF) for all the local models. In fact, such a CQLF might not exist for many fuzzy systems, especially for highly nonlinear complex systems. Therefore, stability analysis and controller synthesis based on CQLF tend to be more conservative. At the same time, a number of methods based on piecewise quadratic Lyapunov function (PQLF) for T-S fuzzy systems have been proposed in [7-14]. The basic idea of these methods is to design a controller for each local model and to construct a global piecewise controller from closed-loop fuzzy control system is established with a PQLF. The authors in [7,13] considered the information of membership function, a novel piecewise continuous quadratic Lyapunov function method has been proposed for stability analysis of T-S fuzzy systems. It is shown that the PQLF is a much richer class of Lyapunov function candidates than CQLF, it is able to deal with a large class of fuzzy systems and obtained results are less conservative.

On the other hand, it is well known that time delay is a main source of instability and bad performance of the dynamic systems. Recently, a number of important analysis and synthesis results have been derived for T-S fuzzy delay systems [4-7, 11, 13]. However, it should be pointed out that most of the time-delay results for T-S fuzzy systems are constant delay or

time-varying delay [4-5, 7, 11, and 13]. In fact, Distributed delay occurs very often in reality and it has been drawing increasing attention. However, almost all existing works on distributed delays have focused on continuous-time systems that are described in the form of either finite or infinite integral and delay-independent. It is well known that the discrete-time system is in a better position to model digitally transmitted signals in a dynamic way than its continuous-time analogue. Generalized $H_2$ control is an important branch of modern control theories, it is useful for handling stochastic aspects such as measurement noise and random disturbances [10]. Therefore, it becomes desirable to study the generalized $H_2$ control problem for the discrete-time systems with distributed delays. The authors in [6] have derived the delay-independent robust $H_\infty$ stability criteria for discrete-time T-S fuzzy systems with infinite-distributed delays. Recently, many robust fuzzy control strategies have been proposed a class of nonlinear discrete-time systems with time-varying delay and disturbance [15-33]. These results rely on the existence CLKF for all local models, which lead to be conservative. It is observed, based on the PLKF, the delay-dependent generalized $H_2$ control problem for discrete-time T-S fuzzy systems with infinite-distributed delays has not been addressed yet and remains to be challenging.

Motivated by the above concerns, this paper deals with the generalized $H_2$ control problem for a class of discrete time T-S fuzzy systems with infinite-distributed delays. Based on the proposed Delay-dependent PLKF(DDPLKF), the stabilization condition and controller design method are derived for discrete time T-S fuzzy systems with infinite-distributed delays. It is shown that the control laws can be obtained by solving a set of LMIs. A simulation example is presented to illustrate the effectiveness of the proposed design procedures.

*Notation*: The superscript "*T*" stands for matrix transposition, $R^n$ denotes the *n*-dimensional Euclidean space, $R^{n \times m}$ is the set of all *n×m* real matrices, *I* is an identity matrix, the notation *P>0(P≥0)* means that *P* is symmetric and positive(nonnegative) definite, *diag*{…} stands for a block diagonal matrix. $Z^-$ denotes the set of negative integers. For symmetric block matrices, the notation * is used as an ellipsis for the terms that are induced by symmetry. In addition, matrices, if not explicitly stated, are assumed to have compatible dimensions.

## 2. Problem Formulation

The following discrete-time T-S fuzzy dynamic systems with infinite-distributed delays [6] can be used to represent a class of complex nonlinear time-delay systems with both local analytic linear models and fuzzy inference rules:

$$R^j : if \ s_1(t) \ is \ F_{j1} and \ s_2(t) \ is \ F_{j2} \ and \cdots and \ s_g(t) \ is F_{jg} , then$$

$$x(t+1) = A_j x(t) + A_{dj} \sum_{d=1}^{\infty} \mu_d x(t-d) + B_{1j} u(t) + D_j v(t)$$

$$z(t) = C_j x(t) + B_{2j} u(t)$$

$$x(t) = \varphi(t) \qquad \forall t \in Z^- \qquad j = 1,2 \cdots r$$

(1)

where $R^j$, $j \in N := \{1,2,\ldots, r\}$ denotes the $j$-th fuzzy inference rule, $r$ the number of the infer‐ence rules. $F_{ji}$ ($i=1, 2,\ldots, g$) are the fuzzy sets, $s(t)=[s_1(t), s_2(t),\ldots, s_g(t)] \in R^s$ the premise varia‐ble vector, $x(t) \in R^n$ the state vector, $z(t) \in R^q$ the controlled output vector, $u(t) \in R^m$ the control input vector, $v(t) \in l_2[0 \ \infty)$ the disturbance input, $\varphi(t)$ the initial state, and ($A_j$, $A_{dj}$, $B_{1j}$, $D_j$, $C_j$, $B_{2j}$) represent the $j$-th local model of the fuzzy system (1).

The constants $\mu_d \geq 0$ ($d =1,2, \ldots$) satisfy the following convergence conditions:

$$\bar{\mu} := \sum_{d=1}^{+\infty} \mu_d \leq \sum_{d=1}^{+\infty} d\mu_d < +\infty$$

(2)

***Remark* 1.** The delay term $\sum_{d=1}^{+\infty} \mu_d x(t-d)$ in the fuzzy system (1), is the so-called infinitely distributed delay in the discrete-time setting. The description of the discrete-time-distribut‐ed delays has been firstly proposed in the [6], and we aim to study the generalized H$_2$ con‐trol problem for discrete-time fuzzy systems with such kind of distributed delays in this paper, which is different from one in [6].

***Remark* 2.** In this paper, similar to the convergence restriction on the delay kernels of infin‐ite-distributed delays for continuous-time systems, the constants $\mu_d$ ($d =1,2, \ldots$)are assumed to satisfy the convergence condition (2), which can guarantee the convergence of the terms of infinite delays as well as the DDPLKF defined later.

By using a standard fuzzy inference method, that is using a center-average defuzzifiers product fuzzy inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be ex‐pressed by the following global model:

$$x(t+1) = \sum_{j=1}^{r} h_j(s(t))[A_j x(t) + A_{dj} \sum_{d=1}^{\infty} \mu_d x(t-d) + B_{1j} u(t) + D_j v(t)]$$

$$z(t) = \sum_{j=1}^{r} h_j(s(t))[C_j x(t) + B_{2j} u(t)]$$

(3)

where $h_j(s(t)) = \dfrac{\omega_j(s(t))}{\sum_{j=1}^{r} \omega_j(s(t))}$, $\omega_j(s(t)) = \prod_{i=1}^{g} F_{ji}(s(t))$, with $F_{ji}(s(t))$ being the grade of mem‐bership of $s_i(t)$ in $F_{ij}$, $\omega_j(s(t)) \geq 0$ has the following basic property:

$$\omega_j(s(t)) \geq 0, \sum\nolimits_{j=1}^{r} \omega_j(s(t)) > 0, j \in N \quad \forall t \tag{4}$$

and therefore

$$h_j(s(t)) \geq 0, \sum\nolimits_{j=1}^{r} h_j(s(t)) = 1, j \in N \quad \forall t \tag{5}$$

In order to facilitate the design of less conservative $H_2$ controller, we partition the premise variable space $\Omega \subseteq R^s$ into $m$ polyhedral regions $\Omega_i$ by the boundaries [7]

$$\partial\Omega_i^v = \{s(t) \,|\, h_i(s(t)) = 1, 0 \leq \underset{0<|\delta|<1}{h_i(s(t+\delta))} < 1, i \in N\} \tag{6}$$

where $v$ is the set of the face indexes of the polyhedral hull with satisfying $\partial\Omega_i = \cup_v (\partial\Omega_i^v)$

Based on the boundaries (6), $m$ independent polyhedral regions $\Omega_l$, $l \in L = \{1, 2 \cdots m\}$ can be obtained satisfying

$$\Omega_l \cap \Omega_j = \partial\Omega_i^v, l \neq j, l, j \in L \tag{7}$$

where $L$ denotes the set of polyhedral region indexes.

In each region $\Omega_l$, we define the set

$$M(l) := \{i \,|\, h_i(s(t)) > 0, s(t) \in \Omega_l, i \in N\}, l \in L \tag{8}$$

Considering (5) and (8), in each region $\Omega_l$, we have

$$\sum\nolimits_{i \in M(l)} h_i(s(t)) = 1 \tag{9}$$

and then, the fuzzy infinite-distributed delays system (1) can be expressed as follows:

$$
\begin{aligned}
x(t+1) &= \sum_{i \in M(l)} h_i(s(t))[A_i x(t) + A_{di} \sum_{d=1}^{\infty} \mu_d x(t-d) + B_{1i} u(t) + D_i v(t)] \\
z(t) &= \sum_{i \in M(l)} h_i(s(t))[C_i x(t) + B_{2i} u(t)] \qquad\qquad s(t) \in \Omega_l
\end{aligned}
\tag{10}
$$

**Remark 3.** According to the definition of (8), the polyhedral regions can be divided into two folds: operating and interpolation regions. For an operating region, the set *M(l)* contains on-ly one element, and then, the system dynamic is governed by the *s*-th local model of the fuzzy system. For an interpolation region, the system dynamic is governed by a convex combination of several local models.

In this paper, we consider the generalized H$_2$ controller design problem for the fuzzy system (1) or equivalently (10), give the following assumptions.

**Assumption 1.** When the state of the system transits from the region $\Omega_l$ to $\Omega_j$ at the time *t*, the dynamics of the system is governed by the dynamics of the region model of $\Omega_l$ at that time *t*.

For future use, we define a set$\Theta$that represents all possible transitions from one region to itself or another regions, that is

$$\Theta = \{(l, j) \mid s(t) \in \Omega_l, s(t+1) \in \Omega_j, \forall l, j \in L\} \tag{11}$$

Here *l = j*, when the system stays in the same region $\Omega_l$, and $l \neq j$, when the system transits from the region $\Omega_l$ to another one $\Omega_j$.

Considering the fuzzy system (10), choose the following non-fragile piecewise state feed-back controller

$$u(t) = -(K_l + \Delta K_l)x(t) \qquad s(t) \in \Omega_l \quad l \in L \tag{12}$$

here $\Delta K_l$ are unknown real matrix functions representing time varying parametric uncer-tainties, which are assumed to be of the form

$$\Delta K_l = E_l U_l(t) H_l, U_l^T(t) U_l(t) \le I, U_l(t) \in R^{l_1 \times l_2} \tag{13}$$

where $E_1$, $H_1$ are known constant matrices, and $U_l(t) \in R^{l_1 \times l_2}$ are unknown real time varying matrix satisfying$\Delta U_l^T(t) \Delta U_l \le I$.

Then, the closed-loop T-S system is governed by

$$x(t+1) = \overline{A}_{cl} x(t) + A_{dl} \sum_{d=1}^{\infty} \mu_d x(t-d) + D_l v(t)$$
$$z(t) = \overline{C}_{cl} x(t) \tag{14}$$

for $s(t) \in \Omega_l$, $l \in L$  where

$$\overline{A}_{cl} = \sum_{i \in M(l)} h_i A_{il}, \quad A_{dl} = \sum_{i \in M(l)} h_i A_{di}, \quad D_l = \sum_{i \in M(l)} h_i D_i, \quad \overline{C}_{cl} = \sum_{i \in M(l)} h_i C_{il}$$

$$A_{il} = A_i - B_{1i} \overline{K}_l, \quad C_{il} = C_i - B_{2i} \overline{K}_l$$

Before formulation the problem to be investigated, we first introduce the following concept for the system (14).

*Definition 1.* [10] Let a constant $\gamma > 0$ be given. The closed-loop fuzzy system (14) is said to be stable with generalized $H_2$ performance if both of the following conditions are satisfied:

• The disturbance-free fuzzy system is globally asymptotically stable.

• Subject to assumption of zero initial conditions, the controlled output satisfies

$$\| z \|_\infty < \gamma \| v \|_2 \tag{15}$$

for all non-zero $v \in I_2$.

Now, we introduce the following lemmas that will be used in the development of our main result.

*Lemma 1.*[6] Let $M \in R^{n \times n}$ be a positive semi-definite matrix, $x_i(t) \in R^n$ and constant

$a_i > 0 (i = 1, 2, \cdots)$, if the series concerned is convergent, then we have

$$\left( \sum_{i=1}^\infty a_i x_i \right)^T M \left( \sum_{i=1}^\infty a_i x_i \right) \le \left( \sum_{i=1}^\infty a_i \right) \sum_{i=1}^\infty a_i x_i M x_i \tag{16}$$

*Lemma 2.* [14] For the real matrices $P_1, P_2, P_3, P_4, A, A_d, B, X_j (j = 1, \cdots, 5)$ and $D_i (i = 1, \cdots, 10)$ with compatible dimensions, the inequalities show in (17) and (18) at the following are equivalent, where $U$ is an extra slack nonsingular matrix.

$$(a) \quad \begin{bmatrix} He\{P_1^T A\} + D_1 & P_1^T A_d + A^T P_2 + D_2 & A^T P_3 + D_3 & A^T P_4 + P_1^T B + D_4 & X_1 \\ * & He\{P_2^T A_d\} + D_5 & A_d^T P_3 + D_6 & A_d^T P_4 + P_2^T B + D_7 & X_2 \\ * & * & D_8 & P_3^T B + D_9 & X_3 \\ * & * & * & He\{B^T P_4\} + D_{10} & X_4 \\ * & * & * & * & X_5 \end{bmatrix} < 0 \tag{17}$$

$$(b) \quad \begin{bmatrix} -He\{U\} & P_1 + U^T A_2 & P_2 + U^T A_d & P_3 & P_4 + U^T B & 0 \\ * & D_1 & D_2 & D_3 & D_4 & X_1 \\ * & * & D_5 & D_6 & D_7 & X_2 \\ * & * & * & D_8 & D_9 & X_3 \\ * & * & * & * & D_{10} & X_4 \\ * & * & * & * & * & X_5 \end{bmatrix} < 0 \quad (18)$$

where $He\{*\}$ stands for $* + *^T$.

## 3. Main Results

Based on the proposed partition method, the following DDPLKF is proposed to develop the stability condition for the closed-loop system of (14).

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

$$V_1(t) = 2x(t)^T \bar{P}_l x(t), \quad V_2(t) = \sum_{d=1}^{\infty} \mu_d \sum_{k=t-d}^{t-1} x(k)^T \bar{Q} x(k) \quad (19)$$

$$V_3(t) = \sum_{d=1}^{\infty} \mu_d \sum_{i=-d}^{-1} \sum_{l=t+i}^{t-1} \eta(l)^T \bar{Z} \eta(l) \quad l \in L$$

where $\bar{P}_l = F^{-T} P_l F$, $\bar{Q} = F^{-T} Q F$, $\bar{Z} = F^{-T} Z F$, and $P_l, Q, Z > 0$, $F$ is nonsingular matrix, and $\eta(t) = x(t+1) - x(t)$.

Then, we are ready to present the generalized H₂ stability condition of (14) in terms of LMIs as follows

**Theorem 1.** Given a constant $\gamma > 0$, the closed-loop fuzzy system (14) with infinite distributed delays is stable with generalized H₂ performance $\gamma$, if there exists a set of positive definite matrices $P_l, Q, Z > 0$, the nonsingular matrix $F$ and matrices $X_{li}, Y_{li}, l \in L$, $i = 1, \cdots, 4$ satisfying the following LMIs:

$$C_{il}^T C_{il} - \gamma^2 P_l < 0 \quad i \in M(l), l \in L \quad (20)$$

$$\Pi_{ill} < 0 \quad i \in M(l), l \in L \quad (21)$$

$$\Pi_{ilj} < 0 \quad i \in M(l), (l, j) \in \Theta \quad (22)$$

where

$$
\Pi_{ilj} = \begin{bmatrix}
-He\{F\} & \Lambda_{ilj} & Y_{l_2} + A_{di}F & P_j + Y_{l_3} & Y_{l_4} + D_i & 0 \\
* & \Sigma_{l1} & \Sigma_{l2} & \Sigma_{l3} & \Sigma_{l4} & X_{l1} \\
* & * & \Sigma_{l5} & \Sigma_{l6} & \Sigma_{l7} & X_{l2} \\
* & * & * & \Sigma_{l8} & \Sigma_{l9} & X_{l3} \\
* & * & * & * & \Sigma_{l10} & X_{l4} \\
* & * & * & * & * & (-\sum_{d=1}^{\infty} d\mu_d)^{-1} Z
\end{bmatrix}
$$

with

$$
\Lambda_{ilj} = P_j + Y_{l1} + A_i F - B_{1i}\overline{K}_l F,
$$

$$
\Sigma_{l1} = \overline{\mu} Q - 2P_l + He\{\overline{\mu} X_{l1} - Y_{l1}\}, \quad \Sigma_{l2} = -X_{l1} + X_{l2}^T - Y_{l2}, \quad \Sigma_{l3} = X_{l3} - Y_{l1}^T - Y_{l3},
$$

$$
\Sigma_{l4} = X_{l4}^T + Y_{l4}, \quad \Sigma_{l5} = \frac{1}{\overline{\mu}} Q - He\{X_{l2}\}, \quad \Sigma_{l6} = -X_{l3}^T - Y_{l2}, \quad \Sigma_{l7} = X_{l4},
$$

$$
\Sigma_{l8} = \sum_{d=1}^{\infty} \mu_d d Z - He\{Y_{l3}\}, \quad \Sigma_{l9} = Y_{l4}^T, \quad \Sigma_{l10} = -I.
$$

**Proof.** Taking the forward difference of (19) along the solution of the system (14), we have
$\Delta V(t) = V(t+1) - V(t) = \Delta V_1 + \Delta V_2 + \Delta V_3$

Assuming that $s(t) \in \Omega_l$, $s(t+1) \in \Omega_j$. The difference of $V_i(t)$, $i=1,2,3$ can be calculated, respectively, showing at the following

$$
\Delta V_1(t) = 2[\overline{A}_{cl} x(t) + A_{dl} \sum_{d=1}^{\infty} \mu_d x(t-d) + D_l v(t)]^T \overline{P}_j[\eta(t) + x(t)] - 2x^T(t)\overline{P}_l x(t) \tag{23}
$$

$$
\Delta V_2(t) = \sum_{d=1}^{\infty} \mu_d \sum_{\tau=t+1-d}^{t} x^T(\tau)\overline{Q}x(\tau) - \sum_{d=1}^{\infty} \mu_d \sum_{\tau=t-d}^{t-1} x^T(\tau)\overline{Q}x(\tau)
$$

$$
= \overline{\mu} x^T(t)\overline{Q}x(t) - \sum_{d=1}^{\infty} \mu_d x^T(t-d)\overline{Q}x(t-d) \tag{24}
$$

From Lemma1, we have

$$
-\sum_{d=1}^{\infty} \mu_d x^T(t-d)\overline{Q}x(t-d) \leq -\frac{1}{\overline{\mu}} (\sum_{d=1}^{\infty} \mu_d x(t-d))^T \overline{Q}(\sum_{d=1}^{\infty} \mu_d x(t-d)) \tag{25}
$$

Substituting (25) into (24), we have

$$
\Delta V_2(t) \leq \overline{\mu} x^T(t)\overline{Q}x(t) - \frac{1}{\overline{\mu}} (\sum_{d=1}^{\infty} \mu_d x(t-d))^T \overline{Q}(\sum_{d=1}^{\infty} \mu_d x(t-d)) \tag{26}
$$

$$\Delta V_3(t) = \sum_{d=1}^{\infty} \mu_d d\eta(t)^T \overline{Z}\eta(t) - \sum_{d=1}^{\infty} \mu_d \sum_{l=t-d}^{t-1} \eta(l)^T \overline{Z}\eta(l) \tag{27}$$

Observing of the definition of $\eta(t)$ and system (14), we can get the following equations:

$$\Xi_1 = 2[x^T(t)\overline{X}_{l1} + \sum_{d=1}^{\infty} \mu_d x^T(t-d)\overline{X}_{l2} + \eta^T(t)\overline{X}_{l3} + v^T(t)X_{l4}U]$$
$$\times [\overline{\mu}x(t) - \sum_{d=1}^{\infty} \mu_d x^T(t-d) - \sum_{d=1}^{\infty} \mu_d \sum_{l=t-d}^{t-1} \eta(l)] = 0 \tag{28}$$

$$\Xi_2 = 2[x^T(t)\overline{Y}_{l1} + \sum_{d=1}^{\infty} \mu_d x^T(t-d)\overline{Y}_{l2} + \eta^T(t)\overline{Y}_{l3} + v^T(t)Y_{l4}U]$$
$$\times [(\overline{A}_{li} - I)x(t) + A_{di} + D_i v(t) - \eta(t)] = 0 \tag{29}$$

where $\overline{X}_{li} = F^{-T}X_{li}F^{-1}(i=1, 2, 3)$

Since $\pm 2a^T b \le a^T Ma + b^T M^{-1} b$ holds for compatible vectors $a$ and $b$, and any compatible matrix $M > 0$, we have

$$-2[x^T(t)\overline{X}_{l1} + \sum_{d=1}^{\infty} \mu_d x^T(t-d)\overline{X}_{l2} + \eta^T(t)\overline{X}_{l3} + v^T(t)X_{l4}U] \times \sum_{d=1}^{\infty} \mu_d \sum_{l=t-d}^{t-1} \eta(l)$$
$$\le \sum_{d=1}^{\infty} d\mu_d \xi^T(t) \begin{bmatrix} \overline{X}_{l1} \\ \overline{X}_{l2} \\ \overline{X}_{l3} \\ X_{l4}U \end{bmatrix} \overline{Z}^{-1} \begin{bmatrix} \overline{X}_{l1} \\ \overline{X}_{l2} \\ \overline{X}_{l3} \\ X_{l4}U \end{bmatrix}^T \xi(t) + \sum_{d=1}^{\infty} \mu_d \sum_{l=t-d}^{t-1} \eta(l)\overline{Z}\eta(l) \tag{30}$$

with $\xi(t) = [x^T(t), \sum_{d=1}^{\infty} \mu_d x^T(t-d), \eta^T(t), v^T(t)]^T$

Then, from (23-30) and considering (14), we have

$$\Delta V(t) - v^T(t)v(t) + v^T(t)v(t) + \Xi_1 + \Xi_2 \le \sum_{i \in M(l)} h_i \xi^T(t)\Psi_{ilj}\xi(t) + v^T(t)v(t) \tag{31}$$

where

$$\Psi_{ilj} = \begin{bmatrix} \Phi_{ilj}^1 & \Phi_{ilj}^2 & \Phi_{ilj}^3 & \Phi_{ilj}^4 \\ * & \Phi_{ilj}^5 & \Phi_{ilj}^6 & \Phi_{ilj}^7 \\ * & * & \Phi_{ilj}^8 & \Phi_{ilj}^9 \\ * & * & * & \Phi_{ilj}^{10} \end{bmatrix} + \sum_{d=1}^{\infty} d\mu_d \begin{bmatrix} \overline{X}_{l1} \\ \overline{X}_{l2} \\ \overline{X}_{l3} \\ X_{l4}U \end{bmatrix} \overline{Z}^{-1} \begin{bmatrix} \overline{X}_{l1} \\ \overline{X}_{l2} \\ \overline{X}_{l3} \\ X_{l4}U \end{bmatrix}^{\mathrm{T}} \tag{32}$$

with

$$\Phi_{l1} = \mathrm{He}\{(\overline{P}_j + \overline{Y}_{l1})^T \overline{A}_{li}\} + \overline{\mu}Q - 2\overline{P}_l + \mathrm{He}\{\overline{\mu}\overline{X}_{l1} - \overline{Y}_{l1}\}$$

$$\Phi_{l2} = (\overline{P}_j + \overline{Y}_{l1})^T A_{di} + \overline{A}_{li}^T \overline{Y}_{l2} - \overline{X}_{l1} + \overline{X}_{l2}^T - \overline{Y}_{l2}, \Phi_{l3} = \overline{A}_{li}^T (\overline{P}_j + \overline{Y}_{l1}) + \overline{X}_{l3} + \overline{Y}_{l1}^T - \overline{Y}_{l3}$$

$$\Phi_{l4} = (\overline{P}_j + \overline{Y}_{l1})^T D_i + \overline{A}_{li}^T U^T Y_{l4} + U^T X_{l4}^T + U^T Y_{l4}$$

$$\Phi_{l5} = \mathrm{He}\{\overline{Y}_{l2}^T A_{di}\} - \frac{1}{\mu}Q - \mathrm{He}\{\overline{X}_{l2}\}, \Phi_{l6} = A_{di}^T(\overline{P}_j + \overline{Y}_{l3}) - \overline{X}_{l3}^T - \overline{Y}_{l2}^T$$

$$\Phi_{l7} = A_{di}^T U Y_{l4} + \overline{Y}_{l2}^T D_i - U X_{l4}^T, \Phi_{l8} = \sum_{d=1}^{\infty} \mu_d d\overline{Z} - \mathrm{He}\{\overline{Y}_{l3}\}$$

$$\Phi_{l9} = (\overline{P}_j + \overline{Y}_{l3})^T D_i - U^T Y_{l4}, \Phi_{l10} = \mathrm{He}\{D_i^T U^T Y_{l4}\} - I$$

Then

$$\Delta V(t) - v^T(t)v(t) < 0 \tag{33}$$

if

$$\Psi_{ilj} < 0 \tag{34}$$

Using lemma 2, (32) is equivalent to (33)

$$\Xi_{ilj} = \begin{bmatrix} -\mathrm{He}\{U\} & \overline{P}_i + \overline{Y}_{l1} + U^T\overline{A}_{li} & \overline{Y}_{l2} + U^T A_{di} & \overline{P}_j + \overline{Y}_{l3} & U^T(Y_{l4} + D_i) & 0 \\ * & \overline{\Sigma}_{l1} & \overline{\Sigma}_{l2} & \overline{\Sigma}_{l3} & U\Sigma_{l4} & \overline{X}_{l1} \\ * & * & \overline{\Sigma}_{l5} & \overline{\Sigma}_{l6} & U\Sigma_{l7} & \overline{X}_{l1} \\ * & * & * & \overline{\Sigma}_{l8} & U\Sigma_{l9} & \overline{X}_{l1} \\ * & * & * & * & \Sigma_{l10} & X_{l4}U \\ * & * & * & * & * & (-\sum_{d=1}^{\infty} \mu_d d)^{-1}\overline{Z} \end{bmatrix} \tag{35}$$

where $\bar{\Sigma}_{li} = F^{-T}\Sigma_{li}F^{-1}$ ($i = 1, 2, 3, 5, 6, 8, 10$)

Let $U = F^{-1}$, $G = diag(F, F, F, F, I, F)$, pre- and post multiplying (35) by $G^T$, $G$ respectively, then $\Xi_{ilj}$ is equivalent to $\Pi_{ilj}$.

Thus, if (21) and (22) holds, (32) is satisfied, which implies that

$$\Delta V(t) < v^T(t)v(t) \tag{36}$$

It is noted that if the disturbance term $v(t) = 0$, it follows from (31) that

$$\Delta V(t) < \sum_{i \in M(l)} h_i \zeta^T(t)\Omega_{ilj}\zeta(t) \tag{37}$$

with $\zeta(t) = [x^T(t), \sum_{d=1}^{\infty} \mu_d x^T(t-d), \eta^T(t)]^T$

$$\Omega_{ilj} = \begin{bmatrix} \Phi_{ilj}^1 & \Phi_{ilj}^2 & \Phi_{ilj}^3 \\ * & \Phi_{ilj}^5 & \Phi_{ilj}^6 \\ * & * & \Phi_{ilj}^8 \end{bmatrix} + \sum_{d=1}^{\infty} d\mu_d \begin{bmatrix} \overline{X}_{l1} \\ \overline{X}_{l2} \\ \overline{X}_{l3} \end{bmatrix} \overline{Z}^{-1} \begin{bmatrix} \overline{X}_{l1} \\ \overline{X}_{l2} \\ \overline{X}_{l3} \end{bmatrix}^T \tag{38}$$

By Schur's complement, LMI (32) implies $\Omega_{ilj} < 0$, then $\Delta V(t) < 0$. Therefore, the closed-loop system (14) with $v(t) = 0$ is globally asymptotically stable.

Now, to establish the generalized H$_2$ performance for the closed-loop system (14), under zero-initial condition, and $v(t) \neq 0$, taking summation for the both sides of (36) leads to

$$V(x(T+1)) < \sum_{t=0}^{T} v^T(t)v(t) \tag{39}$$

It follows from (20) that

$$z^T(t)z(t) = x^T(t)\overline{C}_{cl}^T\overline{C}_{cl}x(t) = \sum_{i \in M(l)} h_i\lambda^T(t) \begin{bmatrix} C_{il}^TC_{il} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\lambda(t)$$

$$< \gamma^2\lambda^T(t) \begin{bmatrix} P & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Z \end{bmatrix}\lambda(t) = \gamma^2 V(t) \tag{40}$$

with

$$\lambda(t)=\left[x(t),\ \sum_{d=1}^{\infty}\mu_d\sum_{\tau=t-d}^{t-1}x(\tau),\ \sum_{d=1}^{\infty}\mu_d\sum_{i=-d}^{-1}\sum_{l=t-d}^{t-1}\eta(l)\right]$$

From (39) and (40), we have

$$\|z(t)\|_{\infty}^2 < \gamma^2\|v(t)\|_2^2 \tag{41}$$

The proof is completed.

The following theorem shows that the desired controller parameters and considered controller uncertain can be determined based on the results of Theorem 1. This can be easily proved along the lines of Theorem 1, and we, therefore, only keep necessary details in order to avoid unnecessary duplication.

***Theorem* 2.** Consider the uncertain terms (12). Given a constant $\gamma > 0$, the closed-loop fuzzy system (14) with infinite-distributed delays is stable with generalized $H_2$ performance $\gamma$, if there exists a set of positive definite matrices $P_l$, $Q$, $Z > 0$, the nonsingular matrix $F$ and matrices $X_{li}$, $Y_{li}$, $M_l$, $l \in L$ , $i = 1,2,3,4$ satisfying the following LMIs:

$$\begin{bmatrix} -P_l & C_iF - B_{2i}M_l & -B_{2i}H_lF \\ * & -\gamma^2 I + \varepsilon_l E_l^T E_l & 0 \\ * & * & -\varepsilon_l I \end{bmatrix} < 0 \quad i \in M(l), l \in L \tag{42}$$

$$\Upsilon_{ill} < 0 \quad i \in M(l), l \in L \tag{43}$$

$$\Upsilon_{ilj} < 0 \quad i \in M(l), (l,j) \in \Theta \tag{44}$$

where

$$_{ilj}=\begin{bmatrix} -He\{F\} & T_{ilj} & Y_{l_2}+A_{di}F & P_j+Y_{l_3} & Y_{l_4}+D_i & 0 & 0 \\ * & \Sigma_{l1} & \Sigma_{l2} & \Sigma_{l3} & \Sigma_{l4} & X_{l1} & -B_{1i}H_lF \\ * & * & \Sigma_{l5} & \Sigma_{l6} & \Sigma_{l7} & X_{l2} & 0 \\ * & * & * & \Sigma_{l8} & \Sigma_{l9} & X_{l3} & 0 \\ * & * & * & * & \Sigma_{l10} & X_{l4} & 0 \\ * & * & * & * & * & \Gamma_l & 0 \\ * & * & * & * & * & * & -\varepsilon_l I \end{bmatrix}$$

with

$$T_{ilj} = P_j + Y_{l1} + A_i F - B_{1i} M_l, \quad \Gamma_l = (-\sum_{d=1}^{\infty} d\,\mu_d)^{-1} Z + \varepsilon_l E_l^T E_l.$$

Furthermore, the control law is given by

$$K_l = M_l F^{-1} \tag{45}$$

**Proof.** In (20) and (21), replace $\overline{K}_l$ with $K_l + \Delta K_l$, and then by S-procedure, we can easily obtain the results of this theorem, and the details are thus omitted.

***Remark* 4.** If the global state space replace the transitions $\Theta$ and all $P_l$s in Theorem 2 become a common $P$, Theorem 2 is regressed to Corollary 1, shown in the following.

***Corollary* 1.** Consider the uncertain terms (12). Given a constant $\gamma > 0$, the closed-loop fuzzy system (14) with infinite-distributed delays is stable with generalized H$_2$ performance $\gamma$, if there exists a set of positive definite matrices $P_l$, $Q$, $Z > 0$, the nonsingular matrix $F$ and matrices $X_{li}$, $Y_{li}$, $M_l$, $l \in L$ , $i = 1,2,3,4$ satisfying the following LMIs:

$$\begin{bmatrix} -P & C_i F - B_{2i} M_l & -B_{2i} H_l F \\ * & -\gamma^2 I + \varepsilon_l E_l^T E_l & 0 \\ * & * & -\varepsilon_l I \end{bmatrix} < 0 \quad i \in M(l), l \in L \tag{46}$$

$$\Upsilon_{il} < 0 \quad i \in M(l), l \in L \tag{47}$$

where

$$il = \begin{bmatrix} -He\{F\} & T_{il} & Y_{l2} + A_{di} F & P_j + Y_{l3} & Y_{l4} + D_i & 0 & 0 \\ * & \Sigma_{l1} & \Sigma_{l2} & \Sigma_{l3} & \Sigma_{l4} & X_{l1} & -B_{1i} H_l F \\ * & * & \Sigma_{l5} & \Sigma_{l6} & \Sigma_{l7} & X_{l2} & 0 \\ * & * & * & \Sigma_{l8} & \Sigma_{l9} & X_{l3} & 0 \\ * & * & * & * & \Sigma_{l10} & X_{l4} & 0 \\ * & * & * & * & * & \Gamma_l & 0 \\ * & * & * & * & * & * & -\varepsilon_l I \end{bmatrix}$$

with

$$T_{il} = P + Y_{l1} + A_i F - B_{1i} M_l, \ \Gamma_l = (-\sum_{d=1}^{\infty} d\mu_d)^{-1} Z + \varepsilon_l E_l^T E_l,$$

$$\Sigma_{l1} = \overline{\mu} Q - 2P + \mathrm{He}\{\overline{\mu} X_{l1} - Y_{l1}\}, \ \Sigma_{l2} = -X_{l1} + X_{l2}^T - Y_{l2}, \ \Sigma_{l3} = X_{l3} - Y_{l1}^T - Y_{l3},$$

$$\Sigma_{l4} = X_{l4}^T + Y_{l4}, \ \Sigma_{l5} = \frac{1}{\mu} Q - \mathrm{He}\{X_{l2}\}, \ \Sigma_{l6} = -X_{l3}^T - Y_{l2}, \ \Sigma_{l7} = X_{l4},$$

$$\Sigma_{l8} = \sum_{d=1}^{\infty} \mu_d dZ - \mathrm{He}\{Y_{l3}\}, \ \Sigma_{l9} = Y_{l4}^T, \ \Sigma_{l10} = -I.$$

## 4. Numerical Examples

In this section, we will present two simulation examples to illustrate the controller design method developed in this paper.

*Example* 1. Consider the following modified Henon system with infinite distributed delays and external disturbance

$$x_1(t+1) = -\{cx_1(t) + (1-c)\sum_{d=1}^{+\infty} \mu_d x_1(t-d)\}^2 + 0.1x_2(t) - 0.5\sum_{d=1}^{+\infty} \mu_d x_2(t-d) + u(t) + 0.1v(t)$$

$$x_2(t+1) = x_2(t) - 0.5x_1(t)$$

$$z_1(t) = (1-c)x_1(t) + u(t)$$  \hfill (48)

$$z_2(t) = 0.2x_2(t)$$

where the constant $c \in [0,1]$ is the retarded coefficient.

Let $s(t) = cx_1(t) + (1-c)\sum_{d=1}^{+\infty} \mu_d x_1(t-d)$. Assume that $s(t) \in [-1,1]$. The nonlinear term $s^2(t)$ can be exactly represented as

$$s^2(t) = h_1(s(t))(-1)s(t) + h_2(s(t))(1)s(t)$$

where the $h_1(s(t)), h_2(s(t)) \in [0,1]$, and $h_1(s(t)) + h_2(s(t)) = 1$. By solving the equations, the membership functions $h_1(s(t))$ and $h_2(s(t))$ are obtained as

$$h_1(s(t)) = \frac{1}{2}(1 - s(t)), \qquad h_2(s(t)) = \frac{1}{2}(1 + s(t))$$

It can be seen from the aforementioned expressions that $h_1(s(t)) = 1$ and $h_2(s(t)) = 0$ when $s(t) = -1$, and that $h_1(s(t)) = 0$ and $h_2(s(t)) = 1$ when $s(t) = 1$. Then the nonlinear system in (48) can be approximately represented by the following T-S fuzzy model:

$R^1: if\ s(t)\ is\ -1,\ then$

$$x(t+1)=A_1x(t)+A_{d1}\sum_{d=1}^{\infty}\mu_d x(t-d)+B_{11}u(t)+D_1V(t)$$

$$z(t)=C_1x(t)+B_{21}u(t)$$

$R^2: if\ s(t)\ is\ 1,\ then$

$$x(t+1)=A_2x(t)+A_{d2}\sum_{d=1}^{\infty}\mu_d x(t-d)+B_{12}u(t)+D_2v(t)$$

$$z(t)=C_2x(t)+B_{22}u(t)$$

where

$$A_1=\begin{bmatrix}0.9 & 0.1\\-0.5 & 1\end{bmatrix},\ A_{1d}=\begin{bmatrix}0.1 & -0.5\\0 & 0\end{bmatrix},\ B_{11}=B_{12}=\begin{bmatrix}1\\0\end{bmatrix},$$

$$A_2=\begin{bmatrix}-0.9 & 0.1\\-0.5 & 1\end{bmatrix},\ A_{2d}=\begin{bmatrix}-0.1 & -0.5\\0 & 0\end{bmatrix},\ D_1=D_2=\begin{bmatrix}0.1\\0\end{bmatrix},$$

$$C_1=C_2=\begin{bmatrix}-0.1 & 0\\0 & -0.2\end{bmatrix},\ B_{21}=B_{22}=\begin{bmatrix}1\\0\end{bmatrix},\ E_1=E_2=[0.05\ \ 0],$$

$$H_1=H_2=[0.1\ \ 0],$$

$$e_1=10, e_2=11,$$

$$V(t)=0.1\cos(t)\times\exp(-0.05t).$$

The subspaces can be described by

$$\Omega_1=\{s(t)\mid -1\leq s(t)\leq 0\},\ \ \Omega_2=\{s(t)\mid 0\leq s(t)\leq 1\}$$

Choosing the constants $c=0.9$, $\mu_d=2^{-3-d}$, $d=10$ we easily find that

$\bar{\mu}=\sum_{d=1}^{\infty}\mu_d=2^{-3}<\sum_{d=1}^{\infty}d\mu_d=2<+\infty$, which satisfies the convergence condition (2).

with the H₂ performance index $\gamma_{min}=0.11$, we solve (42)-(44) and obtain

$$P_1=\begin{bmatrix}0.1944 & 0.0248\\0.0248 & 0.3342\end{bmatrix},\ P_2=\begin{bmatrix}0.1951 & 0.0252\\0.0252 & 0.3358\end{bmatrix},\ Q=\begin{bmatrix}0.2876 & 0.0746\\0.0746 & 0.1636\end{bmatrix},$$

$$Z=\begin{bmatrix}0.0048 & 0.0019\\0.0019 & 0.1275\end{bmatrix},\ F=\begin{bmatrix}0.3939 & 0.1516\\0.0476 & 0.6285\end{bmatrix},\ \ K_1=[-0.0223\ \ 0.1702],$$

$$K_2=[-0.0171\ \ 0.1685].$$

Simulation results with the above solutions for the H₂ controller designs are shown Fig.1 and Fig.2
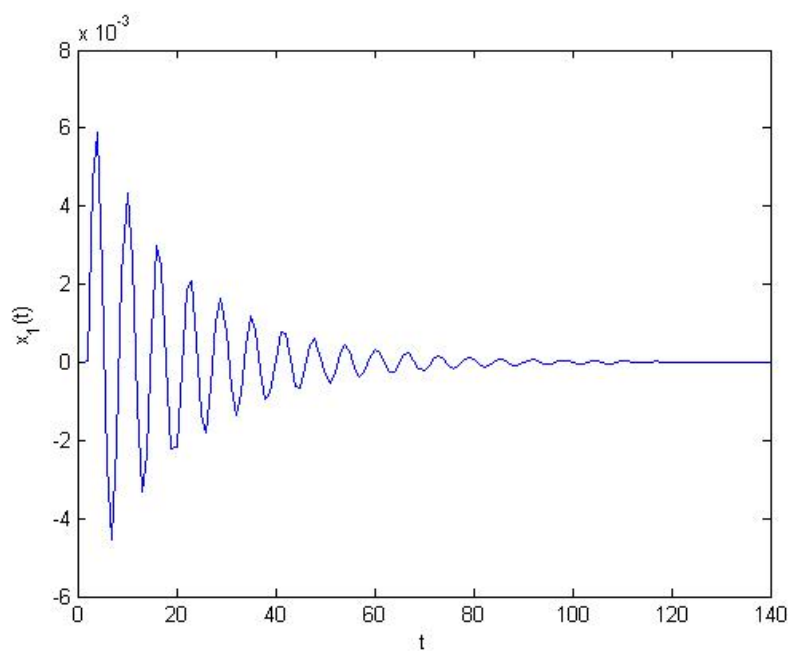
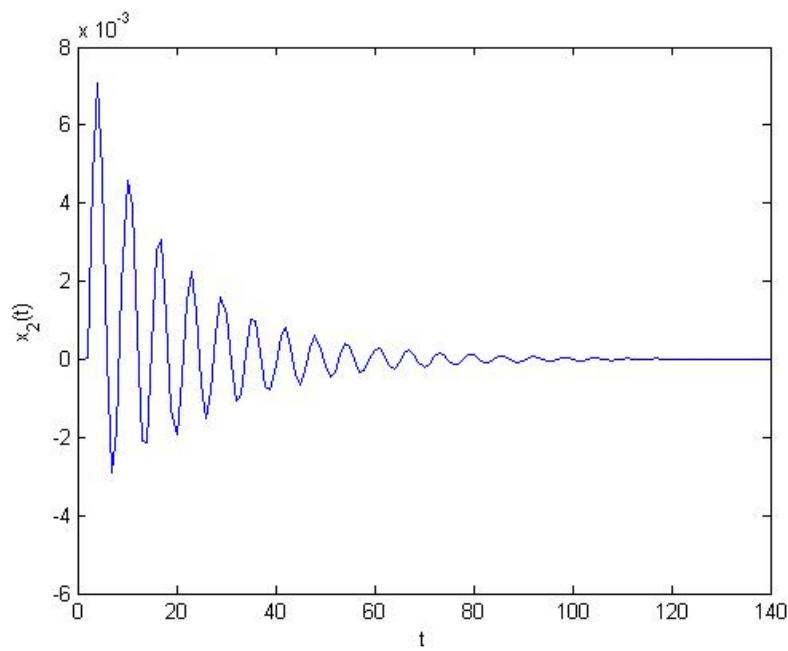**Figure 1.** The state evolution $x_1(t)$ of controlled system.



**Figure 2.** The state evolution $x_2(t)$ of controlled systems.

*Example* **2.** Consider a fuzzy discrete time system with the same form as in Example, but with different system matrices given by

$$A_1 = \begin{bmatrix} -0.986 & 0.1 \\ -0.5 & 1 \end{bmatrix}, \ A_{1d} = \begin{bmatrix} -0.1 & -0.5 \\ 0 & 0 \end{bmatrix}, \ B_{11} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \ B_{12} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.5 & -0.6 \\ 0.6 & 0.5 \end{bmatrix}, \ A_{2d} = \begin{bmatrix} -0.05 & -0.6 \\ 0 & 0 \end{bmatrix}, \ D_1 = D_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.02 & 0 \\ 0 & -0.1 \end{bmatrix}, \ C_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.3 \end{bmatrix}, \ B_{21} = B_{22} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$E_1 = E_2 = [0.05 \quad 0], \ H_1 = H_2 = [0.1 \quad 0], e_1 = 10, e_2 = 11, \ e_3 = 12,$$

$$v(t) = 0.1\cos(t) \times \exp(-0.05t).$$

We expanded the state space from [-1,1] to [− 3,3], the membership functions are given as

$$h_1(s(t)) = \begin{cases} 1 & s(t) \in [-3, \ -1], \\ -0.5s(t) + 0.5 & s(t) \in [-1, \ 1]. \end{cases}$$

$$h_2(s(t)) = \begin{cases} 0.5s(t) + 0.5 & s(t) \in [-1, \ 1], \\ 1 & s(t) \in [1, \ 3]. \end{cases}$$

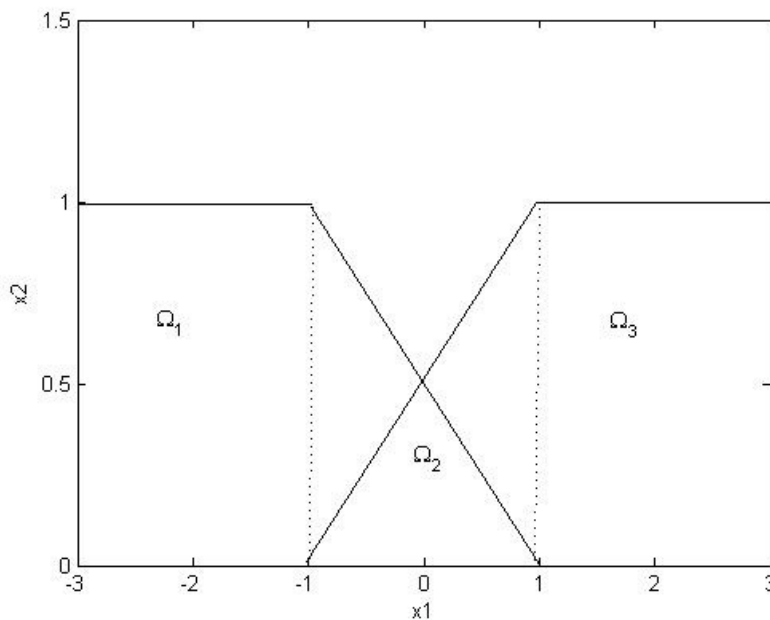The subspaces are given as shown in Fig.3



**Figure 3.** Membership functions and partition of subspaces.

Using the Theorem 2 and Corollary 1, respectively, the achievable minimum performance index for the H$_2$ controller can be obtained and is summarized in Table 1.

| Approach | Performance |
| --- | --- |

| | |
|---|---|
| Common Lyapunov function based generalized H$_2$ performance (Theorem 2) | γ$_{min}$=0,4586 |
| Piecewise Lyapunov function based generalized H$_2$ performance ( Corollary1) | γ$_{min}$=0,3975 |

**Table 1.** Comparison for generalized H$_2$ performance.

By using the LMI toolbox, we have

$$P_1 = \begin{bmatrix} 1.5359 & 0.5771 \\ 0.5771 & 1.4293 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.5254 & 0.6540 \\ 0.6540 & 1.5478 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 1.2754 & 0.5634 \\ 0.5634 & 1.4983 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.8101 & 0.1568 \\ 0.1568 & 0.5915 \end{bmatrix}, \quad Z = \begin{bmatrix} 0.0399 & 0.0285 \\ 0.0285 & 0.4640 \end{bmatrix}, \quad F = \begin{bmatrix} 3.1076 & 0.7119 \\ 0.8671 & 2.5352 \end{bmatrix},$$

$$K_1 = [0.0003 \quad -0.2297], \quad K_2 = [0.1311 \quad -0.0371], \quad K_3 = [-0.1125 \quad -0.0005].$$

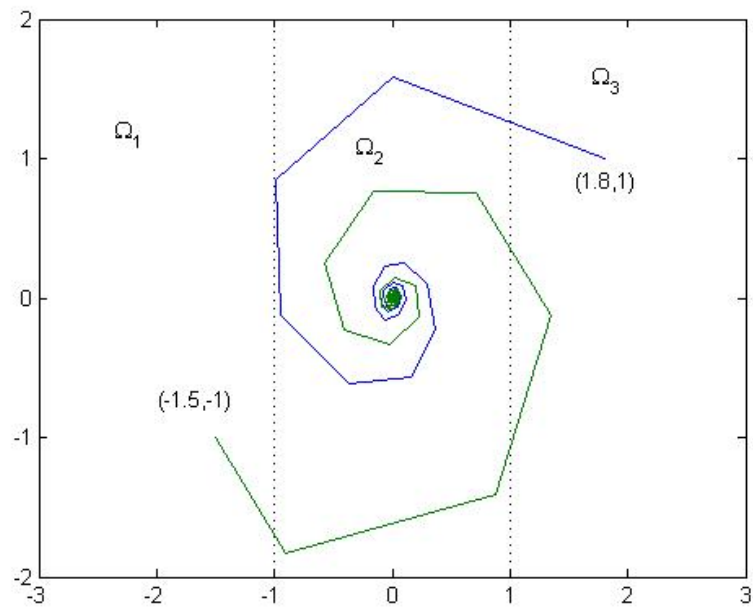The simulation results with the initial conditions are shown Fig.4 and Fig.5



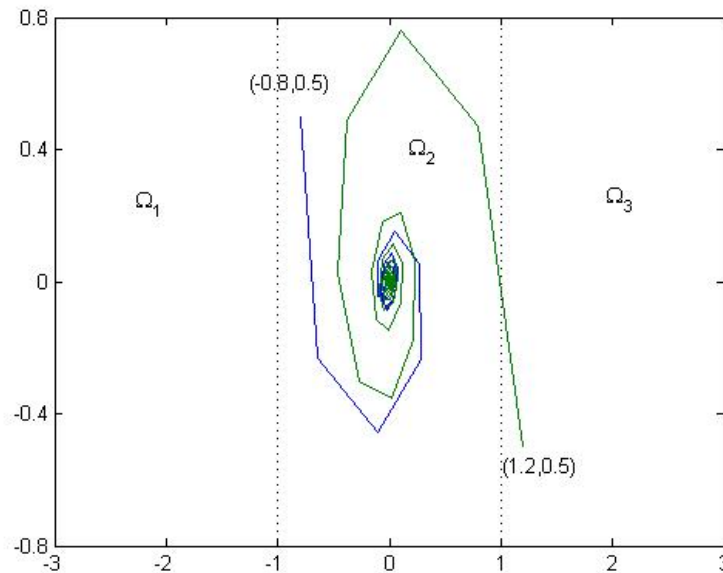**Figure 4.** Trajectories from two initial conditions

**Figure 5.** Trajectories from two initial conditions

## 5. Conclusions

This paper presents delay-dependent analysis and synthesis method for discrete-time T-S fuzzy systems with infinite-distributed delays. Based on a novel DDPLKF, the proposed stability and stabilization results are less conservative than the existing results based on the CLKF and delay independent method. The non-fragile stated feedback controller law has been developed so that the closed-loop fuzzy system is generalized $H_2$ stable. It is also shown that the controller gains can be determined by solving a set of LMIs. A simulation example was presented to demonstrate the advantages of the proposed approach.

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